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INVESTIGATION OF HEAT TRANSFER IN THE PROCESS OF DRYING BY THE REGULAR REGIME METHOD

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The laws governing the change in heat fluxes and mean integral temperatures in the period of the falling rate of drying have been established. Experimental dependences for the rate of heating a moist body and for the coefficient of temperature distribution nonuniformity in a moist body have been obtained.

Keywords: *Kondratiev number, Biot number, thermal diffusivity, rate of heating a moist body, heat transfer coefficient, mean integral temperature.*

Introduction. It is known from the theory and practice of drying that the evaporation intensity, density of heat fluxes, and the mean integral temperature of a moist body change in time exponentially [1, 2]. The problems of heating a moist body in a constant-temperature medium t_{med} = const relate to the problems of G. M. Kondratiev's regular regime, when a change in heat fluxes and mean integral temperatures in the period of the declining drying rate is described by a simple exponential curve.

Statement of the Problem. The regular regime of heating bodies is described by the equation [1, 2]

$$-\frac{d\bar{t}}{d\tau} = m_t\,(t_{\text{med}} - \bar{t})\,. \tag{1}$$

The rate of heating a moist body m_t is determined experimentally from the relation

$$\tan\gamma = \frac{\ln\,(t_{\text{med}} - \bar{t}_0) - \ln\,(t_{\text{med}} - \bar{t})}{\tau - \tau_0} = m_t = \text{const}\,. \tag{2}$$

The reckoning of the time of drying in the period of the declining rate for all the regimes begins from τ_0 = 0 and moisture content $\bar{u}_{\text{cr}}$, whereas the mean integral temperature $\bar{t}_0$ is determined from the mean temperature of the material in the first period. The current values of $\bar{t}$ and $\bar{u}$ correspond to the current value of time τ.

Experimental. Figure 1 shows the change in the excess temperature in time: $\ln\,(t_{\text{med}} - \bar{t}) = f(\tau)$ for asbestos sheets (a) and sole leather (b) in the process of their convective drying in different regimes. It is seen that at the stage of the regular regime the corresponding graphs form a family of straight lines. In the process of heating a moist body at mean integral temperatures the rate of its heating is preserved constant: m_t = const, which is the main characteristic of the regular regime.

A. V. Luikov's main equation of the kinetics of drying has the form [3]

$$q^* = \frac{q_{\text{II}}}{q_{\text{I}}} = N^*\,(1 + \text{Rb})\,. \tag{3}$$

It has been established that the relative rate of drying N^* and the Rebinder number Rb are determined as follows [3–5]:

$$N^* = \exp\,(-\,a_0 N\tau)\,, \tag{4}$$

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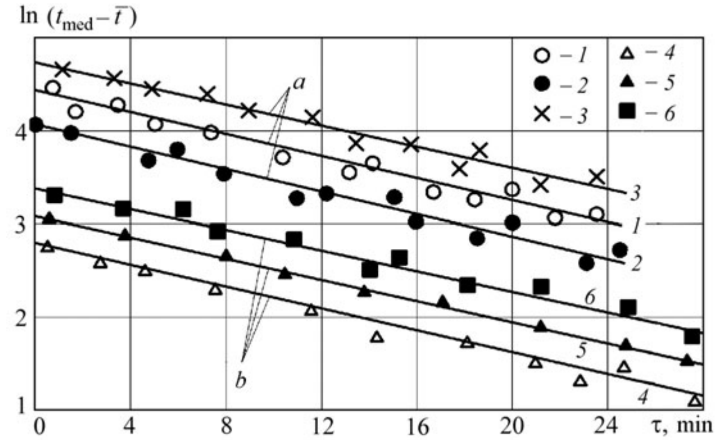

Fig. 1. Time dependence of the mean integral excess temperature in the process of drying asbestos sheets at v = 3–10 m/s, φ = 5% (a) and a sole leather at v = 3–5 m/s, φ = 15% (b): 1) t_{med} = 90°C; 2) 120; 3) 150; 4) 40; 5) 50; 6) 60.

