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INTERACTION OF SOLID PARTICLES WITH VORTEX STRUCTURES AND CONCENTRATION DISTRIBUTION OF SUCH PARTICLES IN A COMBINED VORTEX

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The interaction of solid particles with vortex structures was investigated, and the scattering of such particles by a combined vortex was numerically simulated using the Euler–Lagrange approach. The concentration distributions of particles in the region of a gas flow occupied by a vortex at different instants of time have been obtained. The dependence of the time for which these particles escape from the central zone of the vortex on their size was determined. The results obtained can be used for increasing the efficiency of measurement of the parameters of the disperse phase in a fluid flow by optical methods.

Keywords: *numerical simulation, vortex, particle, concentration, optical method.*

Introduction. Vorticity is an important property of liquid and gas flows, providing the basis for a wide variety of their forms, and it is used for qualitative definition of different effects in fluid mechanics, such as turbulence and the formation and separation of a boundary layer [1, 2]. The addition of solid particles to a fluid flow makes its pattern more complex depending on the properties of the inclusions, in particular, their inertia and concentration. Because of the variety of these properties, different flow regimes are realized.

The concentration of disperse particles in a fluid flow is one of the main physical parameters determining the characteristics of movement of the disperse phase in this flow and its influence on the movement of the carrying medium. In the case where the inertia of the disperse particles in a fluid flow is infinitely small, the velocity fields of the disperse and carrying phases coincide, and, when the initial load of the flow with these particles is homogeneous, the concentration of the disperse phase remains uniform throughout the flow field. A small but finite inertia of the disperse particles in a fluid flow changes their density distribution, with the result that there appear local concentration inhomogeneities of the disperse phase near the kinematic peculiarities of the velocity field of the carrying phase (critical points, discontinuities, local vorticity zones).

In investigations of liquid and gas flows, optical measurement methods, based on the introduction of particles into such a flow and their probing by a monochromatic radiation for determining the flow velocity by optical signals, have a special place. The method of particle image velocimetry (PIV) involves the digital processing of a two-exposure image of tracing particles, which makes it possible to determine not only the velocity of movement of a particle but also its path [4]. In this method, the measurement of the velocity of a particle is based on the recording of its movement in a sectional plane for a definite time. The particles found in a measurement plane are irradiated twice, and particle images are recorded by a digital camera. The subsequent processing of these images makes it possible to calculate the displacement of particles for the time between the flashes of a light source. As a light source, a pulsed solid-state laser generating pulses of small duration (4–10 ns) with a fairly high energy is usually used. The main convenience of the PIV method is that it makes it possible to perform contactless measurements and determine instantaneous distributions of particle velocities in a wide range including supersonic velocities. The method of laser Doppler anemometry (LDA) makes it possible to determine the velocity of movement of particles and their sizes and concentration [5] on the basis of measurement of the Doppler shift of the frequency of the radiation scattered by them, which is a linear function of their velocity. The advantages of this method over the other optical measurement methods is its contactlessness, the wide range of velocities that can be measured (10^{-6}–10^{6} m/s), and the high spatial resolution (as high as 10^{-11} cm^3), time resolution 10^{-7}–10^{-9} s), and accuracy (0.2–3.0%). A condition of applicability of the LDA method for diagnostics of two-phase flows is the occurrence of a fairly large number of particles

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scattering radiation in a measurement region because the intensity of the radiation scattering by a small number of particles can be insufficient for recording the Doppler shift of its frequency [6].

In a dispersion-medium flow there can arise regions free of particles as a result of the fragmentation of its phase volume as well as regions with crossing paths of particles because of the formation of "gathers" and "folds" with a markedly increased concentration of the disperse phase at their boundaries. By a fold is meant a flow region, to one point of which the paths of different particles come so that a part of the flow space becomes covered with several layers of the medium. In the case where the volume concentration of the particles in such a flow is low, the collisions between them can be disregarded because real particles will come to the indicated space point at different instants of time [7]. Therefore, of interest is investigation of the concentration field of the inertial particles near peculiar elements of dispersed gas flows [8–10].

