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LAWS GOVERNING THE FRAGMENTATION OF A DROPLET IN A HIGH-SPEED STREAM

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A common solution of the equations of ablation and motion of a droplet undergoing fragmentation in a homogeneous high-speed gas stream is obtained. Based on the approximate solution of the equation for the quantity of torn off droplets, an expression for the droplet size distribution function corresponding to these laws has been found. A comparison of the results of calculations carried out for the obtained distribution function with the same calculations for the earlier found function with the use of the empirical law of droplet motion points to their satisfactory agreement.

Keywords: *fragmenting droplet, laws of motion and ablation, daughter droplets, size distribution function.*

Introduction. In [1, 2], basic equations of the kinetics of the fragmentation of droplets in high-speed gas streams were derived. With some simplifying assumptions, the law of variation of the mass of a fragmenting droplet $m(t)$ (law of ablation) and the differential size distribution function of the quantity of torn off droplets $f_n(r)$ were obtained. The investigation showed that the kinetic laws of fragmentation are substantially influenced by the law of motion of a droplet in a stream which assigns the change of the factor basic for dispersion, namely, the relative velocity of the gas and droplet $V_\infty - w(t)$. The derivation of the mentioned relations was based on the use, as the law of droplet motion $x_{\text{dr}}(t)$, of dependences that approximate experimental data. In this case, despite the rather good quantitative representation, the possibility of their functional form to influence the result obtained arises. At the same time, the experimental database does not contain sufficient information on the dependence $x_{\text{dr}}(t)$ for the entire set of investigated gas–droplet systems with their diverse physicomechanical properties. The determination of the dependence $x_{\text{dr}}(t)$ for a fragmenting droplet by theoretical methods is complicated by the substantial influence, on the law of its motion, of the laws governing the deformation and ablation, which leads to the necessity of simultaneous solution of the equations for these three processes. We are not aware of attempts at compiling and finding an analytical solution of such a system of nonlinear differential equations, whereas an analysis of a numerical solution would have been difficult because of the multiparameter nature of the problem. At the same time, mathematical simulation of the processes of heat and mass transfer in the spray of a fragmenting droplet, while being of primary importance for investigating such phenomena as heterogeneous detonation, flows of two-phase mixtures in jet engines, and others, requires the finding of the size distribution function of all torn off droplets and its time–space evolution in the aerodynamic wake of the droplet. In the present work, disregarding the influence of droplet deformation, we obtain analytically the basic laws governing the kinetics of fragmentation as solutions of a system of differential equations of ablation, droplet motion, and of the quantity of daughter droplets.

Simultaneous equations of deformation and motion of a droplet that apply to high-speed flows and that take into account the mass loss were derived in [3], but they presuppose the a priori assignment of the law of mass variation $m(t)$. In [4], with application of the method of asymptotic expansions, a joint solution of the equations of deformation and motion was obtained; however, the ablation, which exerts a substantial effect on both processes, was not taken into account. Most often, the deformation of a droplet was considered for low-intensity disintegration regimes ("collapse," "parachute," "claviform"), in particular, in determining the value of the Weber number which is critical for the existence of fragmentation [5, 6], when deformation can attain the values $d \approx (3–4)d_0$. Experimental investigations with application of an X-ray apparatus showed [7] that in high-speed flows, where dispersion is intense (following the "peeling" type), deformation plays a minor role, since it does not exceed the values $d \le 2d_0$.

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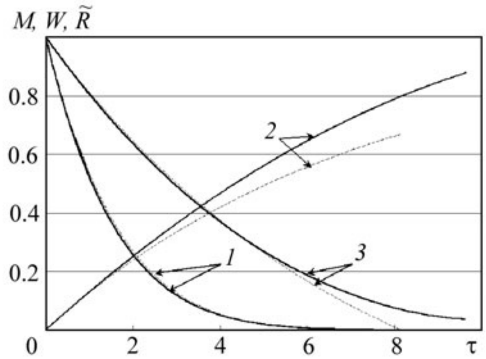

Fig. 1. Functions $M(\tau)$ (curves 1), $W(\tau)$ (curves 2), and $\tilde{R}(\tau)$ (curves 3) at h = 1.5. Solid curves, dependences (3) and dashed curves — (12) from [1].