TABLE 1. Constants A and n for Some Moist Materials

Material	δ, mm	Drying regime			A	n
		t_{med}, °C	v, m/s	φ, %		
Porous ceramics	5–6	90–150	3–10	5	0.5	20
Asbestos sheets	6–8	90–150	3–10	5	0.5	15
Sole leather	3–4	40–60	3–5	15	0.5	8.5
Wool felt	8–18	90–150	3–10	5	0.1	6
Felt	3	50	0.5–0.7	24–74	0.1	10
Board	4–5	90–110	3–10	5	0.025	–3.5

TABLE 2. Values of m_t for Some Moist Materials

Material	δ, mm	Drying regime			m_t, min^{-1}
		t_{med}, °C	v, m/s	φ, %	
Porous ceramics	5–6	90–150	3–10	5	0.1
Sole leather	3–4	4–60	3–5	15	0.04
Asbestos sheets	6–8	90–150	3–10	5	0.06
Clay	10–12	90–150	3–10	5	0.07
Wool felt	8–12	90–150	3–10	5	0.02

$$\mathrm{Rb} = A \exp\left(-n\left(\bar{u} - u_{eq}\right)\right), \tag{5}$$

where $N\tau$ is the generalized time of drying. The constant a_0 for a broad range of moist materials is calculated from the relation

$$a_0 = \frac{8 \cdot 10^{-3}}{\bar{u}_{cr}}, \quad 1‰. \tag{6}$$

The constants A and n in Eq. (5) are determined experimentally, depend on the kind of material, and are independent of the drying regime; they are listed in Table 1 for some materials .

Processing of Experimental Data. The experimental values of the heating rate m_t calculated for some materials from Eq. (2) are presented in Table 2. Substituting Eqs. (4) and (5) into Eq. (3), we obtain

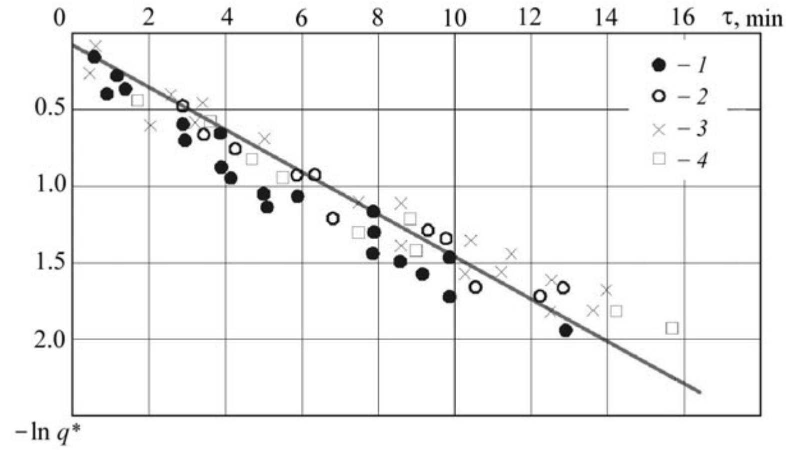

Fig. 2. Time dependence of the relative heat flux in the process of drying porous ceramics (1), asbestos sheets (2), clay (3), and board (4) in different regimes: 1) t_{med} = 90–150°C, v = 3–10 m/s; 2) 90–150 and 3–10; 3) 90–150 and 3–10; 4) 90–110 and 3–10.

$$q^* = \exp(-a_0 N\tau)\,(1 + A\exp(-n(\bar{u} - u_{\text{eq}}))) \,. \tag{7}$$

Consequently, the regularization of the kinetics of heating a moist body occurs not only by temperatures, but also heat fluxes. Equations (4) and (7) also confirm the existence of an analogy between heat and mass transfer in the processes of drying.

Figure 2 presents the experimental dependence of the relative heat flux on time, $\ln q^* = f(\tau)$, in the process of drying porous ceramics, asbestos sheets, clay, and board in different regimes. It is seen that in the regular regime stage all experimental points fit closely to one straight line within the accuracy of experiment. The dependence $\ln q^* = f(\tau)$ is described by the equation

$$q^* = 0.83\exp(-0.12\tau) \,, \tag{8}$$

where τ is the drying time in the second period.

All the curves of the rate of drying for the materials listed in Table 2 refer to type 3 according to A. V. Luikov's classification [3], with their convex surface facing towards the moisture content axis. The curves of the rate of drying fabrics and thin leathers relate to another type and, as experiment shows, the regularization of heat fluxes for such materials obeys other laws. Consequently, the laws governing the occurrence of the regular regime of heating a moist body are determined not only by the shape of the body and the method and regime of drying, but also by the form of the bond between moisture and material.

