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On the social welfare interpretation of growth incidence curves

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Abstract
The Growth Incidence Curve (GIC), introduced in the poverty measurement literature by Ravallion and Chen (Econ. Lett. **78**(1), 93–99, 2003), proved to be a valuable and widely used tool to analyze the impact of growth on poverty and its 'pro-poorness'. Beyond pro-poorness, however, the relationship between the shape of GICs and social welfare is ambiguous. If a declining GIC, together with a positive overall rate of growth, is unambiguously associated with a social welfare gain, such a shape is not the most common and the reciprocal is not necessarily true. This paper analyzes the social welfare properties of GICs, as well as their non-anonymous counterpart (NAGICs), which describe how income growth depends on the initial rank of individuals in the initial income distribution. NAGICs thus account not only for the change in the distribution of income but also for income mobility, and differ conceptually from their anonymous counterpart. However, their social welfare interpretation proves to be very similar.

Keywords Growth · Mobility · Inequality · Social welfare

JEL Classification D3 · H0 · J6

1 Introduction

By the number and breadth of his contributions, by both their academic and policy relevance and, most importantly, by his personal engagement in the monitoring of global poverty, Martin Ravallion played a decisive role in the collective fight against poverty. As a tribute to Ravallion, we elaborate in this paper upon two key concepts or tools that will bear his mark for still a long time to come: the growth incidence curves, introduced in Ravallion and Chen (2003), and the 'pro-poorness' concept as discussed in his widely cited 2004 World Bank

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paper (Ravallion, 2004). Doing so, however, we are aware that we touch upon only a small fraction of his contribution to the economics of poverty.

There are two ways of evaluating changes in the distribution of income in a given population, depending on whether initial income levels are taken into account. If they are ignored, as when working with successive cross sections of the population, the new distribution is compared to the original one without any possibility to account for the identity of income earners. The comparison is thus ‘anonymous’. Poorest people in the initial period are compared to poorest incomes in the final period whoever they may be, the same being true of middle-income earners and of the rich. In the other case, if panel data are available, the comparison can be made between the initial and final distributions conditionally on initial incomes. The comparison is thus ‘non-anonymous’.

Analytical tools adapted to the anonymous case are familiar. In redistribution analysis, the move from the old to the new distribution may be considered as unambiguously increasing social welfare if and only if it corresponds to an upward shift of the Lorenz curve when mean income remains unchanged (or the generalized Lorenz curve otherwise).

Things are less simple when comparing initial and final incomes in a non-anonymous way. The average income of the poorest people may have increased between the two periods, but what should be done about the fact that some of today’s poor were not poor before and have suffered an income loss? Could such losses affect social welfare negatively?

It has become common to represent the anonymous distributional change brought by economic growth by the Growth Incidence Curve (GIC), initially introduced by Ravallion and Chen (2003). This very convenient tool simply shows the rate of income growth of sequential quantiles of the distribution between the initial and final periods. In their initial application, in order to analyze the pro-poorness of growth, Ravallion and Chen (2003) focused on the area below the GIC and to the left of the quantile corresponding to the poverty headcount ratio. More generally, however, the question is whether there is a relationship between the shape of the GIC and an unambiguous increase in social welfare in the usual utilitarian sense. It is easily shown for instance, and it is intuitively obvious, that this is necessarily the case if the GIC is everywhere positive or downward sloping with a positive average growth rate. The reciprocal is not true, however, and monotonically downward-sloping GICs may not be common, especially when defined with some minimum level of granularity. A more rigorous criterion is the positivity of the Poverty Growth Curve, introduced by Son (2004), which may be interpreted as the income weighted mean growth rate of the poorest people, when the poverty headcount goes from 0 to 1.

Non-anonymous Growth Incidence Curves (NAGICs) have been introduced independently in the economic literature by Grimm (2007), Van Kerm (2009) and Bourguignon (2011). They have been widely used since then, especially with the increasing availability of income panel data, *e.g.*, Palmisano and Peragine (2015); Jenkins and Van Kerm (2016). They show the expected growth rate of individual incomes ranked by quantiles of the initial distribution.

It can be seen at the outset that the social-welfare concepts behind the two representations of the distributional impact of growth are different. The NAGICs rely on the joint distribution of initial and final incomes and thus convey two types of information: a) the change in the distribution of income and b) the mobility of income recipients across income levels or ranks. By contrast, the GICs bear only on a). Analyzing NAGICs thus refers to two strands of the income distribution literature: anonymous income distribution comparisons, on the one hand, and income mobility on the other. Because of this, it could be expected that the welfare interpretation of the shape of NAGICs differs from that of GICs’. Yet, this paper will show that the shape of both GICs and NAGICs can be interpreted in a similar way in terms of social welfare, even though they command different social welfare concepts.

As a matter of fact, this paper first shows that, even though the social welfare functions that evaluate GICs and NAGICs are conceptually different, they point to strictly identical relationships between the shape of the two types of curve and the social welfare respectively associated with them.

Secondly, a list of necessary and/or sufficient conditions which relate the shape of the GICs and NAGICs (and other curves directly derived from them) to social welfare is provided. Such conditions could be in absolute terms, *i.e.*, positive, or in relative terms, *i.e.*, in comparison to a benchmark case. Some of these conditions are particularly interesting in the sense that they generalize the 'pro-poorness' concept introduced by Ravallion and Chen (2003) to the whole income range, rather than to the range below some predefined poverty line. With respect to the NAGIC, the analysis in that part of the paper belongs to the recent literature on pro-poor growth that takes explicitly into account individual income paths, including Palmisano and Peragine (2015); Jenkins and Van Kerm (2016); Palmisano and Van de Gaer (2016); Bresson et al. (2019); Lo Bue and Palmisano (2020). That literature establishes normative dominance criteria that permit the comparison of growth spells in a non-anonymous way. This paper shows that these criteria are identical to those obtained from assuming concave social welfare functions under the restriction that the latter exhibit an aversion to inequality greater than unity. This assumption is also found in the literature on normative measures of income mobility, *e.g.*, in Ray and Genicot (2023).

Third, an interesting decomposition of NAGICs is borrowed from Berman and Bourguignon (2023). It splits NAGICs into a pure distribution neutral growth effect, *i.e.*, a change due exclusively to rank mobility, and a change in the marginal distribution of incomes. This decomposition has implications for the shape of NAGICs and the way they relate to social welfare.

The final section of the paper applies the criteria developed in the preceding sections to evaluate income growth in the US, based on data from the Panel Study of Income Dynamics (PSID 2018) during the 1980s. This is a particularly interesting period which witnessed a fast increase of income inequality. In such a context, it is striking that the NAGIC points to an unambiguous increase in social welfare, whereas the GIC points to the opposite. This suggests that the social welfare interpretation of the shape of the NAGIC may implicitly give a weight to its pure mobility component that may be found excessive with other normative criteria. This raises the issue of how to properly balance mobility and inequality. Another practical conclusion is that, although most helpful in describing the distributional patterns of growth spells, the shape of GICs and NAGICs may not be directly interpretable in terms of social welfare. The Cumulative Income-weighted Mean Growth Curves that show the growth rate of the mean income, or the mean growth rate of the poorest incomes, when the poverty line moves along the whole income range, have a more direct interpretation.

