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PROPAGATION OF THE RAYLEIGH WAVE IN AN INITIALLY STRESSED TRANSVERSELY ISOTROPIC DUAL-PHASE-LAG MAGNETOTHERMOELASTIC HALF-SPACE

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The basic equations for the wave on the surface of an initially stressed transversely isotropic dual-phase-lag thermoelastic body subjected to the action of a magnetic field were solved. Particular solutions were applied to the thermally insulated free surface of a half-space to obtain the frequency equation for the Rayleigh wave. This equation was approximated for calculating the numerical values of the dimensionless velocity of the Rayleigh wave. The Rayleigh-wave velocity was represented graphically for the cases of coupled, Lord–Shulman, and dual-phase-lag thermoelasticities. The influence of the dual-phase-lag and the initial stress of a body, the value of the magnetic field, and the frequency of the Rayleigh wave on the velocity of this wave were determined.

Keywords: *dual-phase-lag thermoelasticity, Lord–Shulman theory, Rayleigh wave, frequency equation.*

Introduction. The classical theory of dynamic coupled thermoelasticity was developed by M. A. Biot [1]. H. Lord and Y. Shulman [2], as well as A. E. Green and K. A. Lindsay [3], generalized this theory to include hyperbolic field equations defining heat as a wave. Unlike the Biot theory, the generalized thermoelasticity theory predicts a finite velocity of propagation of heat in a medium. J. Ignaczak and M. Ostoja-Starzewski investigated this phenomenon in detail [4]. R. B. Hetnarski and J. Ignaczak [5] considered generalized thermoelasticity effects. The effect of wave propagation is used in engineering as well as in geophysics, mineral and oil exploration, seismology, and other fields. Various problems on the plane-wave propagation, in the theory of coupled thermoelasticity and the generalized thermoelasticity theory were considered by H. Deresiewicz [6], A. N. Sinha and S. B. Sinha [7], M. I. A. Othman and Y. Song [8], B. Singh [9, 10], and many other authors. D. Y. Tzou [11–13] developed a thermoelastic model of dual-phase lag (DPL), in which the interactions between phonons and electrons on the microscopic level are considered as retarding sources causing a delayed response on the macroscopic level. The DPL model is a modification of the classical thermoelastic model. In it, the Fourier law is replaced by the modified Fourier law with two different time translations: the phase lags of a heat flow and of the temperature gradient. This model is used for investigating the microstructural effect in the process of heat transfer. Recently, A. E. Abouelregal [14] investigated, using the dual-phase-lag model, the propagation of the Rayleigh wave on the surface of an isotropic thermoelastic solid half-space.

The propagation of a wave on the surface of an initially stressed anisotropic, thermoelastic DPL half-space subjected to the action of a magnetic field has not been investigated as yet. In the present work, the anisotropic theory of DPL thermoelasticity was used to investigate the propagation of a Rayleigh wave in a stressed transversely anisotropic, magnetic, thermoelastic solid half-space. The frequency equations for the Rayliegh waves in a thermally insulated space and in an isothermal one have been derived. A numerical example is considered to demonstrate the dependence of the velocity of propagation of the Rayleigh wave in a half-space on the frequency of this wave, the value of the magnetic field acting on the half-space, and its initial stress in the cases of coupled, Lord–Shulman, and dual-phase-lag thermoelasticities.

Basic Equations. The basic equations for a pre-stressed, anisotropic, thermoelastic DPL body subjected to the action of a magnetic field **H** are as follows [11–13, 15]:

the equation of motion

$$\rho \ddot{u}_i = \sigma_{ji,j} + \rho F_i \ , \tag{1}$$

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the strain–stress–temperature relation