The processing of experimental temperature curves of a number of materials in convective drying of thin plane materials has shown that the mean integral temperature of a material in the first period $\bar{t}_0$ takes values below the wet-bulb thermometer temperature t_{w} by 1.5–3°C. For the materials listed in Table 2 the mean integral temperature $\bar{t}_0$ in the first period is a linear function of the medium temperature t_{med} and is expressed by the dependence [6, 7]

$$\bar{t}_0 = 10 + 0.28 t_{\text{med}} \,. \tag{9}$$

Solving Eq. (2) for the mean integral temperature of a body $\bar{t}$ for the second period of drying, we obtain the equation of the temperature curve:

$$\bar{t} = t_{\text{med}} - \frac{t_{\text{med}} - \bar{t}_0}{\exp(m_t \tau_{\text{II}})} \,, \tag{10}$$

where τ_{II} is the running time of drying in the second period reckoned from zero and corresponding to the running value of $\bar{t}_0$. The processing of experimental data on drying various moist materials made it possible to establish the relationship between the relative temperature ΔT^* and the relative rate of drying N^* in the form of the equation [8]

$$\Delta T^* = \frac{t_{\rm med} - t_{\rm sur}}{t_{\rm med} - t_{\rm w}} = N^{*0.43} . \tag{11}$$

Neglecting the difference between $t_{\rm w}$ and t_0, we may write

$$\Delta T^* = \frac{t_{\rm med} - t_{\rm sur}}{t_{\rm med} - \bar{t}_0} . \tag{12}$$

Analytical Solution of the Problem. For the stage of the regular regime of heating bodies of any shape the heat balance equation is valid [1, 2]:

$$c_{\rm m}\rho_{\rm m}V\frac{d\bar{t}}{d\tau} = \bar{\alpha}F\,(t_{\rm med} - t_{\rm sur}) = c_{\rm m}\rho_{\rm m}V\,(t_{\rm med} - \bar{t})\,m_t . \tag{13}$$

From the heat balance equation the rate of heating a moist body m_t is equal to

$$m_t = \frac{\bar{\alpha}F}{c_{\rm m}\rho_{\rm m}V}\,\frac{t_{\rm med} - t_{\rm sur}}{t_{\rm med} - \bar{t}} = \frac{\bar{\alpha}a}{\lambda_{\rm m}R_V}\,\psi = \frac{a}{R_V^2}\,{\rm Kn} , \tag{14}$$

where R_V is the ratio between the body volume and surface (the characteristic size of the body for a plate $R_V = \delta$), $\psi = \dfrac{t_{\rm med} - t_{\rm sur}}{t_{\rm med} - \bar{t}}$, ${\rm Kn} = {\rm Bi}\ \psi$, and ${\rm Bi} = \dfrac{\bar{\alpha}R_V}{\lambda_{\rm m}}$. Kondratiev's number Kn is the basic quantity that determines the character of heat exchange of a moist body with the surrounding medium.

In low-intensity processes of heat exchange between a moist body and the surrounding medium ($\alpha \to 0$, ${\rm Bi} \to 0$) the irregularity coefficient $\psi = 1$; in high-intensity processes (${\rm Bi} \to \infty$, $\alpha \to \infty$) the irregularity coefficient $\psi = 0$. At $\psi = 0$ the nonuniformity of temperature distribution is the highest one ($t_{\rm sur} \to t_{\rm med}$) [1, 2]. We write Eq. (14) in the form

$$m_t = \frac{\bar{\alpha}}{c_{\rm m}\rho_{\rm m}R_V}\,\psi , \tag{15}$$

$$m_t = \frac{a}{R_V^2}\,{\rm Kn} . \tag{16}$$

From Eqs. (15) and (16) it follows that the rate of heating a moist body m_t and Kondratiev's number Kn depend on the body shape, its thermophysical characteristics, and on the intensity of heat- and moisture exchange between the body surface and the environment (Bi number).