2 Growth incidence curves: social welfare and inequality related properties

Consider a two-period (t, t') panel income dataset. Let the joint density of incomes in the two periods be $f(y_t, y_{t'})$ with marginal cdfs $F_t(.)$ and $F_{t'}(.)$, and conditional distribution of terminal income on initial income, $\Phi_{t'}(. \mid y_t)$. Use will be made below of the copula of that distribution defined by:

$$R_f\left[p, p'\right] = f\left[F_t^{-1}(p), F_{t'}^{-1}(p')\right], \tag{2.1}$$

where p and p' in (0, 1) are the unit-normalized rank of incomes respectively at time t and t', and the notation $^{-1}$ stands for inverse functions. The joint distribution $f(.,.)$ is thus fully described by the copula $R_f(.,.)$ and the two marginal distributions $F_\tau(.)$, $\tau = t, t'$. To simplify notations, the quantile function $F_\tau^{-1}(p)$ will be denoted $y_\tau(p)$ when this notation brings no ambiguity.

With these definitions, the non-anonymous growth incidence curve is defined as follows:

Definition 1 (Non-Anonymous Growth Incidence Curve). *The Non-Anonymous Growth Incidence Curve (NAGIC), $G_f(p)$, associated with the joint distribution $f(.,.)$ shows the income growth rate between times t and t' of people at rank p at time t:*

$$G_f(p) = \frac{\int_0^1 R(p, p')y_{t'}(p')dp' - y_t(p)}{y_t(p)}\,. \tag{2.2}$$

In comparison, the standard, anonymous growth incidence curve is defined as follows:

Definition 2 (Anonymous Growth Incidence Curve). *The Anonymous Growth Incidence Curve (GIC), $G_f^a(p)$, associated with the joint distribution $f(.,.)$ shows the income growth rate between incomes at rank p in both periods t and t':*

$$G_f^a(p) = \frac{y_{t'}(p) - y_t(p)}{y_t(p)}\,. \tag{2.3}$$

The NAGIC and the GIC thus differ through the former incorporating personal income mobility. There is no difference between Eqs. 2.2 and 2.3 when no reranking takes place between t and t' so that the copula is the identity copula, *i.e.*, $R(p, q) = 1$ if $p = q$, $R(p, q) = 0$ otherwise.

The potential differences in the shape of the GICs and NAGICs are illustrated in Fig. 1, which shows both curves using the same data from the Panel Study of Income Dynamics (PSID 2018). They cover full-time workers during the period 1980–1990 who were 30–40 years old in 1980. This period saw both fast income growth and rising earning inequality. The GIC accounts solely for the distributional differences between 1980 and 1990, it is clearly upward sloping suggesting an increase in inequality. On the contrary, the NAGIC shows the 1980–1990 growth rate of earnings conditional on the rank of earners in the 1980 distribution. The curve is roughly decreasing – although not monotonically – suggesting an improvement of social welfare, at least if enough weight is put on the poorest. This apparent contradiction between the two curves is, in part, what motivates this paper.

Besides describing the change of the income distribution (GIC) or income changes along the initial income distribution (NAGIC) we are interested in the properties of the two types of growth incidence curves in terms of social welfare. In the anonymous case, the social welfare gain defined on the basis of a social valuation $u(.)$ of individual income is given by

$$\Delta W^u = \int_0^\infty u(y_{t'})dF_{t'}(y_{t'}) - \int_0^\infty u(y_t)dF_t(y_t)\,. \tag{2.4}$$

In the non-anonymous case, and using the same social evaluation function $u(.)$, the variation in social welfare is given by

$$\Delta V^u = \int_0^\infty [u(y_{t'}) - u(y_t)]f(y_t, y_{t'})dy_t dy_{t'}\,. \tag{2.5}$$

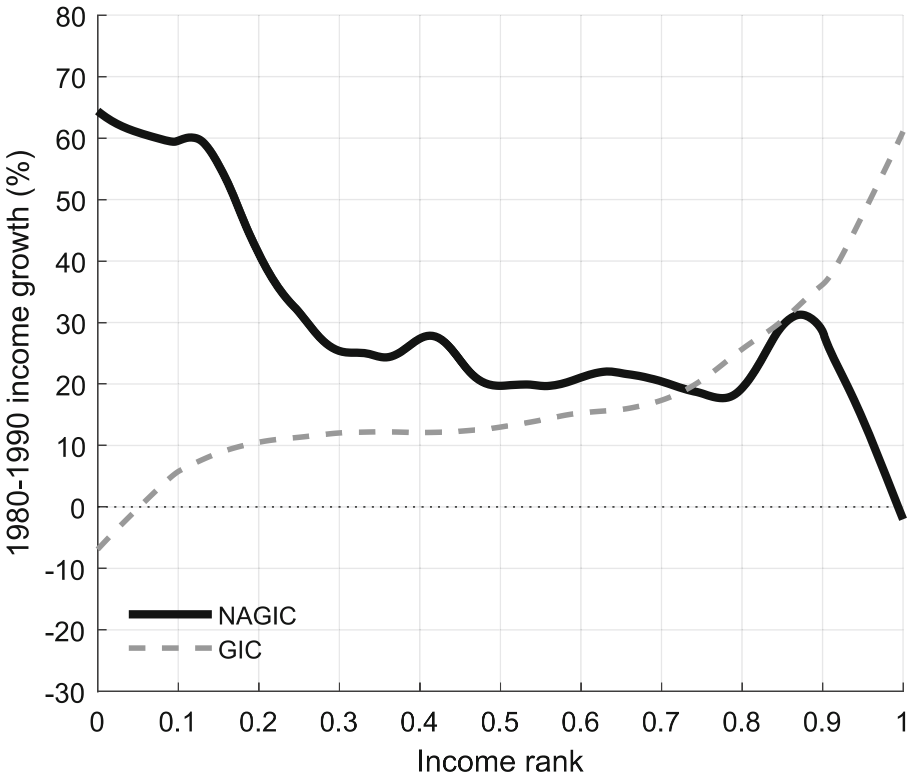

Fig. 1 Anonymous and non-anonymous growth incidence curves for 1980–1990 in the United States. The curves are based on individual annual labor income for 30–40 year-old full-time workers (in 1980) included in the PSID in both the beginning and the end of the period. The curves are smoothed using a LOWESS method, to eliminate the impact of granularity on the shape of the curves (due to the curves being based on individual growth, rather on the division into quantiles such as deciles or percentiles)

If changes in earnings are small enough, which can be obtained by annualizing observed growth rates, a linear approximation of the term in square bracket can be used, namely:[1]

$$\Delta V^u = \int_0^\infty u'(y_t)[y_{t'} - y_t] f(y_t, y_{t'}) dy_t dy_{t'} \,. \tag{2.6}$$

In view of these definitions, a joint distribution of incomes at time t and t' will be said to be *unambiguously social welfare improving in absolute terms* (UIA, resp. na-UIA) if $\Delta W^u \geq 0 \; \forall u(.) \in U$ in the anonymous case (resp. $\Delta V^u \geq 0 \; \forall u(.) \in U$ in the non-anonymous case), where U is the set of non-decreasing and concave functions. It will be *unambiguously social welfare improving in relative terms* (UIR, resp. na-UIR) if ΔW^u (resp. ΔV^u) are greater than what they would be if all incomes had grown as the mean income of the population.

