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HYDROGASDYNAMICS IN TECHNOLOGICAL PROCESSES

DYNAMICS OF WAVES IN MULTIFRACTIONAL BUBBLE LIQUIDS

D. A. Gubaidullin and R. N. Gafiyatov

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A study has been made of pulsed waves in mixtures of a liquid and a dispersed phase consisting of vapor–gas and gas bubbles that differ in radii and thermophysical properties. A system of differential equations has been proposed for description of the motion of such a mixture and a dispersion relation has been derived for it. A comparison has been made of the dynamics of acoustic waves in mixtures of water with vapor–air bubbles, bubbles of carbon dioxide with steam, and helium bubbles, and also in monodisperse mixtures of water with bubbles of one gas. In the investigations, the qualitative composition of the dispersed phase varied due to the buildup in the volume content of the bubbles of one fraction and the relevant decrease in the content of the bubbles of the other fraction, with the total volume content of the bubbles being constant.

Keywords: *acoustic waves, bubble liquid, dispersion relation, heat and mass transfer.*

Introduction. Studies of the wave dynamics of dispersive media are of great current interest. Numerous works on the acoustics of bubble liquids are devoted to a theoretical study of the propagation of harmonic disturbances in monodisperse mixtures. Various problems of the acoustics of mixtures of liquids and gas or vapor bubbles have been considered in the existing monographs [1, 2]. The main features of two-phase media of bubble structure have been described in [3]; works on the propagation of waves in liquids with bubbles of constant mass and on the wave dynamics of liquids containing vapor or soluble-gas bubbles have also been reviewed. A model of propagation of plane small-amplitude pressure waves in a mixture of a liquid and gas bubbles has been presented in [4] and its good performance at subresonant frequencies as applied to mixtures with a volume content of the dispersed phase of 1–2% has been demonstrated. In [5], a study has been made of the propagation of acoustic waves in two-fraction mixtures of a liquid and vapor–gas and gas bubbles of varying size and composition with phase transitions. In [6, 7], consideration has been given to the propagation of acoustic waves in two-fraction mixtures of a liquid and polydisperse gas bubbles of varying composition; a comparison has been made of the theory and the existing experimental data. In [8], a study has been made of the propagation of acoustic waves in multifractional mixtures of a liquid and vapor–gas and gas bubbles of varying size and composition with phase transformations. The present work seeks to study the evolution of pulsed pressure waves in multifractional bubble liquids.

Basic Equations. Consideration is given to the plane one-dimensional motion of a multifractional bubble liquid in an acoustic field in the case where some fractions of the gas bubbles contain vapor and are involved in phase transformations, whereas others consist of insoluble gas. Here, the bubbles of each fraction have dimensions different from the dimensions of the bubbles of the remaining fractions. The gas of which the bubbles of each fraction consist differs in thermophysical properties from the gases and bubbles of the other fractions. The volume contents of the bubbles of each fraction α_{2j} and of the liquid α_1 are determined as

$$\alpha_1 + \sum_{j=1}^{N} \alpha_j + \sum_{i=1}^{M} \alpha_i = 1\,, \quad \alpha_j = \frac{4}{3}\pi a_j^3 n_j\,, \quad \alpha_i = \frac{4}{3}\pi a_i^3 n_i\,, \quad j = \overline{1, N}\,, \quad i = \overline{1, M}\,,$$

and the reduced densities of the carrier and dispersed phases, from the relations

$$\rho_1 = \rho_1^{\text{tr}}\alpha_1\,, \quad \rho_j = \rho_j + \rho_{vj} = \rho_j^{\text{tr}}\alpha_j + \rho_v^{\text{tr}}\alpha_j\,, \quad \rho_i = \rho_i^{\text{tr}}\alpha_i\,, \quad j = \overline{1, N}\,, \quad i = \overline{1, M}\,.$$

Institute of Mechanics and Mechanical Engineering, Kazan Scientific Center of the Russian Academy of Sciences, 2/31 Lobachevskii Str., Kazan, 420111, Russia; Kazan (Volga Region) Federal University, 18 Kremlevskaya Str., Kazan, 420008, Russia; email: gubaidullin@imm.knc.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 90, No. 6, pp. 1506–1511, November–December, 2017. Original article submitted March 31, 2016.