The conditions for the appearance of caustics as a result of the interaction of particles with vortex structures of different configurations were investigated in [11, 12]. It was established that a caustic is formed in the process of interaction of a particle with a point vortex in a fluid flow in the case where, at the initial instant of time, the particles in the flow are found within the circle of radius $r \sim 0.5(\Gamma\tau_p)^{1/2}$. In this case, the Stokes number, determined as $\mathrm{Stk} = \Gamma\tau_p/r^2$ is of the order of unity. Another limiting case with respect to the concentration distribution of the disperse phase in a fluid flow is the formation of flow regions free of particles. A theoretical investigation of the movement of small particles in a combined vortex has been performed in [13]. The scattering of solid particles with a diameter of about 1 μm in the process of their interaction with a combined vortex was considered in [14]. On the basis of numerical calculations, the time for which particles escape from the vortex has been determined. In [15], the movement of discrete inclusions (solid particles, droplets, bubbles) in flows with a concentrated vorticity was simulated, the force factors in the equation of motion of a probe particle were estimated, the results of numerical simulations of the movement of discrete inclusions in the clearance between rotating concentric cylinders and in a vortex flow formed as a result of the rotation of a liquid with a constant angular velocity over an immovable base were compared, and the coordinates of the equilibrium points of a probe particle in a vortex flow were determined. In [16, 17], problems on the selection of the inertia parameters of particles-tracers (their density and sizes) necessary for the visualization and optical diagnostics of vortex flows were considered, an expression for determining the time of dynamic relaxation of the particles tracing the gas flows in vortices of different intensities has been obtained, and an example of deciding on the characteristics of particles used for visualization of a swirl and a cascade of laboratory air vortices is presented.

For increasing the accuracy and reliability of measuring the characteristics of two-phase flows with a concentrated vorticity by optical methods, it is necessary to estimate the time for which the particles found in the flow region occupied by a vortex escape from it and to find the dependence of the concentration distribution of particles in a flow on their sizes. The selection of the optimum parameters of particles-tracers is a complex problem. On the one hand, such particles should be fairly large to reflect radiation with an intensity necessary for its recording by a digital camera, and on the other they should possess a low inertia to trace the flow lines without slip. The difference between the velocities of the particles-tracers in a fluid flow and the velocity of the carrying medium in this flow is responsible for an important component of the error in measuring the velocity of a fluid flow by optical methods.

In the present work, the scattering of monodisperse particles by vortex structures was numerically simulated, and the dependence of the time for which the particles in the central region of a vortex escape from it on their sizes was determined.

Mathematical Model. A gas flow with solid particles is considered. In the case where the influence of the particles on this flow is not taken into account, the computational procedure is split into the calculation of the gas flow and the subsequent calculation of the paths of particles and their concentration in the known gasdynamic field.

Main relations. As characteristic scales, we used the radius R of a vortex for the variables with the dimension of length and the velocity U of the gas in the vortex core for the variables with the dimension of velocity. In the limiting cases of forced and free vortices, the distribution of the tangential velocity of the gas flow in a vortex was defined by the relation

$$u_\theta(r) = \begin{cases} r & \text{at } r << 1\,, \\ 1/r & \text{at } r >> 1\,. \end{cases} \tag{1}$$

It was assumed that the vorticity of the gas in a forced vortex has a definite value, and the vorticity of the gas in a free vortex is equal to zero:

$$\omega_z(r) = \begin{cases} \Omega & \text{at } r << 1\,, \\ 0 & \text{at } r >> 1\,. \end{cases} \tag{2}$$

