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Normative Measures of Tax Progressivity: an International Comparison

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Abstract
The relevance of tax progressivity measures to policymaking depends on whether they help assess the extent to which taxation leads to social welfare gains or losses. The social welfare implications of progressivity measures have yet to be explored adequately in the literature. This paper helps to fill this gap by proposing a social welfare function framework to derive measures of tax progressivity and explore their normative properties. Using the social welfare framework, the paper derives the Kakwani index from Sen's social welfare function as well as a new class of progressivity measures that incorporate a distributional judgment parameter capturing inequality aversion. The paper also discusses the social welfare implications of the Suits measure of tax progressivity and develops a new measure of tax progressivity derived from the Bonferroni social welfare function. The paper derives both relative and absolute measures of tax progressivity from the social welfare function framework. The methodology developed in the paper is applied to make international comparisons of tax progressivity in 32 developed countries. The paper calculates the magnitude of welfare gains and losses due to taxation and the required social rates of return of public investments for governments to break even. This paper finds that the governments in some countries have to generate high social rates of return from their public investments to compensate for losses of social welfare from taxation. It concludes that optimizing social welfare requires designing a progressive tax system, minimizing the administrative costs of collecting taxes, and maximizing the social rates of return by efficiently investing tax revenues.

Keywords Social welfare function · Tax progressivity · Redistribution · Normative analysis · Horizontal inequity

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1 Introduction

Designing a proper taxation system exemplifies the trade-off between efficiency and equity, which are the two fundamental principles of economic analysis. Efficiency deals with the presence of distortion in the economic behavior of agents, while equity is concerned with distributive justice (Duclos et al. 2003). The optimal taxation literature deals with both equity and efficiency issues, mainly with determining the optimal tax structure associated with the maximum social welfare. There is now a sizable literature on optimal taxation, which illuminates the basic structure of the problem and clarifies several issues in relation to the trade-offs between efficiency and equity.[1] However, as Atkinson (1973) pointed out, the literature has not provided definite answers as to how progressive a tax system should be.

This paper focuses on the distributive justice aspects of taxes, building on the pioneering and innovative contributions to the measurement of tax progressivity (regressivity) by Pigou (1949) and Musgrave and Thin (1948). Nearly three decades after their seminal work, Kakwani (1977) and Suits (1977), working independently, revived interest in the measurement of tax progressivity and developed their tax progressivity indices by measuring the extent of a tax system's deviation from proportionality. The Kakwani and Suits indices are related to the concept of tax elasticity, which is equal to one at all income levels when the tax system is proportional (Kakwani 1977). Thus, the indices measure the overall deviation of tax elasticity from one. Since these two indices of progressivity are widely applied to taxation policy, they should not be used merely as statistical devices for measuring how progressive taxes are. Instead, they should incorporate normative judgments implicit in a social welfare function. The Kakwani and Suits indices do not have apparent social welfare interpretations, and the literature on taxation has yet to explore the social welfare implications of the measures of progressivity. This paper helps to fill this gap.

The relationship between inequality indices and social welfare functions is well established (Atkinson 1970; Sen 1974). But, the links between measures of tax progressivity and social welfare functions have yet to be explored. This paper addresses this gap by introducing a social welfare function framework to derive measures of tax progressivity and explore their normative properties. The framework obtains several measures of tax progressivity from alternative social welfare functions proposed in the literature. Every progressivity measure should have an implicit social welfare function that incorporates a society's distributional judgments. The Kakwani index is extensively used to analyze equity in taxation and government expenditures, as well as equity in access to health, education, and essential services. In particular, the index has become a popular tool for analyzing equity in public finance and delivery of health care. The paper derives the Kakwani index of tax progressivity from Sen's social welfare function.[2]

Kakwani (1980) proposed a generalization of the Gini social welfare function that makes it possible to assign higher weights to income transfers at the lower end of the income distribution.[3] In this paper, we derive a class of progressivity measures from the generalized Gini social welfare functions. The Kakwani index is obtained as a particular case of the generalized progressivity measures. This general class of progressivity measures depends on parameter k, which is similar to Atkinson's (1970) inequality aversion parameter that assigns a

[1] See Ramsey (1927), Mirrlees (1971), Atkinson (1973), Sheshinski 1972, Sheshinski 1978), and Itsumi (1974).
[2] For a discussion of the upper bound of the Kakwani index, see Mantovani et al. (2018). Gerber et al. (2019) have examined the relationship between tax progressivity and economic growth based on the Kakwani index.
[3] Yitzhaki published the same generalization of the Gini index later in 1983.

higher weight to the poorer sections of the income distribution. The general social welfare function framework presented in the paper is also used to derive new progressivity measures from Atkinson's class of social welfare functions. The paper also identifies the social welfare implications of Suits' measure of tax progressivity. It demonstrates that the Suits index of progressivity cannot be derived from a meaningful social welfare function. This paper proposes a modification of the Suits measure so that it can be given a social welfare interpretation. Finally, the paper introduces a new measure of tax progressivity derived from the Bonferroni social welfare function (Bonferroni 1930).

The publication of two seminal papers by Kolm (1976a, 1976b) introduced two alternative concepts of relative and absolute inequality measures to the literature. A range of intermediate inequality concepts has been conceived recently, which, as Urban (2019) points out, are referred to as "intermediate" because they reflect a combination of the relative and absolute transformations (Bossert and Pfingsten 1990; Ebert 2004; Bosmans et al. 2014). Measures of tax progressivity have both relative and absolute notions.[4] Relative measures of tax progressivity remain unchanged when everyone's tax is increased or decreased by the same proportion. Similarly, absolute measures of progressivity remain unchanged when everyone's tax is increased or decreased by the same absolute amount. The social welfare functions framework proposed in the paper provides both relative and absolute measures of tax progressivity.[5]

The methodology developed in this paper is applied to international comparisons of tax progressivity. The calculations are based on the income distribution data for 32 countries obtained from the Luxembourg Income Study (LIS) database. This is the largest available income database of about 50 countries in Europe, North America, Latin America, Africa, Asia, and Australasia, spanning five decades.[6]

2 Relative and Absolute Measures of Tax Progressivity

The social welfare framework proposed in this paper requires the usual restriction on a social welfare function—that is, it should be increasing, concave, or quasi-concave in incomes. An additional requirement is that the social welfare framework should be homogeneous of degree one, which implies that if all incomes are increased (decreased) by the same proportion, social welfare should also increase (decrease) by the same proportion. Atkinson's (1970) social welfare function derived from a class of homothetic utility functions is homogeneous of degree one and has become a basis for many empirical studies. The homogeneity requirement is essential to obtain relative measures of inequality from social welfare functions. These measures are mean independent, implying that the value of inequality remains unchanged if the same proportion alters each income. We refer to such social welfare functions as relatively homogeneous of degree one.

