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THERMOPHYSICAL PROPERTIES

MODEL OF THE STRUCTURE OF FIBROUS HEAT-INSULATING MATERIALS FOR ANALYZING COMBINED HEAT TRANSFER PROCESSES

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A model of the structure of a highly porous fibrous material is suggested within the framework of which deformation of a setting semifinished item is considered. An algorithm is suggested for calculating the effective thermal conductivity and its components. It accounts for heat transfer through a solid phase and a gas, as well as by radiation. The Mie theory is used to estimate the radiative heat transfer, which led to a somewhat underestimated result in determining the effective thermal conductivity. To refine the contribution of radiative heat transfer, it is suggested to determine the optical properties of materials by solving the inverse problem of radiation transfer, the initial data for which are furnished by experimentally measured values of the coefficients of radiation transmission through a set of samples of different thickness. As a result, the radiation absorption and diffusion coefficients of a fibrous heat-insulating material have been determined. The dependence of the effective thermal conductivity of a material on temperature has been obtained, which actually coincides with the results of experimental investigations.

***Keywords**: fibrous heat-insulating materials, model of a structure, computational determination of thermal conductivity, radiative component of effective thermal conductivity, optical properties, prediction of properties.*

Introduction. The task confronting material-science laboratories is the creation of materials that can most fully meet the requirements of construction designers. For heat-proof materials, of greatest value is their basic operating characteristic, viz., the thermal conductivity. The characteristics of this class of materials in the time-temperature regimes that correspond to operating conditions may differ appreciably from standard ones measured in the process of certification [1]. The optimization of the composition and structure of a material whose characteristics cannot be estimated under real operating conditions is a problem whose solution is often achieved by the trial and error method. Application of computational methods also proves useful.

In creating an optimal, from the viewpoint of the optimization of the thermal conductivity, material structure, it is necessary to describe the processes of combined heat transfer in a wide temperature range. The solution of the problems associated with the analysis of the influence of the structure and properties of components on the characteristics of the entire material is possible using the technique of the theory of generalized thermal conductivity based on the construction of a model of a structure and on calculation of heat transfer in it. In our opinion, this is the most effective means of estimating the influence of the material structure on its resultant characteristics. The advantage of the technique of the generalized thermal conductivity theory [2] is the relative simplicity of the mathematical apparatus with the simultaneous possibility of taking into account the complex processes of heat transfer. Moreover, the use of the models of structure makes it possible to account for the effect of the characteristic features of the technological process of material production. As a result, the coupling between the technological process parameters and the resultant structure of the material can be revealed. Application of computational methods to determine the thermal conductivity of fibrous materials allows one to justifiably formulate the trends in the optimization of characteristics, aids in reduction of the terms and cost of designing flying vehicles, and upgrades the reliability of a heat-shielding system as a whole.

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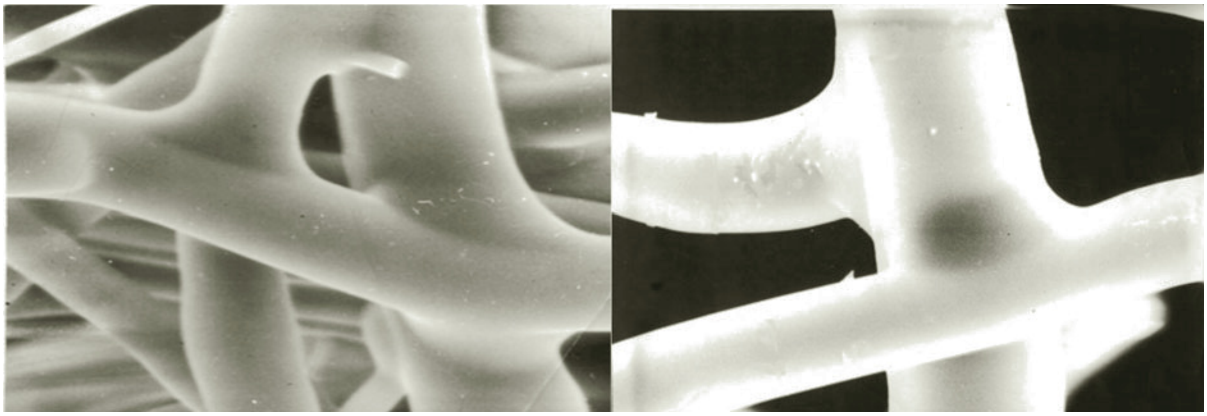


Fig. 1. Contact between fibers [9].

Model of the Structure of a Binder-Containing Highly Porous Fibrous Material. By this time, many models of the structure of a highly porous fibrous material have been devised, mostly based on the principles formulated in [2]. Making no claims to the completeness of this review, we only note part of them [2–8]. Each model has its drawbacks and merits. We will consider some of them.

As a model of the elementary cell of a structure we have selected the variant that was used at one time by M. V. Chebunin and that is based on the fact that in view of the low density of the material the probability of the simultaneous intersection of three fibers at one point is extremely improbable (Fig. 1). This somewhat complicates the calculations but allows one to adopt a more justifiable model of the structure as compared to the most known one [2, 4, 6, 7] in which all fibers or their projections onto the coordinate axes intersect in one contact. According to our data, the replacement of one contact by two in the model did not influence the calculation of thermal conductivity, but this very model was also used for calculating Young's elasticity modulus and of the temperature coefficient of linear expansion in modeling some technological processes where this replacement played a certain role.

A unique model of the structure was suggested by N. A. Bozhkov in [5]. In this model the diameter of the fiber-rod in each direction was adopted equal to the probability of the origination of fibers in this direction, i.e., for an isotropic material the model coincided with the well-known solutions [2]. Such a model of the structure clearly reflected the emergence of a preferable orientation of fibers that appeared after the formation of a semifinished item of a fibrous material.