$$\sigma_{ij} = d_{ijkl}e_{kl} - \beta_{ij}T \ , \tag{2}$$

the strain-displacement relation

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{3}$$

the energy equation

$$-q_{i,i} = \rho T_0 \dot{S} \ , \tag{4}$$

the modified Fourier law

$$-K_{ij}(T_j + \tau_\theta \dot{T}_j) = q_i + \tau_q \dot{q}_i \ , \tag{5}$$

the entropy–strain–temperature relation

$$\rho S = \frac{\rho c_E}{T_0} T + \beta_{ij}e_{ij} \ , \tag{6}$$

the basic equations for the Maxwell electromagnetic field

$$\nabla \cdot \mathbf{h} = \mathbf{j}, \quad \nabla \cdot \mathbf{E} = -\mu_{\mathrm{e}} \frac{\partial \mathbf{h}}{\partial t}, \quad \nabla \cdot \mathbf{h} = 0, \quad \nabla \cdot \mathbf{E} = 0 \ , \tag{7}$$

the equation for the Maxwell stress

$$\bar{\sigma}_{ij} = \mu_{\mathrm{e}}[H_i h_j + H_j h_i - (\mathbf{H}\,\mathbf{h})\delta_{ij}] \ . \tag{8}$$

The DPL theory of thermoelasticity reduces to the coupled thermoelasticity theory in the case where $\tau_\theta = 0$, $\tau_q = 0$, and to the Lord–Shulman generalized thermoelasticity theory when τ_q is replaced by τ_θ $(\tau_\theta = 0)$. Let us assume that $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$ and the perturbed magnetic field $\mathbf{h}$ is so small that the product of $\mathbf{h}$ by $\mathbf{u}$ and their derivatives can be disregarded in the linearization of the field equations. Using Eqs. (1) and (2) and disregarding the forces acting on the body, we obtain

$$\rho \ddot{u}_i = (d_{ijkl}e_{kl} - \beta_{ij}\Theta)_j + (\mathbf{J}\,\mathbf{B})_i \ , \tag{9}$$

where $d_{ijkl} = c_{ijkl} + \delta_{kl}P_{ij}$. Substituting Eqs. (5) and (6) into Eq. (4) and disregarding the heat sources, we obtain

$$\left(1 + \tau_\theta \frac{\partial}{\partial t}\right) K_{ij}\Theta_{ij} = \left(1 + \tau_q \frac{\partial}{\partial t}\right)\left(\rho c_E \dot{\Theta} + T_0 \beta_{ij}\dot{e}_{ij}\right) , \tag{10}$$

where T_0 is the reference uniform temperature of the body, determined from the ratio $\left|\frac{T}{T_0}\right| << 1$.

A transversely isotropic homogenous isotropic infinite medium with an initially uniform temperature distribution is considered in the Cartesian coordinates system (x, y, z). The origin of the coordinate system is positioned on a plane surface, and the z axis is normal to this surface $(z \geq 0)$. It is assumed that the surface $z = 0$ is free of stress and thermally insulated or isothermal. We shall restrict our consideration to the case where the medium is stressed along the x–z plane with a displacement vector $\mathbf{u} = (u_1, 0, u_3)$ and a constant-magnetic-field vector $\mathbf{H}_0 = (0, H_0, 0)$. Using Eqs. (9) and (10), we obtain the following equations of motion and heat conduction:

$$d_{11}u_{1,11} + (d_{13} + d_{44})u_{3,13} + d_{44}u_{1,33} - \beta_1\Theta_1 = \rho\ddot{u}_i - \mu_{\mathrm{e}}H_0^2\left[\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_3}{\partial x \partial z}\right], \tag{11}$$

$$d_{44}u_{3,11} + (d_{13} + d_{44})u_{1,13} + d_{33}u_{3,33} - \beta_3\Theta_3 = \rho\ddot{u}_3 - \mu_e H_0^2\left[\frac{\partial^2 u_1}{\partial x \partial z} + \frac{\partial^2 u_3}{\partial z^2}\right], \tag{12}$$

$$\left(1 + \tau_\theta \frac{\partial}{\partial t}\right)\left[K_1\Theta_{11} + K_3\Theta_{33}\right] = \left(1 + \tau_q \frac{\partial}{\partial t}\right)\left[\rho c_E\dot{\Theta} + \beta_1 T_0\dot{u}_{1,1} + \beta_3 T_0\dot{u}_{3,3}\right], \tag{13}$$

where

$$d_{11} = c_{11} + P_{11}, \quad d_{13} = c_{13}, \quad d_{44} = c_{44} + P_{11}, \quad d_{33} = c_{33} + P_{33},$$

$$K_1 = K_{11}, \quad K_3 = K_{33}, \quad \beta_1 = \beta_{11}, \quad \beta_3 = \beta_{33} .$$