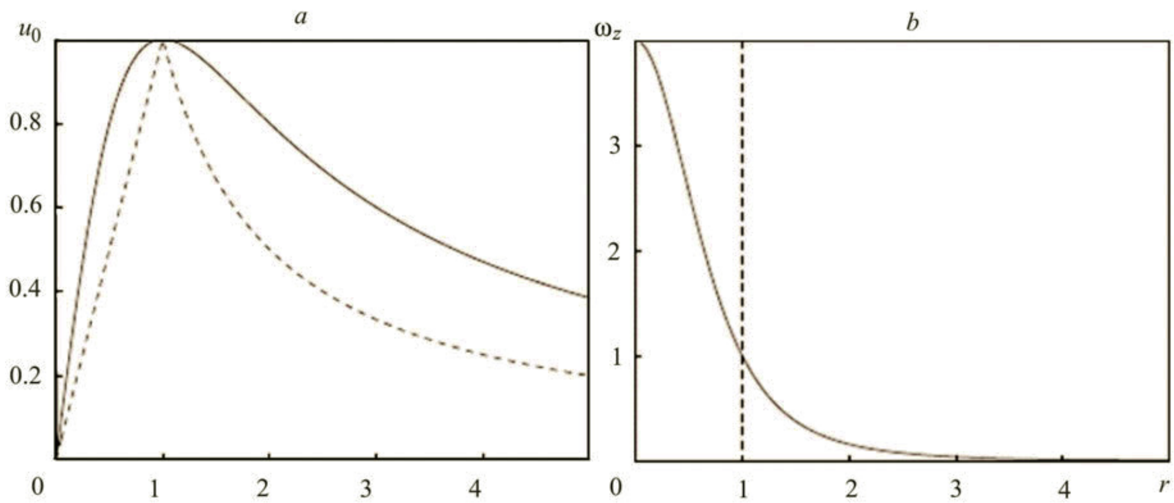

Fig. 1. Distributions of the velocity of the gas flow (a) and its vorticity (b) in a combined vortex, calculated by Eqs. (1) and (2) (dotted lines) and Eqs. (3) and (4) (full lines).

A combined vortex comprises an internal core rotating as a solid body with a constant vorticity Ω and an external region in which the vorticity of the gas tends to zero. At the point $r = 1$ of such a vortex, the distribution of the gas-flow velocity defined by Eq. (1) is nondifferentiable. A smooth distribution of this velocity was constructed using the interpolation formula

$$u_\theta(r) = \frac{2r}{1 + r^2}\,, \tag{3}$$

obeying the ultimate relations (1). The vorticity of the gas in a vortex was determined by the relation

$$\omega_z(r) = \frac{4}{(1 + r^2)^2}\,. \tag{4}$$

The distributions of the velocity and vorticity of the gas flow in a combined vortex, determined by relations (1)–(4), are presented in Fig. 1. It is seen from this figure that, the smaller the radius of a vortex, the smaller the differences between the velocity profiles defined by relations (1), (2), (3) and (4). In the case where the conditions within a combined vortex are isentropic:

$$\frac{p}{p_\infty} = \left(\frac{\rho}{\rho_\infty}\right)^\gamma,$$

the distribution of the density of the gas in it is determined by the relation

$$\frac{\rho}{\rho_\infty} = \left[1 - 2(\gamma - 1)\,\frac{(U/a_\infty)^2}{1 + (r/R)^2}\right]^{1/(\gamma-1)}, \tag{5}$$

which is valid in the limiting cases

$$\rho(r) = \begin{cases} \rho_{\min} & \text{at } r \ll R\,, \\ \rho_\infty & \text{at } r \gg R\,, \end{cases}$$

where $\rho_{\min}$ is the density of the gas at the center of the vortex:

$$\frac{\rho_{\min}}{\rho_\infty} = \left[1 - 2(\gamma - 1)\left(\frac{U}{a_\infty}\right)^2\right]^{1/(\gamma-1)}.$$

At $\Delta\rho/\rho_\infty \ll 1$, the distribution of the gas density in the vortex is determined by the formula

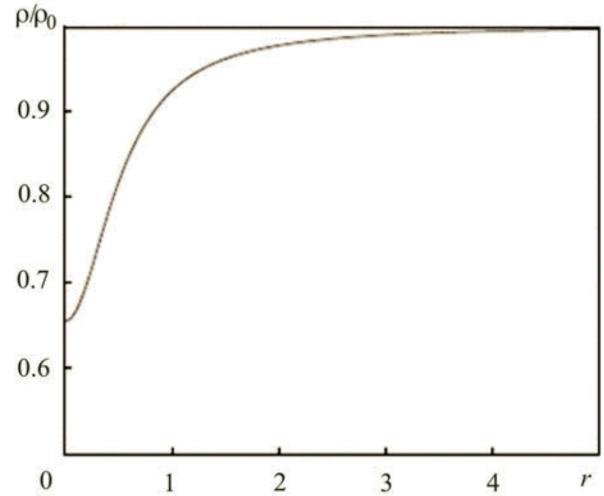

Fig. 2. Distribution of the gas density in a combined vortex.

$$\frac{\rho - \rho_\infty}{\rho_\infty} = -\frac{2(U/a_\infty)^2}{\gamma\left[1 + (r/R)^2\right]} ,$$

where a_∞ is the velocity of sound at infinity. The distribution of the gas density in a combined vortex, defined by relation (5), is shown in Fig. 2. The density of the gas at the center of the vortex is lower than the density of the gas at its periphery.