Law of Ablation of a Droplet Moving under the Action of Aerodynamic Head of the Flow. We write the equation of motion of the spherical droplet mass center under the action of the force of aerodynamic head of a homogeneous gas stream:

$$\rho_{\text{liq}} \frac{4}{3} \pi R^3 (t) \frac{dw}{dt} = C_{\text{d}} \pi R^2 (t) \frac{\rho_{\text{g}} (V_\infty - w(t))^2}{2} . \quad (1)$$

Passing to dimensionless variables, together with the equation of mass entrainment derived in [1] for a spherical droplet ($M = \tilde{R}^3$), we obtain the system

$$\frac{dW}{d\tau} = C \frac{(1 - W)^2}{\tilde{R}} ,$$
$$\frac{dM^{1/3}}{d\tau} = \frac{d\tilde{R}}{d\tau} = -\frac{A}{3} (1 - W) , \quad (2)$$

where A is the characteristic (initial) velocity of mass entrainment, $C = 0.75\sqrt{\alpha}\, C_{\text{d}}$. Excluding the velocity from Eq. (2), we obtain a differential equation of second order for the droplet radius: $\ddot{\tilde{R}} = \frac{\dot{\tilde{R}}^2}{h\tilde{R}}$, where $h \equiv \frac{A}{3C}$ is the parameter that characterizes the ratio of the rates of the processes to for fragmentation — dispersion and relaxation equalization of the gas and droplet velocities [1] — the dot signifying differentiation with respect to τ. The solution of this equation with the initial conditions $\tilde{R} = 1$, $\dot{\tilde{R}} = -\frac{A}{3}$ at $h = 1$ has an exponential form: $\tilde{R} = \exp\left(-\frac{A\tau}{3}\right)$; then Eq. (2) yields $1 - W = \exp(-C\tau)$. Therefore, in the case of $h = 1$ and equality of the rates of the progress of the indicated processes, the laws of the motion of a droplet and of the entrainment of its mass, and, consequently, the size distribution function of droplets coincide identically with analogous ones obtained in [1] with the use of the empirical relaxation law of droplet motion.

At $h \neq 1$, integration of (2) with the initial conditions $\tilde{R}(0) = 1$ and $W(0) = 0$ leads to the exponential dependences:

$$\tilde{R}(\tau) = (1 - C(h-1)\tau)^{h/(h-1)} , \quad W(\tau) = 1 - (1 - C(h-1)\tau)^{1/(h-1)} . \quad (3)$$

Relations (3) are applicable up to the moment of cessation of dispersion τ_{dis}, when, as follows from the expression obtained below for τ_{dis}, the remainder of the droplet is small.

Figure 1 presents the functions $M(\tau)$ (curves 1), $W(\tau)$ (curves 2), and $\tilde{R}(\tau)$ (curves 3) calculated at h = 1.5 for identical initial accelerations of the droplet $C = H$ from Eqs. (3) and from the corresponding equations (12) obtained in [1]. A comparison shows that the higher acceleration of the droplet in the first case gradually leads to the slowing down of mass entrainment, which exerts its influence on the function $\tilde{R}(\tau)$ only at the final stage of fragmentation, when the size and mass of the droplet residue are small.