For binder-containing fibrous materials [9], the quality of the contact between individual fibers is such that its thermal resistance is close to zero (Fig. 1). As to its properties, the material of the contact is close to the material of the fiber, and the interface is not expressed, with no decrease of the solid phase cross section in the zone of contact. For materials without a binder, recommendations of [2] were used in calculating the thermal resistance of the point contact of fibers. The force of the compression of fibers in the zone of contact and the roughness parameters of the surface of fibers were taken into account in this case.

As a result, the elementary cell selected for the structure to analyze the process of heat transfer includes three identical mutually perpendicular rods (Fig. 2) and obviously reveals the anisotropy of properties. For simplicity of calculations of the geometrical parameters of the cell, the rods were selected to have a square cross section.

In using the initially anisotropic cell of a structure, there arises the problem of its coordination with the structure of a highly porous fibrous material that can be isotropic as a whole. For this purpose, the notion of a representative element of the structure constructed of an equal number of cells of all orientations was used (Fig. 3). Since no contact of two cells is possible without rupture of fibers at the boundary, cells of one orientation are collected in small cubes, the size of whose fin is equal to the mean length of a fiber l_f. The representative element of the structure consists of 27 volumetrically uniformly distributed cubes with cells of one orientation. As a result, the representative element of the structure has the shape of a cube with side equal to $3l_f$. With the selected construction of the cell, such a volume of the material that enters into the representative element is the minimum possible for the cells whose properties are isotropic.

In some cases, in binder-containing materials "clotting" of fibers can occur, i.e., formation, in a raw fibrous mass of fiber, of clots having a shape close to spheres (Fig. 4), the so-called globules [9]. In what follows, the parameters that refer to

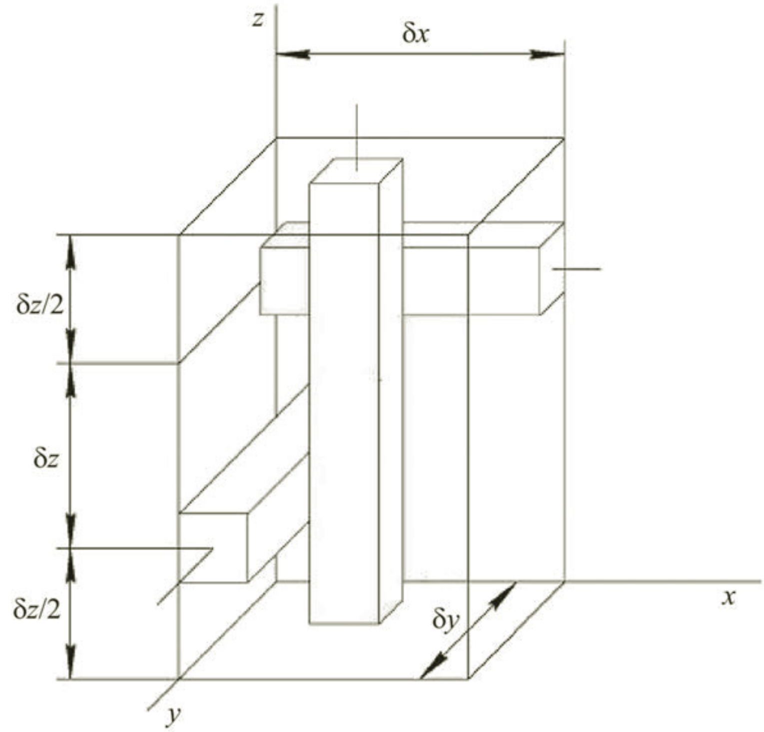


Fig. 2. Model of the elementary cell of material structure.

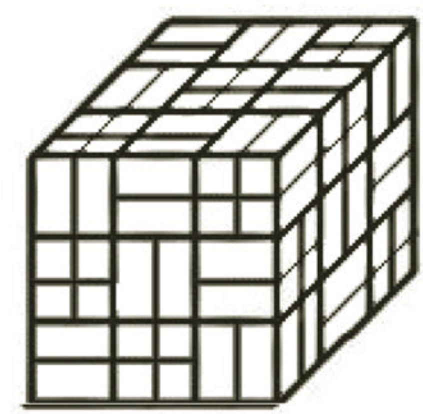


Fig. 3. Representative element of the structure composed of small cubes with cells of one orientation.

a fibrous medium between globules will be labeled by "p," those referring to globules, by "g," those to a binder, by "b," and those to the material as a whole are not denoted by a subscript.

As an example we will consider the influence exerted on the properties of a material by one of the technological processes, i.e., the moulding that greatly determines the parameters of the structure and the degree of the fibrous material anisotropy. Since on compression of a raw fibrous mass (pulp) the repacking of globules can be neglected, we assume that deformation of a preform is equal to the blank thickness-average deformation of a globule Δr_g. Compression of globules under the weight of the fibrous mass is not taken into account, although it is not difficult in principle to consider the pressure variable over the blank thickness and equal to the weight of the corresponding upper layer of the pulp. In calculating Δr_g we use the following expression obtained in [10, 11] for the internal stresses σ appearing in a particle under the action of pressure P:

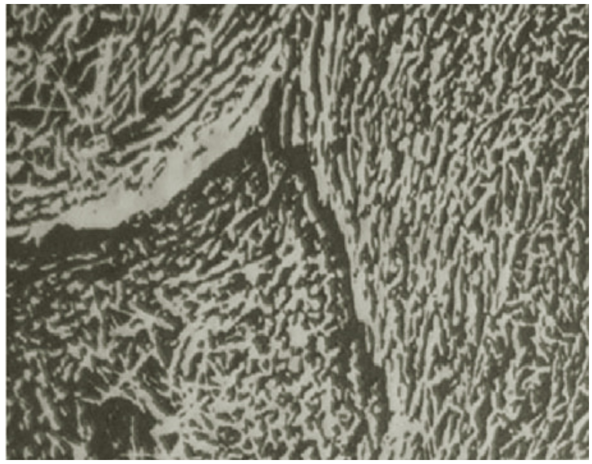


Fig. 4. Contact between globules (magnification of 50) [9].