Movement of a particle. In the trajectory method, the equations of motion of an impurity in a fluid flow are written in Lagrange variables and are integrated along the trajectories of individual particles in the known gasdynamic field calculated in advance. The translational motion of a probe spherical particle in a gas flow is defined by the equation

$$m_\mathrm{p} \frac{d\mathbf{v}}{dt} = \frac{1}{2} C_D \rho S_\mathrm{m} (\mathbf{u} - \mathbf{v}) + m_\mathrm{p}\mathbf{g} . \tag{6}$$

The drag coefficient of the particle is determined from the expression

$$C_D = \frac{24}{\mathrm{Re_p}} f_D(\mathrm{Re_p}) ,$$

where the function f_D accounts for the inertia of the particle [3]. The Reynolds number of the relative movement of the particle and the gas is calculated by the formula

$$\mathrm{Re_p} = \frac{2r_\mathrm{p}\rho\,|\mathbf{u} - \mathbf{v}|}{\mu} .$$

The equation of motion (6) is supplemented with the relation for the radius-vector of the inertia center of the particle

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} . \tag{7}$$

In the general case, the movement of a particle in a gas flow can be defined by the Maxey–Riley equation with account of a number of additional forces acting on the particle [18]. The possibility of using this equation for investigating gasdynamic systems of different classes was considered in [19]. Estimates of different forces acting on a particle in a fluid flow with a concentrated vorticity are given in [15]. The influence of the temperature of a particle on its movement in a fluid flow was taken into account through the introduction of a correction to the drag coefficient of the particle. In many flow regimes, this correction is small and is not used. The concentration distribution of solid particles in a gas flow is determined from the solution of the continuity equation for the disperse phase written in Lagrange variables [3]. Equations (6) and (7) defining the movement of a particle in a gas flow are integrated along the particle path, and they call for the definition of the initial

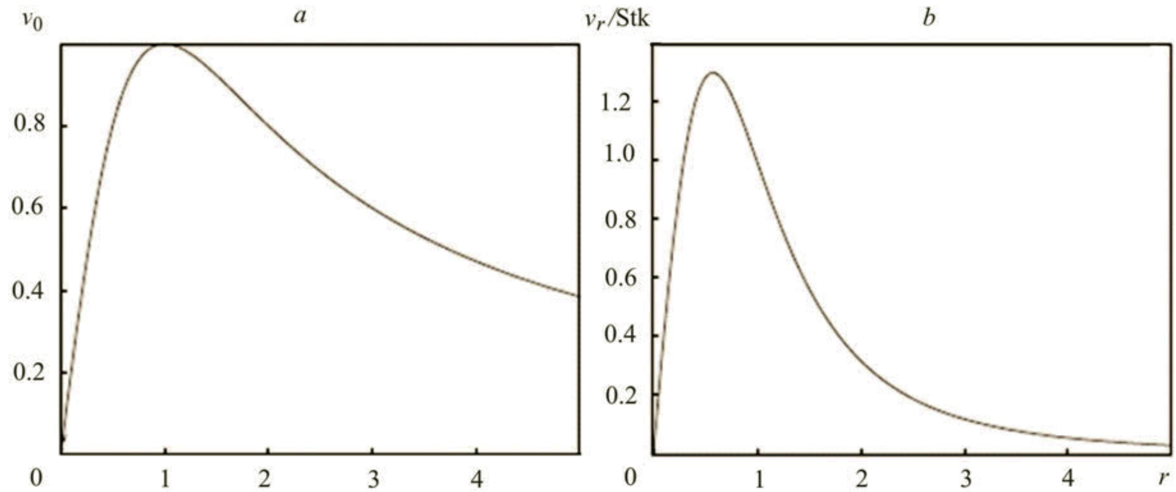

Fig. 3. Distributions of the circumferential (a) and radial (b) velocities of a particle in a combined vortex.