It is assumed that the bubble mixture is initially not perturbed, i.e., its parameters ψ_0 ($\psi \equiv \rho_1, \rho_2, p_1, v, \ldots$) remain constant along the coordinate x and with time t. Let us consider small perturbations of the mixture's parameters ψ':

$$\psi = \psi_0 + \psi', \quad \psi' << \psi_0 .$$

Linearized equations for such one-dimensional perturbed motion of a multifractional bubble liquid follow from the general equations of motion of two-phase media [1]:

$$\frac{\partial \rho_1'}{\partial t} + \rho_{10} \frac{\partial v'}{\partial x} = -J , \quad J = \sum_{j=1}^{N} J_j , \tag{1}$$

$$\frac{\partial \rho_j'}{\partial t} + \rho_{j0} \frac{\partial v'}{\partial x} = J_j , \quad \frac{\partial \rho_{\mathrm{v}j}'}{\partial t} + \rho_{\mathrm{v}j0} \frac{\partial v'}{\partial x} = J_j , \tag{2}$$

$$\frac{\partial \rho_i'}{\partial t} + \rho_{i0} \frac{\partial v'}{\partial x} = 0 , \tag{3}$$

$$\frac{\partial n_j'}{\partial t} + n_{j0} \frac{\partial v'}{\partial x} = 0 , \quad \frac{\partial n_i'}{\partial t} + n_{i0} \frac{\partial v'}{\partial x} = 0 , \tag{4}$$

$$\rho_{10} \frac{\partial v'}{\partial t} + \frac{\partial p_1'}{\partial x} = 0 , \tag{5}$$

$$\rho_{10} c_1 \frac{\partial T_1'}{\partial t} = \sum_{j=1}^{N} n_{j0} q_{1\Sigma j} + \sum_{i=1}^{M} n_{i0} q_{1\Sigma i} , \tag{6}$$

$$\rho_{j0} c_j \frac{\partial T_j'}{\partial t} = \alpha_{j0} \frac{\partial p_j'}{\partial t} + n_{j0} q_{\Sigma j} , \quad \rho_{i0} c_{2i} \frac{\partial T_i'}{\partial t} = \alpha_{i0} \frac{\partial p_i'}{\partial t} + n_{i0} q_{\Sigma i} , \tag{7}$$

$$n_{j0} q_{1\Sigma j} + n_{j0} q_{\Sigma j} = -l_0 J_j' , \quad n_{i0} q_{1\Sigma i} + n_{i0} q_{\Sigma i} = 0 , \tag{8}$$

$$\frac{\partial a_j'}{\partial t} = w_{\mathrm{a}j}' + w_{\mathrm{r}j}' + \frac{J_j'}{4\pi (a_{j0})^2 n_{j0} \rho_{10}^{\mathrm{tr}}} , \tag{9}$$

$$a_{j0} \frac{\partial w_{\mathrm{r}j}'}{\partial t} + \frac{4\nu_1}{a_{j0}} w_{\mathrm{r}j}' = \frac{p_j' - p_1'}{\rho_{10}^{\mathrm{tr}}} , \tag{10}$$

$$\frac{\partial a_i'}{\partial t} = w_{\mathrm{a}i}' + w_{\mathrm{r}i}' , \tag{11}$$

$$a_{i0} \frac{\partial w_{\mathrm{r}i}'}{\partial t} + \frac{4\nu_1}{a_{i0}} w_{\mathrm{r}i}' = \frac{p_i' - p_1'}{\rho_{10}^{\mathrm{tr}}} , \tag{12}$$

$$w_{\mathrm{a}j}' = \frac{p_{2j}' - p_1'}{\rho_{10}^{\mathrm{tr}} C_1 (\alpha_{20j})^{1/3}} , \quad w_{\mathrm{a}i}' = \frac{p_{2i}' - p_1'}{\rho_{10}^{\mathrm{tr}} C_1 (\alpha_{20i})^{1/3}} , \tag{13}$$

$$p_1' = C_1^2 \rho_1'^{\mathrm{tr}} , \tag{14}$$

$$\frac{p_j'}{p_0} = \frac{\rho_j'^{\mathrm{tr}}}{\rho_{j0}^{\mathrm{tr}}} + \Delta R_j k_{\mathrm{v}j}' + \frac{T_j'}{T_0} , \quad \Delta R_j = \frac{R_{\mathrm{v}} - R_{\mathrm{g}j}}{R_j} , \tag{15}$$