Distribution Function. It is of interest to find the size distribution function of the quantity of daughter droplets $f_n(\tilde{r})$ for the case where the mass entrainment and the motion of a droplet obey the laws (3) and to compare this function with the analogous one obtained in [2] for the empirical law of motion of the form $W = 1 - \exp(-H\tau)$. We write the equations for the quantity of torn off droplets n and their radii $\tilde{r}$ that were derived in [1]:

$$\dot{n}' = B_1^3 B_2 \frac{\tilde{R}^2(\tau)\,(1 - W(\tau))}{\tilde{r}^3} \sin^2\varphi\,, \quad B_1 = \frac{4.4\pi k_r \alpha^{1-2\xi}}{\Delta_m(\mathrm{We}_{int})\sqrt{2\mathrm{Re}_\infty}}\,, \tag{4}$$

$$\tilde{r}(\varphi, \tau) = B_1 T(\tau)\,\Psi(\varphi)\,, \quad T(\tau) = \sqrt{\frac{\dot{\tilde{R}}(\tau)}{1 - W(\tau)}}\,, \quad B_2 = \frac{0.21\Delta_m^2(\mathrm{We}_{int})\,\mathrm{Im}\,z_m(\mathrm{We}_{int})\sqrt{2\mathrm{Re}_\infty^3}}{\pi k_r k_t \alpha^{3.5(1-2\xi)}(1 + \alpha^{\xi})}\,, \tag{5}$$

where B_1 and B_2 are the parameters having the meaning of the characteristic scales of dimensions and of the quantity of daughter droplets, respectively; the prime means differentiation with respect to φ. The quantity of droplets contained in the elementary fraction $\Delta\tilde{r}$ is found by integration of Eq. (4) in the band of width $\Delta\tilde{r}(\varphi, \tau)$ that surrounds the line $\tilde{r}(\varphi, \tau)$ = const on the plane (φ, τ). Under the conditions of intense dispersion, when the values of the gradient instability criterion considerably exceed the critical value, $\mathrm{GN} \gg \mathrm{GN}_{cr} \approx 0.3$, we may consider that the wave number Δ_m and the increment of increase in the amplitude of dominant unstable perturbation $\mathrm{Im}(z_m)$ are constant over the greater part of the droplet surface; then B_1, $B_2 \approx$ const [1]. Multiplying Eq. (4) by $\Delta\varphi$ and by $\Delta\tau = \frac{\Delta\tilde{r}}{B_1\dot{T}(\tau)\Psi(\varphi)}$ obtained from Eq. (5) by differentiation at φ = const and B_1 = const and integrating over φ along the line $\tilde{r}(\varphi, \tau)$ = const with the use of Eq. (5), we obtain

$$\Delta n = \frac{6hB_1^3 B_2}{A(h-1)\,\tilde{r}^4} \int_{\varphi_{low}}^{\varphi_{up}} (1 - C(h-1)\,\tau(\varphi))^{\eta} \sin^2\varphi\, d\varphi\, \Delta\tilde{r}\,, \tag{6}$$

where $\eta = \frac{3h}{h-1}$, whereas $\varphi_{low}(\tilde{r})$ and $\varphi_{up}(\tilde{r})$ are selected different for the regions A and B that form the basic and additional ranges of the distribution of daughter droplets [2]. By virtue of the function $\tau(\varphi)$ along the line $\tilde{r}(\varphi, \tau)$ = const following from Eq. (5), the integral in Eq. (6) can be calculated only approximately. Similarly to the approximation technique applied in [2], we approximate the integration path $\tau = \tau(\varphi)$ by the straight line $\tau - \tau_{low} = \frac{\varphi - \varphi_{low}}{a_{ef}}$ with a certain effective value $a_{ef}(\tilde{r})$ of its inclination to the axis τ. In Eq. (6) under the sine sign we pass to a double angle and avail ourselves of the tabulated integral [8] taking place for natural η's:

$$\int P_\eta(\varphi)\cos 2\varphi\, d\varphi = \frac{\sin 2\varphi}{2} \sum_{k=0}^{E(\eta/2)} (-1)^k \frac{P_\eta^{(2k)}(\varphi)}{2^{2k}} + \frac{\cos 2\varphi}{2} \sum_{k=1}^{E((\eta+1)/2)} (-1)^{k-1} \frac{P_\eta^{(2k-1)}(\varphi)}{2^{2k-1}} \equiv F_\eta(\varphi)\,,$$

