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INFLUENCE OF DISSIPATION ON HEAT TRANSFER DURING FLOW OF A NON-NEWTONIAN FLUID IN A POROUS CHANNEL

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A study is made of flow and heat transfer during the motion of a non-Newtonian (power-law) fluid in a plane channel filled with porous material. The Brinkman equation is used as the equation of state, and a one-temperature model, in representing the energy equation. Account us taken of dissipative heat releases. The problem is solved for temperature boundary conditions of the first kind. The authors show the influence of dissipation on the development of the temperature profile, and also on the distributions of the local Nusselt number and the mass-mean temperature along the channel.

Keywords: *non-Newtonian fluid, heat transfer, shear flow, plane channel.*

Introduction. In recent years, a considerable number of works have appeared in the world literature on convective heat transfer during the fluid flow in channels fully or partially filled with porous material. This is due to the wide use of such forms of flow in highly diverse technological processes [1, 2]. We can primarily note the works on modeling viscous-fluid flow though a channel partially filled with porous material [3–8]. In the region of the flowing fluid, flow was described by the Navier–Stokes equation, whereas in the porous layer use was usually made of the Brinkman equation or the Forchheimer law. Both symmetric and nonsymmetric boundary conditions of the second kind (assigned heat flux) were set on channel walls in [3–5]. One wall could be adiabatic. Calculations were mainly reduced to computing the Nusselt number. A model of convective heat transfer was proposed in [6], in accordance with which the temperature on the liquid–porous layer boundary is determined from the condition of equality of heat fluxes, and the velocity on this boundary, from the condition of equality of shear stresses. In [7], a numerical study has been made of laminar forced convection in pipe flow. Consideration was given to three cases of partial filling of the pipe with porous material. The Navier–Stokes equation was used to describe fluid flow in the free region, and the Forchheimer law was adopted in the porous medium. The solution of the thermal problem was based on a model with one equation of state, which assumed a local heat equilibrium between liquid and solid phases. The influence of the Darcy number in a wide range on the velocity profiles, the local and average Nusselt numbers, and also on the pressure difference was studied. The efficiency of free-convective heat transfer in a heated pipe was assessed in [8] for two situations: the porous medium in the pipe′s central zone and the pipe whose wall is coated with a porous-medium layer. The applicability of the assumption of local heat equilibrium was investigated. Conditions were determined where on filling the pipe′s central zone with porous medium, the heat-transfer efficiency is higher than that in the case of the walls coated with a porous-medium layer.

Recent years have also seen a great number of works in which consideration is given to heat transfer during the fluid flow through a channel fully filled with porous material [9–25]. First we should note publications in which the problem is solved without account taken of energy dissipation [9–14]. Thus, in [9, 10], heat transfer during the flow of a power-law fluid in a plane channel has been considered on the basis of the Brinkman–Forchheimer model with thermal boundary conditions of the first [10] and second kind [9]. In the solution, use was made of the classical Kármán–Pohlhausen integral method for the velocity profile. Calculations were mainly reduced to computing local Nusselt numbers. However, work [9] also gives the influence of various parameters (index of flow, the Darcy number, etc.) on the velocity profile. In [11], such a problem has been solved on the basis of the Brinkman model now for a Newtonian fluid. Consideration was given to thermal boundary conditions of the first kind for a plane channel and for a circular channel alike. A classical methodology was used, which is applied in solving the Grätz problem. The authors placed primary emphasis on calculations of the local and average Nusselt numbers, and also on the influence of various parameters on them. Khashan et al. [12] investigated the acceptability

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of the assumption of local thermal equilibrium in forced convective flow of a power-law fluid. Based on numerous calculations, they established the bounds within which the local-thermal-equilibrium assumption is acceptable. The role of the Péclet and Biot numbers, of the index of fluid flow, and of other parameters in this respect was noted. In [13], a study has been made of fully developed forced convection during the fluid flow in rectangular channels. The Brinkman equation was used as the equation of motion. The solution obtained in Fourier series made it possible to obtain temperature and velocity profiles. Use was made of thermal boundary conditions of the first kind. Cekmer et al. [14] modeled, on the basis of the Brinkman equation, steady-state fully developed convective heat transfer during the flow of a Newtonian fluid in a plane channel with asymmetric boundary conditions of the second kind. Numerical calculations of the Nusselt numbers on the upper and lower channel walls were given at different Darcy numbers. The relation of the heat fluxes on the upper and lower walls when the Nusselt number undergoes a discontinuity and changes sign was shown.