[4] Many papers have been written on the impact of taxation on income inequality, exploring the conditions under which tax reduces inequality for alternative concepts of inequality. See Moyes (1988), Pfingsten (1987, 1988), Ebert (2010), Ebert and Moyes (2000), and Urban (2014, 2019).

[5] The distinction can be made between local and global measures of tax progressivity. Local measures relate to progression at a given point in the income scale, whereas the global measures are single indices of overall tax progressivity. Our paper focuses on global measures of tax progressivity. Pfingsten (1987) and Ebert (2010) formulated both relative and absolute indices of local tax progressivity. Urban (2019) dealt with the generalization of both relative and absolute measures of tax progressivity.

[6] Of the 50 countries, we have selected 32 countries based on the availability of comparable tax data around 2013. The list of these 32 countries is provided in the supplementary Excel file.

Alternatively, Kolm (1976a) proposed absolute or leftist measures of inequality, which do not show any change in inequality when each income is increased or decreased by the same amount. The social welfare functions yielding such measures of inequality must satisfy the requirement that if all incomes are increased (decreased) by the same amount, the social welfare function must also increase (decrease) by the same amount. Such social welfare functions are referred to in this paper as absolutely homogeneous functions of degree one. Atkinson's class of social welfare function does not satisfy this requirement.

The literature on taxation distinguishes between relative and absolute measures of tax progressivity (Urban 2019). Relative measures of progressivity remain unchanged if everyone's tax is increased or decreased by the same proportion. Similarly, absolute measures of progressivity remain unchanged when everyone's tax is increased or decreased by the same absolute amount.

The relative measures of tax progressivity indicate the extent to which a given tax system deviates from proportionality when everyone pays tax at the same rate. A tax system is said to be progressive (regressive) when the tax rate rises (falls) with income. An absolute measure of progressivity indicates the extent to which a tax system deviates from a situation where everyone pays the same amount of tax. A tax system is absolutely progressive (regressive) when richer persons pay more tax than the poorer.[7] In this paper, we deal with both the relative and absolute measures of progressivity.

The relative measures of tax progressivity can only be derived from social welfare functions that are relatively homogeneous of degree one. In contrast, the absolute measures of tax progressivity can only be obtained from absolutely homogeneous social welfare functions of degree one.

3 Additive Separable and Rank-Order Social Welfare Functions

Suppose there are n persons in a society, whose pre-tax incomes are given by a vector $\widetilde{x} = (x_1, x_2, \ldots, x_n)$. Given this, a general social welfare function can be written as $W_x = W(\widetilde{x})$. This welfare function qualifies as relatively homogeneous of degree one if $W(\lambda\widetilde{x}) = \lambda W(\widetilde{x})$, implying that if all incomes are increased (decreased) by the same proportion, social welfare should also increase (decrease) by the same proportion. The social welfare function will be called as absolutely homogeneous of degree one if $W(\widetilde{x} + a) = W_x + a$, implying that if all incomes are increased (decreased) by the same amount, the social welfare function must also increase (decrease) by the same amount.

We use two types of social welfare functions to derive the measures of tax progressivity. One is the class of additive separable social welfare functions, defined as

$$u(W_x) = \int_0^\infty u(x) f(x) dx \tag{1}$$

where pre-tax income x is assumed to be a random variable with density function $f(x)$. $u(x)$ is the utility function, which is increasing in x and is concave. W_x is Atkinson's (1970) social welfare function based on the idea of the equally distributed equivalent (EDE) level of income. The EDE income W_x is the income that, if each individual gets it, provides the same level of social welfare as the present distribution.

[7] This is the minimal concept of tax progressivity introduced by Fei (1981). Moyes (1988) demonstrated that a progressive tax under this concept reduces absolute inequality.

The utility function $u(x)$ is said to be homothetic if it has a constant elasticity of marginal utility of income defined by $\frac{u''(x)x}{u'(x)}$, where $u'(x)$ and $u''(x)$ are the first and second-order derivatives of $u(x)$, respectively. If the utility function $u(x)$ in (1) is homothetic, then the social welfare function W_x is relatively homogeneous of degree one.

If the utility function $u(x)$ satisfies either $u(x+a) = u(a) + u(x)$ or $u(x+a) = u(a)u(x)$, then we can show that the social welfare function W_x is absolutely homogeneous of degree one. The social welfare function underlying Kolm (1976a, 1976b) inequality measure is absolutely homogeneous of degree one.

The class of additively separable social welfare functions in (1) has the property that each person's welfare depends only on her income or consumption, and not on the consumption of others in society. Such welfare functions do not capture the relative deprivation suffered by society.

Sen (1974) developed the Gini social welfare function, defined as the weighted average of income levels. A general form of this function is given by[8]

$$W\left(\widetilde{x}\right) = \int_0^\infty x v(F(x))f(x)dx \tag{2}$$

where $f(x)$ s the density function of x, and $F(x)$ is the distribution function of x that measures the proportion of persons who have income less than x. $v(F(x))$ is the weight attached to the income level x such that $v'(F(x)) < 0$, implying weights must decrease monotonically with $F(x)$—greater weights are given to the poorer than the richer—and the total weight adds up to 1:

$$\int_0^\infty v(F(x))f(x)dx = 1 \tag{3}$$

The rank-order social welfare functions are interdependent and are non-additive separable. Since they depend on the ranks of all individuals in society, they provide the measures of horizontal inequity, defined as welfare losses due to changes in ranking.

The most attractive feature of the rank-order social welfare functions is that they are both relatively and absolutely homogeneous of degree one. Hence, they provide both relative and absolute measures of tax progressivity.

4 Social Welfare Framework for Measuring Tax Progressivity

Let $T(x)$ be the tax paid by an individual with income x. The post-tax or disposable income of the individual will then be $y(x) = x - T(x)$. The vector $\widetilde{y} = (y_1, y_2, \ldots, y_n)$ provides the social welfare of the disposable income of n individuals as $W_y = W(\widetilde{y})$. The difference in social welfare between the pre- and post-tax income distributions given by $(W_y - W_x)$ is the contribution of the tax system to social welfare.

A tax system can alter the rankings of the pre- and post-tax income distributions. Feldstein (1976) and Rosen (1978) referred to this as the horizontal inequality in taxation. The classical definition of horizontal inequity is that people with the same economic circumstances are not

[8] This social welfare function was proposed by Yaari (1988).

treated equally. They proposed a measure of horizontal inequity in terms of the rank correlation coefficient between the pre- and post-tax incomes of individuals.[9]

Atkinson (1980) showed that the reranking of individuals based on post-tax income increases the Gini index of the post-tax income distribution. Plotnick (1981, 1982) and Kakwani (1984) have suggested a measure of horizontal inequity based on the increase in the Gini index, which happens when the tax system changes the ranking of individuals.[10]

In the Appendix of this paper, we proved two theorems A.1 and A.2, demonstrating that for a general class of additive separable and rank-order social welfare functions, the change in ranking between the pre- and post-tax incomes will always result in a loss of social welfare. Based on these theorems, we propose to measure horizontal inequity by the loss of social welfare caused due to change in the ranking (instead of an increase in inequality as suggested in the literature).