[1] Another, more general way of obtaining this functional form is to assume that the social valuation of an income profile $(y_t, y_{t'})$ is given by a utility function $v(y_t, y_{t'})$ with standard properties. Then a linear approximation to income growth leads to $v(y_t, y_{t'}) \cong \overline{v}(y_t)(y_{t'} - y_t)$ where $\overline{v}(y_t)$ is the sum of the two derivatives of $v(y_t, y_{t'})$ at (y_t, y_t), which it seems reasonable to assume to be decreasing with y_t.

Having defined the social welfare evaluation criteria, the rest of this paper focuses on the relationship between these social welfare improvement criteria and the shape of the $G_f^a(.)$ and $G_f(.)$ curves associated with a joint distribution $f(.,.)$, or to its two marginal distributions in the case of $G_f^a(.)$.

3 Growth incidence curves and social welfare

Consider first the case of anonymous curves. The UIR criterion is equivalent to normalizing the initial and terminal distributions by their mean in the definition of ΔW^u. Following Atkinson (1970), the UIR criterion is thus equivalent to the Lorenz dominance criterion:

$$L_t(p) = \frac{1}{\overline{y}_t} \int_0^p y_t(q)dq \leq L_{t'}(p) = \frac{1}{\overline{y}_{t'}} \int_0^p y_{t'}(q)dq \, , \tag{3.1}$$

where $L_\tau(p)$, defined on (0, 1) is the Lorenz curve associated with the distribution $F_\tau(.)$. Following Shorrocks (1983), the UIA criterion is equivalent to generalized Lorenz curve dominance or:

$$\mathcal{L}_t(p) = \int_0^p y_t(q)dq \leq \mathcal{L}_{t'}(p) = \int_0^p y_{t'}(q)dq \, , \tag{3.2}$$

where $\mathcal{L}_\tau(p)$ is the generalized Lorenz curve associated with distribution $F_\tau(.)$. Clearly, this dominance criterion combines two differences between the initial and terminal marginal distribution: a) the difference in overall means; b) the difference in the relative distributions, *i.e.*, after normalization by the means, or inequality.

With these definitions, it is possible to relate the shape of the anonymous growth incidence curve and the change in social welfare by introducing $G_f^a(.)$ in the preceding inequalities. So, Eq. 3.1 is equivalent to

$$\int_0^p y_{t'}(q)dq \geq (1+\overline{g}) \int_0^p y_t(q)dq \quad \forall p \in (0,1) \, , \tag{3.3}$$

where $\overline{g}$ is the growth rate of the mean income of the population. To let the GIC appear, rewrite the preceding inequality as:

$$\int_0^p [y_{t'}(q) - y_t(q)]\, dq \geq \overline{g} \int_0^p y_t(q)dq \quad \forall p \in (0,1) \, . \tag{3.4}$$

A necessary and sufficient condition for UIR is thus

$$\mathrm{CIMG}(p) = \int_0^p [y_t(q)/Y_t(p)] G_f^a(q)dq \geq \overline{g} \quad \forall p \in (0,1) \, , \tag{3.5}$$

where $Y_t(p)$ is the total income of the p poorest people in the initial distribution. This curve was initially introduced by Son (2004), under a somewhat different form, who termed it the Poverty Growth Curve. Calling it Cumulative Income-weighted Mean Growth (CIMG) curve seems clearer. It could also be interpreted as the growth rate of cumulative mean income. At $p = 1$, it is equal to the growth rate of the average income of the whole population, $\overline{g}$. Clearly, this income-weighted average differs from the arithmetic average that would be obtained from integrating below the GIC, as initially suggested by Ravallion and Chen (2003) up to the poverty headcount ratio to get the Watts poverty index. This difference between the 'growth rate of the mean' and the 'mean growth rate' is crucial.

Condition Eq. 3.5 refers to the UIR welfare criterion. The UIA criterion is obtained by simply changing the right-hand side to 0. A sufficient (but not necessary) condition for UIA thus is that the GIC is everywhere positive.

It will now be shown that an identical condition holds in the case of the non-anonymous incidence curve. Integrating ΔV^u in Eq. 2.5 with respect to the terminal period income leads to:

$$\Delta V^u = \int_0^\infty u'(y)[\overline{y}_{t'}(y) - y]dF_t(y)\,, \tag{3.6}$$

where $\overline{y}_{t'}(y)$ is the expected income at time t' of those people with income y at time t. The linear approximation of the change in utility in Eq. 2.5 is a simplification found in most of the literature referred to above.[2] However, this neglects the component arising from heterogeneous income changes among people with the same initial income, *i.e.*, horizontal inequality.[3]

Integrating the preceding expression by parts, it is easily shown[4] that the na-UIA criterion is equivalent to

$$\int_0^y \left(\overline{y}_{t'}(v) - v\right) dF_t(v) \geq 0 \;\; \forall y \geq 0\,, \tag{3.7}$$

or, switching to quantile function notation:

$$\int_0^p [\widetilde{y}_{t'}(q) - y_t(q)]dq \geq 0 \;\; \forall p \in [0, 1]\,, \tag{3.8}$$

where $\widetilde{y}_{t'}(q) = \overline{y}_{t'}(y(q))$ is the expected income at time t' of those people ranked q at time t. Clearly, this expression is identical to the generalized Lorenz curve dominance condition Eq. 3.2, where period t' incomes are now expected incomes of people at a given initial income rank. The same transformation that went from Eqs. 3.2 to 3.5 thus applies so that: a) a necessary condition for the NAGIC to be consistent with na-UIA is to be everywhere positive; and b) a necessary and sufficient condition is:

$$\text{CIMG}^*(p) = \int_0^p [y_t(q)/Y_t(p)]G_f(q)dq \geq 0\,, \tag{3.9}$$

where the * stands for the conceptual difference in social evaluation of anonymous and non-anonymous growth. It is easily seen that the na-UIR criteria is obtained by replacing the right-hand side of that inequality by $\overline{g}$.

Despite structural and conceptual differences between GICs, based on cross sections of income data, and NAGICs based on panel data, the social evaluation criteria associated with the two types of curve are rigorously the same.

[2] In effect, most papers focus on weighted averages of growth rates and are thus linear with respect to income change, see, for instance, Jenkins and Van Kerm (2016) and Ray and Genicot (2023).

