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LAMINAR MAGNETOHYDRODYNAMIC BOUNDARY LAYER ON A DISK IN THE PRESENCE OF EXTERNAL ROTATING FLOW AND SUCTION

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The rotation of a conducting viscous medium near a dielectric disk in a homogeneous magnetic field in the presence of an external flow and a suction is considered. On the basis of the Dorodnitsyn transformation, an analytical solution of the system of boundary-layer and heat-conduction equations has been obtained. It is shown that the direction of the radial flow in the boundary layer of the disk can be changed by changing the ratio between the angular velocities of the external flow and the disk and the ratio between the temperatures in the external flow and on the disk as well as by varying the hydrodynamic Prandtl number. The influence of the magnetic field on the intensity of circulation of the viscous medium was investigated.

Keywords: *rotating disk, magnetic field, suction, boundary layer, conducting gas.*

Introduction. The effects of flow and heat transfer in rotating liquid and gaseous media take place in many technological processes, e.g., in centrifugal isotopic enrichment [1]. Considerable recent attention has been focused on the study of the rotation of conducting media. The intensity of the mass exchange between a solid body and such a medium can be controlled with the use of a magnetic field [2]. A rotating conducting gas is used for the separation of the isotopes of elements that have no convenient gaseous compounds (plasma centrifuges) [3]. In this case, of great importance is the stability of rotation of a conducting medium [4]. In the last few years, the problem of rotating media has been actively discussed as applied to the astrophysics and the experimental investigation of the so-called magneto-rotational instability [5].

Investigations of different instabilities under laboratory conditions have shown that the interaction of a rotating gas with the end faces of a setup leads, due to the viscous effects, to the excitation of secondary flows exerting a masking influence on the loss of stability by the setup. The excitation of these flows can be explained by the loss of stability of the rotating gas flow because of the axial inhomogeneity of the centrifugal forces near the retarding surface. The stability of the boundary layer on a rotating body can be also disturbed by the suction of the medium through its porous surface [6].

In the present work, the influence of a magnetic field and a uniform suction on a magnetohydrodynamic (MHD) flow near a rotating dielectric disk in the presence of an external flow rotating as a quasi-solid body is considered.

Formulation of the Problem. A dielectric disk of large radius rotating in a conducting gaseous medium with an angular velocity ω_0 in the presence of a homogeneous axial magnetic field and a flow rotating over the disk with an arbitrary angular velocity ω_1 is considered. The problem on such an unbounded flow is solved frequently in the engineering calculations of rotating flows bounded by immovable and rotating surfaces, e.g., in the analysis of the characteristics of the boundary layer in the region of the nonviscous core of a flow in a cylinder with a retarding cover [7]. Let us assume that there takes place a uniform suction of the boundary layer from the surface of the disk with a rate k and that the homogeneous magnetic field is directed along the axis of the rotating disk. Any additional assumptions are not needed for the closure of the azimuth currents in the medium. The problem on the closure of the radial electric current in the boundary layer is more complex. Let us assume that the radial current is closed through the external circuit and that the temperatures of the disk and the medium are independent of the radial coordinate.

Disregarding the viscous and Joule dissipation and the induced magnetic field, we write the following equations for the magnetohydrodynamic and thermal boundary layers at the disk [8]:

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$$\rho\left(v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\varphi^2}{r}\right)=-\frac{\partial p}{\partial r}+\frac{\partial}{\partial z}\left(\eta\frac{\partial v_r}{\partial z}\right)-\sigma B^2 v_r\ , \tag{1}$$

$$\rho\left(v_r\frac{\partial v_\varphi}{\partial r}+v_z\frac{\partial v_\varphi}{\partial z}-\frac{v_r v_\varphi}{r}\right)=\frac{\partial}{\partial z}\left(\eta\frac{\partial v_\varphi}{\partial z}\right)-\sigma B^2(v_\varphi-\omega_1 r)\ , \tag{2}$$

$$\frac{\partial}{\partial r}(\rho r v_r)+\frac{\partial}{\partial z}(\rho r v_z)=0\ , \tag{3}$$

