DOI 10.1007/s10891-016-1393-9

Journal of Engineering Physics and Thermophysics, Vol. 89, No. 2, March, 2016

PLANE WAVES IN A ROTATING MONOCLINIC MAGNETOTHERMOELASTIC MEDIUM

B. Singh[a] and A. K. Yadav[b]

UDC 536.21

The governing equations for a rotating monoclinic magnetothermoelastic medium are formulated in the context of the Lord–Shulman theory and are solved to yield the velocity equation that points to the existence of three quasi-plane waves. Some particular cases are obtained, i.e., waves in the absence of anisotropy, rotation, and thermal and magnetic fields. A procedure for computing the angles of reflection is carried out. A numerical example is considered to show the dependence of the speeds of various plane waves on the angle of incidence, angle of reflection, rotation rate, and magnetic field strength.

Keywords: *Lord–Shulman thermoelasticity, anisotropy, rotation, magnetic field, plane waves.*

Introduction. The thermoelasticity theory combines the theories of elasticity and heat transfer as well as their coupled effects. Biot [1] studied the theory of thermoelasticity where the diffusion-type heat equation predicts an infinite speed of the thermal signal propagation. Lord and Shulman [2] presented the theory of generalized thermoelasticity where a hyperbolic equation of heat conduction with a relaxation time ensured a finite speed of thermal signals. Using two relaxation times, Green and Lindsay [3] developed another generalized theory of thermoelasticity. A unified treatment of both Lord–Shulman and Green–Lindsay theories was given by Ignaczak and Ostoja-Starzewski [4]. Dhaliwal and Sherief [5] extended the Lord–Shulman generalization of thermoelasticity to an anisotropic case.

Schoenberg and Censor [6] studied the effect of rotation on plane wave propagation in an isotropic medium and considered the propagation of three plane waves in a rotating isotropic medium. Chandrasekharaiah and Srinatiah [7, 8] considered thermoelastic plane waves in a rotating isotropic solid. Ahmad and Khan [9] studied such waves in a rotating isotropic material and showed the existence of four plane waves. None of these waves is dilatational or transverse in character unless special propagation directions are considered.

Keith and Crampin [10] found three types of body waves with mutually orthogonal particle motion that can propagate in an anisotropic elastic solid and are called quasi-P (qP), quasi-SV (qSV), and quasi-SH (qSH) waves. In general, the particle motion is neither purely longitudinal nor transversal. Chattopadhyay and Choudhary [11] studied the reflection of qP waves at the plane free boundary of a monoclinic half-space. Chattopadhyay, Saha, and Chakraboty [12] considered the reflection of qSV waves at a plane free boundary of a monoclinic half-space. Singh [13] published a comment on the above two papers. Singh and Khurana [14] studied the reflection of P and SV waves at the free surface of a monoclinic half-space. Singh [15–17] considered the plane wave propagation in a thermoelastic medium for transvsersely isotropic as well as monoclinic cases. Some other problems of wave propagation in the context of anisotropic thermoelasticity with various parameters were studied by Kumar and Singh [18], Singh and Tomar [19], and Singh and Yadav [20, 21]. In the present work, the governing equations for a homogeneous rotating monoclinic magnetothermoelastic medium will be formulated and solved, showing the existence of three quasi-plane waves.

Formulation and Solution of the Problem. We consider a homogeneous monoclinic magnetothermoelastic medium rotating about the x axis with the rate $\mathbf{\Omega} = (\Omega, 0, 0)$ at the magnetic field strength $\mathbf{H} = (H, 0, 0)$ and the reference temperature T_0. The governing equations in the yz plane are

$$C_{22}\frac{\partial^2 v}{\partial y^2} + C_{44}\frac{\partial^2 v}{\partial z^2} + C_{24}\frac{\partial^2 w}{\partial y^2} + C_{34}\frac{\partial^2 w}{\partial z^2} + 2C_{24}\frac{\partial^2 v}{\partial y \partial z}$$
$$+ \left(C_{23} + C_{44}\right)\frac{\partial^2 w}{\partial y \partial z} - \beta_2\frac{\partial T}{\partial y} + \left(\mathbf{J} \times \mathbf{B}\right)_2 = \rho\left(\frac{\partial^2 v}{\partial t^2} - \Omega^2 v - 2\Omega\frac{\partial w}{\partial t}\right), \quad (1)$$

[a]Department of Mathematics, Post Graduate Government College, Sector 11, Chandigarh, India; email: bsinghgc11@gmail.com; [b]Department of Mathematics, M. M. University Mullana Ambala, India. Published in Inzhenerno-Fizicheskii Zhurnal, Vol. 89, No. 2, pp. 417–427, March–April, 2016. Original article submitted September 29, 2014; revision submitted October 6, 2015.

$$C_{24}\frac{\partial^2 v}{\partial y^2}+C_{34}\frac{\partial^2 v}{\partial z^2}+C_{44}\frac{\partial^2 w}{\partial y^2}+C_{33}\frac{\partial^2 w}{\partial z^2}+2C_{34}\frac{\partial^2 w}{\partial y\partial z}$$

$$+\left(C_{23}+C_{44}\right)\frac{\partial^2 v}{\partial y\partial z}-\beta_3\frac{\partial T}{\partial z}+\left(\mathbf{J}\times\mathbf{B}\right)_3=\rho\left(\frac{\partial^2 w}{\partial t^2}-\Omega^2 w+2\Omega\frac{\partial v}{\partial t}\right). \tag{2}$$

The Maxwell equations are

$$\text{curl }\mathbf{H}=\mathbf{J}\,,\quad \text{curl }\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}\,,\quad \text{div }\mathbf{B}=0\,,\quad \mathbf{B}=\mu_e\mathbf{H}\,. \tag{3}$$

The generalized Ohm's law in deformable continua is

$$\mathbf{J}=\sigma[\mathbf{E}+(\dot{\mathbf{u}}\times\mathbf{B})]\,, \tag{4}$$

where the effect of the temperature gradient on the conduction current **J** is neglected.

We set $\mathbf{H}=\mathbf{H}_0+\mathbf{h}$, where $\mathbf{H}_0=(0, 0, H_0)$. The perturbed magnetic field **h** is so small that the product of **h**, **u**, and their derivatives can be neglected while linearizing the field equations. With the help of Eqs. (3) and (4), Eqs. (1) and (2) become

$$C_{22}\frac{\partial^2 v}{\partial y^2}+C_{44}\frac{\partial^2 v}{\partial z^2}+C_{24}\frac{\partial^2 w}{\partial y^2}+C_{34}\frac{\partial^2 w}{\partial z^2}+2C_{24}\frac{\partial^2 v}{\partial y\partial z}+\left(C_{23}+C_{44}\right)\frac{\partial^2 w}{\partial y\partial z}$$

$$-\beta_2\frac{\partial T}{\partial y}+\mu_e H_0^2\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z}\right)=\rho\left(\frac{\partial^2 v}{\partial t^2}-\Omega^2 v-2\Omega\frac{\partial w}{\partial t}\right), \tag{5}$$

$$C_{24}\frac{\partial^2 v}{\partial y^2}+C_{34}\frac{\partial^2 v}{\partial z^2}+C_{44}\frac{\partial^2 w}{\partial y^2}+C_{33}\frac{\partial^2 w}{\partial z^2}+2C_{34}\frac{\partial^2 w}{\partial y\partial z}+\left(C_{23}+C_{44}\right)\frac{\partial^2 v}{\partial y\partial z}$$

$$-\beta_3\frac{\partial T}{\partial z}+\mu_e H_0^2\left(\frac{\partial^2 v}{\partial y\partial z}+\frac{\partial^2 w}{\partial z^2}\right)=\rho\left(\frac{\partial^2 w}{\partial t^2}-\Omega^2 w+2\Omega\frac{\partial v}{\partial t}\right). \tag{6}$$