Solutions for Surface Waves. The functions u_1, u_3, and Q for thermoelastic waves propagating on the surface of a half-space in the x direction are determined from the relation

$$\{u_1, u_3, \Theta\} = \{\phi_1(z), \phi_3(z), \psi(z)\} \exp\left[\iota k(x - ct)\right] . \tag{14}$$

Substituting Eq. (14) into Eqs. (11)–(13), we obtain the following homogenous system of three equations in terms of ϕ_1, ϕ_3, and ψ:

$$[k^2(\zeta - a_1 - b_1) + D^2]\phi_1 + \iota k(a_2 + b_1 + 1)D\phi_3 - \iota k\psi = 0 , \tag{15}$$

$$\iota k(a_2 + b_1 + 1)D\phi_1 + [k^2(\zeta - 1) + (a_3 + b_1)D^2]\phi_3 - \overline{\beta}D\psi = 0 , \tag{16}$$

$$\iota k^3\varepsilon\zeta\phi_1 + \overline{\beta}\zeta\varepsilon k^2 D\phi_3 + [k^2(\zeta - K_1^*) + K_3^* D^2]\psi = 0 , \tag{17}$$

where

$$\varepsilon = \frac{\beta_1^2 T_0}{\rho^2 c_E c_1{}^2}, \qquad \zeta = \frac{\rho c^2}{d_{44}}, \qquad b_1 = \frac{\mu_e H_0^2}{d_{44}},$$

$$a_1 = \frac{d_{11}}{d_{44}}, \quad a_2 = \frac{d_{13}}{d_{44}}, \quad a_3 = \frac{d_{33}}{d_{44}}, \quad \tau^* = \frac{\tau_q + \dfrac{i}{\omega}}{1 - i\omega\tau_\theta}, \quad K_1^* = \frac{K_1}{c_E d_{44}\tau^*}, \quad K_3^* = \frac{K_3}{c_E d_{44}\tau^*} .$$

For nontrivial solution of Eqs. (15)–(17), it is necessary that

$$D^6 - AD^4 + BD^2 - C = 0 , \tag{18}$$

where

$$A = -k^2\left[(\zeta - a_1 - b_1) + \frac{(\zeta - 1)}{(a_3 + b_1)} + \frac{(\zeta - K_1^*)}{K_3^*} + \frac{(a_2 + b_1 + 1)^2}{(a_3 + b_1)} + \frac{\overline{\beta}^2\varepsilon\zeta}{(a_3 + b_1)K_3^*}\right],$$

$$B = k^4\left[\frac{(\zeta - a_1 - b_1)(\zeta - 1)}{(a_3 + b_1)} + \frac{(\zeta - a_1 - b_1)(\zeta - K_1^*)}{K_3^*} + \frac{(\zeta - 1)(\zeta - K_1^*)}{(a_3 + b_1)K_3^*}\right.$$

$$\left. + \frac{(a_2 + b_1 + 1)^2(\zeta - K_1^*)}{(a_3 + b_1)K_3^*} + \frac{(\zeta - a_1 - b_1)\overline{\beta}^2\varepsilon\zeta}{(a_3 + b_1)K_3^*} + \frac{\varepsilon\zeta(a_2 + b_1 + 1)\overline{\beta}}{(a_3 + b_1)K_3^*} + \frac{\varepsilon\zeta(a_2 + b_1 + 1)\overline{\beta}}{(a_3 + b_1)K_3^*} - \frac{\varepsilon\zeta}{K_3^*}\right],$$

$$C = -k^6\left[\frac{(\zeta - a_1 - b_1)(\zeta - 1)(\zeta - K_1^*)}{(a_3 + b_1)K_3^*} - \frac{\varepsilon\zeta(\zeta - 1)}{(a_3 + b_1)K_3^*}\right].$$

