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## HEAT TRANSFER IN PHASE TRANSFORMATIONS

## EVAPORATION OF WATER DROPLETS IN A HIGH-TEMPERATURE GASEOUS MEDIUM

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A numerical solution of the problem of heat and mass transfer in evaporation of a droplet of water moving in a stream of high-temperature (up to 1200 K) gases is done on the basis of a system of nonlinear nonstationary partial differential equations describing conductive and radiative heat transfer in the droplet, as well as composite heat transfer at the "liquid–gas" interface. The values of the water evaporation rate have been determined. It is shown that the dependence of the evaporation rate on the droplet surface temperature has a nonlinear character. Characteristic relationships between the convective and radiative heat fluxes on the droplet surface (the radiative flux substantially exceeds the convective one; on decrease in the difference between the gas and droplet surface temperatures the difference between the radiative and convective heat fluxes decreases), the lifetimes (total evaporation) of droplets, as well as of the temperature and concentration of steam and gases in the vicinity of droplets have been determined. The calculated characteristics of the water droplet evaporation under conditions of high temperatures of the gas medium differ considerably from those obtained within the framework of the "diffusional" model of evaporation. A comparison of the results of numerical simulation with the experimental data obtained with the use of high-velocity panoramic optical methods of visualization by "tracing particles" is carried out.

Keywords: heat and mass transfer, evaporation, droplet, water, gaseous medium, high temperatures.

**Introduction.** Evaporation of water droplets takes place in the course of implementation of many technologies not only in power engineering [1-4], but also in other branches of industrial production [5-10]. Usually to each technology there corresponds its own temperature range [11]. The tendencies towards the development of heat power engineering presuppose the advisability of a future rise in the working temperatures of many power engineering units and assemblies of thermal power stations [1]. At present, the main laws governing the evaporation of water droplets have been established in full measure only at relatively moderate (arbitrarily up to 500 K) ambient temperatures.

The most commonly used mathematical model of water evaporation in a droplet state [12–14] was developed within the framework of the assumption on the "diffusional" mechanism of evaporation. It is assumed [12–14] that the phase transformation rate on the "water–high-temperature gas medium" interface is determined by the intensity of the process of vapor diffusion in the layer of gases heated to high temperatures and adjacent to the droplet surface. There is also a model of evaporation [15, 16] formulated for the droplets of liquids (including water) in which the phase transformation rate is determined by the interface temperature ("kinetic" regime of vaporization). Of interest is an analysis of the possibilities of each of these models in describing the processes of water evaporation in a droplet state at high (especially at about 1000 K) temperatures of the surrounding gas medium.

The goal of the present work is to investigate the processes of heat and mass transfer in evaporation of water droplets moving in a high-temperature gaseous medium.

**Formulation of the Problem.** We solved the problem of heat and mass transfer for a water droplet moving through a gaseous medium with a known temperature  $T_f$  that substantially exceeds the initial liquid temperature  $T_0$ . Depending on the physical model of evaporation used ("diffusional" or "kinetic") and means of heating (radiant, convective or conductive), the boundary-value conditions of the problem and energy equations for the droplet were varied. The formulations used in [17, 18] were taken as the basic ones.

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Fig. 1. Schematic of the solution domain of the problem of heat and mass transfer of a droplet in the form of a cylindrical disk (a), sphere (b), and ellipsoid (c): 1) high-temperature gases; 2) water droplet.



Fig. 2. Typical images of water droplets (a-c) in the experiments in [19-22].

The problem was solved in a cylindrical coordinate system (Fig. 1). We considered a droplet in the form of a cylinder elongated in the direction of motion (as in [17, 18]). Experiments in [19–22] showed (Fig. 2) that water droplets that move through gas media with moderate velocities (less than 5 m/s) continuously and cyclically change their shape (the time during which the droplet has the form of a sphere or ellipsoid does not exceed 15% of the time of each deformation cycle). It seems expedient to use, in modeling, a less laborious formulation (as compared with those for spheres and ellipsoids) with a droplet of constant shape. The best of the possible variants seems to be a cylinder (Fig. 1a) whose symmetry axis coincides with the direction of the vector of droplet motion. It is always possible to select such dimensions of the cylinder at which its surface area will be equal to the surface area of the droplet.