The next step in the development of the theory of heat transfer in porous channels was entering the term reflecting dissipative heat releases into a mathematical model [15–25]. It has been shown in [15–17] that depending on different theoretical assumptions, the dissipative function in the energy equation can be written variously. Three different forms of representation of the dissipative term were given, but it was noted that, at small Darcy numbers, all the three models yielded identical results. For more significant Darcy numbers, it is necessary to use experimental data to establish which of the models is better suited to concrete flow conditions. Thus, in [15, 16], flow and heat transfer in a plane channel are considered on the basis of the Brinkman equation. In [15], use is made of only thermal boundary conditions of the first kind, but the axial thermal conductivity is taken account of in the energy equation. In [16], consideration is given to thermal boundary conditions of the first kind and the second kind alike. In both works, calculations were done and expressions have been obtained on the distribution of the local Nusselt number along the channel′s dimensionless length at different values of the Darcy, Péclet, and Brinkman numbers. The obvious influence of the dissipation effect on heat-transfer characteristics has been reflected. An analogous problem, but for a rectangular passage, has been solved with boundary conditions of the first and second kind in [18]. The energy equation was solved by the extended weighted residuals method with the use of Green′s functions. Calculations of the heat-transfer coefficients have shown that a fully developed thermal regime of flow can be impractical for very narrow channels with boundary conditions of the second kind. In [19], the problem on convective heat transfer in a circular tube with boundary conditions of the second kind has been solved by the method of separation of variables. In computing the eigenfunctions, ordinary differential equations were numerically solved by the Runge–Kutta method. Results of the temperature-profile calculations have shown that account taken of the energy dissipation makes an appreciable contribution to the solution even at small values of the Brinkman number. The changes in the wall temperature and in the Nusselt number along the tube have also been shown. Unlike the above-given works [15–19] on studying heat transfer with account of dissipation for Newtonian fluids, work [20] considers flow of a power-law fluid in a plane channel with thermal boundary conditions of the second kind. The solution is reduced to numerical integration of a system of ordinary differential equations by the Runge–Kutta method. Fascinating calculations of transverse temperature and velocity profiles are given. The influence of the energy dissipation on stability loss in flow through a plane porous channel with asymmetric thermal boundary conditions is the focus of [21]. The temperature of the upper boundary is constant and the lower boundary is heat-insulated. The flow is described by the Brinkman equation. Furthermore, it is shown that the results obtained within the framework of the Brinkman model in the limiting case of the Darcy parameter tending to infinity (free fluid) differ from the results of solution of the relevant problem within the framework of the Navier–Stokes model. In particular, the first model yield overstated values of temperature throughout the layer thickness except boundary points. Coelho et al. [22] have given the analytical solution for dissipative fully developed hydrodynamic and thermal flow on a very viscous Newtonian fluid in an annulus. Use was made of the form of representation of the dissipative term, which is compatible with the limiting case of flow of a pure (free) fluid at infinitely large Darcy number.

All the works ([15–22]) taking account of dissipative heat releases were based on the so-called one-temperature model using one energy equation and assuming local thermal equilibrium. In this case the internal heat transfer between the fluid and the solid skeleton is rather fast. In [23–25], in describing heat transfer in porous channels, use is made of a model including two energy equations: one describes the temperature field in fluid, and the other (heat-conduction equation), the temperature distribution inside the solid phase. Such models are used in the cases where the low heat transmission on the boundary of the liquid and solid phases leads to a noticeable difference of the temperatures in these phases. Clearly, this leads to a significant complication of the model and cumbersome computations.