$$R_j = k_{\mathrm{g}j0}R_{\mathrm{g}j} + k_{\mathrm{v}j0}R_{\mathrm{v}}\ , \quad j = \overline{1,\ N}\ , \quad i = \overline{1,\ M}\ ,$$

$$\frac{p_i'}{p_0} = \frac{\rho_i'^{\mathrm{tr}}}{\rho_{i0}^{\mathrm{tr}}} + \frac{T_i'}{T_0}\ , \quad i = \overline{1,\ M}\ , \tag{16}$$

$$\frac{T_{\Sigma j}'}{T_0} = E_j k_{\mathrm{v}\Sigma j}' + G_j\ \frac{p_j'}{p_0}\ , \tag{17}$$

$$E_j = \frac{R_{\mathrm{v}}}{R_j}\ \frac{p_0}{l_0\rho_{\mathrm{v}0}^{\mathrm{tr}}}\left(1 - k_{\mathrm{v}j0}\right), \quad G_j = k_{\mathrm{v}j0}E_j\ , \quad j = \overline{1,\ N}\ ,$$

where $k_{\mathrm{g}j} = \rho_{\mathrm{g}j}/\rho_j$ is the mass concentration of the gas and $k_{\mathrm{v}j} = \rho_{\mathrm{v}j}/\rho_j$ is the mass concentration of the vapor in the bubbles of the jth fraction ($k_{\mathrm{g}j} + k_{\mathrm{g}j} = 1$).

Heat transfer and the kinetics of phase transformations in the mixture in question are described by the relations [1, 9]

$$n_{j0}q_{1\Sigma j} = c_1\rho_{j0}\ \frac{T_{\Sigma j}' - T_1'}{\tau_{T1j}}\ , \quad n_{j0}q_{\Sigma j} = c_{2j}\rho_{j0}\ \frac{T_{\Sigma j}' - T_j'}{\tau_{Tj}}\ , \tag{18}$$

$$n_{i0}q_{1\Sigma i} = c_1\rho_{i0}\ \frac{T_{\Sigma i}' - T_1'}{\tau_{T1i}}\ , \quad n_{i0}q_{\Sigma i} = c_i\rho_{i0}\ \frac{T_{\Sigma i}' - T_i'}{\tau_{Ti}}\ , \tag{19}$$

$$J_j = \frac{\rho_{j0}}{1 - k_{\mathrm{v}j0}}\ \frac{k_{\Sigma j}' - k_{\mathrm{v}j}'}{\tau_j}\ , \quad j = \overline{1,\ N}\ , \tag{20}$$

$$\tau_{T1j} = \frac{4c_1\rho_{j0}^{\mathrm{tr}}(a_{j0})^2}{3\mathrm{Nu}_1\lambda_1}\ , \quad \tau_{Tj} = \frac{4c_j\rho_{j0}^{\mathrm{tr}}(a_{j0})^2}{3\mathrm{Nu}_j\lambda_j}\ , \quad \tau_j = \frac{2(a_{j0})^2}{3\mathrm{Sh}D}\ ,$$

$$\tau_{T1i} = \frac{4c_1\rho_{j0}^{\mathrm{tr}}(a_{i0})^2}{3\mathrm{Nu}_1\lambda_1}\ , \quad \tau_{Ti} = \frac{4c_i\rho_{i0}^{\mathrm{tr}}(a_{i0})^2}{3\mathrm{Nu}_i\lambda_i}\ , \quad j = \overline{1,\ N}\ , \quad i = \overline{1,\ M}\ .$$

Dispersion Relation. The solution of the system of equations (1)–(20) is represented in the form of progressive monochromatic waves for perturbations of the parameters of the bubble mixture:

$$\psi' = A_\psi\ \exp\left[i(K_* x - \omega t)\right], \quad K_* = K + iK_{**}\ , \quad C_p = \omega/K\ . \tag{21}$$