On the above requirement, the most general solutions of Eqs. (15)–(17) are as follows:

$$\phi_1(z) = \left[\sum_{i=1}^{3} A_i e^{-m_i z} + \sum_{i=1}^{3} A_i^* e^{m_i z}\right] e^{\iota k(x-ct)}, \tag{19}$$

$$\phi_3(z) = \left[\sum_{i=1}^{3} B_i e^{-m_i z} + \sum_{i=1}^{3} B_i^* e^{m_i z}\right] e^{\iota k(x-ct)}, \tag{20}$$

$$\psi(z) = \left[\sum_{i=1}^{3} C_i e^{-m_i z} + \sum_{i=1}^{3} C_i^* e^{m_i z}\right] e^{\iota k(x-ct)}, \tag{21}$$

where A_i, B_i, C_i, A_i^*, B_i^*, and C_i^* are constants and m_i are the roots of the equation

$$m^6 - Am^4 + Bm^2 - C = 0\,. \tag{22}$$

Equation (22) is cubic with respect to m^2, and its roots m_1^2, m_2^2, and m_3^2 are related as

$$m_1^2 + m_2^2 + m_3^2 = A, \quad m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2 = B, \quad m_1^2 m_2^2 m_3^2 = C\,. \tag{23}$$

In general, the roots $m_i (i = 1, 2, 3)$ are complex; therefore, for the surface waves, it may be suggested without loss of generality that $\mathrm{Re}(m_i) > 0$. We will use only the form of m_i that satisfies the following radiation condition:

$$\phi_1(z), \quad \phi_3(z), \quad \psi(z) \to 0 \quad \text{as} \quad z \to \infty\,. \tag{24}$$

On this condition, relations (19)–(21) reduce to the following solutions for the half-space $z > 0$:

$$\phi_1(z) = \sum_{i=1}^{3} A_i e^{-m_i z} e^{\iota k(x-ct)}, \tag{25}$$

$$\phi_3(z) = \sum_{i=1}^{3} F_i A_i e^{-m_i z} e^{\iota k(x-ct)}, \tag{26}$$

$$\psi(z) = \sum_{i=1}^{3} F_i^* A_i e^{-m_i z} e^{\iota k(x-ct)}, \tag{27}$$

where,

$$F_i = -\iota\,\frac{m_i}{k}\left[\frac{\bar{\beta}\left(\zeta - a_1 - b_1 + \dfrac{m_i^2}{k^2}\right) + (a_2 + b_1 + 1)}{\bar{\beta}\,\dfrac{m_i^2}{k^2}(a_2 + b_1 + 1) - \left\{\zeta - 1 + (a_3 + b_1)\dfrac{m_i^2}{k^2}\right\}}\right], \quad i = 1, 2, 3\,; \tag{28}$$

$$F_i^* = -\frac{\iota\varepsilon k}{\frac{\beta_1}{d_{44}}}\left[\frac{\bar{\beta}(\zeta - a_1 - b_1) + \frac{m_i^2}{k^2}\bar{\beta} + (a_2 + b_1 + 1)}{\bar{\beta}\varepsilon + (a_2 + b_1 + 1)\left(1 - \frac{K_i^*}{\zeta} + \frac{K_3^*}{\zeta}\frac{m_i^2}{k^2}\right)}\right], \quad i = 1, 2, 3 . \tag{29}$$