[3] An exception is Palmisano and Peragine (2015).

[4] We apply here the well-known stochastic dominance result introduced by Hadar and Russell (1969) and used by many authors since then. Note, however, that the following condition requires in addition that $u'(y) \to 0$, when $y \to +\infty$.

Proposition 1 *A necessary and sufficient condition for a GIC to be consistent with UIA (resp. UIR) social welfare criterion, or a NAGIC to be consistent with the na-UIR (resp. na-UIA) criteria is for the Cumulative Income-weighted Mean Growth rate associated with them to be everywhere non-negative (resp. not smaller than the rate of growth of average income).*

What is noticeable in this basic property, besides its remarkable similarity between anonymous and non-anonymous growth incidence curves, is that the relationship between these curves and social dominance relates not to their very shape, but to an integral of these curves. The reason for this is essentially that the concavity of the social valuation function $u(.)$, required in UIA/UIR (resp. na-UIA/na-UIR), implies a kind of second-order stochastic dominance, whereas the non-negativity of the GIC/NAGIC relies on first-order dominance.

4 Additional properties of the growth incidence curves

This section lists various properties of the GICs and NAGICs that can be derived from the preceding proposition and in some way relate to the shape of these curves. We also investigate in some depth the consequences of considering the unweighted counterpart of the CIMG/CIMG*.

This analysis will make use of the following functional property:

Definition 3 (Non-monotonically w()-weighted mean decreasing (or w-NMMD) function). *Given a function* $w()$ *positive and continuous over* $(0, 1)$*, a function* $\varphi(x)$ *on the interval* $[0, 1]$ *is said to be non-monotonically w-weighted mean decreasing if:*

$$\frac{1}{x}\frac{\int_0^x w(v)\varphi(v)dv}{\int_0^x w(v)dv} \geq \frac{1}{1-x}\frac{\int_x^1 w(v)\varphi(v)dv}{\int_x^1 w(v)dv} \qquad \forall x \in [0, 1]\,. \tag{4.1}$$

It can be seen that this definition is indeed consistent with the function $\varphi(v)$ being non-monotonic. Practically, this is clearly the case of the GICs and NAGICs when represented with enough granularity. The w-NMMD feature simply expresses the fact that, weighted by the function $w(.)$, the function is always higher in the bottom part than in the top part of its interval of definition, whatever the cut-off point. It is in that sense that it is 'decreasing'. Note that because the mean is systematically defined over sub-intervals that include the origin or the end of the interval of definition of the function, this property differs from a function that would be decreasing when averaged over sequential intervals, as with the moving average transformation. It will be seen later that this property may be considered as a 'generalized pro-poorness' criterion. A particular case, easier to visualize and interpret is the case where the function $w(.)$ is constant, in which case it will be simply denoted NMMD. Such a function is on average higher in the bottom than in the top part of its interval of definition, whatever the cut-off point.

Based on Proposition 1 and above definitions, the following conditions, which relate the shape of the GICs (resp. NAGIC) and the UIR (resp. na-UIR) criterion, can easily be derived:

Proposition 2 *The following are necessary and/or sufficient conditions for a GIC (resp. NAGIC) to be consistent with UIR (resp. na-UIR):*

1) *A sufficient condition is that the GIC (resp. NAGIC) is everywhere non-increasing*
2) *A necessary condition is that* $\lim_{p\to 0} G_f^a(p) \geq \overline{g} \geq \lim_{p\to 1} G_f^a(p)$*, and the same for* $G_f(p)$

3) A necessary and sufficient condition is that the GIC (resp. NAGIC) is income-weighted NMMD

4) A necessary and sufficient condition is that the GIC (resp. NAGIC) exhibit generalized pro-poorness

Property 1) trivially derives from Proposition 1 and the fact that CIMG(1) (resp. CIMG*(1)) is equal to the growth rate of average income, $\overline{g}$. The entire CIMG curves are thus therefore above or equal to $\overline{g}$ over $(0, 1)$. The proof the three other propositions relies on the following identity that is satisfied by CIMG and CIMG*:

$$s(p)\text{CIMG}(p) + [1 - s(p)]\overline{\text{CIMG}}(p) = \overline{g} \quad \forall p \in (0, 1)\,, \tag{4.2}$$

where $s(p)$ is the share of total income going to the poorest p persons in the initial distribution and $\overline{\text{CIMG}}(p)$ the income-weighted mean growth rate over the interval $(p, 1)$. As Proposition 1 says that UIR is equivalent to $\text{CIMG}(p) \geq \overline{g}$, it follows that $\overline{\text{CIMG}}(p) \leq \overline{g}$. Property 2) follows, whereas Property 3) derives from putting the two preceding inequalities together:

$$\text{CIMG}(p) \geq \overline{\text{CIMG}}(p) \quad \forall p \in (0, 1)\,. \tag{4.3}$$

Finally, Property 4) is a restatement of Property 3), where 'generalized pro-poorness' is defined by the preceding property, *i.e.*, the income weighted mean growth rate of poor people is always higher than the income-weighted mean growth rate of non-poor people, whatever the cut-off point. This is a useful criterion in practice, as it is possible to plot the CIMG alongside its complement and easily observe whether it is satisfied.

This generalized pro-poorness concept differs from the pro-poorness criterion proposed by Ravallion (2004) in two dimensions. First, pro-poorness is defined in relative terms, that is poor incomes grow faster than non-poor incomes, instead of the absolute definition used by Ravallion (2004), *i.e.*, the growth of the mean income of the poor is positive. Second, poverty itself is defined in a relative way since it may include any portion of the population with lowest incomes, whereas Ravallion was referring to a specific poverty line and the associated headcount. From this point of view, the concept of generalized pro-poorness of growth is in the line of Foster and Shorrocks (1988), who proposed a social welfare dominance criterion based on poverty orderings with variable poverty lines. It is also closer to the concept of pro-poorness proposed by Kakwani and Pernia (2000), which involves the whole distribution and is akin to inequality.[5]

An equivalent of Proposition 2 holds for the absolute social welfare dominance criterion UIA or na-UIA on top of Proposition 1 but it is rather trivial. One obvious sufficient condition for UIA (resp. na-UIA) is indeed that the GIC (resp. NAGIC) be everywhere non-negative, whereas an obvious necessary condition is that the GIC (resp. NAGIC) is positive when $p \to 0$. In addition, Properties 3) and 4) in Proposition 2 now are only sufficient, provided that the average income growth rate, $\overline{g}$, is positive.

We now explore the relationship between the income-weighted mean growth curve CIMG, or its NAGIC equivalent CIMG*, and its simpler unweighted counterpart, which Jenkins and Van Kerm (2016) called 'cumulative income growth profile' in the case of NAGIC, and which should logically be termed here Cumulative Mean Growth rate, or CMG (resp. CMG*) curve. We can formulate the following lemma (proof in the appendix):

[5] Kakwani and Pernia (2000) define pro-poor growth as essentially the reduction in income inequality, whereas Ravallion and Chen (2003) define it as a reduction in the Watts poverty measure, which is equivalent to the mean growth rate of the income of the poor being positive.