Substituting expressions of the form (21) into the system of equations (1)–(20), we obtain a system of linear algebraic equations for the amplitudes A_ψ. A dispersion relation governing the dependence of the complex wave number on the oscillation frequency results from the condition of the nontrivial solution existing for the obtained system of linear algebraic equations:

$$\left(\frac{K_*}{\omega}\right)^2 = \frac{1}{C_{\mathrm{f}}^2} + \frac{\rho_{10}}{p_0}\ \frac{\left(\sum_{i=1}^{M} H_{3i} + \sum_{j=1}^{N} H_{3j}\right)\left(\sum_{i=1}^{M} H_{1i} + \sum_{j=1}^{N} H_{1j}\right)}{\sum_{i=1}^{M}\left(\frac{m_i}{\tau_{T1i}} + H_{2i}\right) + \sum_{j=1}^{N}\left(\frac{m_j}{\tau_{T1j}} + H_{2j}\right) - i\omega}$$
$$+ \frac{\rho_{10}}{p_0}\sum_{i=1}^{M}\left(\frac{\alpha_{20i}}{N_{\mathrm{r}i}}\left(1 - \frac{M_{4i}}{M_{3i}}\right)\right) + \frac{\rho_{10}}{p_0}\sum_{j=1}^{N}\left(\frac{\alpha_{20j}}{N_{Rj}}\left(1 - \frac{M_{4j}}{M_{3j}}\right)\right), \tag{22}$$

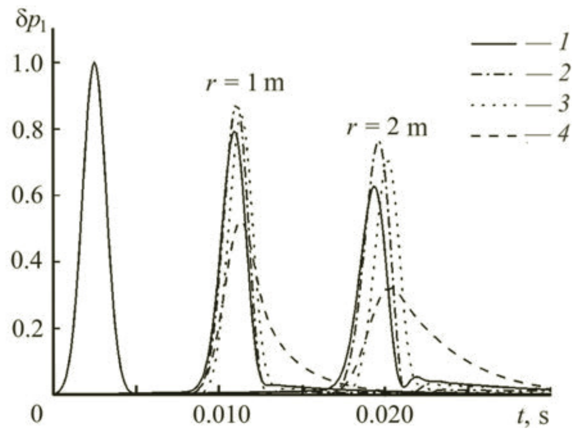

Fig. 1. Evolution of a pulse pressure disturbance in the three-fraction mixture of water and vapor-air bubbles, bubbles of carbon dioxide with steam, and helium bubbles (1) and in monodisperse mixtures of water and vapor-air bubbles (2), bubbles of carbon dioxide with steam (3), and helium bubbles (4):1) $\alpha_2 = \alpha_3 = \alpha_4 = 0.0033$, $a_{20} = 1.5\cdot10^{-3}$ m, $a_{30} = 10^{-3}$ m, and $a_{40} = 2\cdot10^{-3}$ m; 2) $\alpha_2 = 0.01$ and $a_0 = 1.5\cdot10^{-3}$ m; 3) $\alpha_2 = 0.01$ and $a_0 = 10^{-3}$ m; 4) $\alpha_0 = 0.01$ and $a_0 = 2\cdot10^{-3}$ m.

where $C_{\rm f}$ is the frozen velocity of sound ($C_{\rm f} = C_1/\alpha_{10}$), $H_{1j} = \dfrac{m_j}{\tau_{T1j}}\left(\dfrac{M_{1j}M_{4j}}{M_{3j}} + M_{2j}\right)$, $H_{1i} = \dfrac{m_i}{\tau_{T1i}}\left(\dfrac{M_{1i}M_{4i}}{M_{3i}} + M_{2i}\right)$,

$$H_{2j} = \frac{m_j}{\tau_{T1j}}\frac{M_{1j}b_j}{M_{3j}}, \quad H_{2i} = \frac{m_i}{\tau_{T1i}}\frac{M_{1i}b_i}{M_{3i}}, \quad H_{3j} = \frac{\alpha_{j0}}{N_{\rm r\it j}}\frac{b_j}{M_{3j}}, \quad H_{3i} = \frac{\alpha_{i0}}{N_{\rm r\it i}}\frac{b_i}{M_{3i}}, \quad M_{1j} = G_j - M_{2j} - \frac{L_{1j}N_{3j}}{L_{4j} - \delta N_{2j}},$$

$$M_{2j} = \frac{N_{2j}L_{1j}}{N_{\rm r\it j}(L_{4j} - \delta N_{2j})}, \quad M_{3j} = \frac{N_{3j}\delta - L_{2j}N_{3j}}{L_{4j} - \delta N_{2j}} + L_{3j} + M_{4j}, \; M_{4j} = \frac{L_{4j} - L_{2j}N_{2j}}{N_{\rm r\it j}(L_{4j} - \delta N_{2j})}, \quad M_{1i} = -(N_{3i} + M_{2i}),$$