$$\rho c_p\left(v_r\frac{\partial T}{\partial r}+v_z\frac{\partial T}{\partial z}\right)=\frac{\partial}{\partial z}\kappa\frac{\partial T}{\partial z}\ , \tag{4}$$

$$p=\rho\Re T/\mu\ , \tag{5}$$

The system of equations (1)–(4) does not involve the equation of motion in the projection on the z axis, because this equation is used only for determining the dependence of the pressure in the boundary layer on the axial coordinate [8]. The second terms on the right sides of Eqs. (1) and (2) define the azimuth and radial electric currents carried by the boundary layer [9]. At $B \to 0$ and $\frac{\partial p}{\partial r}=0$, Eqs. (1)–(5) are transformed into the system investigated in the work [10], for an ordinary nonconducting medium in the absence of an external rotating flow.

We consider a medium with a small compressibility parameter $\frac{\mu\omega_1^2 r^2}{\Re T^*}$, where T^* is the characteristic temperature of the medium. In this case, the radial redistribution of the density of the medium can be ignored, and system (1)–(5) is solved at the following boundary conditions:

$$z=0\ ,\quad v_r=0\ ,\quad v_\varphi=\omega_0 r\ ,\quad v_z=-k\ ,\quad T=T_0\ , \tag{6}$$

$$z\to\infty\ ,\quad v_r\to 0\ ,\quad v_\varphi\to\omega_1 r\ ,\quad T\to T_1\ , \tag{7}$$

where T_0 is the temperature of the gas on the surface of the disk and T_1 is the temperature of the gas in the external flow.

Methods of Solving the Model Problem. By analogy with [11], we introduce the Dorodnitsyn transformation

$$Z_0=\int_0^z\left(\frac{\rho(z)}{\rho_1}\right)dz\ , \tag{8}$$

where ρ_1 is the density of the medium in the external flow, independent of the radial coordinate. Then the expression for the transformed axial component of the flow velocity will take the form

$$v_{z1}=v_z\rho/\rho_1+v_r\frac{\partial Z_0}{\partial r}\ . \tag{9}$$

Let us transform Eqs. (1)–(4) on the assumption that

$$v_r=rF(Z_0)\ ,\quad v_\varphi=rG(Z_0)\ ,\quad T=T_0+(T_1-T_0)\theta(Z_0)\ . \tag{10}$$

Since the external flow rotates with a constant angular velocity ω_1, we assume that the distribution of the pressure in the boundary layer is identical to the distribution of the pressure in the main flow:

$$\frac{\partial p}{\partial r}=\rho_1\omega_1^2 r\ . \tag{11}$$

This circumstance is of importance in the dynamics of movement of the fluid near the disk, because the force that is due to the pressure gradient is directed to the disk axis and depends on the density of the fluid in the external flow ρ_1 and on the angular velocity ω_1, while the centrifugal force is directed to the periphery of the disk and is determined by the axial distributions of the density $\rho(Z_0)$ and the azimuth velocity $v_\varphi(Z_0)$. We also assume that the dynamic-viscosity and heat-conduction coefficients change in proportion to the first power of the temperature ($\eta = \eta_1 T/T_1$, $\kappa = \kappa_1 T/T_1$, η_1 and κ_1 are the dynamic-viscosity and heat-conduction coefficients of the external flow [11]). As in [12], we assume that the conductivity of the gas medium is in inverse proportion to the temperature: $\sigma = \sigma_1 \left(\frac{T_1}{T}\right)$, where σ_1 is the electric conduction of the gas at a large distance from the disk surface.

In view of (8)–(11), Eqs. (1)–(4) take the form allowing one to find a self-similar solution of the problem

$$F^2 + v_{z1}F' - G^2 = -\omega_1^2 \frac{T}{T_1} + \nu_1 F'' - \frac{\sigma_1 B^2}{\rho_1} F \,, \tag{12}$$

$$2FG + v_{z1}G' = \nu_1 G'' - \frac{\sigma_1 B^2}{\rho_1}(G - \omega_1) \,, \tag{13}$$

$$2F + v_{z1}' = 0 \,, \tag{14}$$

$$v_{z1}\theta' = \chi_1 \theta'' \,, \tag{15}$$

where $\nu_1 = \eta_1/\rho_1$ is the kinematic-viscosity coefficient, and the prime denotes the differentiation with respect to Z_0. Thus, the system of partial differential equations (1)–(4) is transformed to the system of ordinary differential equations (12)–(15) that is identical in form to the system used for an incompressible medium. Note that we consider the "compressibility" associated with the change in the temperature of the medium but not with the action of the external-force field.