conditions, i.e., the coordinates and velocity of the particle at the instant of time $t = 0$. A characteristic parameter of the problem is the Stokes number representing the ratio between the relaxation time of a particle and the characteristic time scale: $\mathrm{Stk} = \dfrac{2\rho_p r_p^2 U}{9\mu R}$, where ρ_p is the density of the particle material. Assuming that $\rho_p = 10^3$ kg/m^3, $d_p = 1$ μm, $R = 0.1$ m, and $U = 10$ m/s, we obtain $\mathrm{Stk} = 10^{-3}$.

The Cauchy problem is solved using methods allowing one to separate its rapidly and slowly decaying components [3]. The difference schemes developed are based on the linearization of the initial system of equations by freezing individual terms or parts of equations, their approximation in the form of simplified functional dependences, and the subsequent analytical integration of an approximate equation in each time step.

Results of Calculations. The equations of motion of a particle in a combined vortex in a gas flow were integrated over its known velocity field in this vortex.

Velocity distribution. The distributions of the velocity components of a particle in a combined vortex are shown in Fig. 3. In this case, the distributions of the circumferential velocities of the particle and the gas are almost coincident (Fig. 3a), and the distribution of the radial velocity of the particle is practically independent of its size (Fig. 3b). The velocity distributions obtained are in fairly good agreement with those obtained in [13].

The distribution of the velocities of a particle in the regions of a combined vortex $r \sim O(\mathrm{Stk})$, $O(1)$, and $O(\mathrm{Stk}^{-1})$ [13] were analyzed. Since $\mathrm{Stk} = O(10^{-3})$, the results obtained for the outer region $r \sim O(\mathrm{Stk})$ are of no interest. According to the asymptotic analysis performed in [3], the velocity distribution of a particle in the intermediate region is defined as

$$\mathbf{v} = \mathbf{u} + \mathrm{Stk}\left(\frac{3}{2}\delta - 1\right)(\mathbf{u}\cdot\nabla\mathbf{u} - \mathbf{g}) + O(\mathrm{Stk}^{3/2}),$$

where δ is the ratio between the densities of the gas and dispersed phases ($\delta = \rho/\rho_p$). The radial and circumferential velocities of the particle, projected onto the cylindrical coordinate axes, are determined from the relations

$$v_r(r, \theta) = \mathrm{Stk}\left(\frac{3}{2}\delta - 1\right)\left[-\frac{u_\theta^2(r)}{r} - g_r(\theta)\right] + O(\mathrm{Stk}^{3/2}),$$

$$v_\theta(r, \theta) = u_\theta(r) - g_\theta(\theta)\,\mathrm{Stk}\left(\frac{3}{2}\delta - 1\right) + O(\mathrm{Stk}^{3/2}).$$

The orders of the summands in these relations allow the conclusion that the gravitational forces weakly influence the movement of the particle, as compared to the centrifugal forces. The order of the summands defining the gravity effect is $O(10^{-5})$. The error of the asymptotic analysis performed is of the order of $O(10^{-4})$. In this case, the distributions of the circumferential

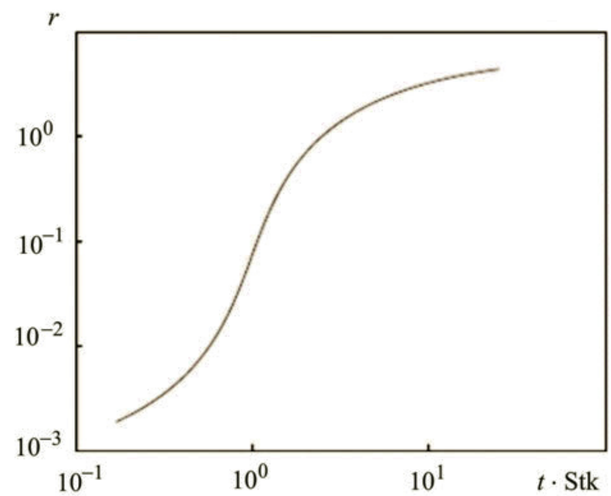

Fig. 4. Time dependence of the radial coordinate of a particle in a combined vortex.