We first define the absolute measure of horizontal inequity by the absolute loss of welfare caused due to change in ranking as

$$H_A = W_y - \widehat{W}_y < 0 \tag{4}$$

where $\widehat{W}_y$ is the pseudo-social welfare function of the post-tax income (discussed in the Appendix). It is defined as the social welfare of post-tax income, obtained using the pre-tax income distribution weights.

We also propose a relative measure of horizontal inequity as

$$H_R = \frac{W_y - \widehat{W}_y}{\overline{T}} < 0 \tag{5}$$

where $\overline{T}$ is the average tax collected by the government. H_R is the loss of social welfare due to the change in ranking for an average of one dollar of tax collected by the government.[11]

A tax system is said to be proportional if every person pays taxes at the same rate $e = \frac{\overline{T}}{\overline{x}}$, where $\overline{x}$ is the average pre-tax income and $\overline{T}$ is the average tax paid by society. Under the proportional tax system, the homogeneity requirement implies that the post-tax social welfare is given by $\widetilde{W}_y = (1-e)W_x$, which leads to the following decomposition.

[9] The literature distinguishes between the classical formulation of horizontal inequity and rank change. A change in ranking occurs when a richer person becomes poorer and a poorer person becomes richer, which can only occur when persons with the same pre-tax income are not treated equally, implying there is violation of horizontal equity. But, when there is violation of horizontal equity, rank changes may or may not happen. For a detailed discussion of the two notions of horizontal inequity, see Duclos et al. (2003). If any two persons interchange their positions, the ranking by incomes does not change; hence, social welfare does not change because social welfare functions are anonymous with respect to the identity of individuals.

[10] Atkinson's (1980) formulation of the horizontal inequity index is basically the same as the one developed by Plotnick (1981) and Kakwani (1984), and is therefore called the "Atkinson-Plotnick" or "Atkinson-Plotnick-Kakwani index of reranking. See Van De Ven et al. (2001) and Monti et al. (2012).

[11] See Jenkins (1988), Jenkins and Lambert (1999), Kaplow (1989), and Lambert and Ramos (1997) for their work relating to horizontal inequity.

$$\begin{aligned}\left(W_y - W_x\right) &= H_A + \left(\widetilde{W}_y - W_x\right) + \left(\widehat{W}_y - \widetilde{W}_y\right) \\ &= H_{\mathrm{A}} - eW_x + \left(\widehat{W}_y - (1-e)W_x\right)\end{aligned} \tag{6}$$

This equation implies that the welfare impacts of the tax system have three components. The first component is the loss of social welfare due to horizontal inequity as caused by rank change. The second is a loss of social welfare when there is no change in income distribution due to taxes—that is, when the tax system is proportional. The third component is the difference between the pseudo-social welfare of the post-tax income and the social welfare under the counterfactual if the tax system were to be proportional. This term can be either positive or negative as it measures the gain (loss) of social welfare when the tax system is progressive (regressive).

The progressivity indices proposed in the literature do not tell policymakers the extent of gains or losses in social welfare a tax system contributes. The framework presented here derives the progressivity indices that have direct social welfare implications for tax systems. Dividing (6) by the average tax collected from society gives

$$\frac{W_y - W_x}{\overline{T}} = H_R - \frac{W_x}{\overline{x}} + \frac{\widehat{W}_y - (1-e)W_x}{\overline{T}} \tag{7}$$

The term on the left-hand side of (7) is the welfare contribution of the tax system when the government collects an average tax of one dollar from society. This term is the sum of three contributions, as shown in the equation. The first contribution is the relative horizontal inequity, which is the loss of social welfare due to a change in ranking between pre- and post-tax incomes. The second term on the right-hand side of Eq. (7) is the loss of social welfare if there are no changes in income distribution because of taxation—that is, when the tax system is proportional. The third contribution is the gain (loss) of social welfare contributed by a progressive (regressive) tax system when the government collects an average tax of one dollar per person from society. We, therefore, propose a general measure of relative tax progressivity

$$\theta_R = \frac{\widehat{W}_y - (1-e)W_x}{\overline{T}}. \tag{8}$$

These are relative measures of progressivity because they remain unchanged if the tax paid by everyone in a society increases or decreases by the same proportion. They measure the gain or loss of social welfare when the tax system deviates from proportionality; the gain (loss) signifies the tax system is progressive (regressive).

A tax system is absolutely progressive if the richer pay more tax than the poorer. Thus, an absolute tax progressivity measure indicates the extent of the overall deviation of a tax system from a situation when everyone pays the same absolute amount of tax. If everyone pays the same tax equal to $\overline{T}$, the after-tax social welfare will be given by

$$W_y = W\left(\widetilde{x} - \overline{T}\right). \tag{9}$$

If the social welfare function is absolutely homogeneous of degree one, then

$$W\left(\widetilde{x} - \overline{T}\right) = W\left(\widetilde{x}\right) - \overline{T} \tag{10}$$

must always hold, which from (9) yields

$$W_y = W_x - \overline{T} \tag{11}$$

So a general measure of absolute progressivity is the deviation of $\widehat{W}_y$ from W_y

$$\theta_A = \widehat{W}_y - W_x + \overline{T} \tag{12}$$

which will be positive (negative) if tax is absolutely progressive (regressive), and 0 if everyone pays the same tax. Similar to inequality measures, these progressivity measures remain unchanged when everyone's tax is increased or decreased by the same amount.

The loss of social welfare due to taxation is related to the absolute measure of progressivity by

$$W_y - W_x = H_A - \overline{T} + \theta_A \tag{13}$$

where H_A is the absolute measure of horizontal inequity. This equation demonstrates that the absolutely progressive (regressive) tax increases (decreases) social welfare. If everyone pays the same amount of tax—that is, $\theta_A = 0$—the ranking of individuals does not change, with the loss of social welfare equal to the average tax paid by society. The larger the tax that the government collects, the larger is the welfare loss. We subsequently derive the progressivity measures from particular social welfare functions in Sections 5 to 7.

5 Redistributive Effect of Taxation

Musgrave and Thin (1948) proposed a measure of progressivity obtained from the difference between the inequality indexes of the pre- and post-tax income distributions. Their measure of progressivity indicates the extent to which a given tax system leads to a reduction in income inequality. The progressive tax is associated with a decrease in income inequality, whereas a regressive tax is related to an increase in income inequality (Jacobsson 1976). Musgrave and Thin (1948) and Jacobsson (1976) measured the redistributive effect of taxation and not tax progressivity. Tax progressivity deals with an equity principle of taxation, which suggests that richer persons must pay more taxes or even at a higher rate. The redistributive effect of taxation is an outcome of the equity principle.[12] While the two concepts are related, they are distinct. This paper explains the relationship between the two as follows.