Following Lord and Shulman [2], we write the heat conduction equation as

$$K_2\frac{\partial^2 T}{\partial y^2}+K_3\frac{\partial^2 T}{\partial z^2}-T_0\left[\beta_2\left(\frac{\partial^2 v}{\partial y\partial t}+\tau_0\frac{\partial^3 v}{\partial y\partial t^2}\right)\right]+\beta_3\left(\frac{\partial^2 w}{\partial z\partial t}+\tau_0\frac{\partial^3 w}{\partial z\partial t^2}\right)$$

$$=\rho C_E\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right). \tag{7}$$

We assume that the solutions for plane wave are

$$\nu=A\exp\left\{ik\left(y\sin\theta+z\cos\theta-Vt\right)\right\},$$

$$w=B\exp\left\{ik\left(y\sin\theta+z\cos\theta-Vt\right)\right\},$$

$$T=C\exp\left\{ik\left(y\sin\theta+z\cos\theta-Vt\right)\right\} \tag{8}$$

and

$$\nu = A \exp\left\{ik\left(y \sin\theta - z\cos\theta - Vt\right)\right\},$$

$$w = B \exp\left\{ik\left(y \sin\theta - z\cos\theta - Vt\right)\right\},$$

$$T = C \exp\left\{ik\left(y \sin\theta - z\cos\theta - Vt\right)\right\}. \tag{9}$$

Using Eqs. (8) and (9) in Eqs. (5)–(7), we obtain

$$\left(D_1 - \Omega^*\zeta\right) A + \left(D_2 + 2i\,\frac{\Omega}{\omega}\,\zeta\right) B + i\,\frac{\beta_2}{k}\sin\theta\, C = 0\,, \tag{10}$$

$$\left(D_2 - 2i\,\frac{\Omega}{\omega}\,\zeta\right) A + (D_3 - \Omega^*\zeta)B \pm \frac{i\beta_3}{k}\cos\theta\, C = 0\,, \tag{11}$$

$$\varepsilon\zeta\sin\theta\, A \pm \bar{\beta}\varepsilon\zeta\cos\theta\, B + i\,\frac{\beta_2}{k}\,(D_5 - \zeta)C = 0\,, \tag{12}$$

where

$$\zeta = \rho V^2\,,$$

$$D_1 = C_{22}\sin^2\theta + C_{44}\cos^2\theta \pm 2C_{24}\sin\theta\cos\theta + \mu_e H_0^2\sin^2\theta\,,$$

$$D_2 = C_{24}\sin^2\theta + C_{34}\cos^2\theta \pm (C_{23} + C_{44})\sin\theta\cos\theta \pm \mu_e H_0^2\sin\theta\cos\theta\,,$$

$$D_3 = C_{44}\sin^2\theta + C_{33}\cos^2\theta \pm 2C_{34}\sin\theta\cos\theta + \mu_e H_0^2\cos^2\theta,$$

$$D_5 = \frac{D_4}{\tau^* C_E}\,, \quad D_4 = K_2\sin^2\theta + K_3\cos^2\theta\,,$$

$$\tau^* = \frac{i}{\omega} + \tau_0\,, \quad \Omega^* = 1 + \left(\frac{\Omega}{\omega}\right)^2, \quad \varepsilon = \frac{{\beta_2}^2 T_0}{\rho C_E}\,, \quad \bar{\beta} = \frac{\beta_3}{\beta_2}\,, \quad \omega = kv\,.$$

Here the upper sign corresponds to incident waves and the lower one, to reflected waves.

A nontrivial solution of Eqs. (10)–(12) exists if

$$A_0\zeta^3 + A_1\zeta^2 + A_2\zeta + A_3 = 0\,, \tag{13}$$

where

$$A_0 = 4\left(\frac{\Omega}{\omega}\right)^2 - \Omega^{*2}\,,$$

$$A_1 = \Omega^*\left(D_1 + D_3 + \varepsilon\sin^2\theta + \bar{\beta}^2\varepsilon\cos^2\theta\right) + \left(\Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2\right) D_5\,,$$

$$A_2 = D_2^2 - D_1 D_3 - \Omega^*\left(D_1 D_5 + D_3 D_5\right)$$

$$- \varepsilon\left(\bar{\beta}^2 D_1\cos^2\theta + D_3\sin^2\theta \mp 2\bar{\beta} D_2\sin\theta\cos\theta\right),$$

$$A_3 = \left(D_1 D_3 - D_2^2\right) D_5 \ .$$

The three roots $\zeta_j = \rho V_j^2 (j = 1, 2, 3)$ of Eq. (13) correspond to the complex phase velocities V_j of the three plane waves, namely, of the qP, qSV, and qT waves, respectively. We can write $V_j^{-1} = V_j^{*-1} - i\omega^{-1} q_j$, where V_j^* and q_j are the propagation speeds and attenuation coefficients of the qP, qSV, and qT waves.

Particular Cases. Equation (13) reduces to the following cases for different media:

1) for $C_{24} = C_{34} = 0$, rotating orthotropic magnetothermoelastic;
2) for $C_{24} = C_{34} = 0$, $C_{23} = C_{33} - 2C_{44}$, rotating transversely isotropic magnetothermoelastic;
3) for $C_{22} = C_{33} = \lambda + 2\mu$, $C_{13} = C_{23} = C_{12} = \lambda$, $C_{44} = C_{55} = C_{66} = \mu$, $C_{14} = C_{24} = C_{34} = C_{56} = 0$, $\beta_2 = \beta_3 = \beta$, $K_2 = K_3 = K$, rotating isotropic magnetothermoelastic;
4) for $\Omega = 0$, monoclinic magnetothermoelastic;
5) for $H_0 = 0$, rotating monoclinic thermoelastic;
6) for $\varepsilon = 0$, $D_4 = 0$, rotating monoclinic magnetoelastic;
7) for $H_0 = 0$, $\Omega = 0$, $D_4 = 0$, $\varepsilon = 0$, monoclinic elastic.

Computation of the Angles of Reflection. The reflection coefficient depends on the velocities $V_i(e_i)$, where $i = 1, 2, 3, \ldots, 6$, which are functions of the angles of incidence and reflection. For the incident qP wave, the angle of incidence e_1, and therefore $V_1(e_1)$, is assumed to be known. It is necessary to compute the angles of reflection e_4, e_5, and e_6 for a given value of e_1. Then the velocities $V_4(e_4)$, $V_5(e_5)$, and $V_6(e_6)$ can be computed from explicit algebraic formulas. The procedure is given below for computing e_4, e_5, and e_6: for given e_1 in the case of incident qP waves, for given e_2 in the case of incident qT waves, and for given e_3 in the case of incident qSV waves.