As can be seen from the above-given brief literature review, all the published works, in practice, are devoted to heat transfer in porous channels during the flow of Newtonian fluids. In [20], consideration has been given to power-law-fluid

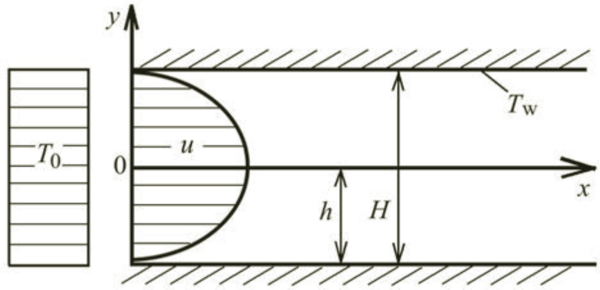


Fig. 1. Diagram of flow in the plane channel.

flow in a plane channel with thermal boundary conditions of the second kind. The flow was considered as fully stabilized thermally, the initial thermal portion was absent, and the temperature profiles, accordingly, were independent of the axial coordinate. Taking into account all the aforesaid, in the present work, we have made an attempt to fill the exiting gap in the field of simulation of flow of a power-law fluid in a porous channel with account of energy dissipation with boundary conditions of the first kind.

The important role of the temperature dependence of viscosity is worthy of separate note. However, all the published works, in practice, are devoted to heat transfer in porous channels at the fluid′s constant physical properties, including viscosity. But the fact that it is precisely the temperature dependence of viscosity that is the most substantial for all kinds of fluids is a matter of common knowledge. The number of works on nonisothermal (i.e., with a varying viscosity) fluid flow in porous channels is obviously insufficient. One example of such publications, which are few in number, can be [26] where the analytical solution has been obtained to the steady-state problem of forced convection in a rectangular duct with boundary conditions of the second kind. Viscosity was considered to be inversely proportional to temperature. However, the Darcy equation was used as the equation of motion, and the pressure gradient was assumed constant. Of course, all this renders the mathematical model rather far from reality. Therefore, taking into consideration the aforesaid, we will make account of the temperature dependence of viscosity the focus of our next publication.

Mathematical Model. In formulating a mathematical model, we make a number of universally adopted assumptions, which were already used earlier in describing free-fluid flow in channels [27–29]. The fluid viscosity is considered fairly high, so that the flow is implemented at low values of the Reynolds number ($\mathrm{Re} \le 0.01$). As a result, the hydrodynamic initial portion is absent, in practice, and the velocity profile at the channel inlet may be assumed developed. Furthermore, this enables us to disregard inertial terms in the equation of motion. Consideration is given to fluids with a low thermal conductivity. In this case, during the flow in the channel the condition for the Péclet number $\mathrm{Pe} \ge 100$ is satisfied, which makes it possible to disregard the transfer of heat by heat conduction along the channel axis compared to the transfer by convection. The flow is implemented with boundary conditions of the first kind when the temperature of the channel wall T_w is considered constant. It is assumed that the temperature of the fluid at the channel inlet is distributed uniformly over the cross section and is equal to T_0 (Fig. 1).

As the equation of motion, we use the modified Brinkman equation [10, 20]

$$-\frac{\eta}{k}u^n + \frac{\eta}{\varepsilon^n}\frac{\partial}{\partial y}\left[\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right] = \frac{\partial p}{\partial x}\,. \tag{1}$$

The mathematical model of transfer of energy is based on the so-called one-temperature model when one energy equation is used [9–22]. This approach assumes local heat equilibrium between the liquid and solid phases. Consequently, the energy equation is of the form

$$\rho c_p u\frac{\partial T}{\partial x} = \lambda\frac{\partial^2 T}{\partial y^2} + \Phi(x,\ y)\,. \tag{2}$$

The effective value of the thermal conductivity of a heterogeneous composite medium λ can be computed by different methods [11, 30]. In the present work, use is made of the following expression [31]:

$$\lambda = \lambda_b\left(\varepsilon^2\varsigma + \frac{4\varsigma\varepsilon(1-\varepsilon)}{1+\varsigma} + (1-\varepsilon)^2\right). \tag{3}$$

The function $\Phi(x, y)$ reflects dissipative heat releases and may be written in various forms [17]. It has been noted in [15, 16] that at small values of the Darcy number all forms of representation yield an identical result. In the present work, the dissipative term is written in a form compatible with the limiting case of flow of a pure (free) fluid at infinitely large Darcy numbers:

$$\Phi = \frac{\eta u^{n+1}}{k} + \eta \left| \frac{\partial u}{\partial y} \right|^{n-1} \left(\frac{\partial u}{\partial y} \right)^2 . \tag{4}$$