Given that the social welfare function is relatively homogeneous of degree one, we can derive the relative measure of inequality of the pre-tax income distribution as[13]

$$W_x = \overline{x}(1 - I_x) \tag{14}$$

where $\overline{x}$ is the average of pre-tax income, and I_x is pre-tax inequality (Sen 1974; Atkinson 1970). Similarly, the post-tax social welfare is related to the post-tax inequality as

[12] Kakwani and Lambert (1998) have provided a detailed discussion of equity in taxation and have demonstrated that violations of these principles exert negative influences on the redistribution effect of taxation. These principles deal with the fairness of taxation, which has an impact on redistribution of income due to taxation.

[13] A relative measure of inequality is defined as the proportionate loss of social welfare due to inequality.

$$W_y = (1-e)\bar{x}\left(1-I_y\right) \tag{15}$$

where I_y is the inequality of post-tax income distribution. Similarly, using the pseudo-social welfare function for post-tax income distribution when the ranking does not change (as defined in the Appendix), we obtain a pseudo-post-tax inequality measure $\widehat{I}_y$ from

$$\widehat{W}_y = (1-e)\bar{x}\left(1-\widehat{I}_y\right) \tag{16}$$

As we have proven in the Appendix, $\widehat{W}_y > W_y$, which implies $\widehat{I}_y < I_y$.

Substituting (14) and (16) into (8) yields

$$\theta_R = \frac{(1-e)}{e}\left[I_x-\widehat{I}_y\right] \tag{17}$$

that can be expressed in terms of change in relative inequality

$$\varphi_R = I_y-I_x = \left(I_y-\widehat{I}_y\right)-\frac{\theta_R e}{(1-e)} \tag{18}$$

which is a relative measure of income redistribution due to taxation.[14] The first term on the right-hand side of Eq. (18) is positive—implying the change in ranking between pre- and post-tax incomes always contributes to increases in inequality. The second term on the right-hand side of Eq. (18) demonstrates that the tax system reduces (raises) the relative inequality if the relative measure of tax progressivity θ_R is positive (negative). This implies that the progressive tax system reduces relative inequality, while the regressive tax system increases it. When tax is proportional—that is, $\theta_R = 0$—the ranking of individuals will not change, and taxation will have no impact on inequality.

If the social welfare function is absolutely homogeneous of degree one, we can then calculate the absolute measures of inequality of the pre- and post-tax income distributions—with the difference between them providing an absolute measure of income redistribution.[15]

$$\varphi_A = (1-e)\bar{x}I_y - \bar{x}I_x = (1-e)\bar{x}\left(I_y - \widehat{I}_y\right) - \theta_A \tag{19}$$

The first term on the right-hand side of Eq. (19) shows that the change in ranking between the pre- and post-tax incomes always increases the absolute inequality, while the second term indicates that the tax system reduces (increases) the absolute inequality if the absolute measure of progressivity θ_A is positive (negative). This result implies that an absolutely progressive tax system (that is, when the richer pay higher taxes than the poorer) reduces absolute inequality. In contrast, the absolutely regressive tax system (that is, when the poorer pay higher taxes than the richer) raises absolute inequality. When everyone pays the same tax, absolute inequality due to taxation does not change.

We can, therefore, conclude that the progressivity and the redistribution effect of taxation are related but distinct concepts. The three factors that affect the relative redistribution effect of tax are the progressivity of taxation, the change in ranking due to taxation, and the average tax

[14] This result is similar to the one derived by Kakwani (1984) for the Kakwani index of progressivity. The first term in this equation is the horizontal inequity and the second term is the vertical inequity. The net effect of the two inequities is called the redistributive effect of taxes. The second term in the equation is the concentration index of the post-tax income, which Reynolds and Smolensky (1977) termed a measure of tax progressivity.

[15] An absolute measure of inequality is defined by the product of mean income and relative income inequality.

rate. On the other hand, the absolute redistribution is affected by the two factors: the change in ranking and the absolute progressivity.

6 A General Class of the Gini Social Welfare Functions

Kakwani (1980) proposed a generalization of the Gini index that makes it possible to assign higher weights to the income of the poor. The general social welfare function implicit in the generalized Gini index is

$$W_x(k) = (k+1)\int_0^\infty x[1-F(x)]^k f(x)dx = \bar{x}(1-G_x(k)) \tag{20}$$

where G_k is the generalized Gini index. The parameter k is similar to the inequality aversion parameter introduced by Atkinson (1973). When $k = 0$, $W_x(k) = \bar{x}$, which is the mean income of the pre-tax distribution. In such a case, the income of everyone receives the same weight, which reflects an inequality-neutral attitude—that is, society does not care about inequality at all and focuses solely on enhancing economic growth. When $k = 1$, $W_x(k)$ equals the Gini social welfare function. The larger the value of k, the higher is the relative weight given to the lower end of the income distribution. A higher value of k, therefore, would be appropriate if a society desires to give greater importance to transfers of income to its poorer sections.

The generalized Gini social welfare for the post-tax income distribution is given by

$$W_y(k) = (k+1)\int_0^\infty y\left[1-F^*(y)\right]^k f^*(y)dy = \bar{y}\left(1-G_y(k)\right) \tag{21}$$

where $y = x - T(x)$ and $G_y(k)$ is the generalized Gini index of the post-tax income distribution.

The pseudo-social welfare function of the post-tax income is derived using weights of the pre-tax income. Hence, the pseudo-social welfare for the generalized Gini social welfare function is given by

$$\widehat{W}_y(k) = (1+k)\int_0^\infty [x-T(x)][1-F(x)]^k f(x)dx = \bar{y}\left(1-C_y(k)\right) \tag{22}$$

where $C_y(k)$ is the generalized concentration index of the post-tax income distribution. As demonstrated in the Appendix, $\widehat{W}_y(k) > W_y(k)$ when there is a change in ranking. Hence, we measure the absolute horizontal inequity in taxation by the index

$$H_A(k) = W_y(k) - \widehat{W}_y(k) < 0 \tag{23}$$

The relative measure of horizontal inequity as defined in (5) is obtained as

$$H_R(k) = \frac{H_A(k)}{\overline{T}} < 0 \tag{24}$$

Similarly, the pseudo-social welfare function of tax is given by

$$W_T(k) = (k+1)\int_0^\infty T(x)[1-F(x)]^k f(x)dx = \overline{T}(1-C_T(k)) \tag{25}$$

where $C_T(k)$ is the generalized concentration index of tax. It is easy to verify from (20), (22) and (25) that

$$\widehat{W}_y(k) = W_x(k) - W_T(k) \tag{26}$$

which on substituting into (8) and utilizing (20) and (25) gives a relative measure of tax progressivity

$$\theta_R(k) = K(k) = C_T(k) - G_x(k) \tag{27}$$

that is the generalized Kakwani index of tax progressivity. When $k = 1$, $\theta_R(k) = K = C_T - G_x$, which is the Kakwani index of progressivity. When k is greater than 1, a higher weight is given to transfers among those who have income less than the mode. Combining (20), (21), and (22) and (24) with (27) yields

$$\frac{W_y(k) - W_x(k)}{\overline{T}} = H_R(k) - (1 - G_x(k)) + K(k) \tag{28}$$

where H_k is the measure of horizontal inequity for the generalized Gini social welfare function.