velocities of the particle and the gas are almost identical ($v_\theta = u_\theta$). Unlike the gas, the radial velocity of the particle is nonzero and depends on its inertia. In the inner region of the vortex where $r \sim O(\mathrm{Stk}^{-1})$, the distributions of the velocity components of the particle are determined by the relations

$$v_r(r,\,\theta) = \mathrm{Stk}\left(\frac{3}{2}\,\delta - 1\right)\left[-4r - g_r(\theta)\right] + O(\mathrm{Stk}^{5/2})\,,$$

$$v_\theta(r,\,\theta) = 2r - g_\theta(\theta)\mathrm{Stk}\left(\frac{3}{2}\,\delta - 1\right) + O(\mathrm{Stk}^{5/2})\,.$$

The gravitation effects in the inner region of the vortex play a more important role compared to those in its intermediate region. The gravitational forces mainly influence the distribution of the radial velocity of the particle, with the result that it begins to execute a periodic motion with a small amplitude. In this case, $v_r/v_\theta \sim O(10^{-2})$ and, therefore, the shift of the particle in the radial direction is of the order of $\Delta r \sim O(10^{-5})$.

The displacement of the particles-tracers in a complex vortex in the radial direction results in that they do not trace the lines of flow in it. The quantitative measure of the deviation of these particles from the flow lines is the angle of rotation of their velocity vector appearing due to the movement of the particles in the radial direction. The lines of the gas flow in a combined vortex have the form of concentric circles. Small particles move along these flow lines and, in so doing, gain a velocity in the radial direction, with the result that their path becomes spiral. The time change in the radial coordinate of a particle in a combined vortex is presented in the logarithmic scale in Fig. 4. At the initial instant of time, the particle is found in the neighborhood of the vortex center ($r_{p0} = \mathrm{Stk}$), which prevents the appearance of a singularity in it. Trajectory calculations of probe particles made it possible to determine the time for which a particle escapes the boundaries of a vortex. The influence of the initial position of a particle in a vortex on the time it takes for the particle to reach a definite radial coordinate is demonstrated in Fig. 5. For example, at $U = 25$ m/s, a particle of diameter 1.5 μm ($\mathrm{Stk} = 3.5 \cdot 10^{-3}$) escapes from the core of a vortex of radius $R = 0.05$ m (the region $r < R$) for 0.3 s.

Concentration distribution. Let us determine the concentration distribution of the particles in the cylindrical region of radius r and height l of a combined vortex. The spatial and time distributions of the concentration of small particles $C = dN/(2\pi r dr l)$ was determined from the solution of the continuity equation written in the axisymmetric form without regard for the diffusion transfer of these particles:

$$\frac{\partial C}{\partial t} + \frac{1}{r}\,\frac{\partial (rCv_r)}{\partial r} = 0\,. \qquad (8)$$

It was assumed that, at the initial instant of time, the particles are distributed uniformly in the indicated space: $C = 1$ at $t = 0$, and that the concentration of the particles at the axis of the vortex has a definite value. Assuming that $Z = 2\pi r l C$, from (8) we obtain the equation

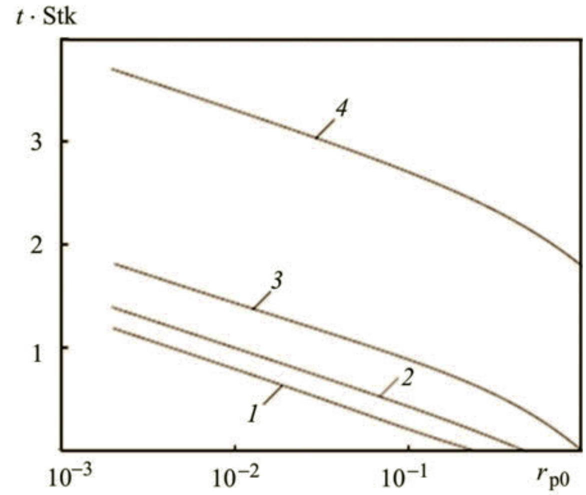

Fig. 5. Influence of the radial position of a particle in a combined vortex on the time for which the particle reaches the coordinates $r = 0.25$ (1), 0.5 (2), 1 (3), and 2 (4).