$$\sigma = P\rho gH(1 - \xi)/G . \quad (1)$$

The condition of deformation termination is determined from the equality of internal stresses appearing in a globule under the action of compression forces at the contact spot of radius r_{cg} that form the left-hand side of Eq. (2) and the resistance forces of the binder surface tension σ_{st} appearing in deformation of the contact spot with the perimeter L_c that in a linear approximation represent the right-hand side of Eq. (2):

$$\frac{\sigma_{st} L_c N_c}{V_g} = 4\pi \int_0^{r_{cg}} \sigma dr L_c . \quad (2)$$

As a result we can obtain an expression for the globule relative deformation equal, under the adopted assumptions, to the relative deformation of the blank with a binder that had not been solidified:

$$\xi = 1 - \frac{8\sigma_{st}}{3PGH} \frac{dr}{\rho g \delta^3} . \quad (3)$$

This allows one to estimate the characteristics of the spot of contact between globules. Assuming that the contact spot has the shape of a circle of radius r_{cg}, we obtain

$$r_{cg} = r_g \sqrt{1 - \left(1 - \xi^2\right)} . \quad (4)$$

When calculating the change in the volume between the globules in the course of material moulding, we will consider the process of deformation of a sphere with the shell surrounding it. The shell consists of a less dense fibrous mass that was not incorporated into the globules. We determine it as a porous medium with a volume V_p. In accordance with the recommendations given in [2], we assume that the globules pack among themselves with the coordination number $N_c = 6–8$. Using the relationship between the coordination number and porosity, we can determine the change occurring in the latter depending on the characteristics of deformation of a raw fibrous mass:

$$\frac{1 - \Pi_0}{1 - \Pi} = \frac{1 - \frac{N_c}{4}\left[1 - \frac{r_g}{r_p}(1 + \xi)\right]^2 \left[2 + \frac{r_g}{r_p}(1 + \xi)\right]}{\left[1 - \frac{N_c}{4}\left(1 - \frac{r_g}{r_p}\right)^2 \left(2 + \frac{r_g}{r_p}\right)\right]\left[1 + \frac{N_c}{4}\xi^2(3 - \xi)\right]} . \quad (5)$$

The mean distance between fibers is calculated from the volume contents of the components:

$$\delta = d_{\mathrm{f}} \sqrt{\frac{\pi}{2}\left(1 + \nu_{\mathrm{b}} \frac{\rho_{\mathrm{b}}}{\rho_{\mathrm{f}}}\right)\frac{\rho_{\mathrm{f}}}{\rho}} . \tag{6}$$

In the course of moulding, directions of the preferred orientation of fibers are formed. We assume that the fraction of fibers in the ith direction, N_{f_i}, is proportional to the radius of a globule in this direction r_{g_i}, i.e., $N_{\mathrm{f}_i}/r_{\mathrm{g}_i}$ = const. Then, for the distances between the contacts in the cell we obtain

$$\delta_z = \delta_x (1 - \xi_z) , \tag{7}$$

when compression occurs along the z axis, where $\delta_x = \delta_y = \delta\sqrt{\frac{1}{2}\left(\frac{2 - \xi_z}{1 - \xi_z}\right)}$,
and

$$\delta_z = \delta_y = \delta\sqrt{\frac{1}{4}(4 - \xi_z)(1 - \xi_z)} , \tag{8}$$

when compression is along the x axis. The expression for the y axis is similar to Eq. (8).

The proposed algorithm for constructing the model of the structure allows one not only to determine the distance between the contacts of individual fibers, but also to relate them to the characteristics of a raw fibrous mass and to the technological parameters, which makes it possible to analyze the change in the structure and subsequently in the thermal conductivity over the thickness of the moulded tablet in various directions. This, in turn, allows one to determine the optimum direction of the orientation of an anisotropic fibrous material when casting an element of a heat shielding system.

As a result, we have obtained a relationship between the cell parameters, semifinished-item properties, and the parameters of the technological process that forms the material structure. The use of a representative element of the structure makes it possible, also without altering the model of the elementary cell, to carry out the calculation of the elasticity moduli of a material as of a deformable rod system, which makes it possible to determine the thermal coefficient of the linear expansion of a fibrous material

Heat Transfer in the Elementary Cell of a Fibrous Material. The general scheme of calculation of the effective thermal conductivity with the use of the apparatus of the theory of generalized conductivity in the case of the globular structure of a highly porous fibrous material is presented in Fig. 5. First the conductive transmission of heat through the fibrous frame of the material λ_{fr} and molecular heat conduction through the gas medium in the cell λ_{mp} are determined. According to our results and foreign author estimates [12–14], we can neglect the convective component in the heat transfer balance for the fibrous materials considered. The total thermal conductivity λ_{cond} was calculated as a combination of the thermal resistances connected in parallel and successively [2]. Thereafter we determined the thermal resistance of the contact between globules and the conductive component of the effective thermal conductivity of a fibrous material. As the size of a globule is larger than that of the representative element of the structure, the thermal conductivity of the fibrous medium in the globule is taken equal to the thermal conductivity of the starting material of corresponding density. Otherwise it is necessary to determine the thermal conductivity of the material as mathematical expectation in an ergodic homogeneous random field of the local values of the thermal conductivity of these globules [15]. Next the radiative component of the thermal conductivity λ_{rad} and the effective thermal conductivity of the fibrous material as a whole λ_{eff} are determined.