For solving system (12)–(15), we introduce the quantity $v_{z0} = v_{z1} + k_1$, where $k_1 = \rho_0 k/\rho_1$ and ρ_0 is the density of the conducting gas on the surface of the disk. For a strong suction ($v_{z1} << k$), we have

$$-\frac{G^2}{\nu_1} + \frac{\omega_1^2}{\nu_1 n}\left[1 + (n-1)\theta\right] - F'' - \frac{F'}{l} + \frac{F}{l_1^2} = 0 \,, \tag{16}$$

$$G'' + \frac{G'}{l} - \frac{G}{l_1^2} + \frac{\omega_1}{l_1^2} = 0 \,, \tag{17}$$

$$\theta'' + \frac{\theta'}{l_2} = 0 \,, \tag{18}$$

where $l = \frac{\nu_1}{k_1}$, $l_1 = \sqrt{\frac{\nu_1\rho_1}{\sigma_1 B^2}}$, and $l_2 = \frac{\chi_1}{k_1}$ are dimensional parameters (meters). Introducing the dimensionless functions $g = \frac{G}{\omega_0}$ and $f = \frac{F}{\omega_0}$, the variable $Z = Z_0\sqrt{\frac{\omega_0}{\nu_1}}$, and the dimensionless parameters $S = \frac{\sigma_1 B^2}{\rho_1\omega_0}$, $m = \frac{\omega_1}{\omega_0}$, and $K_1 = \frac{k_1}{\sqrt{\nu_1\omega_0}}$, from (16)–(18) we obtain

$$-g^2 + \frac{m^2}{n}\left[1 + (n-1)\theta\right] - f'' - K_1 f' + Sf = 0 \,, \tag{19}$$

$$g'' + K_1 g' - Sf + mS = 0 \,, \tag{20}$$

$$\theta'' + K_1 \,\mathrm{Pr}\, \theta' = 0 \,. \tag{21}$$

Integration of the system of equations (19)–(21) gives

$$\theta(Z) = 1 - \exp\left(-K_1 \,\mathrm{Pr}\, Z\right) , \tag{22}$$

$$g(Z) = m + (1-m)\exp\left(\left(-\frac{K_1}{2} - \sqrt{\frac{K_1^2}{4} + S}\right) Z\right) . \tag{23}$$

The function $f(Z)$ is determined, in accordance with (22) and (23), from the differential equation

$$f'' + K_1 f' - Sf = m^2 \left[\frac{1}{n} + \left(1 - \frac{1}{n}\right) \left(1 - \exp\left(-K_1 \text{ Pr } Z\right)\right) \right]$$

$$- \left[m + (1 - m) \exp\left(\left(-\frac{K_1}{2} - \sqrt{\frac{K_1^2}{4} + S} \right) Z \right) \right]^2 .$$

A solution of this equation has the form

$$f(Z) = \exp\left(-K_0 Z\right) \left\{ \frac{(1-m)^2}{(2K_1 K_0 + 3S)} \left[1 - \exp\left(-K_0 Z\right)\right] + \frac{2m(1-m)Z}{\sqrt{K_1^2 + 4S}} \right.$$

$$\left. + \frac{m^2 (n-1)}{(\Pr(\Pr - 1) K_1^2 - S) n} \left[1 - \exp\left(K_0 Z - \Pr K_1 Z\right)\right] \right\} , \qquad (24)$$

where $K_0 = \dfrac{K_1}{2} + \sqrt{\dfrac{K_1^2}{4} + S}$.