Lemma 1 *The cumulative mean growth curve (CMG, resp. CMG*) is*

$$\overline{G}(p) \equiv \frac{1}{p}\int_0^p g(q)dq\,, \tag{4.4}$$

where $g(q)$ is the income growth rate of people at rank q in the initial income distribution, is related to the cumulative income-weighted mean growth rate CIMG (resp. CIMG) curve through:*

$$CIMG(p) \geq \lambda \;\; \forall p \in [0,1] \quad \Longrightarrow \quad CMG(p) \geq \lambda \quad \forall p \in [0,1] \text{ and } \forall \lambda \geq 0\,, \tag{4.5}$$

and the same for $CIMG^*(p)$ and $CMG^*(p)$.

In turn, this Lemma implies the following:

Proposition 3 *The following conditions hold:*

1) *A necessary condition for the joint distribution of incomes at times t and t' to be UIR (resp. na-UIR) is for the CMG of the GIC (resp. the CMG* of the NAGIC) to be greater than the growth rate of the overall mean income of the population, $\overline{g}$, for all p in $(0, 1)$ (resp. to be non-negative).*
2) *A necessary condition for the joint distribution of incomes at times t and t' to be UIA (resp. na-UIA) is for the CMG of the GIC (resp. the CMG* of the NAGIC) to be non-negative for all p in $(0, 1)$.*

It is important to stress that in the case of UIR/na-UIR, the mean value of the GIC/NAGIC over an interval $(0, p)$ is compared to the growth rate of the mean income, rather than the mean growth rate of individual incomes in the whole population. Therefore, no property equivalent to the NMMD of CIMG/CIMG* in Proposition 3 can be established in case of the CMG curve. At the same time, however, if the CMG and CIMG curves (resp. CMG* and CIMG*) are typically close to one another in practice, we could potentially infer on whether the CIMG is UIR from observing the CMG. While such qualitative similarity is not guaranteed, empirical evidence suggests that in practice it could be the case, as demonstrated below using US data.

Proposition 3 seems to be in contradiction with the proposition in Jenkins and Van Kerm (2016)[6] which implies that in the case of a NAGIC, a CMG* curve that is everywhere above $\overline{g}$ (resp. non-negative) characterizes a growth spell that is socially better than uniform growth at rate $\overline{g}$ (resp. no growth), given their definition of social preferences. These preferences require a growth spell to be socially preferable to another if the weighted average growth rates in the former is higher than in the latter, for all weights that are positive and declining with the income rank. It is easily proven that these social preferences are the same as those assumed in this paper when inequality aversion is restricted to be greater than unity.

The case of high inequality aversion and NAGICs

In the definition of social welfare associated with NAGICs, define $h(y) \equiv yu'(y)$ and rewrite Eq. 2.6 so as to have the growth rate of income appearing in the definition of aggregate social welfare, namely:

$$\Delta V^u = \int_0^\infty h(y)\overline{g}(y)dF_t(y)\,, \tag{4.6}$$

6 Proposition 2, page 685.

where $\overline{g}(y)$ is the expected growth rate of incomes of those people with initial income y, *i.e.*, the NAGIC in the income space. The function $h(.)$ thus appears as a social weight given to the growth rate of people with some given initial income.[7] A key point thus is whether this weight is increasing or decreasing with initial income, *i.e.*, whether more or less weight is given to the growth of low than high incomes.

From its definition, the function $h(.)$ is increasing or decreasing depending on the elasticity of the marginal utility of the original utility function $u()$:

$$h'(y) = u'(y)(1 - \varepsilon)\,, \tag{4.7}$$

where $\varepsilon = -yu''(y)/u'(y)$ representing the aversion of the social observer to inequality, Atkinson (1970). An elasticity greater than unity implies that the function $h(.)$ is decreasing, so that more weight is given to the growth rate of low incomes. This property is also associated with the concept of 'growth sensitivity' of inequality or poverty measures – see Kraay et al. (2023). It is central in the axiomatization of the measurement of upward income mobility by Ray and Genicot (2023). The opposite assumption, namely that the weight given to growth rates increases with income, does not violate the basic properties of social welfare measurement, but it turns out that it is only in presence of a high inequality aversion that the relationship between social welfare and the shape of the NAGIC is stronger than in Proposition 3 for the general case.

With the specification Eq. 4.6 of the social welfare of individual income gains, the na-UIA criterion is:

$$\Delta V^u = \int_0^\infty h(y)\overline{g}(y)dF_t(y) \geq 0 \tag{4.8}$$

for all positive and decreasing functions $h(.)$.

Integrating by parts, it is easily shown that social welfare increases with growth for all positive and decreasing functions $h(.)$ iff:

$$\int_0^y \overline{g}(v)dF_t(v) \geq 0 \;\; \forall y \geq 0\,, \tag{4.9}$$

or, using quantile functions:

$$\overline{G}(p) = \int_0^p G_f(q)dq \geq 0 \;\; \forall p \in (0, 1)\,, \tag{4.10}$$

where $G_f(q)$ is the NAGIC.

It is thus the case that, with high inequality aversion the necessary condition in Proposition 3 is also sufficient in the case of the NAGIC.

Proposition 4 *With aversion to inequality greater or equal to unity, a necessary and sufficient condition for the joint distribution of incomes at times t and t' to be na-UIA (resp. na-UIR) is for the cumulative mean of the NAGIC – the CMG* curve – to be non-negative (resp. not below the mean growth rate $\overline{g}$) over any interval* $(0, p)$ *with p in* $(0, 1)$.

In the case of high inequality aversion, there thus is an equivalence between the CIMG* and the CMG* curves in indicating whether social welfare unambiguously improves with

[7] This is precisely the formulation used by Jenkins and Van Kerm (2016) or Palmisano and Peragine (2015).

growth or not. The important point, however, is that the CMG* does not necessarily exhibit the NMMD property found with CIMG* in the na-UIR case. Indeed the former does not satisfy the identity Eq. 4.1 used to prove the NMMD feature of the latter.

4.1 A convenient decomposition of the NAGIC

A simple decomposition based on the definition of the NAGIC allows to identify the role of the two dimensions of panel income data, *i.e.*, the change in the marginal distribution of income, and therefore of inequality, and income mobility. The NAGIC can then be broken down into two NAGIC-type curves corresponding to these two effects. Analyzing the shape of these curves inform on the shape of the NAGIC itself.