The system of nonlinear nonstationary partial differential equations has the form  $(0 < t < t_d)$ :

the Poisson equation for a mixture of gases and steam

$$0 < r < R_{\rm L}, \quad 0 < z < Z_{\rm I}; \quad R_{\rm I} < r < R_{\rm L}, \quad Z_{\rm I} < z < Z_{\rm 2}; \quad 0 < r < R_{\rm L}, \quad Z_{\rm 2} < z < Z_{\rm L};$$
$$\frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} = -r\omega; \qquad (1)$$

the vorticity equation for a vapor-gas mixture

$$0 < r < R_{\rm L} \ , \quad 0 < z < Z_1 \ ; \quad R_1 < r < R_{\rm L} \ , \quad Z_1 < z < Z_2 \ ; \quad 0 < r < R_{\rm L} \ , \quad Z_2 < z < Z_{\rm L} \ ;$$

$$\frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial r} + w \frac{\partial \omega}{\partial z} - \frac{\omega u}{r} = v_1 \left[ \frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} + \frac{\partial^2 \omega}{\partial z^2} - \frac{\omega}{r^2} \right] + \varphi g \frac{\partial T_1}{\partial r}; \qquad (2)$$

the energy equations for a mixture of steam, gases, and for a liquid droplet (with account for the absorption of the radiant heat flux energy)

$$0 < r < R_{\rm L}$$
,  $0 < z < Z_1$ ;  $R_1 < r < R_{\rm L}$ ,  $Z_1 < z < Z_2$ ;  $0 < r < R_{\rm L}$ ,  $Z_2 < z < Z_{\rm L}$ ;

$$\frac{\partial T_1}{\partial t} + u \frac{\partial T_1}{\partial r} + w \frac{\partial T_1}{\partial z} = a_1 \left[ \frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r} \frac{\partial T_1}{\partial r} + \frac{\partial^2 T_1}{\partial z^2} \right],\tag{3}$$

$$0 < r < R_1$$
,  $Z_1 < z < Z_2$ ;

$$C_2 \rho_2 \frac{\partial T_2}{\partial t} = \lambda_2 \left[ \frac{\partial^2 T_2}{\partial r^2} + \frac{1}{r} \frac{\partial T_2}{\partial r} + \frac{\partial^2 T_2}{\partial z^2} \right] + \left[ \frac{\partial H(r)}{\partial r} + \frac{\partial H(z)}{\partial z} \right], \tag{4}$$

$$H(r) = H_{rd} \exp \left[-\chi(R_1 - r)\right], \quad 0 < r < R_1, \quad Z_1 < z < Z_2$$

$$at \ H_{rd} = \varepsilon \sigma \left(T_{3s}^4 - T_{2s}^4\right), \quad r = R_1, \quad Z_1 < z < Z_2;$$
(5)

$$H(z) = H_{zd1} \exp \left[ -\chi \left( z - Z_1 \right) \right], \quad 0 < r < R_1 , \quad Z_1 < z < Z_1 + 0.5(Z_2 - Z_1)$$
  
at  $H_{zd1} = \varepsilon \sigma \left( T_{3s}^4 - T_{2s}^4 \right), \quad z = Z_1 , \quad 0 < r < R_1 ;$   
(6)

$$H(z) = H_{zd2} \exp\left[-\chi \left(Z_2 - z\right)\right], \quad 0 < r < R_1, \quad Z_1 + 0.5(Z_2 - Z_1) \le z < Z_2$$
  
at  $H_{zd2} = \varepsilon \sigma \left(T_{3s}^4 - T_{2s}^4\right), \quad z = Z_2, \quad 0 < r < R_1;$ 
(7)

the equation of steam diffusion

$$\frac{\partial \gamma_{w}}{\partial t} + u \frac{\partial \gamma_{w}}{\partial r} + w \frac{\partial \gamma_{w}}{\partial z} = D_{3} \left[ \frac{\partial^{2} \gamma_{w}}{\partial r^{2}} + \frac{1}{r} \frac{\partial \gamma_{w}}{\partial r} + \frac{\partial^{2} \gamma_{w}}{\partial z^{2}} \right];$$
(8)