Derivation of the Frequency Equation. We set the following mechanical and thermal conditions at the stressed free surface of the body being investigated $z = 0$:

for the normal component of the stress

$$\sigma_{zz} + \bar{\sigma}_{zz} = 0 , \tag{30}$$

for the tangential component of the stress,

$$\sigma_{zx} + \bar{\sigma}_{zx} = 0, \tag{31}$$

for the normal component of the heat flow or the temperature potential

$$\frac{\partial\Theta}{\partial z} + h\Theta = 0 , \tag{32}$$

where $h \to 0$ corresponds to the thermally insulated surface, $h \to \infty$ corresponds to the isothermal surface, and

$$\begin{gathered}\sigma_{zz} = d_{33}u_{3,3} + d_{13}u_{1,1} - \beta_3\Theta, \qquad \sigma_{zx} = d_{44}(u_{1,3} + u_{3,1}) , \\ \bar{\sigma}_{zz} = -\mu_e H_0^2(u_{1,1} + u_{3,3}), \qquad \bar{\sigma}_{zx} = 0 .\end{gathered} \tag{33}$$

The solutions (25)–(27) satisfy the boundary conditions (30)–(32). In the final analysis we obtain the following homogeneous system of three equations in terms of A_1, A_2, and A_3:

$$\sum_{i=1}^{3}\left(d_{33}^* m_i F_i - \iota k d_{13}^* + \beta_3 F_i^*\right) A_i = 0 , \tag{34}$$

$$\sum_{i=1}^{3}\left(m_i - \iota k F_i\right) A_i = 0 , \tag{35}$$

$$\sum_{i=1}^{3} F_i^*(m_i - h)A_i = 0 , \tag{36}$$

where $d_{33}^* = d_{33} - \mu_e H_0^2$ and $d_{13}^* = d_{13} - \mu_e H_0^2$.

The nontrivial solution of Eqs. (34)–(36) yield

$$(m_1 - h)X_1 + (m_2 - h)X_2 + (m_3 - h)X_3 = 0 , \tag{37}$$

where

$$X_1 = F_1^* \left[(d_{33}^* m_2 F_2 - \iota k d_{13}^* + \beta_2 F_2^*)(m_3 - \iota k F_3) - (d_{33}^* m_3 F_3 - \iota k d_{13}^* + \beta_3 F_3^*)(m_2 - \iota k F_2) \right],$$

$$X_2 = F_2^* \left[(d_{33}^* m_3 F_3 - \iota k d_{13}^* + \beta_3 F_3^*)(m_1 - \iota k F_1) - (d_{33}^* m_1 F_1 - \iota k d_{13}^* + \beta_3 F_1^*)(m_3 - \iota k F_3) \right],$$

$$X_3 = F_3^* \left[(d_{33}^* m_1 F_1 - \iota k d_{13}^* + \beta_3 F_1^*)(m_2 - \iota k F_2) - (d_{33}^* m_2 F_2 - \iota k d_{13}^* + \beta_3 F_2^*)(m_1 - \iota k F_1) \right].$$

Equation (37) is the required frequency equation for the Rayleigh wave in a stressed transversely isotropic, thermoelastic DPL half-space subjected to the action of a magnetic field.

Particular Cases. *Thermally insulated surface.* For a thermally insulated surface, we put $h \to 0$ in Eq. (37) and obtain the following frequency equation:

$$m_1 X_1 + m_2 X_2 + m_3 X_3 = 0\,. \tag{38}$$

Isothermal surface. For an isothermal surface, we put $h \to \infty$ in Eq. (37). In this case, Eq. (37) reduces to the relation

$$X_1 + X_2 + X_3 = 0\,. \tag{39}$$

Generalized thermoelasticity. In the case of Lord–Shulman generalized thermoelasticity, τ_θ reduces to 0 and τ_q is considered only.

Coupled thermoelasticity. In the case of coupled thermoelasticity, $\tau_q = \tau_\theta = 0$.

Uncoupled thermoelasticity. In the case of uncoupled thermoelasticity, $\tau_q = \tau_\theta \to 0$ and $\varepsilon = 0$.