The continuity equation, under the assumption of one-dimensionality of the flow, degenerate into the equation of a constant flow rate

$$\bar{u} = \frac{1}{h} \int_0^h u dy . \tag{5}$$

Hydrodynamic and thermal boundary conditions are written in the following manner:

$$y = h, \quad u = 0, \quad T = T_\mathrm{w} ; \tag{6}$$

$$y = 0 , \quad \frac{\partial u}{\partial y} = 0 , \ \frac{\partial T}{\partial y} = 0 ; \tag{7}$$

$$x = 0 , \quad T = T_0 . \tag{8}$$

On passage to dimensionless variables, the fundamental equations (1), (2), and (5) will take the form

$$- \frac{U^n}{Da} + \frac{\partial}{\partial Y} \left[\left| \frac{\partial U}{\partial Y} \right|^{n-1} \frac{\partial U}{\partial Y} \right] = \frac{dP}{dX} , \tag{9}$$

$$U \frac{\partial \theta}{\partial X} = \frac{1}{\mathrm{Pe}} \frac{\partial^2 \theta}{\partial Y^2} + \frac{\mathrm{Br}}{\mathrm{Pe}} \frac{1}{\mathrm{Da}} U^{n+1} + \frac{\mathrm{Br}}{\mathrm{Pe}} \left| \frac{\partial U}{\partial Y} \right|^{n-1} \left(\frac{\partial U}{\partial Y} \right)^2 , \tag{10}$$

$$\int_0^1 U dY = 1 , \tag{11}$$

where

$$Y = \frac{y}{h} , \quad X = \frac{x}{h} , \quad U = \frac{u}{\bar{u}} , \quad \mathrm{Da} = \frac{k}{h^{n+1} \varepsilon^n} , \quad P = \frac{p h^n \varepsilon^n}{\bar{u}^n \eta} ,$$

$$\mathrm{Pe} = \frac{\bar{u} h}{a} , \quad \mathrm{Br} = \frac{\eta \bar{u}^{n+1}}{\lambda (T_0 - T_\mathrm{w}) h^{n-1}} , \quad \theta = \frac{T - T_\mathrm{w}}{T_0 - T_\mathrm{w}} .$$

The values of the medium′s mass-mean temperature in dimensional and dimensionless form are determined in the following manner:

$$T_\mathrm{m} = \frac{1}{h \bar{u}} \int_0^h T u dy , \quad \theta_\mathrm{m} = \int_0^1 \theta U dY . \tag{12}$$

Knowing the distribution of the mass-mean temperature, it is easy to find the local Nusselt number referred to the local temperature difference:

$$\mathrm{Nu}_\mathrm{m} = \frac{2 h \alpha_\mathrm{m}}{\lambda} = \frac{-2}{\theta_\mathrm{m}} \left(\frac{\partial \theta}{\partial Y} \right)_{Y=1} . \tag{13}$$

The problem was solved numerically by the finite-difference method using iterations and the marching method.

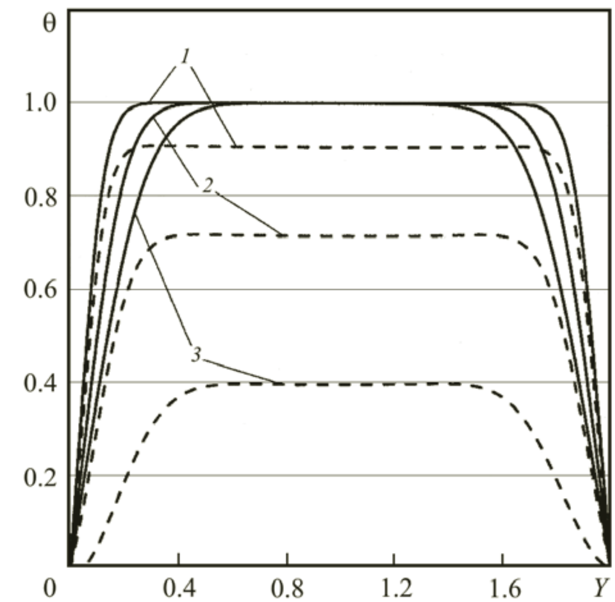
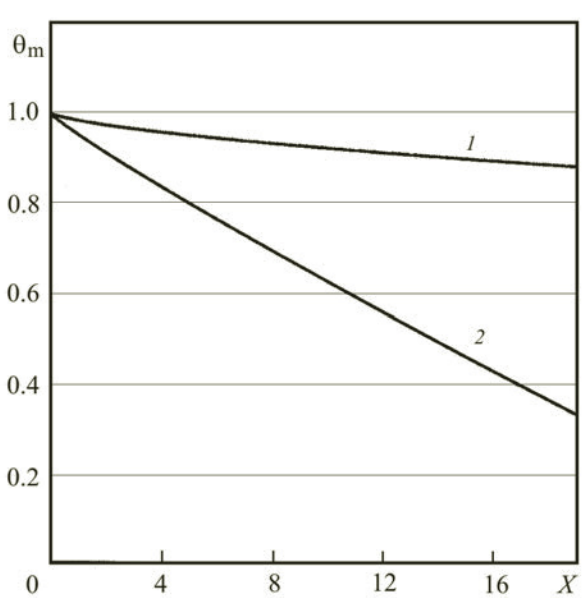