A key concept in the definition Eq. 2.2 is the expected income, $y^R(p; x)$ of people ranked p in the initial distribution when the final marginal distribution of income is given by the quantile function $x(p)$ and the expected rank in the final distribution is given by the copula $R(.,.)$:

$$y^R(p; x) = \int_0^1 R(p, q)x(q)dq \,. \tag{4.11}$$

With this definition, a simple decomposition of the NAGIC is:

$$G_f(p) = \frac{(1+\overline{g})y^R(p; y_t) - y_t(p)}{y_t(p)} + \frac{y^R(p; y_{t'}) - (1+\overline{g})y^R(p; y_t)}{y_t(p)} = \frac{RR(p)}{y_t(p)} + \frac{DC(p)}{y_t(p)} \,. \tag{4.12}$$

The NAGIC thus comprises 2 components: a) uniform growth with pure reranking, $RR(.)$, a change that keeps the marginal distribution of incomes constant, up to the homothetic growth factor $(1 + \overline{g})$ where $\overline{g}$ is, as before, the growth rate of the mean income; b) change in the marginal distribution of income, $DC(.)$, once the initial distribution has been adjusted for the growth of the mean income. If the marginal distribution of income does not change between time t and t', then $DC = 0$ and the NAGIC describes the effect of reranking individuals within the same distribution of income, after moving up all incomes by the growth factor $(1+\overline{g})$. Conversely, no reranking would simply make the NAGIC identical to the GIC since $RR(p)$ would be zero and no-reranking would imply $y^R(p; x) = x(p)$ so that the $DC(p)$ term would become

$$\frac{y^R(p; y_{t'}) - (1+\overline{g})y^R(p; y_t)}{y_t(p)} = G_f^a(p) - \overline{g} \,. \tag{4.13}$$

What can be said about the shape of these various components? It is first shown in what follows that the reranking cumulative growth effect represented by $RR(.)$ (resp. $RR(.)/y_t(.)$) correspond to CIMG* (resp. NAGIC), curves that are NMMD. Second, the shape of the distributional change effect $DC(.)$ or $DC(.)/y_t(.)$ depends on the change in the marginal distributions from $(1 + \overline{g})y_t(p)$ to $y_{t'}(p)$, as well as on the copula of the joint income distribution. Yet, when restricting moderately the shape of the copula it can be shown that any improvement in the marginal distribution of income between time t and t' in the sense of Lorenz dominance also leads to $DC(p)$ and $DC(p)/y_t(p)$ curves that are NMMD.

Proposition 5 *The CIMG* curve corresponding to the reranking cumulative growth income change $RR(.)$ is everywhere non-negative and the NAGIC curve associated with the growth rate $RR(.)/y_t(.)$ is NMMD.*

Proof Any rank switch between two individuals necessarily produces a non-negative mean income change in the bottom part of the distribution. Suppose that somebody ranked i switches income with somebody ranked j ($\geq i$). This (i, j) switch does not change the mean income of the poorest p people, if $p \leq i$ or if $p \geq j$, but it necessarily increases it if $p \in (i, j)$. As any reranking can be decomposed into a number of this kind of bilateral switches, the reranking effect produces a non-negative change in the mean income of the poorest p people, whatever p in (0, 1). The same result holds with the NAGIC defined by $RR(.)/y_t(.)$.

In the preceding argument, the cumulative mean of $RR(.)/y_t(.)$ over the poorest p people is unchanged for $p \leq i$, it increases for $p \in (i, j)$ and it is positive – rather than zero as before – for $p \geq j$. This is because the positive rate of growth of income of people at $p = i$ is greater in absolute value than the negative rate of growth of people at $p = j$.

To analyze the distributional change effect, $DC(p)$, some regularity assumption is needed on the copula $R(p, q)$.

Assumption A

For all i and j in (0, 1) *such that $i < j$, there exists $p^*(i, j)$ in* (0, 1) *such that $R(p, i) \geq R(p, j)$ if $p \leq p^*$ and vice-versa.*

It is thus assumed that the probability of reranking at the lower level, i, is higher than the probability of reranking at the higher level j when the initial rank, p, is low, *i.e.*, below some value that depends on the terminal ranks i and j being considered. In other words, people starting in low ranks are more likely to be reranked at the lowest of the two ranks, i and j, the opposite being true of people starting higher in the income scale. This seems a reasonable and not unduly restrictive assumption that is simply making extreme rank changes relatively less likely.

With this assumption, it is possible to establish the following proposition:

Proposition 6 *Under assumption A, a Lorenz-dominant change in the marginal distribution of income $y_{t'}(.)$ with respect to $y_t(.)$, leads to a CIMG* curve associated with $DC(.)$ that is everywhere non-negative and a NAGIC curves associated with $DC(.)/y_t(.)$ that is NMMD.*

Proof Although the analysis proceeds with continuous functions on the interval [0, 1], it is simpler to prove this proposition using a discrete argument. It goes along the lines of the proof of the preceding proposition when taking into account that a Lorenz dominant change in an income distribution can always be decomposed into a sequence of so-called Pigou-Dalton income transfers, that is transfers from an individual at some level, j of the income scale to an individual at rank i below j.[8] Thus, assume that the terminal income distribution is the same as the initial distribution, after scaling it up by the growth factor $(1 + \overline{g})$, except for an infinitesimal income transfer of size ε, from an individual ranked j to an individual ranked i ($< j$). It is thus the case that:

$$\begin{aligned} y_{t'}(p) &= (1 + \overline{g})y_t(p) \;\; for \;\; p \neq i, j \\ y_{t'}(i) &= (1 + \overline{g})y_t(i) + \varepsilon \\ y_{t'}(j) &= (1 + \overline{g})y_t(j) - \varepsilon \,. \end{aligned}$$

[8] This is central theorem of the measurement of inequality relies on the well-known Hardy-Littlewood-Polya theorem. It was first formulated by Dasgupta et al. (1973).

When switching to expected incomes after reranking in terminal period t' we obtain

$$y^R(p; y_{t'}) - (1+\overline{g})y^R(p; y_t) = DC(p) = [R(p,i) - R(p,j)]\varepsilon\,. \tag{4.14}$$

When integrating over $(0, p)$, assumption A and the copula property $\int_0^1 R(p,i)dp = 1\ \forall i \in (0,1)$ ensure that the cumulative mean of $DC(.)$ is everywhere non-negative. Indeed, it is non-negative for $p \leq p^* \in (i,j)$. If it were negative for some $p > p^* \in (i,j)$, then $R(p,i) - R(p,j)$ would need to be positive to ensure $\int_0^1 [R(p,i) - R(p,j)]dp = 0$. But this is in contradiction with assumption A.

As the cumulative mean of $DC(.)$ is everywhere non-negative for a Pigou-Dalton transfer, this property will still hold when combining any number of such transfers so that the terminal marginal distribution of income $y_{t'}(.)$ Lorenz dominates the growth augmented initial one, $(1+\overline{g})y_t(.)$. A CIMG* curve can be built on the cumulative mean of DC by dividing it by the incomplete mean of the initial income distribution. The non-negativity of the cumulative mean of $DC(.)$ and the fact that the full mean of DC is zero shows that the CIMG* defined on $DC(.)$ is non-negative.