The left-hand side of Eq. (28) is the change in social welfare when society pays an average of one dollar of tax. The right-hand side of Eq. (28) shows that the total welfare impact of taxation is the sum of three contributions: (i) the loss of welfare when there is a change in ranking, (ii) the loss of welfare when the tax system is proportional, and (iii) the gain (loss) of welfare when the tax system is progressive (regressive).

The taxation system is progressive when the generalized Kakwani index $K(k)$ is positive, which means that the progressive tax contributes to a gain in social welfare. Conversely, if $K(k)$ is negative, the tax system is regressive and contributes to a welfare loss. We can now, therefore, ascribe a social welfare interpretation to the generalized Kakwani index. The value of $K(k)$ measures the magnitude of the gain (loss) of social welfare if the tax system is progressive (regressive) when society pays an average of one dollar in tax. Even if taxes are proportional, society suffers a loss of social welfare equal to $(1 - G_x(k))$. Since Kakwani developed his index by measuring the deviation of a tax system from proportionality, the index had seemingly no social welfare implication. We have now shown that the Kakwani index has a useful social welfare interpretation.

The relative redistributive effect for the generalized social welfare functions, derived from (18), is given by

$$\varphi_R(k) = \left[G_y(k) - C_y(k)\right] - \frac{e\,K(k)}{(1-e)} \tag{29}$$

which leads to the Kakwani (1984) decomposition when $k = 1$. The first term in (29) measures the horizontal inequity, and the second term measures the vertical inequity. The first term is always positive when there is horizontal inequity, implying that horizontal inequity always increases income inequality, and there is no change in inequality when there is no horizontal inequity. The second term is negative (positive) when tax is progressive (regressive).

Aronson et al. (1994) and Duclos et al. (2003) proposed another version of this decomposition in their models of horizontal inequity[16]

[16] We are grateful to one of the referees for drawing our attention to the decomposition in (29) used in numerous studies.

$$-\varphi_R(k) = \left[G_x(k) - G_y(k)\right] = \frac{e}{(1-e)} K(k) - \left[G_y(k) - C_y(k)\right] \tag{30}$$

Note that (19) and (30) are identical but have different interpretations. The first term on the right-hand side of (30) is the "vertical effect," which represents the "potential" redistributive effect that would be achieved in the absence of horizontal inequity. The term $G_y(k) - C_y(k) \geq 0$ is the reranking effect that represents the reduction of the redistributive impact due to the presence of horizontal inequity.

Similarly, we derive the absolute measure of tax progressivity by substituting (25) into (12) and utilizing (25) as

$$\theta_A(k) = C_T(k) \tag{31}$$

where $C_T(k)$ is the generalized concentration index of tax. A tax system is absolutely progressive when the rich pay more tax than the poor, in which case $C_T(k) > 0$. When $k = 1$, $C_T(k)$ equals the concentration index of tax C_T. The change in social welfare due to tax will thus be

$$W_y(k) - W_x(k) = H_k - \overline{T} + C_T(k) \tag{32}$$

If everyone pays the same amount of tax, there will be no change in ranking. Hence, society suffers a loss of social welfare equal to the average tax that society pays.

The absolute measure of the redistribution effect of taxes will be given by

$$\varphi_k = (1-e)\overline{x}G_y(k) - \overline{x}G_x(k) = -H_k - C_T(k) \tag{33}$$

which shows that if taxes are absolutely progressive, absolute inequality declines because of taxation. If $C_T(k) = 0$, everyone pays the same tax, and the ranking of individuals will not change. Hence, absolute inequality does not change.

7 A Class of Atkinson and Kolm Social Welfare Functions

Atkinson's (1970) general class of social welfare functions given in (1) are additive separable. As discussed in Section 3, he derived it based on the idea of the equally distributed equivalent level of income. He assumed that the social welfare function is utilitarian, and every individual has the same utility function that is increasing and is concave in income. The utility function implicit in his social welfare function is homothetic, given by

$$\begin{aligned} \varphi(x) &= \frac{x^{(1-\epsilon)}}{1-\epsilon} && if \epsilon \neq 1 \\ &= ln(x) && if \epsilon = 1 \end{aligned} \tag{34}$$

which on substituting into (1) gives the social welfare function for the pre-tax income distribution as

$$W_x(\in) = \left[\int_0^\infty x^{1-\in} f(x)dx\right]^{\frac{1}{(1-\in)}} \qquad if \in \neq 1$$
$$= exp\left[\int_0^\infty ln(x) f(x)dx\right] \qquad if \in = 1 \tag{35}$$

This class of Atkinson's social welfare functions is relatively homogeneous of degree one, and hence we can derive from it a class of relative tax progressivity measures. $\in$ measures the degree of inequality aversion—that is, the relative sensitivity to income transfers at different income levels.

The social welfare function for the post-tax income distribution from (35) is given by

$$W_y(\in) = \left[\int_0^\infty (x-T(x))^{(1-\in)} f^*(y)dy\right]^{\frac{1}{(1-\in)}} \qquad if \in \neq 1 \tag{36}$$
$$= exp\left[\int_0^\infty ln(x-T(x)) f^*(y)dy\right] \qquad if \in = 1$$

The pseudo-social welfare function for the post-tax income distribution obtained by using the pre-tax weights is given by

$$\widehat{W}_y(\in) = \left[\int_0^\infty (x-T(x))^{(1-\in)} f(x)dx\right]^{\frac{1}{(1-\epsilon)}} \qquad if\ \epsilon \neq 1 \tag{37}$$
$$= \ exp\left[\int_0^\infty ln(x-T(x)) f(x)dx\right] \qquad if \in = 1$$

Under the proportional tax system, everyone pays tax at the same rate. Since the social welfare function is homogeneous of degree one, the social welfare function of the post-tax distribution when the tax system is proportional is $(1-e)W_x(\in)$, where e is the average tax rate of society. Substituting (34) and (37) into (8) yields a class of progressivity measures based on Atkinson's social welfare functions as

$$\theta_\in = \frac{\widehat{W_y}(\in)-(1-e)W_x(\in)}{\overline{T}} \tag{38}$$

Equation (38) defines a measure of tax progressivity for a given value of the inequality aversion parameter. The inequality aversion parameter in the context of tax progressivity measures the relative sensitivity of tax rates at different income levels. As $\in$ rises, a higher weight is assigned to tax rates at the lower end of the income distribution and a lower weight to tax rates at the top end of the distribution. If $\in = 0$, all individuals pay the same tax rate, and the tax system is thus proportional. These are relative measures of tax progressivity. Since Atkinson's social welfare functions are not absolutely homogeneous of degree one, they do not lend to absolute measures of tax progressivity.