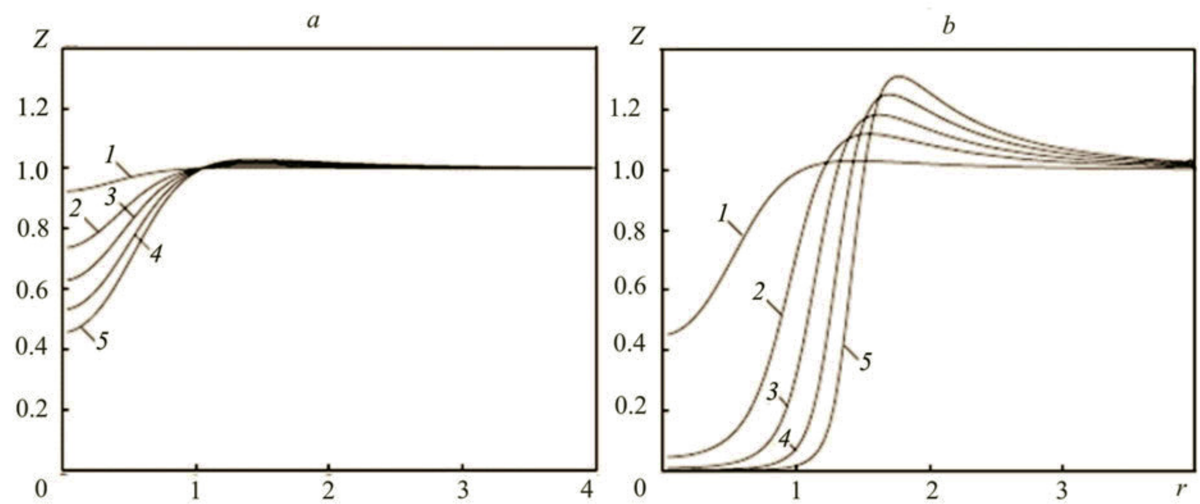

Fig. 6. Concentration distribution of particles with $d_p = 0.5$ (a) and 1.5 μm (b) along the radial coordinate of a combined vortex at different instants of time: $t = 0.1$ (1), 0.4 (2), 0.6 (3), 0.8 (4), and 1.0 (5).

$$\frac{\partial Z}{\partial t} + \frac{1}{r}\frac{\partial (Z v_r)}{\partial r} = 0 . \quad (9)$$

As the boundary condition for this equation, the condition of symmetry at the vortex axis $\partial Z/\partial r = 0$ is used. Equation (9) was solved by the finite difference method. Its discretization with respect to the time was performed by the Euler scheme of the first order of accuracy, and the discretization of his equation with respect to the radial coordinate was performed using the centered finite-difference formulas of the second order of accuracy. Numerical simulation of the concentration distribution of particles in the region $[-2, +2]^2$ of the vortex was performed, and the distributions of particles in both the core of the vortex and outside it were determined.

Figure 6 shows the concentration distributions of particles of different sizes with Stokes numbers differing by an order of magnitude in a vortex of radius $R = 0.05$ m in which the velocity of the gas flow is equal to $U = 25$ m/s and the ratio between the densities of the gas and disperse phases is $\delta = 10^{-3}$. While the concentration of the small particles at the center of the vortex decreases fairly slowly (Fig. 6a), large particles escape the central region of the vortex for a comparatively short time (Fig. 6b). The concentration distribution of large particles along the radius of the vortex is nonmonotonous, and it