Isotropic elasticity. In the case of isotropic elasticity, $P_{11} = P_{33} = 0$, $H_0 = 0$, $c_{11} = c_{33} = \lambda + 2\mu$, $c_{13} = \lambda$, $c_{44} = \mu$, $\beta_1 = \beta_3 = \beta$, and $K_1 = K_3 = K$. In this case, Eq. (37) takes the form

$$(m_1^* - h)X_1^* + (m_2^* - h)X_2^* + (m_3^* - h)X_3^* = 0\,, \tag{40}$$

where m_i^* is obtained from Eq. (23) and X_i^* is determined by X_i. Then, if we put $\beta = 0$, $K = 0$, and $\varepsilon = 0$ in Eq. (40), it is transformed into the relation

$$\left(2 - \frac{c^2}{c_2^2}\right)^2 = 4\sqrt{\left(1 - \frac{c^2}{c_1^2}\right)\left(1 - \frac{c^2}{c_2^2}\right)}\,, \tag{41}$$

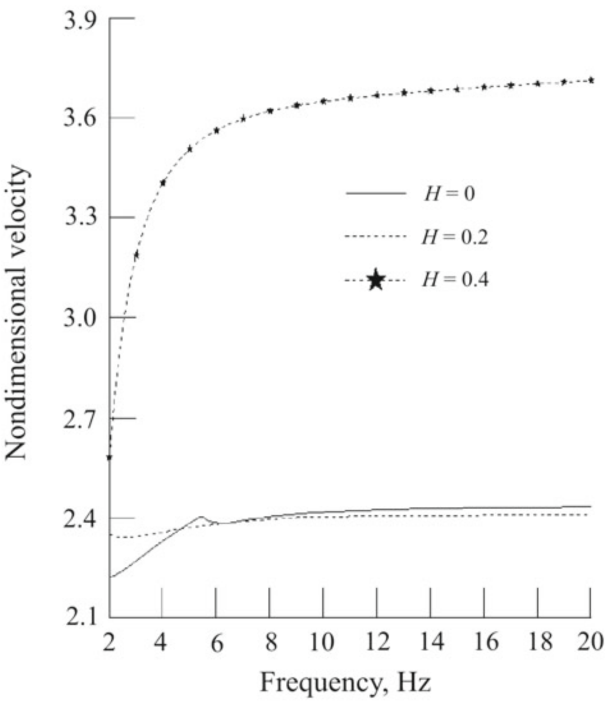
representing the velocity equation for the Rayleigh wave in an isotropic, elastic solid half-space.

Numerical Results and Discussion. For the majority of materials, ε is small at a normal temperature. For $\varepsilon << 1$, Eq. (23) has the following approximate roots:

$$\frac{m_1^2}{k^2} \approx -(\zeta - a_1 - b_1) = \frac{d_{11} - \rho c^2 + \mu_e H_0{}^2}{d_{44}}\,, \tag{42}$$

$$\frac{m_2^2}{k^2} \approx -\frac{(\zeta - 1)}{a_3 + b_1} = \frac{d_{44} - \rho c^2}{d_{33} + \mu_e H_0^2}\,, \tag{43}$$

$$\frac{m_3^2}{k^2} \approx -\frac{(\zeta - K_1^*)}{K_3^*} = \frac{K_1 - \rho c^2 c_E T^*}{K_3}\,. \tag{44}$$