Fig. 2. Transformation of the temperature profile along the channel: 1) X = 3, 2) 9, and 3) 19; solid line, Br = 0, dot-dash line, Br = –0.3.



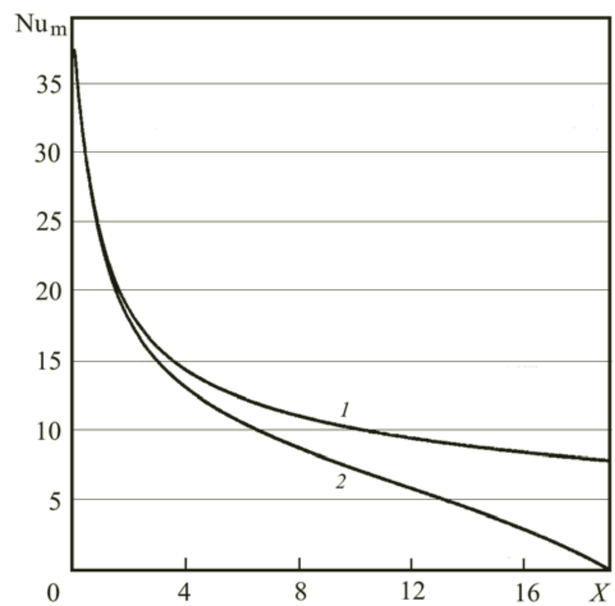


Fig. 3. Influence of dissipation on the distribution of the mass-mean temperature along the channel: 1) Br = 0 and 2) Br = –0.3.

Fig. 4. Influence of dissipation on the distribution of the local Nusselt number along the channel: 1) Br = 0 and 2) Br = –0.3.

Discussion of Results. Figure 2 shows the influence of the Brinkman number on the development of the temperature distribution along the channel. It can be seen that taking account of dissipation is fundamental in character and totally changes the entire temperature field in the channel. Also, this substantially alters the distribution of the mass-mean temperature (Fig. 3) and of the Nusselt number (Fig. 4) along the channel. However, it should be noted that account taken of the temperature dependence of viscosity must compensate to a degree for the influence of dissipation. Clearly, the dissipative heating will cause the viscosity to decline, and this in turn will reduce dissipative heat releases. As has already been noted above, this formulation of the problem will be analyzed in the next publication.

Conclusions. Thus, in the present work, we have formulated and solved the problem on flow and heat transfer of a non-Newtonian fluid in a plane channel with boundary conditions of the first kind. The mathematical model is based on the modified Brinkman equation and the one-temperature energy equation. Consideration has been given to the initial thermal portion when the inlet temperature of the fluid and the temperature of channel walls are not coincident. In this case we have an active development of the temperature profiles along the channel. The calculations show the substantial influence of dissipation on the entire process of heat transfer in the channel.

NOTATION

a, thermal diffusivity; Da, Br, and Pe, Darcy, Brinkman, and Péclet numbers, respectively; h, half-height of the plane channel; k, permeability coefficient of the porous material; n, index of flow; p, pressure; T, temperature; T_w, temperature of channel walls; T_0, initial temperature of the fluid at the channel inlet; u, flow velocity; $\overline{u}$, average flow velocity in the channel; x, y, longitudinal and transverse coordinates; α_m, local heat-transfer coefficient; λ and λ_b, thermal conductivity of the fluid and of the solid skeleton of the porous material respectively; ρ and c_p, density and heat capacity of the fluid respectively; η, consistency of the fluid; ε, porosity; $\varsigma = \lambda/\lambda_b$.

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