the equation of vapor-gas (binary) mixture balance

$$\gamma_{\rm f} + \gamma_{\rm w} = 1 \,. \tag{9}$$

The initial conditions (t = 0) are

$$T = T_0 \text{ at } 0 < r < R_1 , \quad Z_1 < z < Z_2 ; \quad T = T_f , \quad \gamma_f = 1 , \quad \gamma_w = 0 , \quad \psi = 0 , \quad \omega = 0$$

$$\text{at } 0 < r < R_L , \quad 0 < z < Z_1 ; \quad R_1 < r < R_L , \quad Z_1 < z < Z_2 ; \quad 0 < r < R_L , \quad Z_2 < z < Z_L .$$

$$(10)$$

The boundary conditions at  $0 \le t \le t_d$  are

$$r = 0, \quad 0 < z < Z_1, \quad Z_2 < z < Z_L : \quad \frac{\partial T_1}{\partial r} = 0, \quad \frac{\partial \gamma_w}{\partial r} = 0, \quad \frac{\partial \psi}{\partial r} = 0;$$
 (11)

$$r = 0, \quad Z_1 < z < Z_2 : \quad \frac{\partial T_2}{\partial r} = 0 ;$$
 (12)

$$r = R_{1} , \quad Z_{1} < z < Z_{2} : \quad T_{1} = T_{2} , \quad -\lambda_{1} \frac{\partial T_{1}}{\partial r} = -\lambda_{2} \frac{\partial T_{2}}{\partial r} - Q_{e}W_{e} + \varepsilon\sigma(T_{3s}^{4} - T_{2s}^{4}) ,$$
$$-\rho_{3}D_{3} \frac{\partial \gamma_{w}}{\partial r} = W_{e} , \quad \frac{W_{e}}{\rho_{3}} = -\frac{1}{r} \frac{\partial \Psi}{\partial r} ; \qquad (13)$$

$$r = R_{\rm L}$$
,  $0 < z < Z_{\rm L}$ :  $\frac{\partial T_1^2}{\partial r} = 0$ ,  $\frac{\partial \gamma_{\rm w}^2}{\partial r} = 0$ ,  $\frac{\partial \psi}{\partial r} = 0$ ; (14)

$$z = 0$$
,  $z = Z_{\rm L}$ ,  $0 < r < R_{\rm L}$ :  $\frac{\partial T_1^2}{\partial z} = 0$ ,  $\frac{\partial \gamma_{\rm w}^2}{\partial z} = 0$ ,  $\frac{\partial \psi}{\partial z} = 0$ ; (15)

$$z = Z_{1}, \quad 0 < r < R_{1}: \quad T_{1} = T_{2}, \quad -\lambda_{1} \frac{\partial T_{1}}{\partial z} = -\lambda_{2} \frac{\partial T_{2}}{\partial z} - Q_{e}W_{e} + \varepsilon\sigma(T_{3s}^{4} - T_{2s}^{4}),$$
$$-\rho_{3}D_{3} \frac{\partial \gamma_{w}}{\partial z} = W_{e}, \quad \frac{W_{e}}{\rho_{3}} = \frac{1}{r} \frac{\partial \Psi}{\partial z}; \qquad (16)$$

$$z = Z_2 , \quad 0 < r < R_1 : \quad T_1 = T_2 , \quad -\lambda_1 \frac{\partial T_1}{\partial z} = -\lambda_2 \frac{\partial T_2}{\partial z} - Q_e W_e + \varepsilon \sigma (T_{3s}^4 - T_{2s}^4) ,$$
$$-\rho_3 D_3 \frac{\partial \gamma_W}{\partial z} = W_e , \quad \frac{W_e}{\rho_3} = \frac{1}{r} \frac{\partial \Psi}{\partial z} . \tag{17}$$

It should be noted that at the "liquid droplet-high-temperature gaseous medium" interface, account is made of radiant heat transfer, phase change energy absorption, and expenditure of the gas energy on heating the droplet.