The molecular component of the conductivity in a pore in the ith direction was calculated with the use of the notion of the thermal conductivity of a gas in an infinite volume λ_{gas}, the parameter of the structure $C_i = \frac{d_{\mathrm{f}}}{\delta_i}$, and the Knudsen number Kn [16]:

$$\lambda_{\mathrm{mp}_i} = \frac{\lambda_{\mathrm{gas}}}{1 + \frac{6}{\sqrt{\pi}}\left(\frac{C_i^2\,(1 - C_i)}{\Pi}\right)\mathrm{Kn}} . \tag{9}$$

The analysis carried out in [16] shows that calculation from (9) yields better agreement with experimental data than from well-known Prasolov′s formula (2). For modeling radiation transfer in partially transparent media use was made of the optical

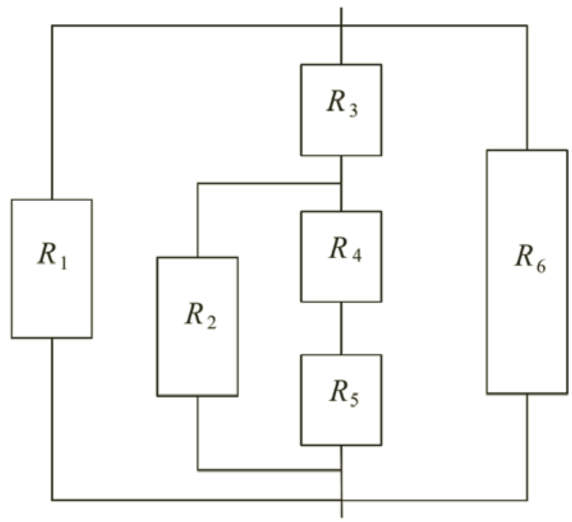


Fig. 5. Scheme of calculation of the total thermal resistance of the elementary cell of material structure.

dense medium approximation (the Rosseland approximation) [17] in which the radiative component of the effective thermal conductivity of the material is represented in the form

$$\lambda_{rad} = \frac{16}{3}\frac{\sigma_0 n^2}{\kappa}T^3 . \quad (10)$$

Here the model of a "gray" medium was used. The assumption was based on the data of [18–20] where the problem of the applicability of this approach for materials from amorphous silicon oxide fibers is considered.

To calculate the spectral dependence of the attenuation factor κ_λ and of its components, the absorption k_λ and scattering σ_λ coefficients, we used the Mie theory that describes the interaction of a single spherical particle with a plane electromagnetic wave propagating in an isotropic infinite dielectric medium. The objection to the application of such an approach to highly porous fibrous materials is well known (for example, from [18]). At the same time, some research works demonstrate good numerical agreement of the calculation by the Mie theory with experiments on a group of particles. For example, in [19] scattering and absorption of radiation were measured in a suspension of polystyrene microspheres enclosed between two parallel glass plates. Good agreement is obtained between the results of measurements and calculations by the Mie theory with account for a single scattering when the mean distance between the spheres is greater than 0.3Λ. Layers of optical thickness from 0.25 to 3000 m^{-1} were investigated in that work.

The attenuation factor of radiation (with a wavelength Λ_{max}) that corresponds to the maximum of intrinsic emission has the following form with the use of the dimensionless attenuation factor Γ_p :

$$\kappa_\lambda = \Gamma_p n S_f = \frac{8}{3\pi}\Gamma_p \frac{\rho_m}{\rho_f}(1-\nu_b)\frac{1}{d_f} , \quad (11)$$

$$S_f = d_f l_f . \quad (12)$$

The attenuation factor $\Gamma_p \approx 1.35$ was calculated in accordance with the Mie theory in the approximation of scattering on a single dielectric sphere [20–23], which for a material from amorphous silicon oxide fibers with a density of 144 kg/m^3 and with a mean diameter of a fiber of 1.5 μm yielded $\kappa_{\Lambda max} \approx 45{,}000\ m^{-1}$ at 1500 K. Next, we took the average of the spectral dependence of the attenuation factor using the radiation functions of the second kind [24]. As a result, for a temperature of 1500 K we obtained that $\kappa = (10–15)\cdot 10^3\ m^{-1}$. The well-known experimental and calculated data in this case are at the level $(5–8)\cdot 10^3\ m^{-1}$ [25–28].

Thus, calculation of the radiative component of thermal conductivity by means of the employed algorithm yields an underestimated result for the radiative component of the effective thermal conductivity. However, despite the simplicity of the mathematical technique used, the coupling between the technological process parameters and the characteristics of a semifinished item with the structure and properties of a finished material has been revealed.

Determination of the Optical Properties of Fibrous Materials. In our opinion, the problem of the discrepancy between the calculated and experimentally measured values of the optical properties consists in the incomplete agreement between the implications of the Mie theory and the real characteristics of fibrous materials. This primarily concerns the assumption on the independent character of radiation scattering on individual fibers. Here account for multiple scattering leads to the necessity of constructing complex algorithms that in turn require the availability of initial data on the optical properties not only of a fiber but also of a binder and of the material of globules, and this usually requires special experimental research. Therefore to elevate the accuracy with which the radiative component of the effective thermal conductivity could be found, we decided to determine the optical properties of studied materials from the experimentally measured transmission coefficients of a set of samples of different thickness. Such an approach also calls for carrying out of supplementary experiments but without doubt cuts down on the whole the time needed for investigation.

We have developed a method of determining the optical properties (coefficients of radiation absorption and diffusion) from the results of measurement of radiation transmission by a set of samples of different thickness [27, 28]. Theoretically the method is based on the solution of inverse problems of radiation transfer.

It was considered that a test sample in the form of a disk or a plate of thickness Δ is illuminated at the front by a monochromatic radiation flux $q_{w,R}$ of directional-diffusive character. The temperature within the sample does not change during the experiment, which allows us to consider the optical properties of the material constant. The optical properties of the boundaries of the sample are assumed to be known. Because the samples are not very thick (1–20 mm) and there is a considerable fraction of the directed component in the radiative flux incident on the frontal surface of it, the transfer equation was solved using a modification of the diffusion approximation [29, 30] in which the directed (singular) component of the radiation flux is considered separately. The application of this approach makes it possible to substantially increase the accuracy of calculating the radiation field.