where $P_\eta(\varphi) = (b + c\varphi)^\eta$, $P_\eta^{(k)}$ is its kth derivative, $b = 1 - C(h-1)\left(\tau_{low} - \frac{\varphi_{low}}{a_{ef}}\right)$, $c = -\frac{C(h-1)}{a_{ef}}$, and $\mathrm{E}(\eta)$ is the integral part of η. Then, for the differential distribution function we obtain

$$f_n(\tilde{r}) = \frac{\Delta n}{\Delta \tilde{r}} = \frac{3hB_1^3B_2}{A(h-1)\tilde{r}^4}\left[\frac{1}{c(\eta+1)}P_{\eta+1}(\varphi) - F_\eta(\varphi)\right]_{\varphi_{\text{low}}}^{\varphi_{\text{up}}}. \quad (7)$$

The dependence on $\tilde{r}$ within the squared brackets consists of the values $b(\tilde{r})$, $c(\tilde{r})$, $\varphi_{\text{low}}(\tilde{r})$, and $\varphi_{\text{up}}(\tilde{r})$ and is attributable to their dependence on $a_{\text{ef}}(\tilde{r})$.

A number of discrete values of h, $1 < h = \frac{\eta}{\eta - 3} \le 4$, correspond to the natural values of $\eta > 3$. For integer-valued $\eta < 0$ we obtain a number of values of h belonging to he interval $0.25 \le h < 1$ of incomplete fragmentation regimes; in this case the integral in (6) is expressed via the integral sine and cosine [8]. The indicated set of values of η rather completely covers the entire practically important range of the values of h.

The principle of the determination of $a_{\text{ef}}(h, \tilde{r})$ remains unchanged and at $h > 1$ leads to the same equation for the region A on the plane (φ, τ) that forms the basic range, just as in the case of the empirical law of droplet motion in [2]:

$$a_{\text{ef}} = \frac{\left(h - 1 + \frac{1}{h^2} + \frac{k(2h-1)\sqrt{|h-1|}}{h^2}\right)a_{\text{av}}a_{\text{low}}}{(h-1)a_{\text{low}} + \frac{a_{\text{av}}}{h^2} + \frac{(a_{\text{av}} + a_{\text{low}})\sqrt{|h-1|}}{h}} \quad (8)$$

with the value of $k = 1.08$. In the region B, where in the later stage of fragmentation finely divided fractions are formed, the dependence is somewhat different:

$$a_{\text{ef}} = \frac{(k_1h + k_2)\,a_{\text{av}}a_{\text{low}}}{ha_{\text{low}} + a_{\text{av}}}. \quad (9)$$

The difference is due to the changes in the droplet acceleration and correspondingly in the kinetics of mass entrainment in the later stage, as shown in Fig. 1; the values $k_1 = 1.133$ and $k_2 = 0.867$ for $h > 1$ remain unchanged.

For more practically interesting regimes of complete fragmentation at $h > 1$ Eq. (7) is simplified at the upper limit $\varphi_{\text{up}} = \frac{\pi}{2}$, where $\sin\varphi_{\text{up}} = 0$, $\cos\varphi_{\text{up}} = -1$, and at the lower limit for the basic range, where $\tau_{\text{low}} = 0$ and $P_\eta = (\varphi_0) = 1$. The following are the final formulas for $f(\tilde{r})$ at $h = 1.50$ $(\eta = 9)$ in the region A:

$$f_n(\tilde{r}) = \frac{3hB_1^3B_2}{2A(h-1)\tilde{r}^4}\left[\frac{A_1^{10} - 1}{10c} - \left(\frac{9}{2}cA_1^8 - 63c^3A_1^6 + \frac{945}{2}c^5A_1^4 - \frac{2835}{2}c^7A_1^2 + \frac{2835}{4}c^9\right)\right.$$
$$\left. - \left(\frac{9}{2}c - 63c^3 + \frac{945}{2}c^5 - \frac{2835}{2}c^7 + \frac{2835}{4}c^9\right)\cos 2\varphi_{\text{low}} - \left(1 - 18c^2 + 189c^4 - 945c^6 + \frac{2835}{2}c^8\right)\sin 2\varphi_{\text{low}}\right], \quad (10)$$