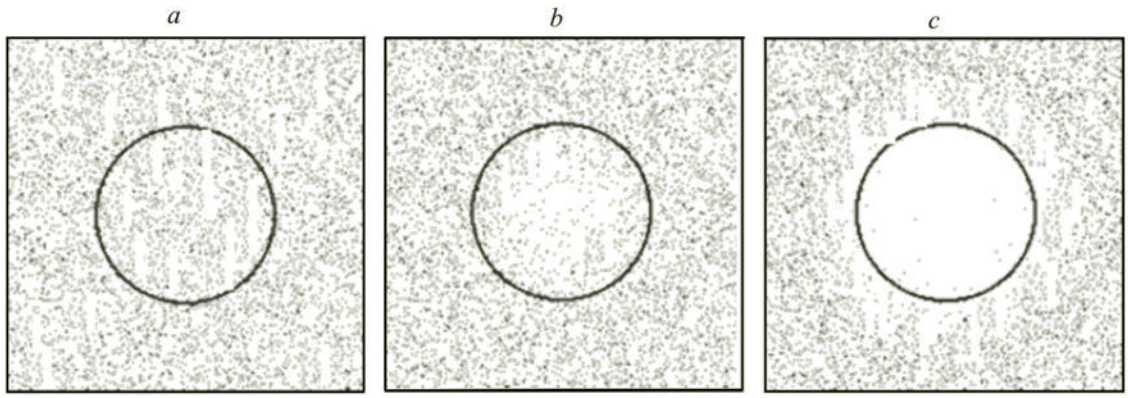

Fig. 7. Concentration distribution of monodisperse particles in a combined vortex at $t = 0$ (a), 0.3 (b), and 1 s (c).

reaches a maximum in the peripheral region of the vortex. With time this maximum shifts in the direction from the center of the vortex.

The instantaneous concentration distributions of monodisperse particles with $d_p = 1$ μm in a vortex in which their total number is equal to 10^5 are shown in Fig. 7 where the full line defines the vortex core. At the initial instant of time, the concentration distribution of particles in the vortex is homogeneous (Fig. 7a). Then particles are thrown out of the core of the vortex to its peripheral regions under the action of the centrifugal forces and are accumulated at the boundary of the computational region (Fig. 7b). At the final instant of time, the vortex core is almost free of particles (Fig. 7c).

Conclusions. When nonstationary vortex flows arising spontaneously and changing with space and time are investigated experimentally, the selection of the sizes of particles-tracers takes on great significance because of the centrifugal inertial force acting on the particles in one direction in a vortex (from its rotation center). The results obtained allow one to select the sizes of tracing particles providing a reliable measurement of their concentration in the region of a flow occupied by a vortex. The large particles in the core of a vortex escape from it for a fairly short time, which makes the measurement of their parameters by the PIV method difficult. The measurement of the parameters of small particles by optical methods is inefficient because of the scattering of light by them (the scattering cross section of a particle is proportional to d_p^2). At $\pi d_p/\lambda << 1$, the Rayleigh scattering of light by a particle takes place.

A numerical simulation of the scattering of monodisperse particles in a gas flow by a combined vortex in it has been performed. The dependence of the time for which these particles escape the central region of the vortex on their size and the change in the concentration distribution of particles in the flow region occupied by the vortex with time have been determined. The results obtained can be used for increasing the efficiency of measurements of the parameters of the disperse phase in a gas flow by the PIV and LDA methods.

NOTATION

C, concentration, m^{-3}; C_D, drag coefficient; d, diameter, m; f_D, correction function; g, free fall acceleration, m/s; l, length, m; m_p, mass of a particle, kg; p, pressure, Pa; r_p, radius of a particle, m; **r**, radius-vector, m; R, radius of a vortex, m; Re, Reynolds number; r and θ, cylindrical coordinates; S_m, area of the midsection of a particle, m^2; Stk, Stokes number; t, time, s; u, velocity of a gas, m/s; U, velocity of the gas in the core of a vortex, m/s; v, velocity of a particle, m/s; x, y, and z, Cartesian coordinates, m; Z, normalized concentration of particles; γ, ratio between the specific heat capacities of the gas at a constant pressure and a constant volume; Γ, circulation of the gas-flow velocity, m^2/s; λ, wavelength, m; μ, dynamic viscosity, kg/(m·s); ν, kinematic viscosity, m^2/s; ρ, density of the gas, kg/m^3; τ_p, time of the dynamic relaxation of a particle, s; τ_f, time for which a particle escapes the core of a vortex, s; ω, vorticity, 1/s. Subscripts: f, final; p, particle; 0, initial instant of time.

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