The mathematical model of the process of radiative transfer in this case has the form

$$D_\nu \frac{\partial^2 U_\nu(x)}{\partial x^2} - k_\nu U_\nu(x) = -k_\nu B_\nu^*(x, T) - \left(\frac{1}{D_\nu} - 3k\right) q_{w,R}\eta_\delta(1 - r_\delta)\ \exp\ \left(-\frac{x}{3D_\nu}\right) , \quad x \in \Omega; \tag{13}$$

$$-\frac{3}{2}\left(\frac{1}{3} + \tilde{R}_{11,\nu}\right) D_\nu \frac{\partial U_\nu(x)}{\partial x} + \frac{1}{2}\left(\frac{1}{2} - \tilde{R}_{01,\nu}\right) U_\nu(x) = 2q_{w,R}\eta_m\tilde{Q}_{01,\nu} , \quad x \in \Gamma_1 ; \tag{14}$$

$$\frac{3}{2}\left(\frac{1}{3} + \tilde{R}_{11,\nu}\right) D_\nu \frac{\partial U_\nu(x)}{\partial x} + \frac{1}{2}\left(\frac{1}{2} - \tilde{R}_{01,\nu}\right) U_\nu(x) = 0 , \quad x \in \Gamma_2 ; \tag{15}$$

$$B_\nu^* T(x, \tau) = 4\pi n_\nu^2(T) B_\nu T(x, \tau) ,$$

$$\tilde{R}_{mn,\nu}(T) = \int_0^1 \mu^n \int_0^1 r_\nu(\mu, \mu', T)(\mu')^m d\mu' d\mu , \tag{16}$$

$$\tilde{Q}_{mn,\nu}(T) = \int_0^1 \mu^n \int_0^1 q_\nu(\mu, \mu', T)(\mu')^m d\mu' d\mu .$$

A set of N samples of different thicknesses ($N > 2$) are investigated. In each experiment the radiation flux $q_{\Delta,R}$ transmitted through the sample is measured. From these data we are to determine the radiation absorption and diffusion coefficients of a partially transparent material. For solving the inverse problems of radiation transfer, we used an extreme formulation, i.e., we determined the vector of the unknown parameters $\mathbf{u} = \{D, k\}$ that minimized the functional of the discrepancy between the experimental and predicted values of the transmission coefficient:

$$F_i = \frac{q_{\Delta,R,i}}{q_{w,R,i}}, \quad i = \overline{1, N},$$

$$S(\mathbf{u}) = \frac{1}{2}\sum_{i=1}^{N}\left(\frac{F_i}{F_i^{e}} - 1\right)^2, \tag{17}$$

where F_i and F_i^{e} are the calculated and experimentally measured values of the transmission coefficient in the ith experiment. Thus, to calculate the value of the functional it is necessary to solve system (13)–(16) N times. The value of the transmission coefficient for each experiment was calculated as

$$F = \frac{1}{2q_{w,R}} U(\Gamma_2)\left(\tilde{H}_{01,\nu} + \tilde{H}_{11,\nu}\frac{\frac{1}{2} - \tilde{R}_{01,\nu}}{\frac{1}{3} + \tilde{R}_{11,\nu}}\right) + (1 - r_\delta)^2\ \eta_\delta \exp\left(-\frac{\Delta_i}{3D_\nu}\right),$$

$$H_{mn,\nu}(T) = \int_0^1 \mu^n \int_0^1 h_\nu(\mu, \mu', T)(\mu')^m\, d\mu' d\mu. \tag{18}$$

The functional (17) was minimized by the conjugate-gradient method. To find the gradient of the functional, the problem conjugate of (13)–(16) is solved N times. It has the form

$$D_\nu \frac{\partial\varphi_{\nu,i}^2(x)}{\partial x^2} - k_\nu\ \varphi_{\nu,i}(x)$$

$$+ \frac{1}{2q_{w,R}}\left(\frac{F_i}{F_i^{e}} - 1\right)\left(\tilde{H}_{01,\nu} + \tilde{H}_{11,\nu}\frac{\frac{1}{2} - \tilde{R}_{01,\nu}}{\frac{1}{3} + \tilde{R}_{11,\nu}}\right)\delta(x - \Delta_i) = 0, \qquad x \in \Omega; \tag{19}$$

$$-\frac{3}{2}\left(\frac{1}{3} + \tilde{R}_{11,\nu}\right) D_\nu \frac{\partial\varphi_{\nu,i}(x)}{\partial x} + \frac{1}{2}\left(\frac{1}{2} - \tilde{R}_{01,\nu}\right)\varphi_{\nu,i}(x) = 0, \qquad x \in \Gamma_1; \tag{20}$$

$$\frac{3}{2}\left(\frac{1}{3} + \tilde{R}_{11,\nu}\right) D_\nu \frac{\partial\varphi_{\nu,i}(x)}{\partial x} + \frac{1}{2}\left(\frac{1}{2} - \tilde{R}_{01,\nu}\right)\varphi_{\nu,i}(x) = 0, \quad x \in \Gamma_2. \tag{21}$$

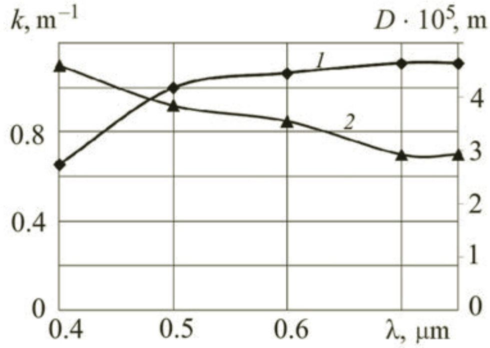
After the solution of problem (19)–(21) the components of the residual functional gradient vector are calculated as