Kolm (1976a, 1976b) proposed an absolute measure of income inequality as

$$I_{KOM} = \frac{1}{\beta} ln\left[\int_0^\infty exp\{\beta(\mu-x)\} f(x)dx\right], \tag{39}$$

where $\beta > 0$ is the parameter and μ is the mean income. An absolute measure of inequality is related to social welfare as

$$I_K = \mu - W_{KOM} \tag{40}$$

where W_{KOM} is the social welfare function underlying the Kolm inequality measure. It is easy to verify that W_{KOM} is given by

$$-exp(-\beta W_{KOM}) = \int_0^\infty -exp(-\beta x) f(x) dx \tag{41}$$

The utility function implicit in this social welfare function is given by

$$u(x) = -exp(-\beta x) \tag{42}$$

which on substituting in (1) give the Kolm social welfare function (41). This demonstrates that the Kolm social welfare function belongs to the general class of social welfare functions in (1). The utility function $u(x)$ in (42) is increasing in x and is concave. The utility function also satisfies $u(x + a) = u(a)\ u(x)$, which from (1) implies that the social welfare function in (41) is absolutely homogeneous of degree one; if everyone's income is increased (decreased) by the same amount, the social welfare function also increases (decreases) by the same amount.

It is interesting to note that Pollak (1971) proposed precisely the same social welfare function as in (41), but in the context of multiple commodity utility functions. This social welfare function is popularly known as Kolm-Pollak social welfare function (Blackorby et al. 1999; Elbert 1988; Gajdos 2001; Mas-Colell et al. 1995).

Utilizing (12) in conjunction with (41) yields the class of progressivity measures underlying the social welfare function in (41) as

$$\theta_{KP} = -\frac{1}{\beta} ln\left[\int_0^\infty exp(-\beta x) \cdot exp(\beta T(x)) f(x) dx\right] + \frac{1}{\beta} ln\left[\int_0^\infty exp(-\beta x) f(x) dx\right] + \overline{T} \tag{43}$$

which is a new measure of absolute tax progressivity underlying the Kolm-Pollak social welfare function. If we substitute $T(x) = a$ for all x in (43), $\theta_{KP} = 0$; if everyone pays the same tax, the progressivity index is equal to zero.

Blackorby and Donaldson (1984) attempted to derive measures of tax progressivity by comparing the actual post-tax social welfare with the social welfare that would be achieved by an equal-yield proportional tax. They defined their index of progressivity as

$$BD_\in = \frac{W_y(\in) - (1-e) W_x(\in)}{(1-e) W_x(\in)}. \tag{44}$$

Atkinson's before- and after-tax inequality measures are given by

$$I_x(\in) = 1 - \frac{W_x(\in)}{\overline{x}} \text{ and } I_y(\in) = 1 - \frac{W_y(\in)}{\left(1-\overline{e}\right)\overline{x}} \tag{45}$$

respectively, which on substituting in (44) yields

$$BD = \frac{I_x(\in) - I_y(\in)}{1 - I_x(\in)} \tag{46}$$

which is the change in inequality as a percentage of equality of pre-tax income. This equation shows that Blackorby and Donaldson (1984) measured the redistributive effect of tax and not tax progressivity. The tax progressivity measures the distribution of tax burden according to people's ability to pay.

8 Suits' Measure of Tax Progressivity

Suppose $C(p)$ is the proportion of taxes paid by the bottom p proportion of individuals, and $L(p)$ is the proportion of their income. The graph of $C(p)$ and $L(p)$ is called the relative concentration curve of taxes with respect to income (Kakwani 1980). Suits (1977) proposed a measure of progressivity, which is equal to 1 minus twice the area under the relative concentration curve:

$$S = 1-2\int_0^1 C(p)dL(p) \tag{47}$$

When $C(p)=L(p)$ for all p, all individuals pay the same share of taxes, which is equal to their share of incomes, implying that the tax system is proportional and thus, $S=0$. When $C(p)=0$ for all p (when the richest person in society pays all the taxes), $S=1$; this scenario shows extreme progressivity. When $C(p)=1$ for all p (the poorest person in society pays all the taxes), $S=-1$. Hence, S lies between -1 and $+1$.

The social welfare function that provides the Suits measure of progressivity is somewhat unclear. To determine this, we evaluated the integral in (47) as

$$S = 1-\frac{2}{\overline{T}}\int_0^\infty T(x)[1-F_1(x)]f(x)dx \tag{48}$$

where $F_1(x)$ is the cumulative proportion of incomes of individuals with income less than or equal to x.

Kakwani (1980) introduced the following social welfare function, which for the pre-tax income is given by

$$W_x^S = \frac{2}{(1+G_x)}\int_0^\infty x[1-F_1(x)]f(x)dx \tag{49}$$

where G_x is the Gini index of the pre-tax income distribution. This social welfare function is the weighted average of income levels. The weight given to an individual with income x is $\frac{2}{(1+G_x)}[1-F_1(x)]$, which adds up to 1 for the whole population. When integrating (49) by parts, W_x^S simplifies to $\frac{\overline{x}}{(1+G_x)}$. Similar to Sen's social welfare function, this function captures the relative deprivation suffered by a society. The extent of deprivation experienced by an individual with income x is proportional to the total income of individuals in a society who are richer than the person with income x. In Sen's social welfare function, the deprivation is proportional to the number of individuals richer than the person with income x. This social welfare function has different normative judgments from that of Sen. Although Kakwani (1980) proposed this social welfare function, we refer to it as the Suits social welfare function for convenience. The post-tax social welfare of this function would be

$$W_y^S = \frac{2}{\left(1+G_y\right)} \int_0^\infty y\left[1-F_1^*(y)\right] f^*(y)dy \tag{50}$$

where G_y is the Gini index of the post-tax income distribution. The pseudo-social welfare of the post-tax income distribution uses the pre-tax weights and is given by

$$\widehat{W}_y^S = \frac{2}{\left(1+G_y\right)} \int_0^\infty [x-T(x)][1-F_1(x)]f(x)dx \tag{51}$$

Thus, the relative measure of horizontal inequity for the Suits social welfare function would be

$$H_S = \frac{W_y^S - \widehat{W}_y^S}{\overline{T}} < 0 \tag{52}$$

Using (49) and (50) in conjunction with (51) and (52) yields the following decomposition

$$\frac{W_y^S - W_x^S}{\overline{T}} = H_S - \frac{1}{(1+G_x)} + \frac{S}{(1+G_x)} \tag{53}$$

where S is the Suits index of tax progressivity. This decomposition shows that the total welfare impact of taxation when the government collects an average of one dollar of tax from society has three components: (i) the loss of welfare when there is a change in ranking, (ii) the loss of welfare when the tax system is proportional, and (iii) the gain (loss) of welfare when the tax system is progressive (regressive). A taxation system is progressive when the Suits index (S) is positive—that is, tax is contributing to a gain in welfare. Conversely, if S is negative, taxation is regressive and contributes to a welfare loss.