With the use of Eq. (23), we find the moment of the viscous forces acting on one side of the disk of radius R:

$$M = 2\pi\eta_0 \int_0^R r^2 \left| \frac{\partial v_\varphi}{\partial z} \right|_{z=0} dr = \frac{\pi}{2} \rho_1 R^4 (\nu_1 \omega_0^3)^{1/2} (1 - m) \left(\frac{nK}{2} + \sqrt{\frac{n^2 K^2}{4} + S} \right) , \qquad (25)$$

where $K = \dfrac{k}{\sqrt{\nu_1 \omega_0}}$ and η_0 is the dynamic viscosity of the medium at a temperature T_0. In the limiting case of absence of magnetic field ($S \to 0$), expression (25) is identical to the dependence obtained in the work [13]. As $S \to 0$, $n \to 0$, and $m \to 0$, the solution obtained is transformed into the known relation presented in the works [14, 15].

Discussion of the Results Obtained. We restrict our consideration to the influence of suction and magnetic field on the intensity of the radial flow in the boundary layer near the surface of a rotating dielectric disk. According to the results obtained in the works [9, 14], if the change in the density of the gas along the coordinate Z is not taken into account ($n = 1$, the case of equal temperatures on the disk and in the external flow), the direction of the flow in the boundary layer at the disk changes sign depending on whether the parameter $m = \dfrac{\omega_1}{\omega_0}$ is larger or smaller than unity. At $m < 1$ (the disk rotates more rapidly than the external flow) the centrifugal force is larger than the radial pressure gradient and the radial flow is positive. If the disk rotates slowlier than the external flow does ($m > 1$), the force caused by the pressure gradient exceeds, in absolute value, the centrifugal force and the flow near the disk is directed to its axis. This pattern is demonstrated by the results of calculations by relation (24), presented in Fig. 1a where the full lines represent the profiles of the radial velocity of the flow near the disk for different values of the parameter m at $n = 2$ in the absence of magnetic field ($S = 0$). The fact that the velocity of the radial flow does not turn to zero at $m = 1$ is explained by the change in its density along the axial coordinate at $n = 2$. If the parameter n is equal to unity, the radial flow is absent at $m = 1$. At a reverse ratio between the temperatures in the external flow and at the disk ($n = 0.5$), the density of the medium near the disk is smaller than the density of the external flow and, at $m = 1$, the radial flow is opposite in direction (Fig. 1b).

The results obtained point to the fact that, even at $m \neq 1$, the radial flow can be substantially attenuated by varying the thermal regime. This result is of practical importance because, in this case, the influence of the end faces of a body on the rotation of the medium in the spaces bounded by solid retarding surfaces decreases. The calculation data represented by the dotted lines in Fig. 1 allow the conclusion that the magnetic field influences the intensity of the radial flow. It is seen that the magnetic field aids in the deceleration of the secondary flow, which is due to the action of the decelerating radial electromagnetic force $[\mathbf{j}, \mathbf{B}]_r$ in the MHD boundary layer. Note that the suction, along with the magnetic field, decreases the radial flow in the boundary layer independently of whether the parameter n is larger or smaller than unity. This statement is illustrated by the results of calculations of the profiles of the radial flow velocity in the case where the medium rotates more rapidly than the disk does and the parameter $\Pr = 1$, presented in Fig. 2. Figure 2a shows the profiles of the radial flow velocity