$$S'_D = \sum_{i=1}^{N}\left(\int_0^{\Delta_i}\left(\frac{dU_{\nu,i}(x,\tau)}{dx}\frac{d\varphi_{\nu,i}(x,\tau)}{dx} + \varphi_{\nu,i} q_{w,R}\eta_\delta(1 - r_\delta)\exp\left(-\frac{x}{3D_\nu}\right)\left(\frac{1}{D_\nu^2} - \left(\frac{1}{D_\nu} - 3k_\nu\right)\frac{x}{3D_\nu^2}\right)\right)dx\right.$$

$$\left. + \frac{\Delta_i}{3D_\nu^2}(1 - r_\delta)^2\eta_\delta \exp\left(-\frac{\Delta_i}{3D_\nu}\right)\right), \tag{22}$$

$$S'_k = -\sum_{i=1}^{N}\int_0^{\Delta_i}\varphi_{\nu,i}(x,\tau)\left(U_{\nu,i}(x,\tau) - B_\nu^*(x,T) - 3q_{w,R}\eta_\delta(1 - r_\delta)\exp\left(-\frac{x}{3D_\nu}\right)\right)dx. \tag{23}$$

The inverse problems of radiation transfer were tested by a method of mathematical simulation. In the course of simulation in a wide range of variation of the optical properties of the material $k = 0.1$–1000 m^{-1}, $D = 10^{-6}$–10^{-2} m, bound-



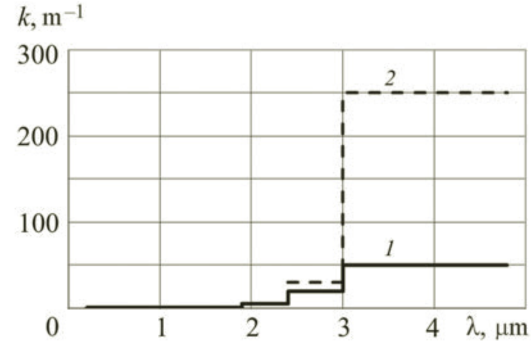


Fig. 6. Spectral dependence of the optical properties of the material based on amorphous silicon oxide fibers of density 144 kg/m^3: 1) radiation absorption coefficient; 2) radiation diffusion coefficient.

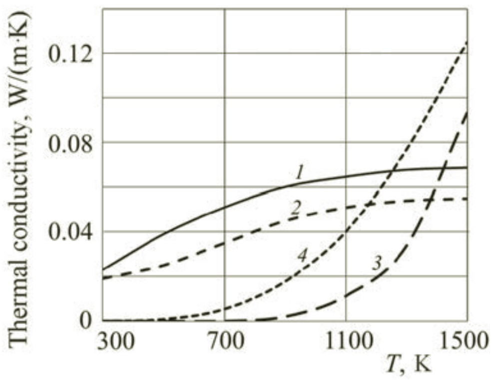
Fig. 7. Model of the spectral dependence of the absorption coefficient of the material based on amorphous silicon oxide fibers of density 144 kg/m^3: 1) at 293 K; 2) at 1500 K.

ary surfaces r = 0.01–0.95, and sample thickness Δ = 0.001–0.1 m; the accuracy for which the inverse problems of radiation transfer were solved was not more than 0.1%. The cases of the solution nonuniqueness were not noted. We also analyzed the stability of solution of the inverse problems of radiation transfer with respect to random errors in the experimentally measured transmission coefficients. The calculations showed that 10% of the random errors introduced into the values of the transmission coefficients leads to not more than 5% of errors in the results of the solution of inverse heat transfer problems in a broad range of simulation parameters. It has been revealed by numerical experiments that with the optical properties determined from the results of two to three experiments and with the use of perturbed data, the error of solution of the inverse radiation transfer problems may reach 20%. When a larger number of experiments were carried out (with a number of samples of different thicknesses from 5 to 10), the error of solution of these problems decreased to 5%. This permits us to draw a conclusion on the necessity of carrying out experimental investigations with samples no fewer than five.

Experimental investigations of the optical properties of amorphous silicon oxide fibers of density 144 kg/m^3 were carried out at the N. É. Bauman Moscow State Technical University with the use of a commercial SF-14 spectrophotometer within the range 0.4–0.75 μm [28]. In the process of experiments, the frontal surface of samples was illuminated by a directed monochromatic radiation flux of known density. An integrating hemisphere was installed at the rear of the samples for recording the radiation flux transmitted through a sample. As a result of a series of experiments, the values of the transmission coefficient were obtained for 25 samples of thickness from 1.4 to 14.8 mm to be used subsequently for solving the inverse radiation transfer problems. Figure 6 presents the optical properties of the material based on the amorphous silicon oxide fibers. They correlate well with the results of [25–28].

Because of the lack of data on the spectral range of the optical properties of the studied material and their unavailability for high temperatures, a spectral model of optical properties has been developed (Fig. 7). In constructing the model it was assumed that the absorption coefficient and the range of partial transparency of the material corresponded to the absorption coefficient and to the range of partial transparency (0.2–4.8 μm) of quartz glass KV. Since the radiation diffusion coefficient is mainly determined by the structure of the material, which does not change within the temperature range 300–1500 K, we may assume that the radiation diffusion coefficient is constant in this spectral region and is equal to $4.75 \cdot 10^{-5}$ m.

Determination of the Thermal Conductivity of a Fibrous Material. The developed model for the structure of a material was used for calculating the thermal conductivity of a material based on the amorphous silicon oxide fibers of density 144 kg/m^3 (Fig. 8). Figure 9 displays a comparison of the calculated data on the thermal conductivity of a material with the data obtained in processing the results of experimental investigations by means of an absolute stationary method [31]. It is seen that the data are in good agreement, which confirms the adequacy of the proposed models of the structure and calculation methods. Figure 10 presents the calculated dependence of the effective thermal conductivity and of its components for a material based on amorphous silicon oxide fibers of density 144 kg/m^3 on the fiber diameter. It was concluded that the optimal value for the fiber diameter is 1.5–2.0 μm.