The gain (loss) of social welfare is $\frac{S}{(1+G_x)}$, when the tax system is progressive (regressive), and society pays an average of one dollar of tax. Like other progressivity measures, a social welfare interpretation cannot be directly ascribed to the Suits index unless it is normalized by $(1 + G_x)$. We, therefore, propose a modified Suits measure of progressivity as $\frac{S}{(1+G_x)}$, which has a social welfare interpretation. This proposed modification is the first in the literature. Furthermore, the Suits social welfare function is not absolutely homogeneous of degree one, making it impossible to obtain the absolute measures of progressivity.

9 A New Progressivity Index Based on Bonferroni Social Welfare Function

In 1930, Carlo Emilio Bonferroni proposed a curve similar to the Lorenz curve based on the cumulative means of income distribution. This curve is defined as $B(p) = \frac{L(p)}{p}$. Based on this curve, Son (2011) derived a social welfare function, which for the pre-tax income distribution is written as

$$W_x^B = -\int_0^\infty x ln(F(x))f(x)dx = \overline{x}(1-B_x) \tag{54}$$

where $F(x)$ is the probability distribution function. This social welfare function is the weighted average of income levels. The weight given to an individual with income x is $-ln(F(x))$, which decreases monotonically with income. The total weight adds up to 1 for the entire population. The inequality measure implicit in this social welfare function is B_x, which is the Bonferroni index of inequality of the pre-tax income distribution.

The social welfare function of the post-tax income $y(x) = x - T(x)$ is given by

$$W_y^B = -\int_0^\infty (x - T(x)) ln\left(F^*(y)\right) f^*(y) dx = \overline{x}\,(1-e)\left(1-B_y\right) \tag{55}$$

where B_y is the Bonferroni inequality index for the post-tax income. The Bonferroni pseudo-social welfare function of the post-tax income distribution is given by

$$\widehat{W}_y^B = -\int_0^\infty (x - T(x)) ln(F(x)) f(x) dx = \overline{x}\,(1-e)\left(1-C_y^B\right) \tag{56}$$

where C_y^B is the concentration index of the post-tax income for the Bonferroni social welfare function.

Similarly, the Bonferroni the pseudo-social welfare function for taxes is

$$W_T^B = -\int_0^\infty T(x) ln(F(x)) f(x) dx = \overline{T}\left(1-C_T^B\right) \tag{57}$$

where C_T^B is the concentration index of taxes. Horizontal inequity is measured by the loss of social welfare due to the change in ranking and is given by

$$H_B = W_y^B - \widehat{W}_y^B < 0 \tag{58}$$

From (54), (56) and (57), it is easy to verify that

$$\widehat{W}_y^B = W_x^B - W_T^B \tag{59}$$

which upon substituting in (8) and using (54) and (57) yields a new tax progressivity index based on the Bonferroni social welfare function as

$$\theta_B = C_T^B - B_x \tag{60}$$

which we will refer to as the Bonferroni index of tax progressivity. The tax system is progressive (regressive) when θ_B is positive (negative).

Substituting (59) into (7) and using (54), (55), (58), (59), and (60) gives the decomposition:

$$\frac{W_y^B - W_x^B}{\overline{T}} = \frac{H_B}{\overline{T}} - (1 - B_x) + \theta_B \tag{61}$$

which shows that the total welfare impact of taxation when society pays an average of one dollar tax is the sum of three components: (i) the loss of social welfare when there is a change in ranking between the pre- and post-tax incomes, (ii) the loss of social welfare when the tax system is proportional, and (iii) the gain (loss) of social welfare when the tax system is progressive (regressive). The progressivity index θ_B is interpreted as the welfare contribution of tax progressivity when society pays an average of one dollar of tax. The progressive tax system, therefore, contributes to a gain in social welfare, while the regressive tax system contributes to a loss of social welfare.

The Bonferroni social welfare function is absolutely homogeneous of degree one, so we obtain the absolute measure of tax progressivity as

$$\theta_A = C_T^B \tag{62}$$

which upon substituting into (53) yields the change in social welfare as

$$W_y^B - W_x^B = H_B - \overline{T} + C_T^B \tag{63}$$

When $C_B = 0$, everyone pays the same amount of tax, and there is no change in ranking. The resulting loss of social welfare is thus equal to the average tax paid by society.

10 International Comparison of Tax Progressivity

This section discusses an international comparison of tax progressivity using the measures developed in this paper. We use the income distribution data for 32 countries obtained from the Luxembourg Income Study database, which contains the most comprehensive income data available for around 50 countries in Europe, North America, Latin America, Africa, Asia, and Australasia, spanning five decades. We selected the 32 countries based on (i) the availability of comparable tax data (observed only for industrialized countries) that allows for the comparisons of tax progressivity across countries, and (ii) the availability of household surveys conducted around 2013.

Gross household income is defined as total monetary and non-monetary current income, gross of income taxes, and social security contributions. Disposable income is the total household gross income net of taxes and social security contributions. We equalized household incomes and taxes (including social security payments) by dividing them by the square root of the number of household members. This equalizing procedure accounts for different needs of household members and economies of scale that occur in larger households. To make international comparisons, incomes and taxes are measured based on the 2011 purchasing power parity (PPP) international dollars.

Linear regression and correlation techniques are commonly used to measure relationships between variables. The relationships involving social welfare functions and tax progressivity are often non-linear. Thus, the correlation coefficients that measure a deviation from linearity may invariably show that the variables are not significantly or only weakly related. Given the non-linear nature of variables, some analysts have estimated linear regressions after applying a non-linear transformation to the original data. Since the exact forms of non-linear relationships are unknown, incorrect conclusions on the significance of relationships may emerge. In these situations, rank-correlation methods are more robust (Iman and Conover 1978). For this paper, we have used the Spearman correlation coefficient to test whether there is a significant relationship between variables. The following t statistics are used to test the significance of relationships.

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \tag{64}$$

where r is the Spearmen rank correlation, and that is distributed approximately as Student's t distribution with $(n-2)$ degrees of freedom. Pitman (1937) proposed this test procedure, which performs better than the usual normal approximation (Iman and Conover 1978).