Putting $\zeta = \rho V^2$ in Eq. (13), we obtain

$$A_0 \left(\rho V^2\right)^3 + A_1 \left(\rho V^2\right)^2 + A_2 \rho V^2 + A_3 = 0 \ . \tag{14}$$

We define the dimensionless apparent velocity $\bar{V}$ as

$$\bar{V} = \frac{V_a}{\beta} = \frac{V}{P_2 \beta} \ , \tag{15}$$

where $\beta = \sqrt{\dfrac{C_{44}}{\rho}}$. From Eq. (15) we have $\rho V^2 = P_2^2 C_{44} \bar{V}^2$ and then Eq. (14) results in

$$A_0 P_2^6 C_{44}^3 \bar{V}^6 + A_1 P_2^4 C_{44}^2 \bar{V}^4 + A_2 P_2^2 C_{44} \bar{V}^2 + A_3 = 0 \ . \tag{16}$$

Dividing Eq. (16) by $P_2^6 C_{44}^3$ and putting

$$\bar{C}_{ij} = \frac{C_{ij}}{C_{44}} \ , \quad \bar{K}_2 = \frac{K_2}{C_{44}} \ , \quad \bar{K}_3 = \frac{K_3}{C_{44}} \ , \quad \bar{\varepsilon} = \frac{\varepsilon}{C_{44}} \ , \quad \bar{\mu}_e = \frac{\mu_e}{C_{44}} \ , \quad V = \bar{V} P_2 \beta \ ,$$

we obtain

$$A_0 \bar{V}^6 + \bar{A}_1 \bar{V}^4 + \bar{A}_2 \bar{V}^2 + \bar{A}_3 = 0 \ , \tag{17}$$

where

$$A_0 = 4 \left(\frac{\Omega}{\omega}\right)^2 - \Omega^{*2} \ ,$$

$$\bar{A}_1 = \Omega^* \left(\bar{D}_1 + \bar{D}_3 + \bar{\varepsilon} + \bar{\varepsilon}\bar{\beta}^2 p^2\right) + \bar{D}_5 \left(\Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2\right),$$

$$\bar{A}_2 = \bar{D}_2^2 - \bar{D}_1\bar{D}_3 - \Omega^*\left(\bar{D}_1 + \bar{D}_3\right)\bar{D}_5 - \bar{\varepsilon}\left(\bar{\beta}^2\bar{D}_1 p^2 + \bar{D}_3 - 2\bar{\beta}\bar{D}_2 p\right),$$

$$\bar{A}_3 = \left(\bar{D}_1\bar{D}_3 - \bar{D}_2^2\right)\bar{D}_5\,, \quad \bar{D}_1 = \bar{C}_{22} + p^2 + 2\bar{C}_{24}p + \bar{\mu}_e H_0^2\,,$$

$$\bar{D}_2 = \bar{C}_{24} + \bar{C}_{34}p^2 + \left(1 + \bar{C}_{23}\right)p + \bar{\mu}_e H_0^2 p\,,$$

$$\bar{D}_3 = 1 + \bar{C}_{33}p^2 + 2\bar{C}_{34}p + \bar{\mu}_e H_0^2 p^2, \quad \bar{D}_5 = \frac{1}{\tau^* C_E}\left(\bar{K}_2 + \bar{K}_3 p^2\right),$$

$$p = \frac{P_3}{P_2}\,, \quad P_2 = \sin\theta\,, \quad P_3 = \cos\theta\,.$$

We have for incident waves: qP, $p = -\cot e_1$; qT, $p = -\cot e_2$; qSV, $p = -\cot e_3$; for reflected waves: qP, $p = -\cot e_4$; qT, $p = \cot e_5$; qSV, $p = -\cot e_6$.

For a given value of p, Eq. (17) may be solved, and its three roots correspond to the qP, qT, and qSV waves. For a given value of $\bar{V}$, Eq. (17) is a six-degree equation in p for the incident qP, qT, and qSV waves and for the reflected ones, where the positive and negative roots correspond to the reflected and incident waves, respectively.

Substituting the values of $\bar{D}_1$, $\bar{D}_2$, $\bar{D}_3$, and $\bar{D}_5$ into Eq. (17), after simplification we obtain a six-degree equation in p which can be written as

$$g_0 p^6 + g_1 p^5 + g_2 p^4 + g_3 p^3 + g_4 p^2 + g_5 p + g_6 = 0\,. \tag{18}$$

The expressions for g_i, where $i = 1, 2, 3, \dots, 6$, are given in the Appendix. After introducing $q = 1/p$, Eq. (18) becomes

$$g_6 q^6 + g_5 q^5 + g_4 q^4 + g_3 q^3 + g_2 q^2 + g_1 q + g_0 = 0\,. \tag{19}$$

For the angles of incidence for which all three reflected qP, qSV, and qT waves exist, Eq. (19) has three positive roots. The smaller positive root, say q_6, corresponds to the reflected qT waves, the root q_5, to the reflected qSV waves, and the larger positive root q_4, to the reflected qP waves. We have

$$e_4 = \tan^{-1}(q_4)\,, \quad e_5 = \tan^{-1}(q_5)\,, \quad e_6 = \tan^{-1}(q_6)\,. \tag{20}$$

For an isotropic thermoelastic medium, putting in Eq. (18)

$$\bar{C}_{11} = \bar{C}_{22} = \bar{C}_{33} = \frac{\lambda + 2\mu}{\mu}\,, \quad \bar{C}_{14} = \bar{C}_{24} = \bar{C}_{34} = 0\,,$$

$$\bar{C}_{44} = \bar{C}_{55} = \bar{C}_{66} = 1\,, \quad \bar{C}_{12} = \bar{C}_{13} = \bar{C}_{23} = \frac{\lambda}{\mu}\,, \quad \bar{K}_2 = \bar{K}_3 = \bar{K}\,, \quad \bar{\beta} = 1\,, \quad \gamma = \frac{\lambda + 2\mu}{\mu}\,,$$

we obtain

$$g_0' p^6 + g_1' p^5 + g_2' p^4 + g_3' p^3 + g_4' p^2 + g_5' p + g_6' = 0\,, \tag{21}$$

where the expressions for g_i' ($i = 0, 1, 2, \dots, 6$) are given in the Appendix. Taking

$$\frac{\bar{K}}{\tau^* C_E}\left(\gamma + \bar{\mu}_e H_0^2\right) = S\gamma\,, \quad \gamma + \bar{\varepsilon} + \mu_e H_0^2 + \frac{1}{\tau^* C_E}\bar{K}\Omega^* = r\,,$$

$$\frac{r}{S\gamma} + \Omega^* = d_1\,, \quad \frac{1}{S\gamma}\left\{\Omega^* + \Omega^* r - 4\frac{1}{\tau^* C_E}\bar{K}\left(\frac{\Omega}{\omega}\right)^2\right\} = d_2\,, \quad \frac{1}{S\gamma}\left\{4\left(\frac{\Omega}{\omega}\right)^2 - \Omega^{*2}\right\} = d_3\,,$$