From the solution of the differential equation for the condition of heating a moist body of simplest shape at $t_{\rm med}$ = const [1], the dependence for the rate of its heating was obtained:

$$m_t = \frac{a}{R_V^2}\,\mu_1^2 . \tag{17}$$

The first root of the characteristic equation μ_1 is determined by the relation [1]

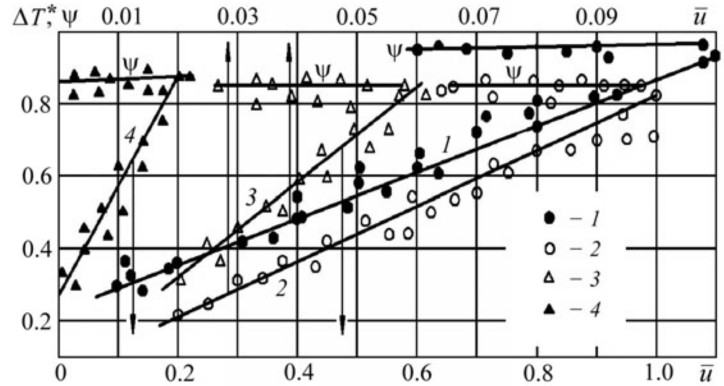

Fig. 3. Relative temperature ΔT^* and the coefficient of irregularity in temperature distribution ψ vs. moisture content $\bar{u}$ in the process of convective drying of porous ceramics (1), clay (2), sole leather (3), and asbestos sheets (4) in different regimes: 1) t_{med} = 90–150°C, v = 3–10 m/s; 2) 90–150 and 3–10; 3) 40–60 and 3–5; 4) 90–150 and 3–10.

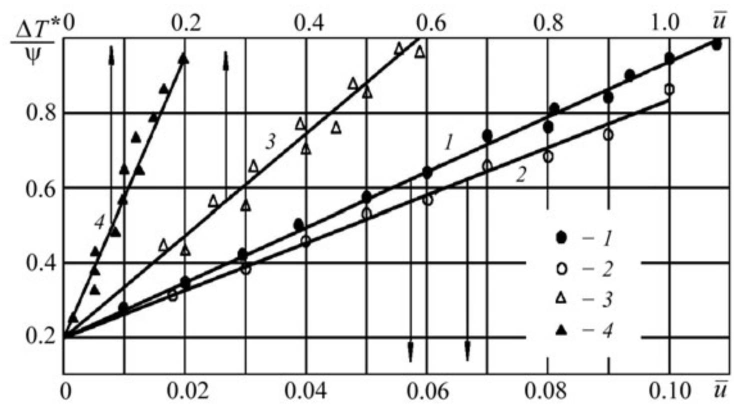

Fig. 4. Ratio $\Delta T^*/\psi$ vs. moisture content $\bar{u}$ in the process of drying porous ceramics, clay, sole leather, and asbestos sheets in different regimes. Symbols 1–4 are same as in Fig. 3.

$$\mu_1^2 = (\mu_1^2)_\infty \left(\frac{1}{1 + A_0/\mathrm{Bi}^k} \right), \tag{18}$$

where $(\mu_1)_\infty$ is the value of μ_1 at Bi = ∞ equal to $\pi/2$ for a plate. The constants A_0 and k for a limited plate take values A_0 = 2.24, k = 1.02 [1].

Discussion of Results. Neglecting the 1.5–3°C-difference between t_w and $\bar{t}_0$, for the relative temperatures, we can write the following expressions:

$$\Delta T^* = \frac{t_{med} - t_{sur}}{t_{med} - \bar{t}_0}, \quad \psi = \frac{t_{med} - t_{sur}}{t_{med} - \bar{t}}. \tag{19}$$

Figures 3 and 4 present the dependences of the relative temperatures ΔT^*, ψ, and $\Delta T^*/\psi$ on the moisture content $\bar{u}$ in convective drying of porous ceramics, clay, sole leather, and asbestos sheets in different regimes. It is seen that the characters of variation of ΔT^* and ψ differ considerably. The irregularity coefficient ψ, practically at the beginning of the second period of drying (regular regime stage), takes values from ψ = 0.75–0.8 to ψ = 0.85–0.94 irrespective of the kind of material and regime of drying.