here $A_1 = 1 + c\left(\frac{\pi}{2} - \varphi_0\right)$, and in the region B:

$$f_n(\tilde{r}) = \frac{3hB_1^3B_2}{2A(h-1)\tilde{r}^4}\left[\frac{A_2^{10} - A_3^{10}}{10c} - \left(\frac{9}{2}cA_2^8 - 63c^3A_2^6 + \frac{945}{2}c^5A_2^4 - \frac{2835}{2}c^7A_2^2 + \frac{2835}{4}c^9\right)\right.$$
$$- \left(\frac{9}{2}cA_3^8 - 63c^3A_3^6 + \frac{945}{2}c^5A_3^4 - \frac{2835}{2}c^7A_3^2 + \frac{2835}{4}c^9\right)\cos 2\varphi_{\text{low}}$$

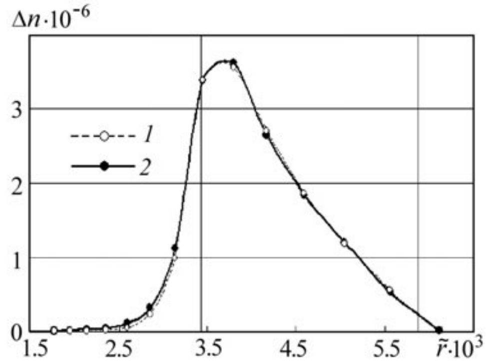

Fig. 2. Distribution of the quantity of torn off droplets $\Delta n(\tilde{r})$ calculated at h = 1.5 from Eqs. (7)–(9) (1) and from Eqs. (4)–(8) of [2] (2). The vertical straight lines, the limits of the basic range $\tilde{r}_{0\text{left}} = 3.43\cdot10^{-3}$, $\tilde{r}_{0\text{right}} = 5.87\cdot10^{-3}$.

$$-\left(A_3^9 - 18c^2A_3^7 + 189c^4A_3^5 - 945c^6A_3^3 + \frac{2835}{2}c^8A_3\right)\sin 2\varphi_{\text{low}}\Bigg]. \tag{11}$$

Here $A_2 = 1 + c\left(\tau_1 a_{\text{ef}} + \frac{\pi}{2} - \varphi_1\right)$, $A_3 = 1 + c\tau_1 a_{\text{ef}}$, $\varphi_1 = \varphi_1(\tau)$ is the equation of the left boundary of the dispersion region [2]. In conformity with the general procedure, $\tau_{\text{low}} = \tau_1$ in the region B is defined as the moment of the intersection of the lines $\varphi_1(\tau)$ and $\tilde{r}(\varphi, \tau)$ = const and can be found from the system of equations (5)–(6) of [2].

Similarly to [2], Eq. (7) can yield formulas for the intermediate distribution of droplets torn off up to the arbitrary moment τ_c; for this purpose, it is necessary to assume that $\varphi_{\text{up}} = \varphi_{\text{up}}(\tau_c)$.

Using Eqs. (7)–(11), we calculated the distributions for the values h = 1.5, h = 2.0, and h = 4.0. Figure 2 presents the size distributions of all torn off droplets $\Delta n(\tilde{r})$, calculated in conformity with the two techniques discussed in the present work, of determining the law of droplet motion. A comparison shows good agreement, whereas an insignificant discrepancy in the range of finely dispersed fractions is explained by the above-mentioned difference in the relative velocity $1 - W$ in the late stage, when the finely dispersed fractions are just formed at $h > 1$ [2].