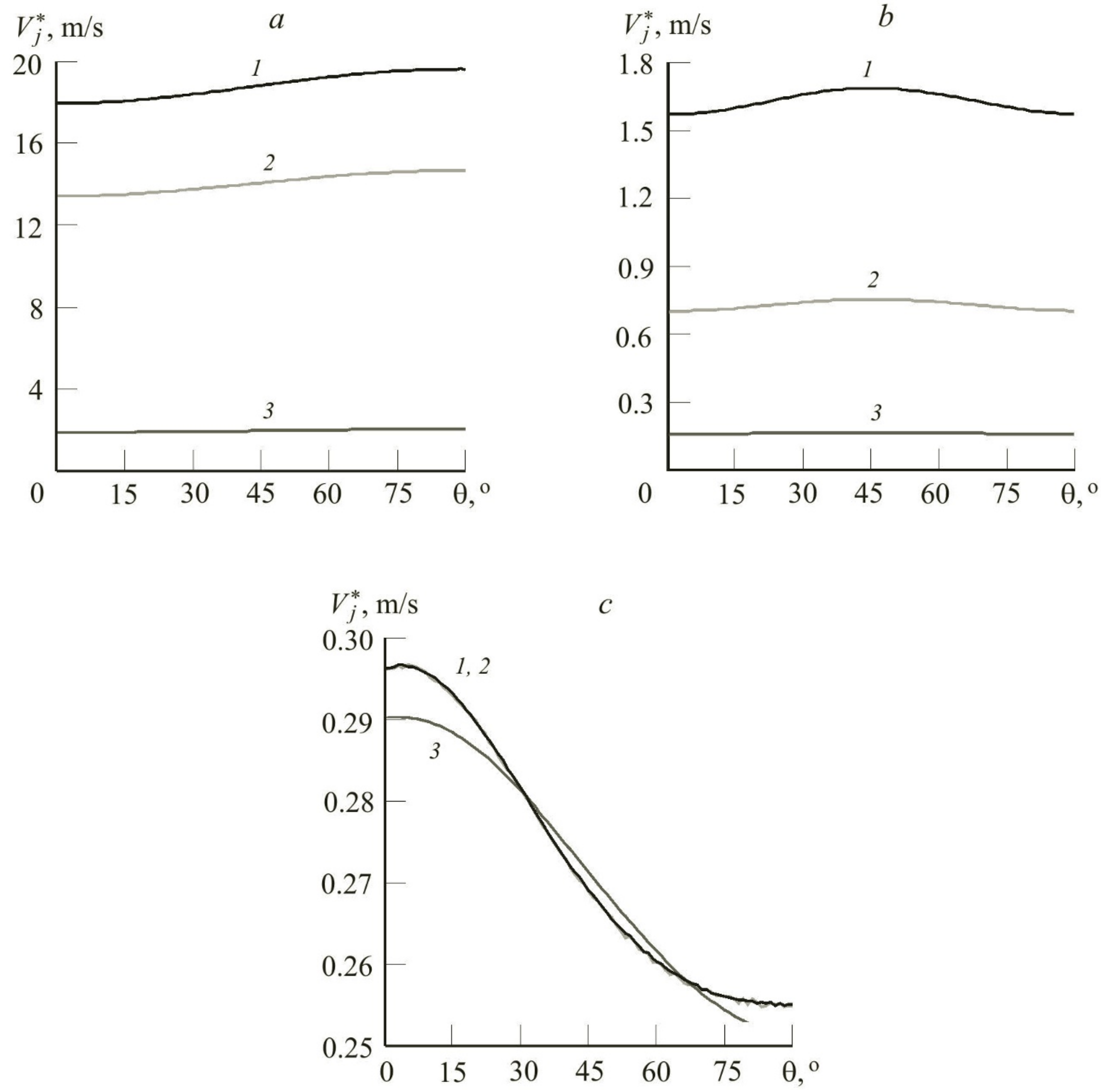

Fig. 1. Variations of the speed of the plane qP (a), qSV (b), and qT (c) waves against the angle of incidence at H_0 = 10 A/m and Ω/ω = 0 (1), 2 (2), and 10 (3).

we rewrite Eq. (21) as

$$S\gamma\left(p^2-\delta_1^2\right)\left(p^2-\delta_2^2\right)\left(p^2-\delta_3^2\right)=0\,, \tag{22}$$

where

$$\delta_1^2+\delta_2^2+\delta_3^2=d_1\bar{V}^2-3\,,$$

$$\delta_1^2\delta_2^2+\delta_2^2\delta_3^2+\delta_3^2\delta_1^2=d_2\bar{V}^4-2d_1\bar{V}^4-2d_1\bar{V}^2+3\,,$$

$$\delta_1^2\delta_2^2\delta_3^2=-\left(d_3\bar{V}^6+d_2\bar{V}^4-d_1\bar{V}^2+1\right).$$

In this case Snell′s law becomes

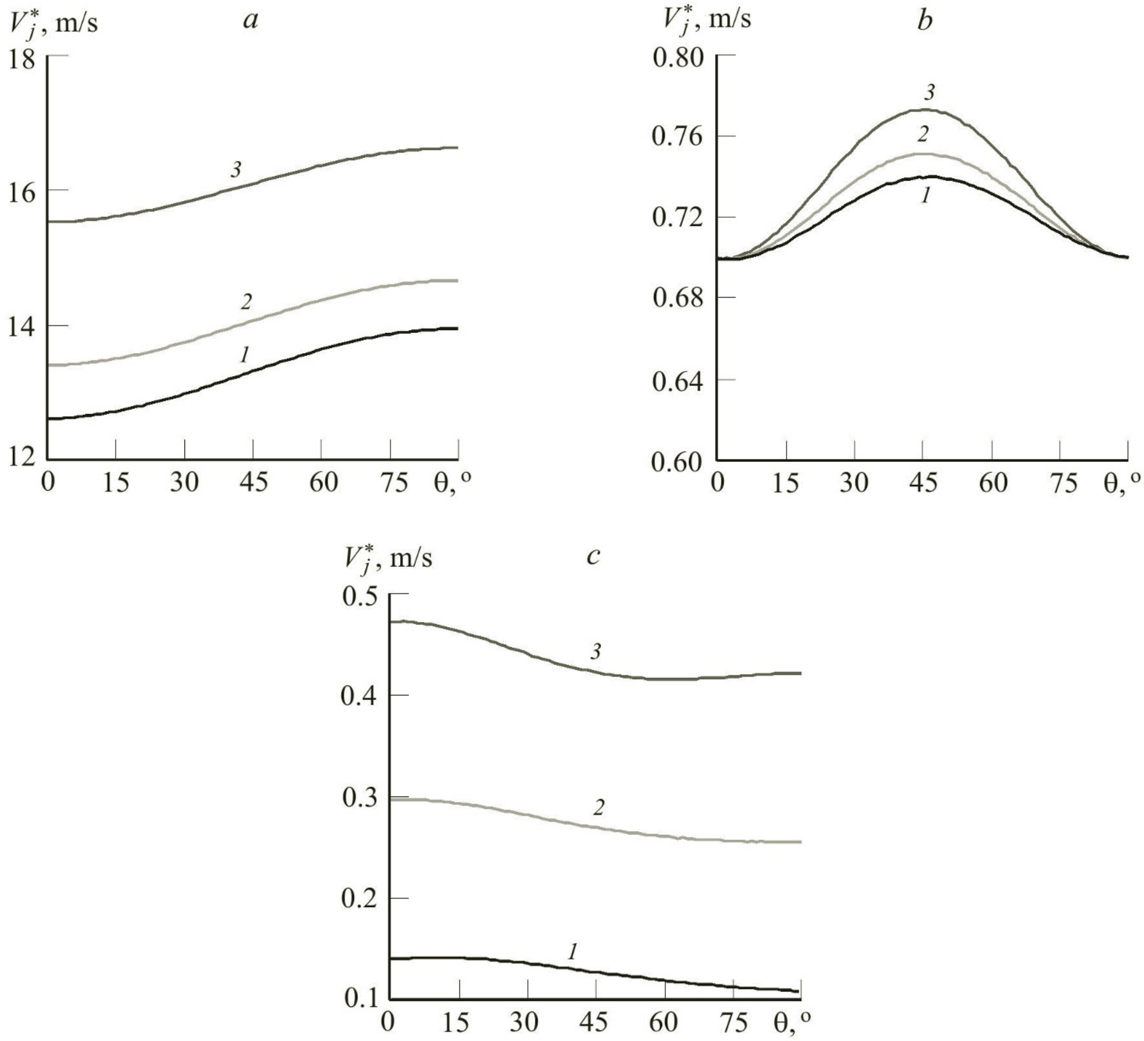

Fig. 2. Variations of the speed of the plane qP (a), qSV (b), and qT (c) waves against the angle of incidence at $\Omega/\omega = 2$ and $H_0 = 0$ (1), 10 (2), and 20 A/m (3).