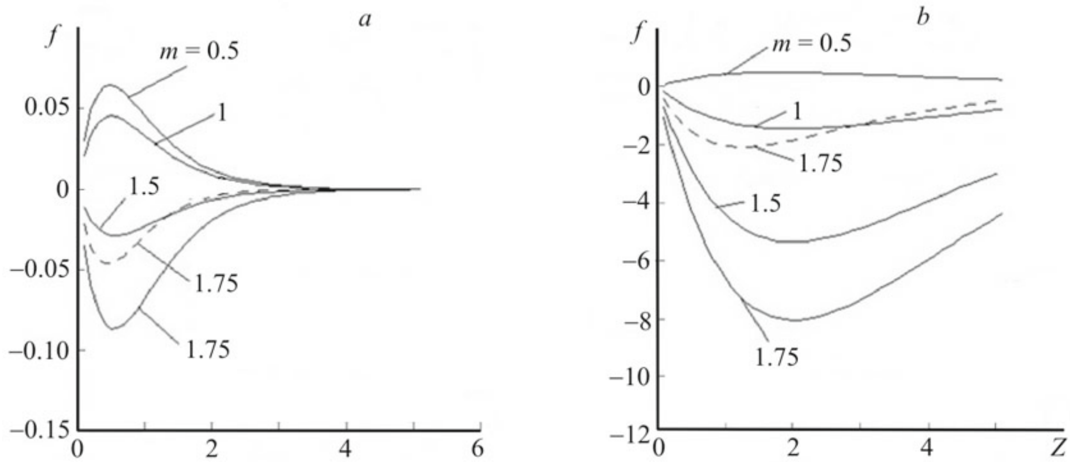

Fig. 1. Dependence of the dimensionless radial component of the flow velocity f on the coordinate Z at $S = 0$: a) $K = 1$, $n = 2$, Pr = 1; b) 1, 0.5, 1.

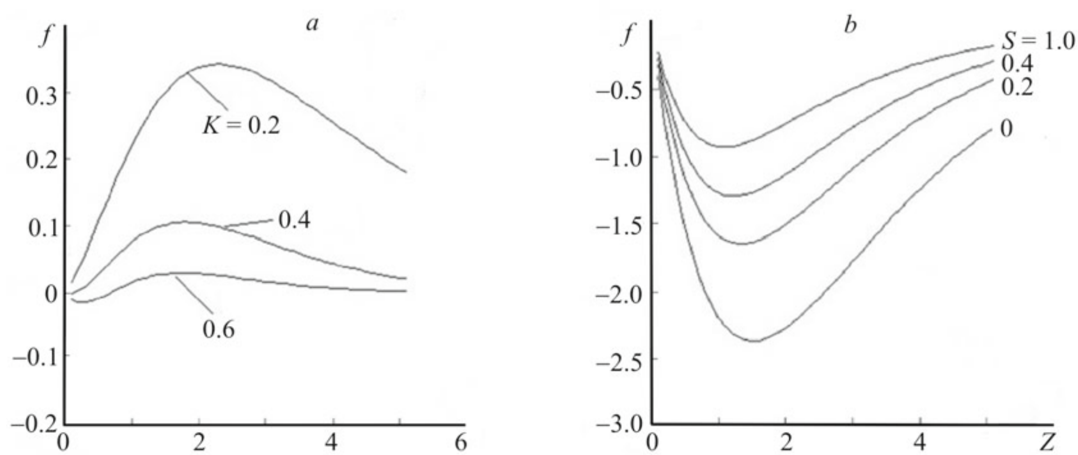

Fig. 2. Profiles of the velocity of the radial flow near the disk for different values of the suction parameter K at $m = 1.75$, $n = 2$, Pr = 1 and $S = 1$ (a) and different values of the MHD-effect parameter S at $K = 1$, $m = 1.5$, $n = 0.5$, and Pr = 1 (b).

$f(Z)$ for different values of the suction parameter K at $S = 1$. The dependences presented in Fig. 2b demonstrate the evolution of the profile $f(Z)$ with change in the parameter S at $K = 1$. These data not only give information on the dependence of the intensity of the radial flow on the suction and the magnetohydrodynamic effect but also show the influence of the temperature effects on the radial movement of the medium near the disk. For example, at $n = 2$, because of the increased density of the medium near the surface of the disk, the centrifugal force dominates over the force that is due to the pressure gradient, and the gas flow is directed to the periphery (Fig. 2a). In the case where $n = 0.5$ (the density of the medium near the disk is smaller than the density of the external flow), there takes place a directly opposite pattern (Fig. 2b). The above-indicated dependences have been obtained for the case where the Prandtl number is equal to unity and the thicknesses of the thermal and hydrodynamic boundary layers are equal. For an understanding of the reasons for one or another behavior of the flow with change in the Prandtl number, we note that the thickness of the thermal boundary layer δ_T is in inverse proportion to this number ($\delta_T \sim 1/\mathrm{Pr}$). Figure 3a shows the profiles of the radial flow velocity, calculated for different Prandtl numbers at $m = 0.5$ (the disk rotates more rapidly than the external flow does) and $n = 2$ (the medium near the disk is more dense as compared to the density of the main flow). It is seen from the dependences obtained that a decrease in the Prandtl number leads to an increase in the velocity of the flow in the boundary layer, which is due to the widening of the zone of increased density of the gas near the disk. In the case where the temperature of the gas near the disk is higher than the temperature of the main flow