In assessing the boundary conditions for the vorticity equation (2), the Woods formula [23] was used:

$$\begin{split} \omega_{0,j} &= -\frac{1}{r} \left. \frac{\partial^2 \psi}{\partial r^2} \right|_{0,j} + \frac{1}{r^2} \left. \frac{\partial \psi}{\partial r} \right|_{0,j} = \frac{-8\psi_{1,j} + \psi_{2,j} + 7\psi_{0,j}}{2r_0 h_r^2} + \left( \frac{3}{r_0 h_r} + \frac{1}{r_0^2} \right) \frac{\partial \psi}{\partial r} \right|_{0,j}, \\ \omega_{i,0} &= -\frac{1}{r_i} \left. \frac{\partial^2 \psi}{\partial z^2} \right|_{i,0} = \frac{-8\psi_{i,1} + \psi_{i,2} + 7\psi_{i,0}}{2r_i h_z^2} + \frac{3}{r_i h_z} \left. \frac{\partial \psi}{\partial z} \right|_{i,0}, \end{split}$$

where i, j are the numbers of steps over the coordinates r and z;  $h_r$  and  $h_z$  are the steps over the coordinates r and z, m. The components of the vapor–gas mixture velocity vector u and w were determined by the formulas [23]

$$u = \frac{1}{r} \frac{\partial \psi}{\partial z}$$
,  $w = -\frac{1}{r} \frac{\partial \psi}{\partial r}$ ,  $\omega = \frac{\partial w}{\partial r} - \frac{\partial u}{\partial z}$ .

The rate of water evaporation was calculated from the expression [16]

$$W_{\rm e} = \frac{\beta}{1 - k_{\beta}\beta} \frac{P^n - P}{\sqrt{2\pi RT_{\rm e}/M}}$$

The velocity of blowing (flowoff) of vapors from the droplet surface was calculated (as in [17, 18]) by the formula

$$V_{\rm e} = W_{\rm e}/\rho_3$$
 .

According to the notions expressed in [24–33] and generalized in [11], the equation of droplet motion in the conditions of vaporization with account for the action of the forces of resistance, gravity, and others has the following form:

$$\frac{dV_{\rm d}}{dt} = \frac{3\rho_3}{4\rho_2 2R_{\rm d}} c_{\chi} \left| V_{\rm d} - V_{\rm e} \right| \left( V_{\rm d} - V_{\rm e} \right) + g \, ,$$

where  $V_d(0) = V_0$ .

The dimensionless resistance coefficient  $c_{\chi}$ , which generally depends on the surface configuration of the body and its position relative to the flow motion, as well as on a group of other factors, was determined according to the theory outlined in [27–33]. Thus, the resistance coefficient  $c_{\chi}$  vs. the Reynolds number Re dependence, which characterizes the influence of the velocity of droplet motion  $V_d$  and of the size  $R_d$  on the resistance force, when used to approximate the Shiller–Neuman and Oseen correlation curves for the possible range of Re numbers in the system considered (Fig. 1), has the form [11]

$$c_{\gamma} = 24.3 \text{Re}^{-0.635}$$

To determine the resistance coefficient with account for the droplet nonsphericity, its nonstationary motion, evaporation, and for the convective flows inside of the droplet immersed in a gas flow, the following expression [11] was used:

$$c_{\chi} = 24.3k_{\rm g} \frac{1}{B+1} \frac{1+\frac{2}{3}\frac{\mu_2}{\mu_3}}{1+\frac{\mu_2}{\mu_3}} (A+1)^{1.2\pm0.03} \text{ Re}^{-0.635}$$

The geometric coefficient  $k_g$  characterizes the deviation of the shape of the droplet immersed in a gas flow from a sphere [29]. The ratio 1/(B + 1) represents a coefficient that describes the influence of the process of droplet evaporation on the resistance force  $(B = C_2(T_{3s} - T_{2s})/Q_e + q_i/W_e$ , where  $q_i$  is the heat flux to the droplet, W/m<sup>2</sup>) [30]. The expression  $\left(1 + \frac{2}{3} \frac{\mu_2}{\mu_3}\right) / \left(1 + \frac{\mu_2}{\mu_3}\right)$  is a coefficient that accounts for the possible convective flows inside of the droplet [31]. The expression  $(A + 1)^{1.2\pm0.03}$  characterizes the accelerated motion of the droplet  $(A = \frac{d}{V_d^2} \left(\frac{dV_d}{dt}\right)$  is a dimensionless complex that describes the relative acceleration [32]).