$$\frac{\sin e_1}{V_{\text{qP}}} = \frac{\sin e_2}{V_{\text{qSv}}} = \frac{\sin e_3}{V_{\text{qT}}} . \tag{23}$$

Therefore, the roots $p^2 = \delta_1^2 = \cot^2 e_3$, $p^2 = \delta_2^2 = \cot^2 e_1$, and $p^2 = \delta_3^2 = \cot^2 e_2$ correspond to the qSV, qP, and qT waves, respectively. The quantities $q_1 = -\tan e_1$, $q_2 = -\tan e_2$, $q_3 = -\tan e_3$, $q_4 = \tan e_1$, $q_5 = \tan e_2$, and $q_6 = \tan e_3$ are the six roots of Eq. (19). This choice will act as a guiding factor in computing the angles of reflection of the qP, qT, and qSV waves in a rotating monoclinic magnetothermoelastic medium. For an orthotropic medium, it can be shown that $q_1 = q_3 = q_5 = 0$. Therefore, Eq. (19) is reduced to a cubic equation in q^2. Thus, we can choose $q_1 = -q_4$, $q_2 = -q_5$, $q_3 = -q_6$. Therefore, the angles of reflection of the qP, qT, and qSV waves are equal to the angles of incidence of these waves. This is not true for the monoclinic case. Following this procedure, one can compute the angles of reflection for a particular incident wave.

Numerical Results and Discussion. For numerical computations of the speeds of plane waves, we consider the following relevant physical constants:

$$C_{33} = 24.9 \cdot 10^9 \ \text{N} \cdot \text{m}^{-2} , \quad C_{22} = 19.8 \cdot 10^9 \ \text{N} \cdot \text{m}^{-2} , \quad C_{44} = 6.67 \cdot 10^9 \ \text{N} \cdot \text{m}^{-2} ,$$

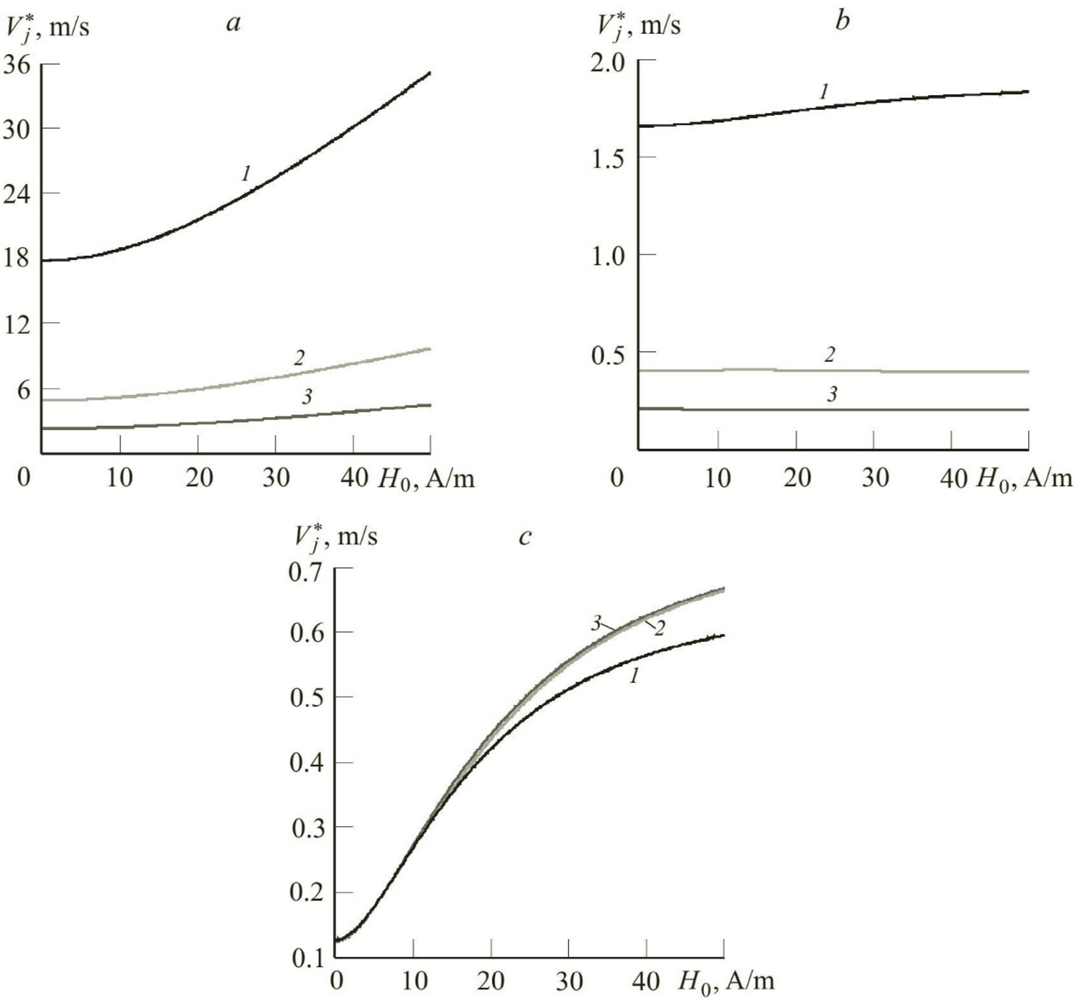

Fig. 3. Variations of the speed of the plane qP (a), qSV (b), and qT (c) waves against the magnetic field strength at $\theta = 45^o$ and $\Omega/\omega = 0$ (1), 4 (2), and 8 (3).

$$C_{23} = 7.8 \cdot 10^9 \ \mathrm{N \cdot m^{-2}}, \quad C_{34} = C_{44}/5, \quad C_{24} = C_{44}/5, \quad \rho = 2.714 \cdot 10^3 \ \mathrm{kg \cdot m^{-3}},$$

$$C_E = 3.9 \cdot 10^2 \ \mathrm{J \cdot kg^{-1} \cdot deg^{-1}}, \quad K_2 = 1.24 \cdot 10^2 \ \mathrm{W \cdot m^{-1} \cdot deg^{-1}},$$

$$K_3 = 1.34 \cdot 10^2 \ \mathrm{W \cdot m^{-1} \cdot deg^{-1}}, \quad \beta_2 = 5.75 \cdot 10^6 \ \mathrm{N \cdot m^{-2} \cdot deg^{-1}},$$

$$\beta_3 = 5.17 \cdot 10^6 \ \mathrm{N \cdot m^{-2} \cdot deg^{-1}}, \quad T_0 = 296 \ \mathrm{K}, \quad \tau_0 = 0.05 \ \mathrm{s}, \quad \omega = 5 \ \mathrm{Hz}.$$

Equation (13) is solved numerically to obtain the real speeds V_j^* of the propagation of plane waves in a rotating monoclinic magnetothermoelastic medium.