The moment τ_{dis} is determined from the condition of disappearance of the dispersion region $\varphi_1(\tau_{\text{dis}}) = \frac{\pi}{2}$ [2], which leads to the equation $\tilde{R}(\tau_{\text{dis}})(1 - W(\tau_{\text{dis}}))^3 = \left(7.9\cdot10^{-4}\frac{(1 + (\alpha\mu)^{1/3})^2}{\alpha\text{GN}}\right)^2 \equiv Z$. Using Eq. (3), we obtain

$$\tau_{\text{dis}} = \frac{3h}{A\,(h-1)}\left(1 - Z^{(h-1)/(h+3)}\right). \tag{12}$$

From Eq. (4), integrating over the entire region $\varphi_{10} < \varphi \le \frac{\pi}{2}$, $0 < \tau < \tau_{\text{dis}}$, we find the upper estimate for the total number $N = \sum_{\Delta\tilde{r}=0}^{\tilde{r}_{0\text{int}}} \Delta n$ of torn off droplets:

$$N \approx B_2 \int_0^{\tau_{\text{dis}}} \sqrt{\tilde{R}\,(1 - W(\tau))^5} \int_{\varphi_{10}}^{\pi/2} \frac{\sin^2\varphi}{\Psi^3(\varphi)}\, d\varphi\, d\tau\,, \tag{13}$$

Here $\varphi_{10} = \varphi_1(0)$. Using the approximation of the function $\Psi^3(\varphi) = \left(\frac{13.59\sin^2\varphi}{\sin^2(1.53\varphi)}\right)$ proposed in [2] and Eqs. (3), we find

$$N \approx 0.047 \frac{2hB_2}{A\,(h+1)} \left(1 - Z^{3(h+1)/2(h+3)}\right)\left(1.45 - 0.76\varphi_{10} + 0.25\,\sin\,(3.05\varphi_{10})\right). \tag{14}$$

At $h = 1$ Eq. (14) practically coincides with the analogous one obtained in [1] for the case $h = 1$.

The value of Z is small, and at $\mathrm{GN} >> \mathrm{GN}_{cr} \approx 0.3$ it is negligibly small, allowing one to simplify Eqs. (12) and (14). It should be noted that at GN > 3 the quantity Z can be expressed only via the initial position of the critical point on the droplet surface: $Z \approx 0.34\varphi_{10}^4$.

Conclusions. The investigation carried out has shown that the two techniques determining the law of droplet motion, based on the empirical and theoretical methods, result in close distribution functions. The absence of a complete, at least to some extent, experimental database on the functions $x_{dr}(t)$ noted in the Introduction allows one to prefer those obtained in the present work by solving a system of differential equations of motion, ablation, and of the number of torn off droplets to approximate theoretical formulas for the distribution function which can be used for various combinations of the physicomechanical properties of gas–droplets systems. These equations allow one to find all the main statistical characteristics of the nonstationary spray of a droplet required for constructing its mathematical model and describing the kinetics of subsequent processes of evaporation of the whole set of torn off droplets, convection of vapors, and mixture formation in the aerodynamic wake of a fragmenting droplet.