**Methods of Solution.** The system of nonlinear nonstationary partial differential equations (1)–(17) was solved by the finite-difference method [33], the difference analogs of the initial differential equations were solved by the methods of locally one-dimensional and variable directions, and the nonlinear equations, by the method of iterations. The pivot method with the use of an implicit four-point difference scheme was employed for solving the one-dimensional difference equations [33].

The adequacy of the model (1)–(17) and of the results of numerical simulation was estimated while checking the conservativeness of the employed difference schemes (the algorithm is analogous to that used in [12]]).

**Results and Discussion.** Numerical investigations were carried out at typical values of the parameters: initial temperatures of the water droplet  $T_0 = 300$  K and of gases  $T_f = 500-1200$  K; heat of water evaporation  $Q_e = 2.26$  MJ/kg; dimensions of the solution domain  $R_L = 10$  mm and  $Z_L = 10$  m; molar mass of water M = 18 kg/kmole; dimensionless coefficient of evaporation  $\beta = 0.1$ ; reduced emissivity of the droplet  $\varepsilon = 0.85$ ; absorption coefficient of water  $\chi = 0.7$ ; initial dimensions of the droplet  $R_d = 0.025-0.25$  mm and  $Z_d = 0.1-1$  mm, rate of motion  $V_0 = 0.5$  m/s. The thermophysical characteristics of water, steam, gases, as well as the vapor–gas mixture corresponded to the formulations made in [17, 18].

Using the results of the numerical simulation, we calculated the temperatures and concentrations of the gases and vapors in the small vicinity of liquid droplets, the rates of evaporation, as well as the lifetimes of droplets. Thus, for example, Fig. 3 presets the isotherms and lines of constant concentrations of steam for one of the typical variants of the heating conditions considered. It is seen that the temperature gradients are at their maximum near the front boundary of the droplet





TABLE 1. Dependence of the Evaporation Rate of Water Droplets on Their Surface Temperature

T <sub>e</sub> , K	300	310	320	330	340	350	360	370
$W_{\rm e}, {\rm kg}/({\rm m}^2 \cdot {\rm s})$	0.005	0.009	0.025	0.042	0.078	0.136	0.204	0.261

(Fig. 3a). Near the side surface the temperature of the gases decreases substantially relative to the front "water-external medium" interface. The temperature of the gases in the droplet wake is still smaller. Such a character of its distribution in the vicinity of the "cold" (relative to the environment) droplet is due not so much to the motion of the latter as to the endothermal phase transformations. The rates of droplet evaporation are maximal on the front and side surfaces. Due to the motion of the droplet, the injected vapors are shifted to the droplet wake and displace the high-temperature gases. The temperature of the droplet wake decreases, and therefore the rates of evaporation on the rear boundary of the droplet are minimal.

Table 1 lists the established values of the rates of water evaporation in the considered conditions depending on the surface temperature of droplets  $T_e$ . The nonlinear character of the dependence  $W_e = f(T_e)$  should be noted. It has also been established that due to the significantly differing conditions of heat and mass transfer near the front, side, and rear boundaries of the droplet (Fig. 3), the temperatures  $T_e$  differ rather substantially (by 25–45 K). As a consequence, the rates of evaporation on these boundaries at identical values of the gas temperatures may change multiple times (Table 1). Moreover, as a result of numerical simulation, it has been revealed that the rates of droplet evaporation vary in time because of the heating of the droplet and reduction of its size.



Fig. 4. Temperature distribution in a water droplet at the initial dimensions of the droplet  $R_d = 0.25$  mm and  $Z_d = 1$  mm and at the temperature of gases at different instants of time.