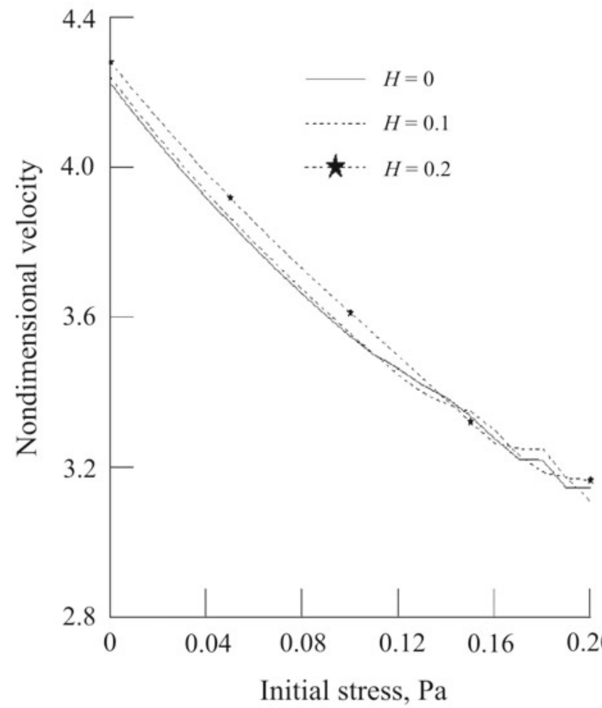


Fig. 1. Dependence of the dimensionless velocity of the Rayleigh wave on its frequency in the DPL case at $P = 0.5$ Pa.

Fig. 2. Dependence of the dimensionless velocity of the Rayleigh wave on the initial stress of the body P in the DPL case at $w = 5$ Hz.

We numerically calculated the nondimensional velocity of propagation of the Rayleigh wave only on the thermally insulated surface of a half-space and approximated Eq. (38) with the use of Eqs. (42)–(44). The velocity of propagation of the Rayleigh wave on this surface was determined for definite ranges of change in its initial stress and in the value of the magnetic field acting on the half-space. Following P. Chadwick and L. T. C. Seet ([16], we used the physical constants of a single zinc crystal for simulation of the half-space: $c_{11} = 1.628 \cdot 10^{11}$ N·m^{-2}, $c_{33} = 1.628$ N·m^{-2}, $c_{13} = 0.508 \cdot 10^{11}$ N·m^{-2}, $c_{44} = 0.385 \cdot 10^{11}$ N·m^{-2}, $\beta_1 = 5.75 \cdot 10^6$ N·m^{-2}·deg^{-1}, $\beta_3 = 5.17 \cdot 10^6$ N·m^{-2}·deg^{-1}, $K_1 = 1.24 \cdot 10^2$ W·m^{-1}·deg^{-1}, $K_3 = 1.34 \cdot 10^2$ W·m^{-1}·deg^{-1}, $C_v = 3.9 \cdot 10^2$ J·kg^{-1}·deg^{-1}, $\rho = 7.14 \cdot 10^3$ kg·m^{-3}, $T_0 = 296$ K, $\tau_q = 0.005$ s, $\tau_\theta = 0.0005$ s, $P_{11} = P_{33} = P$.

In the DPL case, the dimensionless velocity of propagation of the Rayleigh wave ($\rho c/d_{44}$) was determined in the frequency range $4Hz \le \omega \le 20Hz$ at $P = 0.5$ Pa and $H = 0$, 0.2, and 0.4 Oe. Comparison of the solid curve $H = 0$, the dashed curve $H = 0.2$, and the dashed curve $H = 0.4$, presented in Fig. 1, shows the dependence of the frequency of the velocity of the Rayleigh wave on its frequency at different values of the magnetic field. It is seen that, at $H = 0$, the dimensionless velocity of the Rayleigh wave increases sharply with increase in its frequency. However, at higher magnetic-field strengths, this velocity increases more slowly.

The dimensionless velocity of propagation of the Rayleigh wave $(\rho c^2/d_{44})$ was also calculated in the range of change in the initial-stress parameter $0 \le P \le 0.2$ Pa at $\omega = 5$ Hz and $H = 0$, 0.1, and 0.2 Oe. Comparison of the solid and dashed curves in Fig. 2 shows the effect of the initial stress of the body being investigated on the velocity of the Rayleigh wave in it at different values of the magnetic field. For all the values of H, the dimensionless velocity of this wave decreased with increase in P in the indicated range of its values.