TABLE 3. Values of Constants C and n_0

Material	C	n_0	Material	C	n_0
Wool felt	0.435	0.5	Sole leather	0.8	0.25
Asbestos sheets	0.75	0.5	Porous ceramics	0.75	0.5
Clay	0.45	0.9			

TABLE 4. Comparison between Experimental and Predicted Values of the Rate of Heating Moist Bodies Determined from Eqs. (16)–(18) and (2)

Material	$a\cdot 10^6$, m^2/s	Bi	$\bar{u}$, kg/kg	$\bar{t}$, °C	ρ_0, kg/m^3	$m_{t\text{pr}}$, s^{-1} (16)	$m_{t\text{pr}}$, s^{-1} (17)	$m_{t\text{pr}}$, s^{-1} (18)	$m_{t\text{exp}}$, s^{-1} (2)
Asbestos sheets, δ = 6–8 mm	0.22	0.15–0.2	0.1–0.2	30–50	770	0.0012	0.00129	0.0011	0.001
Porous ceramics, δ = 5 mm	0.83	0.03–0.05	0.08–0.1	40–55	1560	0.0017	0.00154	0.00145	0.0016
Sole leather, δ = 3–4 mm	0.05	0.2–0.3	0.4–0.6	30–45	900	0.0007	0.0007	0.00077	0.00067
Clay, δ = 10–12 mm	0.51	0.2–0.4	0.08–0.2	30–90	1960	0.001	0.0011	0.00182	0.00116

Consequently, in the regular regime stage at Bi < 0.4 and heat transfer coefficient $\alpha \approx 20$–40 W/($m^2\cdot$°C), a rapid averaging of temperatures over the surface and bulk of the body occurs, and the mean integral temperature $\bar{t}$ tends to the surface temperature t_{sur}. The dependences $\Delta T^* = f(\bar{u})$ have the form of straight lines. The mean bulk temperature $\bar{t}$ tends monotonically to t_{sur}.

The linear dependences depicted in Fig. 4 have a common point from which they spread out fanlike and represent the dependences reciprocal of the irregularity coefficient ψ $\left(\frac{\Delta T^*}{\psi} = \frac{1}{\psi}\right)$.

For the materials presented in Fig. 4, the dependence $\Delta T^*/\psi = f(\bar{u})$ is expressed by the equation

$$\frac{\Delta T^*}{\psi} = K\bar{u} + 0.2\,, \quad u_{\text{eq}} < \bar{u} \le u_{\text{cr}}\,. \tag{20}$$

The parameter K in Eq. (20) for the given moist materials is calculated from the relation

$$K = 9.65 \exp\left(-4.3\bar{u}_{\text{cr}}\right)\,. \tag{21}$$

Let us analyze the analytical solution of the problem on heating a moist body in a medium with t_{med} = const. For all the investigated moist materials, in the regular regime stage (the period of the declining drying rate) Bi ≤ 0.06 for porous ceramics, Bi ≤ 0.25 for sole leather, and Bi ≤ 0.4 for clay and asbestos.

Thus, it would suffice if already at Bi < 0.4 the irregularity coefficient ψ could take the values (Fig. 3) $\psi \to 1$ and, consequently, the Kondratiev number Kn ≈ Bi. In this case, the rate of heating m_t (at $\psi = 1$) is equal to

$$m_t = \frac{\bar{\alpha}}{c_{\text{m}}\rho_{\text{m}}R_V}\,. \tag{22}$$

The heat transfer coefficient $\bar{\alpha}$ was calculated from the formula [6, 8]

$$\text{Nu} = C\text{Re}^{0.5}\left(\frac{T_{\text{med}}}{T_{\text{w}}}\right)^2\left(\frac{\bar{u}}{\bar{u}_{\text{cr}}}\right)^{n_0}, \tag{23}$$

where T_{med} and T_{w} are the absolute temperatures of the medium and of the wet-bulb thermometer, K. The correction $(\bar{u}/\bar{u}_{\text{cr}})^{n_0}$ takes into account the decrease in the heat transfer coefficient $\bar{\alpha}$ with the moisture content. The values of the constants C and n_0 from Eq. (23) are given in Table 3.