NOTATION

A, B, characteristic regions on the plane of events (φ, τ) [2]; A, dimensionless initial velocity of mass entrainment [1]; A_1, A_2, A_3, auxiliary parameters; $a_{ef}(\tilde{r})$, effective value of the inclination of the curve $\tilde{r}(\varphi, \tau)$ = const; b, c, formal parameters; B_1, B_2, dimensionless parameters; C, characteristic dimensionless acceleration of a droplet; C_d, coefficient of aerodynamic resistance; d, droplet deformation; f_n, differential size distribution function of a droplet; $\mathrm{GN} \equiv \mathrm{We}_\infty/\sqrt{\mathrm{Re}_\infty}$, criterion of the appearance of gradient instability on the droplet surface; H, dimensionless acceleration [1]; $\mathrm{Im}(z_m)$, increment of the growth of the amplitude of a dominating perturbation; k_r, k_t, proportionality factors [1]; m, mass of a droplet; $M = m/m_0$; n, quantity of daughter droplets; N, total number of droplets; $\dot{n}'$, rate of production of droplets on an elementary area of the droplet surface; R_0, initial radius of a droplet; R, current radius of a droplet; $\tilde{R} = R/R_0$; r, radius of a torn off droplet; $\tilde{r} = r/R_0$; $\tilde{r}_{0\mathrm{left}}$, $\tilde{r}_{0\mathrm{right}}$, left and right boundaries of the basic range of distribution; $\mathrm{Re}_\infty = \rho_g V_\infty 2R_0/\mu_g$, Reynolds number for a paternal droplet; $T(\tau)$, dimensionless function; t, time; t_{dis}, time of dispersion cessation; $t_{ch} = 2R_0/\sqrt{\alpha}\, V_\infty$, characteristic time of the process; V_∞, gas flow velocity; w, droplet velocity; $W = w/V_\infty$; $\mathrm{We}_\infty = \rho_g V_\infty^2 2R_0/\sigma$, Weber number for a paternal droplet; $\mathrm{We}_{int} = \rho_g V_\infty^2 2R_0/\sigma$, "surface" Weber number [1]; x_{dr}, displacement of a droplet; Z, auxiliary parameter; z, unstable root of the characteristic equation for perturbations; $\alpha = \rho_g/\rho_{liq}$, density ratio; $\Delta_m \equiv k_m\delta_{liq}$, dimensionless wave number; Δn, number of torn off droplets in an elementary range; Δr, δ_{liq}, boundary layer thickness in a liquid; $\eta = 3h/(h-1)$, parameter; $\mu_{g,liq}$, dynamic coefficients of the viscosity of media; $\mu = \mu_g/\mu_{liq}$; $\xi = \log_\alpha(\alpha\mu)^{1/3}$, parameter; ρ, density; σ, surface tension coefficient; $\tau = t/t_{ch}$, dimensionless time; τ_c, current moment of time; φ = polar angle under which the elementary area is seen form the droplet center; $\varphi_1 = \varphi_1(\tau)$, equation of the left boundary of the region of dispersion [2]; $\Psi(\varphi) \equiv \sqrt{(6\varphi - 4\sin 2\varphi + 0.5\sin 4\varphi)/\sin^5\varphi}$. Indices: av, average; c, current moment; ch, characteristic; cr, critical conditions; d, aerodynamic resistance; dis, dispersal; dr, parental droplet; ef, effective; g, gas parameters; int, value on the gas–liquid interface surface; liq, parameters of a liquid; low, lower limit of integration; m, parameter of dominating perturbation; r, radius; t, time; up, upper limit of integration; 0, initial; ∞, values of parameters of an incoming flow.

REFERENCES

1. A. G. Girin, Equations of the kinetics of droplet fragmentation in a high-speed gas flow, *Inzh.-Fiz. Zh.*, **84**, No. 2, 248–254 (2011).
2. A. G. Girin, Distribution of dispersed droplets in fragmentation of the drop in a high-velocity gas flow, *Inzh.-Fiz. Zh.*, **84**, No. 4, 805–812 (2011).
3. V. V. Mitrofanov, Equation of liquid droplet deformation in a gas flow behind a shock wave, in: *Dynamic Problems of Continuum Mechanics*, Issue 39, 76–87, Novosibirsk (1979).

4. G. A. Simons, Acceleration and deformation of a droplet, *Raketn. Tekh. Kosmonavt.*, **14**, No. 2, 178–180 (1976).
5. K. A. Gordin, A. G. Istratov, and V. B. Librovich, Concerning the kinetics of liquid droplet deformation and disintegration in a gas flow, *Izv. Akad. Nauk SSSR*, *Mekh. Zhidk. Gaza*, No. 1, 8–16 (1969).
6. M. S. Volynskii and A. S. Lipatov, Deformation and disintegration of liquid droplets in a gas flow, *Inzh.-Fiz. Zh.*, **18**, No. 5, 838–843 (1970).
7. W. G. Reinecke and G. D. Waldman, Shock layer shattering of cloud drops in reentry flight, *AIAA Paper*, No. 152 (1975).
8. I. S. Gradshtein and I. M. Ryzhik, *Tables of Integrals, Sums, Series, and Products* [in Russian], Nauka, Moscow (1971).