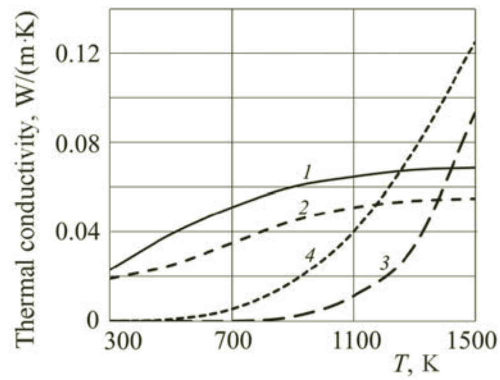


Fig. 8. Temperature dependence of the thermal conductivity of the material based on amorphous silicon oxide fibers of density 144 kg/m^3: 1) λ_{cond}; 2) λ_{mp}; 3) λ_{rad} determined by the Mie theory; 4) λ_{rad} determined by solving the inverse problem of radiation transfer.

Fig. 9. Temperature dependence of the thermal conductivity of the material based on amorphous silicon oxide fibers of density 144 kg/m^3: λ_{eff}, experiment, stationary method [31]; 2) λ_{eff}, calculation, inverse problem of heat conduction [31]; 3) λ_{eff}, calculation, generalized thermal conductivity theory.

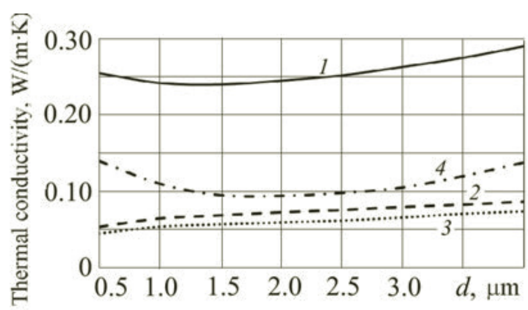


Fig. 10. Thermal conductivity of the material based on amorphous silicon oxide fibers of density 144 kg/m^3 vs. the fiber diameter at a temperature of 1500 K and normal atmospheric pressure: 1) λ_{eff}; 2) λ_{cond}; 3) λ_{mp}; 4) λ_{rad}.

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NOTATION

B, radiation intensity of a black body, W/(m^2·sr); $C_i = \dfrac{d_f}{\delta_i}$, parameter of a structure; D, radiation diffusion coefficient, m; d_f, fiber diameter, m; F_i and F_i^e, calculated and experimentally measured values of the transmission coefficient in the ith experiment; G, weight of the fill layer per unit area, kg; H, height of the fill layer per area unit, m; h, coefficient of radiation transmission to the outside of a sample; Kn, Knudsen number; k_λ, absorption coefficient, m^{-1}; $L_c = 4d_f$, perimeter of the spot of contact between fibers, m; l_f, mean length of a fiber, m; $N_c = V_g/\delta^3$, number of contacts between fibers at a mean distance between them δ; n, refractive index; q, coefficient of radiation transmission inside a sample; $q_{w,R}$, incident radiation flux, W/(m·K); $q_{\Delta,R,t}$, transmitted radiation flux,W/(m·K); r, reflection coefficient of the sample surface; r_δ, reflection coefficient of the frontal surface in relation to directed radiation; T, temperature, K; U, radiation energy density, W/m^2 ; $V_g = \dfrac{4}{3}\pi r_{cg}^3$, volume of a globule, m^3; x, coordinate, m; Γ_1 and Γ_2, frontal and rear surfaces of a sample; Δ, thickness of a sample, m; δ, delta function; δ_i, distance between contacts in the direction of the *i* axis, m; η_m, fraction of a diffusive radiation flux at the frontal boundary of a sample; η_δ, fraction of directed radiation flux at the frontal boundary of a sample; κ, attenuation factor, m^{-1}; Λ, radiation wavelength, μm; λ, thermal conductivity, W/(m·K); λ_{fr}, conductive heat transmission through fibrous material frame, W/(m·K); λ_{mp}, molecular heat conduction through the gas medium in a cell, W/(m·K); λ_{cond}, total conductive heat transmission in a cell, W/(m·K); λ_{rad}, radioactive component of heat conduction in a cell, W/(m·K); λ_{eff}, effective thermal conductivity of fibrous material, W/(m·K); λ_{gas}, thermal conductivity of a gas in an infinite volume, W/(m·K); μ, μ′, angles of radiation incidence and reflection (transmission); $\xi = \Delta r_g/r_g$, relative deformation of globules; ρ, density of a pulp, kg/m^3; σ_0, Stefan–Boltzmann constant, $W/(m^2{\cdot}K^4)$; σ_λ, scattering coefficient, m^{-1}; Ω, region of a sample. Indices: λ, η, refer to the spectral and frequency dependences.