In Fig. 1 the speeds of the qP, qSV, and qT waves are plotted against the angle of incidence for $H_0 = 10$ A/m and different values of Ω/ω. The speed of the qP waves is 17.93 $\mathrm{m \cdot s^{-1}}$ at $\theta = 0^o$ for $\Omega/\omega = 0$. Then it increases slowly up to 19.6 $\mathrm{m \cdot s^{-1}}$ at $\theta = 90^o$. With increase in the rotation rate, it decreases for each angle of incidence. The speed of the qSV waves is 1.567 $\mathrm{m \cdot s^{-1}}$ at $\theta = 0^o$ and $\theta = 90^o$ for $\Omega/\omega = 0$. It increases to a maximum value of 1.682 $\mathrm{m \cdot s^{-1}}$ at $\theta = 45^o$. With increase in the rotation rate, the speed of the qSV waves decreases. The speed of the qT waves is 0.2962 $\mathrm{m \cdot s^{-1}}$ at $\theta = 0^o$ for $\Omega/\omega = 0$. Then it first increases slightly to 0.2967 $\mathrm{m \cdot s^{-1}}$ at $\theta = 4^o$ and thereafter decreases sharply to a minimum

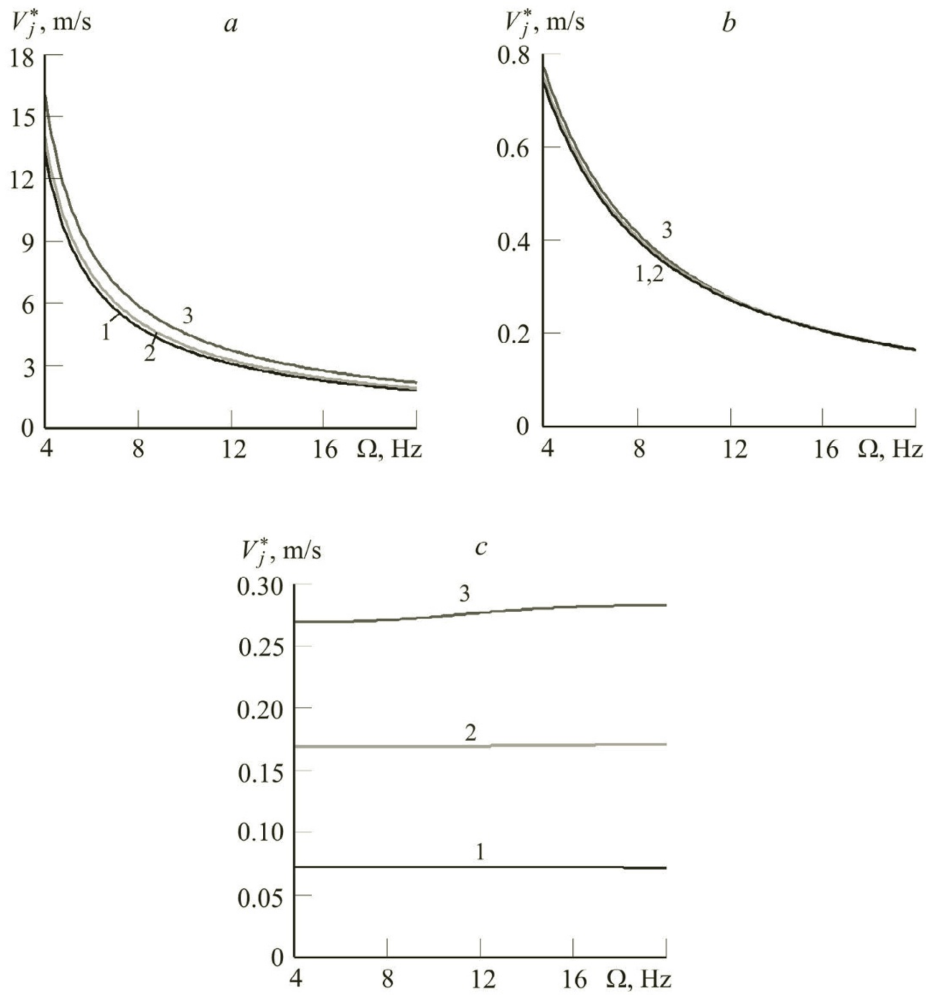

Fig. 4. Variations of the speed of the plane qP (a), qSV (b), and qT (c) waves against the rotation frequency at $\theta = 45^o$, $\omega = 2$ Hz, and $H_0 = 0$ (1), 10 (2), and 20 A/m (3).

value of 0.255 $m \cdot s^{-1}$ at $\theta = 90^o$. It can be seen from Fig. 1c that the effect of rotation on the qT wave speed increases with the rotation rate and is different from those observed in the cases of the qP and qSV waves.

Figure 2 shows the speeds of the qP, qSV, and qT waves against the angle of incidence for $\Omega/\omega = 2$ at different values of H_0. It is seen that the speed of the qP wave is 12.6 $m \cdot s^{-1}$ at $\theta = 0^o$ for $H_0 = 0$. It increases slowly up to 13.9 $m \cdot s^{-1}$ at $\theta = 90^o$. It is also seen that the speed increases with the magnetic field strength. The speed of the qSV waves is 0.699 $m \cdot s^{-1}$ at $\theta = 0^o$ and $\theta = 90^o$ for $H_0 = 0$. It attains a maximum value of 0.740 $m \cdot s^{-1}$ at $\theta = 47^o$. The speed of the qSV waves also increases with the magnetic field strength, except for the values at $\theta = 0^o$ and $\theta = 90^o$. The speed of the qT waves is 0.140 $m \cdot s^{-1}$ at $\theta = 0^o$ for $H_0 = 0$. Then, after a slight increase at $\theta = 3^o$, it decreases slowly to a minimum value of 0.108 $m \cdot s^{-1}$ at $\theta = 90^o$. The speed of the qT waves also increases with the magnetic field strength.

In Fig. 3, the speeds of the qP, qSV, and qT waves are plotted against the magnetic field strength for $\theta = 45^o$ and different values of Ω/ω. It is observed that the effect of rotation increases with the magnetic field strength. The variations of the wave speeds with the angle of reflection are almost similar to those given in Fig. 1. Because of this, these variations are not presented graphically.

In Fig. 4, the speeds of the qP, qSV, and qT waves are plotted against the rotation rate at $\theta = 45^o$, $\omega = 2$ Hz, and different values of H_0. It is seen that these speeds decrease sharply with increasing Ω. For example, at $H_0 = 0$ the speed of the qP waves at $\Omega = 4$ Hz is 13.27 $m \cdot s^{-1}$, and then it decreases to 1.81 $m \cdot s^{-1}$ at $\Omega = 20$ Hz. It follows from Fig. 4 that the speeds of all the waves are affected by the rotation rate and magnetic field strength.

Conclusions. The solutions of the equations for the plane wave propagation in a rotating monoclinic magnetothermoelastic medium are obtained. There exist three plane waves, namely quasi-P, quasi-SV, and quasi-T waves. The speeds of these waves are computed for a particular material modeling a half-space. From numerical results it is observed that the speeds of the waves are significantly affected by the presence of rotation and magnetic field.