$$M_{2i} = \frac{N_{2i}}{N_{\rm r\it i}}, \quad M_{3i} = 1 + N_{3i}(1 + b_i) + M_{4i}, \quad M_{4i} = \frac{1}{N_{\rm r\it i}}\left(1 + N_{2i}(1 + b_i)\right), \quad N_{1j} = \frac{i\omega\tau_{T1j}}{m_j} - 1, \quad N_{1i} = 1,$$

$$N_{2j} = i\omega\tau_{Tj} - 1, \quad N_{2i} = i\omega\tau_{Ti} - 1, \quad N_{3j} = k_{2j}(c_j - R_j) - 1 + G_j, \quad N_{3i} = k_{2i}(c_i - R_i) - 1, \quad L_{1j} = E_j(1 - i\omega\tau_j),$$

$$L_{2j} = -\frac{l_0 k_{2j}}{(1 - k_{{\rm v}j0})T_0} + \Delta R_j - L_{1j}(1 + b_j), \quad L_{3j} = 1 - G_j(1 + b_j), \quad L_{4j} = L_{1j} + \Delta R_j N_{2j}, \quad k_{2j} = \frac{i\omega\tau_{Tj}}{c_j}, \quad k_{2i} = \frac{i\omega\tau_{Ti}}{c_i},$$

$$b_j = \frac{c_1\tau_{Tj}}{c_j\tau_{T1j}}, \quad b_i = \frac{c_1\tau_{Ti}}{c_i\tau_{T1i}}, \quad N_{\rm r\it j} = \frac{-(i\omega)(a_{j0})^2 G_{\rm r\it j}\rho_{10}^{\rm tr}}{3(t_{{\rm a}j}G_{\rm r\it j} + 1)p_0}, \quad N_{\rm r\it i} = \frac{-(i\omega)(a_{i0})^2 G_{\rm r\it i}\rho_{10}^{\rm tr}}{3(t_{{\rm a}i}G_{\rm r\it i} + 1)p_0}, \quad G_{\rm r\it j} = \frac{1}{t_{\rm r\it j}} - i\omega, \quad G_{\rm r\it i} = \frac{1}{t_{\rm r\it i}} - i\omega,$$

$$t_{\rm r\it j} = \frac{(a_{j0})^2}{4\nu_1}, \quad t_{\rm r\it i} = \frac{(a_{i0})^2}{4\nu_1}, \quad t_{{\rm a}j} = \frac{a_{j0}}{C_1(\alpha_{j0})^{1/3}}, \quad t_{{\rm a}i} = \frac{a_{i0}}{C_1(\alpha_{i0})^{1/3}}, \quad m_{j0} = \frac{\rho_{j0}^{\rm tr}}{\rho_{10}^{\rm tr}}, \quad m_{i0} = \frac{\rho_{i0}^{\rm tr}}{\rho_{10}^{\rm tr}}, \quad m_j = \frac{\rho_{j0}}{\rho_{10}}, \text{ and } m_i = \frac{\rho_{i0}}{\rho_{10}},$$

$i = \overline{1, M}$ and $j = \overline{1, N}$. Dispersion dependence (22) of the complex wave number K_* on the oscillation frequency ω governs the propagation of acoustic perturbations in multifractional mixtures of a liquid and vapor–gas and gas bubbles, which include bubble fractions with gas bubbles of different initial radii, volume contents, and thermophysical properties.

Calculation Results. Consideration was given to an evolution (of the Gauss-curve type) of pressure pulses of the initial form $p(0, t) = \exp[-((t - t_*)/N^2]$, where t_* is half the phase duration and N is the parameter determining the peak width of the pulses created on the boundary of the bubble medium. The calculations were carried out using dispersion relation (22) and fast-Fourier-transformation subprograms according to the procedure presented in [11].

Figure 1 compares the evolution of a pressure pulse in various mixtures of water and gas bubbles at the initial pressure and temperature of the mixture $p_0 = 0.1$ MPa and $T_0 = 327$ K. The calculations were carried out for distances of 1 m and 2 m from the site of initiation of the pulse. It can be seen that the decay of pressure pulses in the three-fraction mixture is larger than that in monodisperse mixtures of water and vapor–air bubbles and bubbles of carbon dioxide with steam and smaller than that in the mixture with helium bubbles.