Then, that part of proposition 6 that refers to the NAGIC built upon $DC(.)/y_t(.)$ is proven using Lemma 1 above. **QED**

It follows from the two preceding propositions that the NAGIC is necessarily NMMD if the terminal marginal distribution of income Lorenz dominates the initial one. In the opposite case, the decomposition shown in this section suggests that nothing can be said about the shape of the NAGIC, as the reranking and the distributional change effects go in opposite directions. This implies in particular that the GIC defined on the basis of the initial and terminal marginal distributions may be upward sloping, because the terminal distribution is Lorenz-dominated by the initial one, whereas the NAGIC is downward sloping in a rough sense, *i.e.*, NMMD. This will be the case if the reranking effect dominates the effect of the increasing inequality in the marginal income distribution.

5 An illustration based on US data

To demonstrate the discussions in the previous sections we use panel data from the Panel Study of Income Dynamics (PSID 2018). Specifically, we consider workers in the United States and their earnings in the period 1980–1990. As discussed above, this period is a period of specific interest as it was characterized by both fast wage growth and substantial increase in inequality.

Figure 2 presents several of the curves discussed above: the GIC, CIMG (and its complement – *i.e.*, when integrating from p to 1 – $\overline{\text{CIMG}}$), and CMG (and its complement, $\overline{\text{CMG}}$). The figure presents these curves for several groups of workers: for all adult workers (aged 20–100 in 1980); including only 30 year old (in 1980) full-time workers; including only 35 year old (in 1980) full time workers. The reason for restricting the sample to a specific age cohort and only full-time workers is to exclude mechanical effects that create regression to the mean, such as life cycle effects, and workers moving from part-time to full-time work and vice versa.

As expected, in a period of substantial increase in inequality, the GIC for all workers is upward sloping. In this case, the CIMG is always below its complement. This indicates no UIR. In addition, the CMG and its complement behave in a qualitatively similar way to the CIMG. For 30 year-old workers, the picture is also similar. The sample is smaller, which

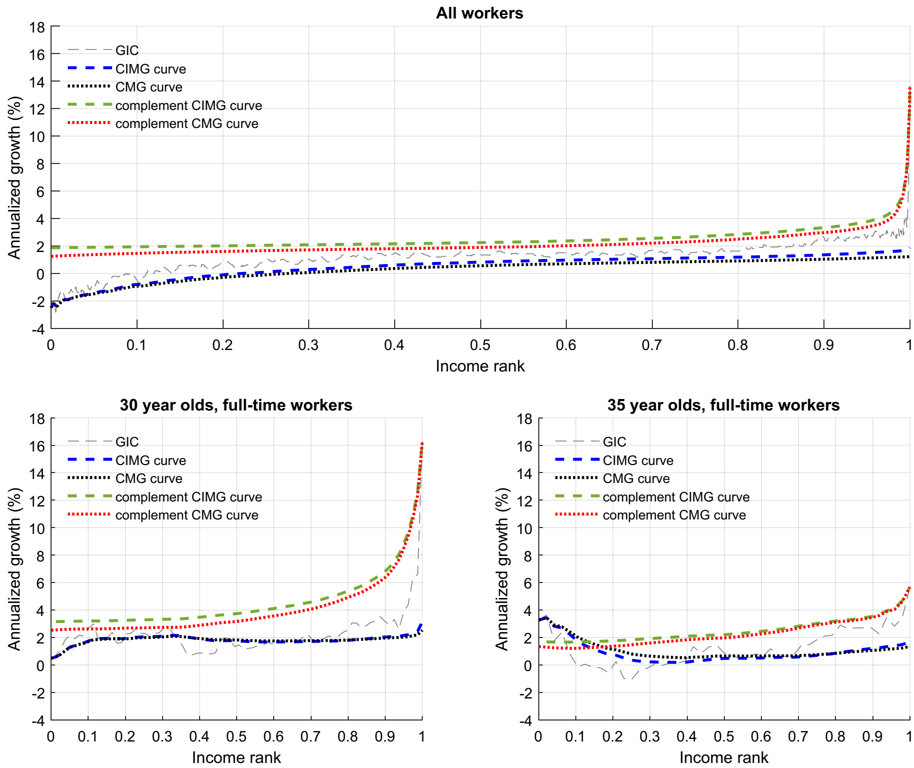

Fig. 2 Pro-poorness and growth for 1980–1990 in the United States. The figure shows the GIC, CIMG and CMG curves (and their complements) for the period 1980–1990 using earnings data from the PSID (PSID 2018). The different panels present the curves for this period given three different samples: all adult workers; only 30 year old (in 1980) full-time workers; only 35 year old (in 1980) full time workers. In all cases the sample includes workers present in the data in both 1980 and 1990

creates a less smooth GIC, however, the CIMG and CMG and their complements, are almost qualitatively identical to the case of all workers. The 35 year olds slightly differ. The growth of the lower ranks was higher than for the middle ranks. For this reason, we get that the CIMG crosses its complement. It is not above the complement for all income ranks, and thus, it is not indicating pro-poor growth. As in the other cases, the CMG provides the same qualitative picture as the CIMG.

Figure 3 shows a similar analysis for the non-anonymous case. It depicts the NAGIC, CIMG* curve, and CMG* curve (and the complements of the latter two curves) for the same period and same samples as Fig. 2.

The first clear observation is that the NAGIC fluctuates substantially in all specifications. This is simply because income ranks are taken here at the individual level: an analysis by decile or percentile would create a smoother curve. However, the fluctuations at this granular level are indicative of a limitation in treating the NAGIC as very informative from a welfare perspective. Even if its slope could be linked to social welfare criteria, it cannot practically be monotonic at such granularity. The CIMG* curve, however, being an integral over individual growth rates, is much smoother. In addition, like in the anonymous case, the CIMG* curve and