NOTATION

B, magnetic induction; C_E, specific heat at constant strain; C_{ij}, elastic constants; e_{ij}, components of the strain tensor; **E**, electric field strength; e_1, e_2, e_3, angles of incidence; e_4, e_5, e_6, angles of reflection; **H**, total magnetic field strength; **J**, electric current density; k, wave number; K_2, K_3, thermal conductivities; q, attenuation coefficient; T, temperature; T_0, reference uniform temperature; t, time; $\mathbf{u}(v, w)$, displacement vector; V, phase velocity; V_j^*, speed of wave propagation; y, z, coordinates; β_2, β_3, thermal coefficients; θ, angle of propagation; λ, μ, Lame constants; μ_e, magnetic permeability; ρ, density; σ, electric conductivity; τ_0, relaxation time; ω, circular frequency; Ω, rotation rate.

REFERENCES

1. M. A. Biot, Thermoelasticity and irreversible thermodynamics, *J. Appl. Phys.*, **27**, 249–253 (1956).
2. H. W. Lord and Y. Shulman, A generalized dynamical theory of thermo-elasticity, *J. Mech. Phys. Solid,* **15**, 209–309 (1967).
3. A. E. Green and K. A. Lindsay, Thermoelasticity, *J. Elast.*, **2**, 1–7 (1972).
4. J. Ignaczak and M. Ostoja-Starzewski, *Thermoelasticity with Finite Wave Speeds*, Oxford University Press, Oxford (2009).
5. R. S. Dhaliwal and H. H. Sherief, Generalized thermoelasticity for anisotropic media, *Quart. Appl. Math.*, **33**, 1–8 (1980).
6. M. Schoenberg and D. Censor, Elastic waves in rotating media, *Quart. Appl. Math.*, **31**, 115–125 (1973).
7. D. S. Chandrasekharaiah and K. R. Srikantiah, Thermoelastic plane waves in a rotating solid, *Acta Mech.*, **50**, 211–219 (1984).
8. D. S. Chandrasekharaiah and K. R. Srikantiah, Thermoelastic plane waves without energy dissipation in a rotating body, *Mech. Res. Commun.*, **24**, 551–560 (1984).
9. F. Ahmad and A. Khan, Thermoelastic plane waves in rotating isotropic medium, *Acta Mech.*, **136**, 243–247 (1999).
10. C. M. Keith and S. Crampin, Seismic body waves in anisotropic media: Reflection and refraction at a plane interface, *Geophys. J. Roy. Astron. Soc.*, **49**, 181–208 (1977).
11. A. Chattopadhyay and S. Choudary, The reflection phenomena of P-waves in a medium of monoclinic type, *Int. J. Eng. Sci.*, **33**, 199–207 (1995).
12. A. Chattopadhyay, S. Saha, and M. Chakraboty, The reflection of SV-waves in a monoclinic medium, *Indian J. Pure Appl. Math.*, **27**, 1029–1042 (1996).
13. S. J. Singh, Comments on the reflection phenomena of SV-waves in a medium of monoclinic type, *Int. J. Eng. Sci.*, **37**, 407–410 (1986).
14. S. Singh and S. Khurana, Reflection of P and SV waves at the free surface of a monoclinic elastic half-space, *Proc. Indian Acad. Sci.*, **80**, 401–412 (1986).
15. B. Singh, Wave propagation in an anisotropic generalized thermoelastic solid, *Indian J. Pure Appl. Math.*, **34**, 1479–1485 (2003).
16. B. Singh, Dispersion relations in a generalized monoclinic thermoelastic solid half-space, *J. Tech. Phys.*, **47**, 119–129 (2006).
17. B. Singh, Reflection of plane waves at the free surface of a monoclinic thermoelastic solid half-space, *Eur. J. Mech. Solids*, **29**, 911–916 (2010).
18. R. Kumar and M. Singh, Effects of rotating and imperfection on reflection and transmission of plane waves in anisotropic generalized thermoelastic media, *J. Sound Vibr.*, **324**, 773–797 (2009).
19. S. S. Singh and S. K. Tomar, Quasi-P waves at a corrugated interface between two dissimilar monoclinic elastic half-spaces, *Int. J. Solids Struct.*, **44**, 197–228 (2007).
20. B. Singh and A. K. Yadav, Plane waves in a transversely isotropic rotating magneto-thermo-elastic medium, *J. Eng. Phys. Thermophys.*, **85**, 1226–1232 (2012).
21. B. Singh and A. K. Yadav, Reflection of plane waves in a rotating transversely isotropic magneto-thermoelastic solid half-space, *J. Theor. Appl. Mech. (Sofia)*, **42**, 33–60 (2012).

APPENDIX

$$g_0 = \frac{1}{\tau^* C_E} \overline{K}_3 \left(\overline{C}_{33} - \overline{C}_{34}^2 + \overline{\mu}_e H_0^2\right),$$

$$g_1 = \frac{2}{\tau^* C_E} \overline{K}_3 \left(\overline{C}_{24}\overline{C}_{33} - \overline{C}_{34}\overline{C}_{23} + \overline{C}_{24}\overline{\mu}_e H_0^2 - \overline{C}_{34}\overline{\mu}_e H_0^2\right),$$

$$g_2 = \left\{\overline{C}_{34}^2 - \overline{C}_{33} - \overline{\varepsilon}\overline{\beta}^2 - \overline{\mu}_e H_0^2 - \frac{1}{\tau^* C_E} \overline{K}_3 \left(\Omega^* + \overline{C}_{33}\Omega^* + \Omega^* \overline{\mu}_e H_0^2\right)\right\} \overline{V}^2$$

$$+ \frac{1}{\tau^* C_E} \overline{K}_3 \Big(\overline{C}_{22}\overline{C}_{33} + 2\overline{C}_{24}\overline{C}_{34} + 1 - \left(1 + \overline{C}_{23}\right)^2 + \overline{C}_{22}\overline{\mu}_e H_0^2 + \overline{C}_{33}\overline{\mu}_e H_0^2$$

$$- 2\overline{\mu}_e H_0^2 - 2\overline{C}_{23}\overline{\mu}_e H_0^2\Big) + \frac{1}{\tau^* C_E} \overline{K}_2 \left(\overline{C}_{33} - \overline{C}_{34}^2 + \overline{\mu}_e H_0^2\right),$$

$$g_3 = \Big\{2\overline{C}_{34}\overline{C}_{23} - 2\overline{C}_{24}\overline{C}_{33} + 2\overline{C}_{34}\mu_e {H_0}^2 - 2\overline{C}_{24}\mu_e {H_0}^2$$

$$- \frac{2}{\tau^* C_E} \overline{K}_3 \left(\Omega^* \overline{C}_{34} + \Omega^* C_{24}\right) + 2\overline{\varepsilon} \left(\overline{\beta}\overline{C}_{34} - \overline{\beta}^2 \overline{C}_{24}\right)\Big\} \overline{V}^2$$

$$+ \frac{2}{\tau^* C_E} \overline{K}_3 \left(\overline{C}_{22}\overline{C}_{34} - \overline{C}_{24}\overline{C}_{23} + \overline{C}_{34}\overline{\mu}_e H_0^2 - \overline{C}_{24}\overline{\mu}_e H_0^2\right)$$