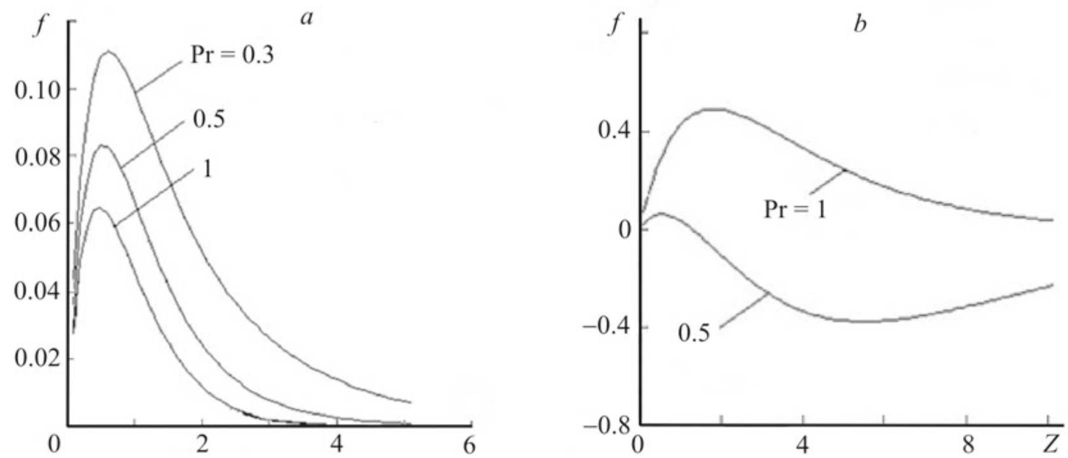

Fig. 3. Profiles of the velocity of the radial flow near the disk for different values of the Prandtl number: a) $S = 0$, $K = 1$, $m = 0.5$; b) $S = 1$, $K = 1$, $m = 0.5$, $n = 5$.

($n = 0.5$), the opposite pattern of the flow takes place (Fig. 3b). In this case, at Pr = 0.5, the velocity of the flow near the outer boundary of the thermal boundary layer changes its direction.

CONCLUSIONS

1) The direction of the radial flow in the boundary layer of a rotating dielectric disk in the presence of an external flow can be changed or (if necessary) this flow can be substantially decelerated by varying the thermal regime of the MHD flow rotating near the disk.

2) The axial magnetic field exerts a stabilizing action on the rotating flow and, in so doing, decreases the intensity of the secondary flows circulating near the disk.

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NOTATION

B, magnetic induction, T; c_p, specific heat capacity at constant pressure, J/(kg·K); F and G, functions dependent on the dimensional coordinate Z_0, 1/s; M, moment of the friction forces acting on one side of the disk; $n = \frac{T_1}{T_0}$, ratio between the temperatures in the external flow and on the disk; p, pressure, Pa; $\mathrm{Pr} = \frac{\nu_1}{\chi_1}$, Prandtl number for the external flow; $\mathfrak{R}$, universal gas constant; r, radial coordinate, m; R, radius of the disk, m; $S = \frac{\sigma_1 B^2}{\rho_1 \omega_0}$, parameter of the magnetohydrodynamic effect; T, temperature, K; v_φ, v_r, and v_z, azimuth, radial, and axial velocities of the medium, m/s; v_{z1}, transformed axial component of the flow velocity, m/s; z, axial coordinate measured from the disk surface, m; Z_0, Dorodnitsyn variable, m; δ_T, thickness of the thermal boundary layer, m; η, dynamic-viscosity coefficient, Pa·s; κ, heat-conduction coefficient, W/(m·K); μ, molecular weight of the gas, kg/mole; ν, kinematic-viscosity coefficient, m^2/s; ρ, density, kg/m^3; σ, electrical-conduction coefficient, S/m; $\chi_1 = \kappa/\rho_1 c_p$, thermal diffusivity of the medium, m^2/s; ω_0, angular velocity of rotation of the disk, rad/s.

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