It is of interest to analyze the studied process (Fig. 1) under conditions of convective-conductive heat exchange at the "liquid–gas" interface [14]. There is a group of correlational expressions for heat transfer coefficients on the surface of an evaporating droplet [12, 14, 34, 35]. The most common are Ranz–Marshall's relations [12], in particular, the expression

$$Nu = 2 + 0.6Re_1^{1/2}Pr_1^{1/3}.$$
 (18)

For the considered, rather typical conditions of the process realization, the maximum possible value of the Nusselt number in accordance with expression (18) is 25. The maximum values of the heat transfer coefficient  $\alpha$  reach in this case 100. The values of the water evaporation rate within the framework of model [14] can be found from the expression

$$\chi_{\rm w} Q_{\rm e}/S_{\rm e} = \alpha (T_{\rm f} - T_{\rm e}) , \qquad (19)$$

where  $\chi_w$  is the mass flow, kg/s, and  $S_e$  is the evaporation area, m<sup>2</sup>. In Eq. (19) the ratio  $\chi_w/S_e$  represents the evaporation rate  $W_e$ . At  $T_f = 1100$  K,  $T_e = 300$ ,  $Q_e = 2.26 \cdot 10^6$  J/kg and at the heat transfer coefficient  $\alpha = 100$  W/(m<sup>2</sup>·K), the evaporation rate is  $W_e = 0.045$  kg/(m<sup>2</sup>·s).

It is important to point out that the well-known refinements [34, 35] of the expression for the Nusselt number (18) that characterize the conditions of intensive blowing of vapors from the droplet surface in the form of  $Nu = (2 + 0.6 \text{ Re}_1^{1/2} \text{ Pr}_1^{1/3})/((1 + b))$  at b = 1/Ku (Ku is the Kutateladze number) for the considered conditions of heat and mass transfer process realization lead to an insignificant (less than 5%) change in  $W_e$  relative to the above-given maximum value.

Within the framework of the model (1)–(17) ("kinetic" according to [15, 16]), several times larger values of  $W_e$  were established in identical conditions (Table 1). This seems to be due to the more intense regime of heat transfer realizable in vaporization at high temperatures of gases. Correspondingly, the application of the expressions of the form Nu = 2 + f(Re, Pr) does not allow one to describe, in full measure, the conditions of heat supply to the interface. The established high concentrations of steam in the vicinity of droplets (Fig. 3b) also illustrate the absence of conditions for the realization of the "diffusional" regime of evaporation.

Figure 4 presents the temperature distributions in an evaporating water droplet at different instants of time. It is seen that the water temperature (and also the surface temperature  $T_e$ ) increases significantly with time due to its heating. An analysis of Fig. 4 and Table 1 shows that the process of heat transfer in a droplet exerts a great influence on the liquid evaporation rate  $W_e = f(T_e)$ . The change in the temperature inside the droplet and on its surface (Fig. 4) is not taken into account in Eqs. (18) and (19) — the droplet is assumed to be nonconducting.



Fig. 5. Rate of water droplet evaporation ( $R_d = 0.25 \text{ mm}$ ,  $Z_d = 1 \text{ mm}$ ) on its front surface at t = 0.25 s vs. the external gas medium temperature: 1) "diffusional" regime of evaporation and convective-conductive heat transfer on the droplet surface; 2) "kinetic" regime of evaporation, radiative and conductive heat transfer on the "liquid–gas" interface; 3) "kinetic" regime of vaporization, combined heat transfer on the "liquid–gas" interface, radiative and conductive heat transfer in the droplet; 4) experimental values of [19–22]; 5) experimental values of [14]

The numerical investigations carried out made it possible to establish that the radiant heat flux to the droplet surface  $q_1$  substantially exceeds the convective flux  $q_c$ . Thus, for example, at a temperature of gases of about 1100 K,  $q_1$  exceeds  $q_c$  by more than seven times. On decrease in the difference between the temperatures of gases and droplet surface, the differences between  $q_1$  and  $q_c$  decrease. At the same time, at a gas temperature of about 500 K, the relation  $q_1/q_c > 1$  holds. These results allow the conclusion on the determining role of radiative heat transfer on the droplet surface at high environmental temperatures. Correspondingly, a conclusion can be drawn on the limitations in using the convective models of heat transfer under the conditions of evaporation of liquid droplets [12, 14, 34, 35].

Figure 5 presents the dependences of the rates of water evaporation on the front boundary of the droplet at a fixed moment on the temperature of the external gaseous medium. These dependences were obtained with the use of three models: 1) "diffusional" regime of evaporation and convective-conductive heat transfer on the droplet surface; 2) "kinetic" regime of evaporative, radiative, and conductive heat transfer on the "liquid–gas" interface; 3) a model analogous to that under point 2 but accounting for the radiative and conductive heat transfer in a droplet. Substantial differences in the rates of water evaporation in the three different interpretations of heat transfer conditions are seen. Moreover, the deviations increase nonlinearly with increase in the gas medium temperature (for the reasons indicated above).