REFERENCES

1. S. V. Reznik, P. V. Prosuntsov, and A. V. Zuev, Features of inverse methods for determination of heat transfer in porous materials at high heating velocity, in: *Proc. 2nd Int. Symp. on Inverse Problems, Design and Optimization*, Miami (Florida, USA) (2007), pp. 493–500.
2. G. N. Dul'nev and Yu. P. Zarichnyak, *Thermal Conductivity of Mixtures and Composite Materials* [in Russian], Énergiya, Leningrad (1974).
3. S. M. Kats, *High-Temperature Heat-Insulating Materials* [in Russian], Metallurgiya, Moscow (1981).
4. E. Ya. Litovskii, S. L. Bondarenko, Yu. A. Polonskii, and N. I. Ganichev, Concerning the effect of the fiber diameter on the effective thermal conductivity of fireproof heat insulation, *Teplofiz. Vys. Temp.*, **17**, No. 5, 997–1000 (1979).
5. O. M. Alifanov and N. A. Bojkov, Les Methodes des Previsions in Formatiquers des Materaux Composities de Haute Porosite et de l'Analise des Systems de la Protection Thermique a Leur Basc, in: *Proc. Conf. on Spacecraft Structures, Materials and Mechanical Testing*, 27–29 March, 1996, Noordwijk, the Netherlands (1996). ESA SP–386, June, 1996. P. 73.
6. O. M. Alifanov and V. V. Cherepanov, Mathematical simulation of highly porous fibrous materials and determination of their physical properties, *Teplofiz. Vys. Temp.*, **47**, No. 3, 463–472 (2009).
7. O. M. Alifanov, S. A. Budnik, A. V. Nenarokomov, and V. V. Cherepanov, Experimental and theoretical investigation of the process of heat transfer in highly porous materials, *Tepl. Prots. Tekh.*, **3**, No. 2, 53–65 (2011).
8. V. I. Bol'shakov and N. V. Shpirko, Improvement of fibrous heat-insulating materials, *Vestn. Pridneprovsk. Gos. Akad. Stroit. Arkhitekt.*, No. 3, 5-8 (2009).
9. V. N. Gribkov, G. T. Mizyurina, B. V. Shchetanov, and V. V. Lyapin, The possibilities of the fibrous heat shielding, in: *Proc. 1st Int. Aviacosm. Conf. "Man–Earth–Space,"* Moscow, 1992, Vol. 5, Moscow (1995), pp. 223–231.
10. M. P. Volarovich, N. B. Demkin, and I. I. Berkovich, *Experimental Investigation of Friction and Compaction of Fibrous Materials. Problems of Physicochemical Mechanics of Fibrous and Porous Disperse Structures and Materials* [in Russian], Zinatne, Riga (1967).
11. G. M. Zhdanovich, *Some Problems of the Theory of the Metal Powder Pressing Process* [in Russian], Izd. Belorus. Politekh. Inst., Minsk (1960).
12. V. N. Bykov, Experimental investigation of the effective thermal conductivity of insulation in compressed gases, *Teplofiz. Vys. Temp.*, **10**, No. 4, 788–794 (1972).
13. P. J. Burns and C. L. Tien, Natural convection in porous media bounded concentric spheres and horizontal cylinders, *Int. J. Heat Mass Transf.*, **22**, 929–939 (1979).
14. Doon and Dyan′, Radiation heat transfer in fibrous insulations. Pt. 1. Analytical investigation, in: *Proc. ASME*, *Heat Transf.*, **105**, No. 1, 73–80 (1983).
15. V. V. Bolotin and V. N. Moskalenko, Macroscopic coefficients of heat conduction and diffusion in microinhomogeneous solid bodies, *Prikl. Mekh. Tekh. Fiz.*, No. 6, 7–13 (1967).
16. I. G. Chumak and V. G. Pogontsev, Investigation of the mechanism of heat transfer at the gas–fiber interface, *Inzh.-Fiz. Zh.*, **36**, No. 1, 17–22 (1979).
17. N. A. Rubtsov, *Radiative Heat Transfer in Continuous Media* [in Russian], Nauka, Novosibirsk (1984).
18. N. V. Komarovskaya, *Investigation of Radiative Heat Transfer in Disperse Heat Insulators and Heat Insulating Structures*, Candidate′s Dissertation (in Engineering), LII, Moscow (1990).
19. Doon, Yan and Dyan′, Radiative heat transfer in fibrous heat insulation. Pt. 2. Experimental investigation, in: *Proc. ASME*, *Heat Transf.*, **105**, No. 1, 80–86 (1983).

20. Dayan and Dyan′, Heat transfer in a plane layer of gray medium with linear anisotropic scattering, in: *Proc. ASME*, *Heat Transf.*, *Ser. C*, **97**, No. 3, 78–84 (1975).
21. V. M. Kostylev, *Macroscopic Kinetics of the Photon Gas. Radiative Heat Transfer in Disperse Media* [in Russian], Mashinostroenie, Moscow (2000).
22. H. Hottel, A. Serofim, I. Wassalos, and D. Dalzel, Multiple scattering. Comparison between theory and experiment, in: *Proc. ASME*, *Heat Transf.*, **92**, No. 2, 77–83 (1970).
23. H. C. Van de Hulst, *Light Scattering by Small Particles* [Russian translation], IIL, Moscow (1961).
24. M. N. Ozisik, *Combined Heat Transfer* [in Russian], Mir, Moscow (1976).
25. V. S. Dozhdikov and V. A. Petrov, Radiation characteristics of heat protective materials of the Buran orbital spacecraft, *Inzh.-Fiz. Zh.*, **73**, No. 1, 26–30 (2000).
26. A. V. Kondratenko, A. S. Moiseev, V. A. Petrov, and S. V. Stepanov, Experimental determination of the optical properties of fibrous quartz ceramics, *Teplofiz. Vys. Temp.*, **29**, No. 1, 134–138 (1991).
27. V. A. Petrov, *Radiation Diffusion Model of Radiative-Conductive Heat Transfer in High-Temperature Semitransparent Scattering Heat-Insulating Materials* [in Russian], Izd. MGTU MIRÉA, Moscow (2012).
28. P. V. Prosuntsov and S. V. Reznik, Investigation of the volumetric optical properties of partially transparent scattering materials by solving the inverse problem of heat transfer, in: *Proc. 2nd Int. Conf. "Identification of Dynamic Systems and Inverse Problems,"* Vol. 2, D–12–1–9 (1994).
29. A. Ishimaru, *Propagation and Scattering of Waves in Accidentally Inhomogeneous Media* [Russian translation], Mir, Moscow (1981).
30. V. A. Tovstonog, Identification of thermal and radiative characteristics of light-scattering materials, *Izv. Sib. Otd. Akad. Nauk SSSR*, *Ser. Tekh. Nauk*, No. 7, Issue 2, 16–21 (1987).
31. A. V. Zuev, P. V. Prosuntsov, and I. A. Maiorova, Computational-experimental investigation of heat transfer processes in highly porous fibrous heat insulating materials, *Tepl. Prots. Tekh.*, **6**, No. 9, 410–419 (2014).