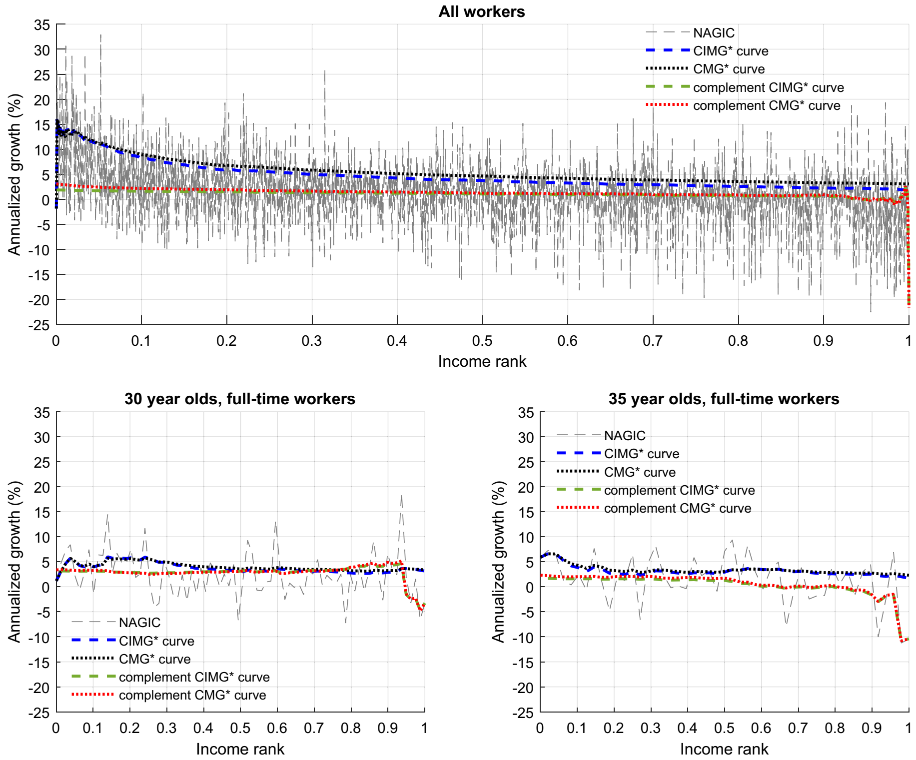

Fig. 3 Pro-poorness and growth for 1980–1990 in the United States. The figure shows the NAGIC, CIMG* and CMG* curves (and their complements) for the period 1980–1990 using earnings data from the PSID (PSID 2018). The different panels present the curves for this period given three different samples: all adult workers; only 30 year old (in 1980) full-time workers; only 35 year old (in 1980) full time workers. In all cases the sample includes workers present in the data in both 1980 and 1990

the CMG* curve are qualitatively similar in all specifications, as well as their complements. For the 30 year-old full-time workers we get that the CIMG* curve is everywhere above its complement $\overline{\text{CIMG}}^*$, which indicates that the NAGIC is income-weighted NMMD or generalized pro-poor.

This is not the case for the 35 year-old workers, where these curves cross. Figures 2 and 3 thus demonstrate together that whether a growth spell can be considered 'generalized pro-poor' or not (in the NMMD sense) can differ when considered anonymously or non-anonymously. In addition, it matters which sample is considered, since there could be a substantial difference between the social welfare interpretation of the same growth spell for different groups.

While the CIMG and CMG curves, as well as the CIMG* and CMG* curves, seem very close to one another, in practice they could diverge substantially. Qualitatively, as shown above, they might demonstrate similar properties with regards to social welfare interpretation. However, they quantitatively differ. This is shown in Fig. 4, where these curves are presented for the case of 35 year old (in 1980) full time workers, for the period 1980–1990, using the same data as above but a finer scale. This shows that the annual growth rates in these curves

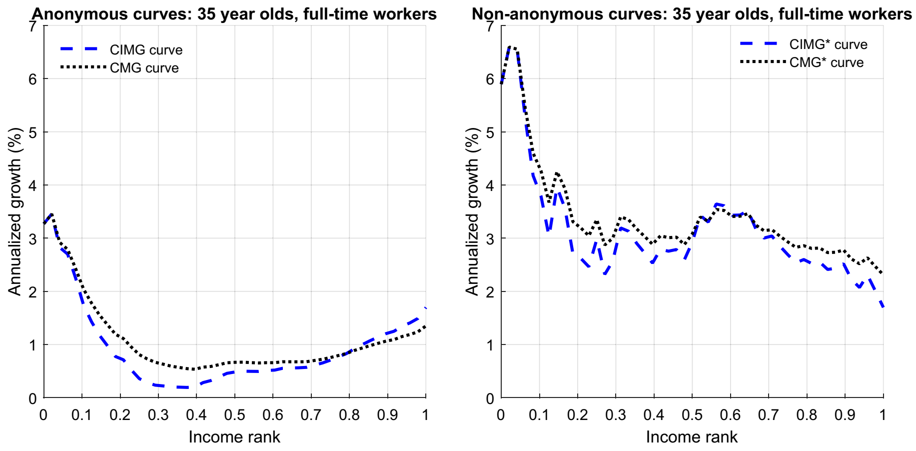

Fig. 4 The the CIMG and CMG curves (left), and the CIMG* and CMG* curves (right) for 1980–1990 in the United States. The curves are the same as in Figs. 2 and 3, yet zoomed-in

differ by up to 0.64 percentage points, which is considerable, given the mean income growth rate being about 2% per year over this time period.

6 Conclusion

This paper returns to two concepts introduced by Martin Ravallion in the analysis of growth: the growth incidence curve and the concept of pro-poorness. It delves deeper into the normative interpretation to be given to the shape of growth incidence curves, both anonymous (*i.e.*, repeated cross sections of income data) and non-anonymous (*i.e.*, panel data), and into the concept of pro-poorness. Criteria for the social welfare dominance of a given growth spell over uniform growth in the population are derived, which lead to the concept of Non-Monotonically Mean Decreasing or generalized pro-poorness of the incidence curve.

Several points should be noted in conclusion. First, despite being based on fundamentally different social valuation concepts, the social welfare interpretation of the shape of both anonymous and non-anonymous incidence curves, as well as the corresponding criteria for dominance over uniform growth, result to be identical, a somewhat unexpected result.

Second, it is unlikely that, with enough granularity, incidence curves will be everywhere downward sloping, an obvious sufficient condition for welfare dominance with respect to uniform growth. The NMMD property, or the generalized pro-poorness, are the criteria that are the closest to this downward-sloping property while being consistent with relative social welfare value judgments. NMMD and pro-poorness are based on the Cumulative Income-weighted Mean Growth rate curve (CIMG) being everywhere above the overall growth rate of income. A close concept is the unweighted Cumulative Mean Growth rate curve (CMG), introduced by Jenkins and Van Kerm (2016). Empirically, both may be close to each other, and they provide an equivalent test of relative social welfare dominance when inequality aversion is restricted to be greater than unity. Yet, the latter does not necessarily satisfy the NMMD and pro-poorness properties.

Third, although not discussed here, both the CIMG and CMG, and their $*$ counterparts (*i.e.*, based on panels rather than cross sections), may be used to compare two different growth spells, provided they start from the same initial distribution. If the initial distributions differ, then GICs and NAGICs of distinct growth spells are not comparable in terms of standard social welfare analysis – they may correspond to widely different incomes and therefore marginal utility levels.

Fourth, our empirical illustration showed that both CIMG and CMG curves and their complements, which are needed for testing the NMMD property, may be quite close to each other. This is to be expected from two curves which are the integrals of the original incidence curve with different weights given to each observation. Yet, they are not identical, as can be seen from the simple fact that the overall income-weighted mean growth rate differs from the arithmetic mean, depending on the shape of the incidence curve and the extent of initial inequality.

Fifth, in the case of NAGICs, the decomposition into a pure income-mobility component and a pure distributional change component shows the extreme importance of the latter in making the whole curve NMMD. This kind of dominance of the income mobility component appears very clearly in the empirical illustration where the non-anonymous incidence curve is NMMD, whereas the anonymous curve is strongly the opposite. This raises the issue of whether a normative criterion associated with non-anonymous curves should not include explicit weights of the mobility and distribution components rather than the implicit weights consistent with conventional social welfare functions.

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