To estimate the validity of the results of numerical simulation, the conservativeness of the difference schemes used was checked as well as the integral characteristics of water evaporation were compared with the experimental data of [19–22] (high-speed cross-correlating cameras and optical methods of "tracing particles" visualization were used). The experiments in [19–22] were carried out with water droplets (the initial dimensions were from 0.5 mm to 5 mm) that moved through combustion products with a temperature of  $1100 \pm 30$  K (a cylindrical channel of length 1 m). Using the results of measurement of the mass and size of droplets at the inlet and outlet of the high-temperature channel, the authors of [19–22] established that the mean rate of water evaporation was about 0.241-0.278 kg/(m<sup>2</sup>·s) (Fig. 5). The numerical investigations carried out have shown that the temperatures of the droplet surfaces  $T_e$  at  $T_f \approx 1100$  K may be equal to 350–370 K (with increase in the time of their motion in a gaseous medium, the temperature  $T_e$  increases nonlinearly). According to Table 1, the rate of evaporation at such temperatures of the droplet surfaces varies from 0.136 to 0.261 kg/(m<sup>2</sup>·s). A conclusion can be drawn on the satisfactory correspondence of the results of mathematical simulation within the scope of the "kinetic" model of evaporation [15, 16] to the experimental data of [19–22]. At low temperatures of the gaseous medium (about 300 K) a good correlation between the results of experiments of [14] and the "kinetic" and diffusional" models of evaporation can be noted (Fig. 5).

It seems advisable to use the terms "kinetic" and "diffusional" regimes of evaporation to emphasize the established physical characteristic feature of phase transformations at the "liquid droplet–high-temperature gases" interface. It is possible to draw an analogy, for example, with the theory of heat shielding [36], when oxidation of carbon-containing materials is realized in two regimes: "kinetic" (the oxidation rate is determined by the surface temperature) and "diffusional" (the process of oxidation is limited by diffusion.) It is shown in the present work that during evaporation of droplets at high temperature, see the regime is implemented that can be called "kinetic" (the rate of evaporation depends exponentially on temperature, see Table 1). In the "kinetic" regime of evaporation the outflow of vapors from the vaporization surface at its temperatures of about 340 K and above occurs with velocities  $V_e$  equal to tens of centimeters per second.

A comparison of the characteristic times of the "diffusional"  $(t_{dif} \approx (2R_d)^2/D_3)$  and "kinetic"  $(t_{kin} \approx 2R_d/V_e)$  processes of water evaporation shows that  $t_{dif} \gg t_{kin}$ . For example, when the droplet is of size  $R_d = 1$  mm, we obtain  $t_{dif} \approx 2$  s and  $t_{kin} \approx 0.001$  s. It may be justifiably concluded that at droplet surface temperatures of above 340 K the "diffusional" regime of evaporation is improbable.

The numerical investigations made it possible to establish the dependence of the time of full evaporation of water droplets  $t_d$  on their dimensions and the temperature of gases. For example, at the initial values of  $R_d = 0.1 \text{ mm}$ ,  $Z_d = 0.4 \text{ mm}$ , and  $T_f = 500 \text{ K}$ , the time  $t_d$  was equal to 1.7 s. When the initial temperature of gases increased up to 1200 K, the value of  $t_d$  reached 0.441 s. After the increase of the initial dimensions to  $R_d = 1 \text{ mm}$  and  $Z_d = 4 \text{ mm}$  and at a constant temperature of gases, the value of  $t_d$  increased up to 4.124 s. For the initial conditions  $R_d = 1 \text{ mm}$ ,  $Z_d = 4 \text{ mm}$ , and  $T_f = 500 \text{ K}$ , the time of complete evaporation of droplets was equal to about 17 s.