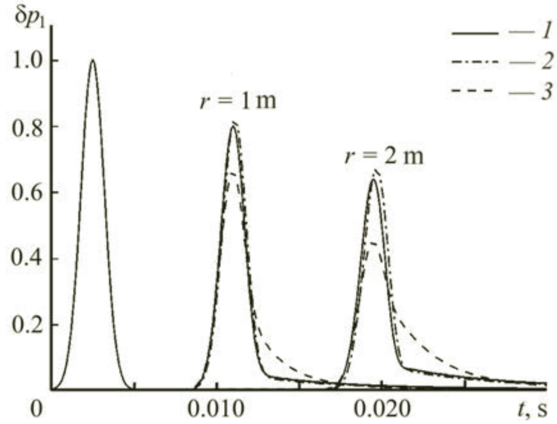

Fig. 2. Evolution of a pulse pressure disturbance in monodisperse mixtures of water and vapor-air bubbles (1), vapor-air bubbles and bubbles of carbon dioxide with steam (2), and vapor-air bubbles, bubbles of carbon dioxide with steam, and helium bubbles (3): 1) $\alpha_2 = 0.01$ and $a_0 = 10^{-3}$ m; 2) $\alpha_2 = 0.0067$, $\alpha_{23} = 0.0033$, and $a_{20} = a_{30} = 10^{-3}$ m; 3) $\alpha_2 = \alpha_3 = \alpha_4 = 0.0033$ and $a_{20} = a_3 = a_4 = 10^{-3}$ m.

Figure 2 compared the evolutions of pressure pulses in monodisperse mixtures of water and gas bubbles with different thermal diffusivities: $\kappa = 2.1 \cdot 10^{-5}$ m^2/s for the mixture of carbon dioxide and steam, $\kappa = 4 \cdot 10^{-5}$ m^2/s for the mixture of air and steam, and $\kappa = 20.9 \cdot 10^{-5}$ m^2/s for helium. It can be seen that the difference in thermophysical properties for the bubbles of different fractions is pronounced even at small volume contents of them in these fractions depending on the thermal diffusivity of the gas contained in the bubbles.

Conclusions. A study has been made of the propagation of an acoustic signal in multifractional bubble liquids including different numbers of bubble fractions with account taken of interphase heat and mass transfer in them. Consideration has been given to bubble media in which bubbles may differ not only in size but also in gas composition, i.e., in thermophysical properties. The results of a numerical comparative experiment show that the degree of dissipation of an acoustic signal in the said liquids is strongly dependent on the qualitative composition of the dispersed phase in them.

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NOTATION

a, bubble radius, m; *c*, specific heat, m^2/(s^2·K); *C*, velocity of sound, m/s; *D*, binary diffusion coefficient, m^2/s; *J*, intensity of phase transition, J; *i*, imaginary unit; K_*, complex wave number, 1/m; K_{**}, linear damping coefficient, 1/m; *k*, mass concentration; *l*, specific heat of vaporization, J/kg; *m* and m^{tr}, ratios of the average and true densities of the dispersed and carrier phases; *n*, number of bubbles per unit volume of the bubble mixture; Nu, Nusselt number; *p*, pressure, Pa; *q*, heat-transfer intensity, J; *R*, gas constant, J/(kg·K); *r*, distance from the site of initiation, m; Sh, Sherwood number; *T*, temperature, K; *t*, time, s; *v*, rate, m/s; w, velocity of motion of bubbles, m/s; *x*, coordinate; α, volume content of bubbles; κ, thermal diffusivity, m^2/s; λ, thermal conductivity, kg·m/(s^3·K); ν, kinematic viscosity, m^2/s; ρ and ρ^{tr}, average and true densities of phases, kg/m^3; τ, relaxation time, s; ω, frequency of acoustic oscillations, Hz. Subscripts and superscripts: a, acoustic addition; f, frozen; f, gas; *i* and *j*, parameters of bubbles without phase transitions and with phase transitions; r, radial motion; tr, true; v, vapor; 0, initial parametric value; ′, perturbed state; 1, carrier phase; Σ, phase boundary.

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