The dimensionless velocity of the Rayleigh wave $(\rho c^2/d_{44})$ was calculated for the DPL, Lord–Shulman (LS), and coupled-thermoelasticity (CT) cases in the frequency range 4 Hz $\le \omega \le$ 20 Hz at P = 0.5 Pa and H = 0.4. In each case, the dimensionless velocity of the Rayleigh wave increased with increase in its frequency. Comparison of the solid and dashed

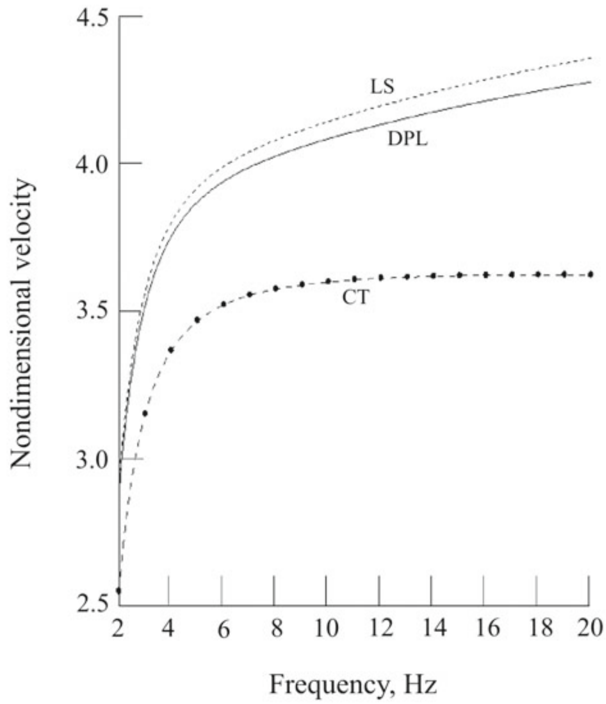


Fig. 3. Effect of the dual-phase-lag on the dimensionless velocity of the Rayleigh wave against its frequency at $P = 0.5$ Pa and $H = 0.4$ Oe.

curves in Fig. 3 shows the effect of the dual-phase-lag on the dimensionless velocity of the Rayleigh wave. It is seen that this effect is enhanced with increase in the Rayleigh-wave frequency.

Conclusions. As a result of our investigations, we determined the influence of the dual- phase-lag and the initial stress of a transversely isotropic magnetothermoelastic body as well as the frequency of the Rayleigh wave on its surface and the value of the magnetic field acting on the body on the dimensionless velocity of propagation of the Rayleigh wave in it. The study of the propagation of such a wave in a pre-stressed thermoviscoelastic porous medium with account for its microstructure and temperature provides vital information useful for seismologists-experimenters in correcting their estimates of earthquakes and for many other applications because pre-stressed materials are used in geophysics, automatics, in the oil, aerospace, and military industries, in the study of biological tissues (lung, tendon, etc.) representing nonlinear pre-stressed viscoelastic composites, and for other purposes.

NOTATION

$\mathbf{B} = \mu_e \mathbf{H}$, magnetic induction vector; $c_{E,}$ specific heat at a constant strain; c_{ijkl}, elasticity tensor; e_{ij}, components of the strain tensor; $F_i = (\mathbf{J}\,\mathbf{B})_i$, components of the vector of the electromagnetic force directed to the body; H, magnetic field; $\mathbf{H}_0$, vector of the constant magnetic field; $\mathbf{h}$, vector of the disturbed magnetic field; $\mathbf{j}$, density vector of the electric current; K_{ij}, components of the thermal conductivity tensor; P, initial stress; P_{ij}, components of the initial stress; q_i, components of the heat conduction vector; S, entropy per unit mass; T, absolute temperature of the body; u_i, components of the displacement vector; β_{ij}, thermal-expansion coefficients; δ_{kl}, Kronecker delta; $\Theta = T - T_0$, small temperature increment; μ_e, magnetic permeability; ρ, mass density; σ_{ij}, components of the stress tensor; τ_q, phase-lag of heat flux; τ_θ, phase-lag of the temperature gradient $(0 \le \tau_\theta \le \tau_q)$; ω, frequency of the Rayleigh wave. Subscripts: e, electromagnetic.

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