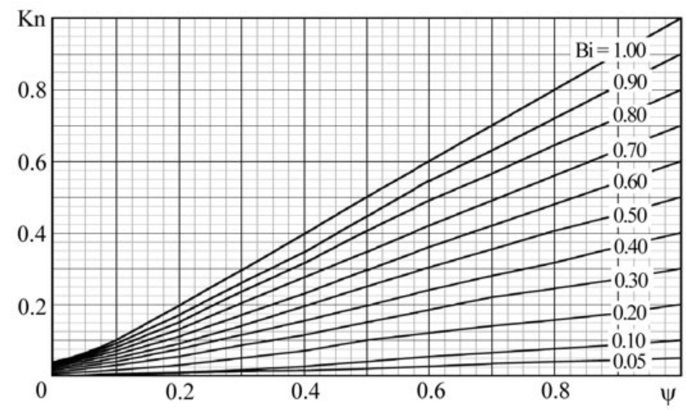

Fig. 5. The Kondratiev number Kn vs. the coefficient of irregular temperature distribution in a body ψ at different values of the Biot number.

A comparison of experimental and predicted values of the rate of heating moist bodies m_t from Eqs. (2), (15)–(17) can be carried out only with the use of accurate and reliable data on the thermal diffusivity $a_m = f(\bar{t}, \bar{u})$ and thermal conductivity $\lambda_m = f(\bar{t}, \bar{u})$, since the dependences of the heat transfer coefficients on the temperature and moisture content of a moist body are complex and nonlinear. In comparing the experimental and predicted values of the rate of heating m_t for the materials listed in Table 4, use was made of the data of [3, 9–11].

A comparison of the experimental and predicted values of the rate of heating m_t of moist bodies from Eqs. (2), (15)–(17) is given in Table 4, from which it is seem that with the use of reliable data on heat transfer coefficients, the analytical solutions also yield quite satisfactory results.

The results of the analytical solution of Eq. (15) on the basis of experimental data are presented in Fig. 5, from which it follows that at Bi < 0.4 the coefficient of irregularity in the distribution of temperature $\psi \to 1$ and the Kondratiev number Kn $\approx$ Bi.

These results of the analytical solution of the problem are confirmed experimentally. At ψ = 0.85–0.92 and Bi < 0.4 for thin moist materials in the regular regime stage the Kn number becomes virtually equal to the number Bi.

Conclusions. The analysis and processing of experimental data on heat transfer in the process of drying moist materials by the regular regime method allow one to establish the laws governing the variation of heat fluxes and mean integral temperatures in the period of declining drying rate. It has been established that the rate of heating a moist body m_t is independent of the drying regime and is a constant quantity, m_t = const. The analytical solution of the problem on the rate of heating moist bodies agrees well with the experiment, confirming its main trends. The use of this method imparts a generalized character to the study of the process of drying at a minimum number of constants A, A_0, a, a_0, C, n, n_0, k, and K determined experimentally.

NOTATION

a, thermal diffusivity, m^2/s; Bi, Biot number; c_m, heat capacity of a moist material, kJ/(kg·^{o}C); F, surface area of a moist material, m^2; Kn, Kondratiev number; m_t, rate of heating a moist body, min^{-1}, s^{-1}; N, rate of drying in the first period, min^{-1}; N^*, relative rate of drying; Nu, Nusselt number; $q_{\rm I}$ and $q_{\rm II}$, heat flux densities in the first and second periods of drying, W/m^2; q^*, relative flux; Re, Reynolds number; $\bar{t}_0$ and $\bar{t}$, mean integral temperature of a moist material in the first period and its mean integral running temperature, ^{o}C; t_{med}, t_w, and t_{sur}, temperature of the medium, wet bulb, and of the material surface, ^{o}C; $d\bar{t}/d\tau$, rate of change in the material temperature, ^{o}C/s; ΔT^*, relative temperature of material; $\bar{u}_{cr}$, u_{eq}, and $\bar{u}$, critical, equilibrium and running moisture content of material, kg/kg; V, moist body volume, m^3; v, heat carrier velocity, m/s; $\bar{\alpha}$, heat transfer coefficient, W/(m^2·^{o}C); γ, slope of the ln $(t_{med} - \bar{t})$ straight lines to the drying time axis; δ, material thickness, mm; λ_m, thermal conductivity of a moist body, W/(m^2·^{o}C); ρ_0, dry body density, kg/m^3; ρ_m, moist body density, kg/m^3; τ, running time of drying, min; $\tau_{\rm I}$ and $\tau_{\rm II}$, time of drying in the first and second periods, min; φ, relative humidity of air, %; ψ, coefficient of irregularity in temperature distribution over a body. Indices: 0, initial state; cr, critical; w, wet; sur, surface; m, moist; med, medium; eq, equilibrium; exp, experimental; pr, predicted.

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