The nonlinear character of the functions  $t_d = f(T_f)$  and  $t_d = f(Z_d)$  has been established, which is due to the corresponding dependence of the rate of evaporation  $W_e$  on the droplet surface temperature (Table 1) in the case of the kinetic" regime of vaporization [15, 16]. Emphasis should also be laid on the practically comparable effect of the size of liquid droplets and of the temperature of gases on the characteristic of high-temperature evaporation, i.e., on the lifetimes of droplets. The established feature characterizes the nonuniform heating of droplets and the determining effect of this process on the times of complete evaporation.

The results of the performed numerical simulation in the system "water droplet-high-temperature gases" can be used in developing a large group of heat power engineering technologies (in a wide range of temperatures). They can be applied for prognostic estimation of the needed parameters of heat transfer agents [3, 4], polydisperse gas- and vapor-liquid fireextinguishing droplet flows [6], as well as high-temperature gas-droplet mixtures for treating the surfaces of structures and materials [10].

**Conclusions.** An analysis of the results of the theoretical investigations carried out allows the conclusion that the "diffusional" models of phase transformations [11–14] in the system "liquid droplet–gaseous medium" correspond well to the results of experimental investigations only in limited temperature ranges. This is due to the different regimes of evaporation at relatively moderate (arbitrarily up to 500 K) and high (above 500 K) temperatures. In particular, it has been established that the characteristics of evaporation of water droplets obtained with the use of the approach that presupposes the realization of the "diffusional" regime of vaporization differ substantially from the experimental values obtained at high temperatures (at about 1000 K). At such temperatures, it is worthwhile to use the "kinetic" model of evaporation [15, 16]. In this case, it is necessary to account for the conductive and radiative heat transfer in a droplet, as well as the combined heat transfer at the "liquid–gas" interface.

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### NOTATION

A, dimensionless complex characterizing the relative acceleration of droplets; *a*, thermal diffusivity, m<sup>2</sup>/s; *B*, Spalding number; *C*, specific heat, J/(kg·K); *D*, diffusion coefficient, m<sup>2</sup>/s; *g*, free fall acceleration, m/s<sup>2</sup>; *H*, radiation flux density, W/m<sup>2</sup>;  $k_g$ , dimensionless geometric coefficient accounting for the nonsphericity of droplets;  $k_\beta$ , dimensionless empirical constant; *M*, molar mass, kg/mole; *P*, pressure of steam near the evaporation boundary, N/m<sup>2</sup>; *P<sup>n</sup>*, pressure of saturated steam, N/m<sup>2</sup>; *Q<sub>e</sub>*, heat of water evaporation, J/kg; *R<sub>L</sub>*, *Z<sub>L</sub>*, dimensions of the solution domain, m; *R*, universal gas constant, J/(mole·K); *r*, *z*, coordinates of a cylindrical system, m; *T*, temperature, K; *T<sub>2s</sub>*, *T<sub>3s</sub>*, temperature of water and steam at the "liquid–gas" interface, K; *T<sub>e</sub>*, temperature of the droplet surface, K; *t*, time, s; *t<sub>d</sub>*, lifetime of droplet, time of its complete evaporation, s; *u*, *w*, velocity components of gases and steam, m/s; *V<sub>e</sub>*, velocity of the flowoff of vapors from the droplet

surface, m/s;  $W_e$ , vaporization rate, kg/(m<sup>2</sup>·s);  $\beta$ , dimensionless coefficient of evaporation (condensation);  $\gamma_f$ , dimensionless concentration of gases;  $\gamma_w$ , dimensionless concentration of steam;  $\varepsilon$ , reduced emissivity;  $\lambda$ , thermal conductivity, W/(m·K);  $\mu$ , dynamic viscosity, kg/(m·s);  $\nu$  kinematic viscosity, m<sup>2</sup>/s;  $\rho$ , density, kg/m<sup>3</sup>;  $\sigma$ , Stefan–Boltzmann constant, W/(m<sup>4</sup>·K<sup>4</sup>);  $\varphi$ , coefficient of thermal expansion, K<sup>-1</sup>;  $\chi$ , dimensionless coefficient of radiation absorption;  $\psi$ , stream function, m<sup>3</sup>/s;  $\omega$ , vorticity function, s<sup>-1</sup>. Indices: 1, mixture of gases and stream; 2, water droplet; 3, steam; g, geometric; e, evaporation; L, location; s, at the "liquid–gas" interface (surface); d, droplet; f, concentration of high-temperature gases (furnace).

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