$$+ \frac{2}{\tau^* C_E} \overline{K}_2 \left(\overline{C}_{24}\overline{C}_{33} - \overline{C}_{34}\overline{C}_{23} + \overline{C}_{24}\overline{\mu}_e H_0^2 - \overline{C}_{34}\overline{\mu}_e H_0^2\right),$$

$$g_4 = \left\{\Omega^* + \overline{C}_{33}\Omega^* + \Omega^* \overline{\varepsilon}\overline{\beta}^2 + \Omega^* \overline{\mu}_e H_0^2 + \frac{1}{\tau^* C_E} \overline{K}_3 \left(\Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2\right)\right\} \overline{V}^4$$

$$+ \Big\{\left(1 + \overline{C}_{23}\right)^2 - 1 - \overline{C}_{22}\overline{C}_{33} - 2\overline{C}_{24}\overline{C}_{34} + 2\overline{\mu}_e H_0^2 + 2\overline{C}_{23}\overline{\mu}_e H_0^2$$

$$- \overline{C}_{22}\overline{\mu}_e H_0^2 - \overline{C}_{33}\overline{\mu}_e H_0^2 - \overline{C}_{22}\overline{\varepsilon}\overline{\beta}^2 - \overline{\varepsilon}\overline{C}_{33} + 2\overline{\varepsilon}\overline{\beta}$$

$$+ 2\overline{C}_{23}\overline{\varepsilon}\overline{\beta} - \overline{\varepsilon}\overline{\beta}^2 \overline{\mu}_e H_0^2 - \overline{\varepsilon}\overline{\mu}_e H_0^2 + 2\overline{\varepsilon}\overline{\beta}\overline{\mu}_e H_0^2$$

$$- \frac{1}{\tau^* C_E} \overline{K}_3 \left(\Omega^* + \overline{C}_{22}\Omega^* + \Omega^* \overline{\mu}_e H_0^2\right) - \frac{1}{\tau^* C_E} \overline{K}_2 \Big(\Omega^*$$

$$+ \overline{C}_{33}\Omega^* + \Omega^* \overline{\mu}_e H_0^2\Big)\Big\} \overline{V}^2 + \frac{1}{\tau^* C_E} \overline{K}_2 \Big(\overline{C}_{22}\overline{C}_{33} + 2\overline{C}_{24}\overline{C}_{34} + 1$$

$$- \left(1 + \overline{C}_{23}\right)^2 - 2\overline{\mu}_e H_0^2 - 2\overline{C}_{23}\overline{\mu}_e H_0^2 + \overline{C}_{22}\overline{\mu}_e H_0^2 + \overline{C}_{33}\overline{\mu}_e H_0^2\Big)$$

$$+ \frac{1}{\tau^* C_E} \overline{K}_3 \left(\overline{C}_{22} - \overline{C}_{24}^2 + \overline{\mu}_e H_0^2\right),$$

$$g_5 = 2\Big[\Omega^*\left(\bar{C}_{24}+\bar{C}_{34}\right)\bar{V}^4 + \Big\{\Big(\bar{C}_{24}\bar{C}_{23} - \bar{C}_{22}\bar{C}_{34} + \bar{C}_{24}\bar{\mu}_e H_0^2 - \bar{C}_{34}\bar{\mu}_e H_0^2 + \bar{C}_{24}\bar{\varepsilon}\bar{\beta}$$

$$- \bar{C}_{34}\bar{\varepsilon}\Big) - \frac{1}{\tau^* C_E}\,\bar{K}_2\left(\Omega^*\bar{C}_{34} + \Omega^*\bar{C}_{24}\right)\Big\}\bar{V}^2 + \frac{1}{\tau^* C_E}\,\bar{K}_2\left(\bar{C}_{22}\bar{C}_{34} - \bar{C}_{24}\bar{C}_{23} - \bar{C}_{24}\bar{\mu}_e H_0^2 + \bar{C}_{34}\bar{\mu}_e H_0^2\right)\Big],$$

$$g_6 = \left\{4\left(\frac{\Omega}{\omega}\right)^2 - \Omega^{*2}\right\}\bar{V}^6 + \Big\{\bar{C}_{22}\Omega^* + \Omega^*\bar{\varepsilon} + \Omega^* + \Omega^*\bar{\mu}_e H_0^2$$

$$+ \frac{1}{\tau^* C_E}\,\bar{K}_2\left(\Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2\right)\Big\}\bar{V}^4 + \Big\{\bar{C}_{24}^2 - \bar{C}_{22} - \bar{\mu}_e H_0^2 - \bar{\varepsilon}$$

$$- \frac{1}{\tau^* C_E}\,\bar{K}_2\left(\Omega^* + \bar{C}_{22}\Omega^* + \Omega^*\bar{\mu}_e H_0^2\right)\Big\}\bar{V}^2 + \frac{1}{\tau^* C_E}\,\bar{K}_2\Big(\bar{C}_{22}$$

$$- \bar{C}_{24}^2 + \bar{\mu}_e H_0^2\Big),$$

$$g_0' = \frac{1}{\tau^* C_E}\,\bar{K}\left(\gamma + \bar{\mu}_e H_0^2\right),$$

$$g_1' = 0,$$

$$g_2' = -\left\{\gamma + \bar{\varepsilon} + \bar{\mu}_e H_0^2 + \frac{1}{\tau^* C_E}\,\bar{K}\left(\Omega^* + \gamma\Omega^* + \Omega^*\bar{\mu}_e H_0^2\right)\right\}\bar{V}^2$$

$$+ \frac{3}{\tau^* C_E}\,\bar{K}\left(\gamma + \bar{\mu}_e H_0^2\right),$$

$$g_3' = 0,$$

$$g_4' = \left\{\Omega^* + \gamma\,\Omega^* + \bar{\varepsilon}\Omega^* + \Omega^*\bar{\mu}_e H_0^2 + \frac{1}{\tau^* C_E}\,\bar{K}\left(\Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2\right)\right\}\bar{V}^4$$

$$+ \left\{-2\gamma - 2\bar{\varepsilon} - 2\bar{\mu}_e H_0^2 - \frac{2}{\tau^* C_E}\,\bar{K}\left(\Omega^* + \gamma\Omega^* + \Omega^*\bar{\mu}_e H_0^2\right)\right\}\bar{V}^2,$$

$$+ \frac{3}{\tau^* C_E}\,\bar{K}\left(\gamma + \mu_e H_0^2\right),$$

$$g_5' = 0,$$

$$g_6' = \left\{4\left(\frac{\Omega}{\omega}\right)^2 - \Omega^{*2}\right\}\bar{V}^6 + \left\{\Omega^* + \gamma\Omega^* + \Omega^*\bar{\varepsilon} + \Omega^*\mu_e H_0^2\right.$$

$$\left. + \frac{1}{\tau^* C_E}\bar{K}\left(\Omega^{*2} - 4\left(\frac{\Omega}{\Omega}\right)^2\right)\right\}\bar{V}^4 + \left\{-\gamma - \bar{\varepsilon} - \bar{\mu}_e H_0^2\right.$$

$$\left. - \frac{1}{\tau^* C_E}\bar{K}\left(\Omega^* + \gamma\Omega^* + \Omega^*\mu_e H_0^2\right)\right\}\bar{V}^2 + \frac{1}{\tau^* C_E}\bar{K}\left(\gamma + \bar{\mu}_e H_0^2\right).$$