Table 1 Relative Progressivity and Social Welfare Contributions of Taxation when the Average Tax Collected is $1

Countries	Russia	Korea	UK	Israel	Germany	Finland	Canada	Australia	U.S.
Generalized Gini Social welfare k = 1									
Horizontal Inequity	−0.01	−0.01	−0.03	−0.03	−0.02	−0.01	−0.01	−0.01	−0.01
Proportional tax	−0.62	−0.67	−0.61	−0.6	−0.62	−0.65	−0.61	−0.59	−0.56
Progressivity	0.08	0.09	0.17	0.2	0.15	0.12	0.15	0.22	0.18
Total welfare loss	−0.55	−0.59	−0.47	−0.43	−0.48	−0.53	−0.48	−0.38	−0.39
Generalized Gini Social welfare k = 2									
Horizontal Inequity	−0.01	−0.01	−0.04	−0.04	−0.02	−0.01	−0.01	−0.01	−0.01
Proportional tax	−0.49	−0.53	−0.48	−0.46	−0.48	−0.52	−0.48	−0.46	−0.42
Progressivity	0.11	0.1	0.21	0.19	0.18	0.15	0.18	0.24	0.19
Total welfare loss	−0.39	−0.44	−0.31	−0.31	−0.32	−0.38	−0.31	−0.22	−0.24
Generalized Gini Social welfare k = 3									
Horizontal Inequity	−0.01	−0.01	−0.05	−0.05	−0.02	−0.01	−0.01	−0.01	−0.01
Proportional tax	−0.42	−0.45	−0.41	−0.38	−0.41	−0.44	−0.4	−0.38	−0.35
Progressivity	0.12	0.09	0.22	0.17	0.19	0.16	0.19	0.24	0.18
Total welfare loss	−0.31	−0.36	−0.24	−0.27	−0.24	−0.29	−0.22	−0.15	−0.17
Bonferroni Social Welfare Function									
Horizontal Inequity	−0.01	−0.01	−0.07	−0.12	−0.02	−0.01	−0.01	−0.01	−0.01
Proportional tax	−0.49	−0.52	−0.48	−0.46	−0.49	−0.52	−0.48	−0.46	−0.43
Progressivity	0.1	0.08	0.18	0.14	0.16	0.14	0.16	0.22	0.17
Total welfare loss	−0.41	−0.46	−0.37	−0.44	−0.35	−0.39	−0.33	−0.25	−0.27
Suits Social Welfare Function									
Horizontal Inequity	−0.02	−0.03	−0.03	−0.06	−0.06	−0.06	−0.06	−0.1	−0.08
Proportional tax	−0.68	−0.69	−0.66	−0.6	−0.61	−0.63	−0.59	−0.5	−0.54
Progressivity	0.05	0.08	0.13	0.18	0.12	0.1	0.12	0.18	0.15
Total welfare loss	−0.65	−0.64	−0.56	−0.48	−0.55	−0.58	−0.54	−0.42	−0.46
Atkinson's Social Welfare Function: Inequality aversion parameter = 0.5									
Proportional tax	−0.88	−0.9	−0.88	−0.87	−0.88	−0.9	−0.88	−0.86	−0.84
Progressivity	0.04	0.12	0.1	0.16	0.08	0.06	0.09	0.14	0.12
Total welfare loss	−0.84	−0.78	−0.77	−0.71	−0.8	−0.83	−0.79	−0.72	−0.72
Atkinson's Social Welfare Function: Inequality aversion parameter = 1.0									
Proportional tax	−0.77	−0.79	−0.77	−0.75	−0.77	−0.8	−0.75	−0.74	−0.7
Progressivity	0.09	0.15	0.17	0.22	0.15	0.12	0.16	0.23	0.2
Total welfare loss	−0.69	−0.64	−0.6	−0.52	−0.62	−0.68	−0.59	−0.51	−0.5
Atkinson's Social Welfare Function: Inequality aversion parameter = 1.5									
Proportional tax	−0.68	−0.68	−0.65	−0.62	−0.68	−0.71	−0.59	−0.53	−0.49
Progressivity	0.16	0.26	0.25	0.26	0.16	0.1	0.23	0.21	0.21
Total welfare loss	−0.51	−0.42	−0.4	−0.36	−0.52	−0.6	−0.36	−0.32	−0.27

Note: Results for 32 countries are available in the supplementary Excel file

Source: Authors' calculations

Our purpose is not to establish a causal relationship between the variables, which would require a highly complex general-equilibrium model. Our aim is limited to determining whether there are significant monotonic relationships between the variables. We, therefore, carried out the rank-correlation analysis using the data on 32 countries, setting the statistical significance level at 1%. If the rank-correlation coefficients among the variables were significant, we could conclude that the relationships among them would exist with a high degree of confidence.

10.1 Relative Measures of Tax Progressivity

Table 1 presents the relative measures of tax progressivity for the nine wealthiest countries of the 32 selected for this study. In the generalized Gini social welfare function, when $k = 0$, the social welfare function collapses to the mean income of society; that is, everyone has the same income and pays the same tax rate, in which case taxes will be proportional. For every dollar of average tax collected by the government, society loses one dollar per person of social welfare.

When $k = 1$, the progressivity index derived from the generalized Gini social welfare function is the Kakwani index based on Sen's social welfare function. Taking Australia as an example, the value of the index is 0.22, implying that the Australian tax system is progressive and contributes to a social welfare gain of 22 cents. If the tax system were proportional, there would be a loss of social welfare equal to 59 cents. Thus, the net loss of social welfare contributed by the Australian tax system is 37 cents. The Australian tax system also incurs a welfare loss of 0.01 cents due to horizontal inequity. The Australian tax system, therefore, contributes to a total social welfare loss of 38 cents. The Australian government mobilizes revenues of one dollar from tax that it can invest in the provision of public goods, public services such as education and health, and welfare programs. For the Australian government to break even, it has to generate a social rate of return of 38% from its investments.

The social welfare lost due to taxation in other countries is much larger. For instance, the social welfare loss in Korea is 59 cents, which implies that the Korean government has to generate a social rate of return of 59% when it invests its tax revenues in public investments. The main reason for this high loss of social welfare is that the Korean tax system is relatively less progressive.

Almost all governments around the globe mobilize a significant proportion of their tax revenues from indirect taxation, which is always regressive. Hence, the loss of social welfare from taxation would be much larger. Suppose tax is mildly regressive with the value of progressivity index $K = -0.10$. Under this scenario, the total loss of social welfare for Korea would be 78 cents for every dollar of tax collected. Hence to break even, the government needs to generate a social rate of return of 78% from its public programs. Society suffers a net loss in social welfare due to taxation if the Korean government fails to generate such high returns.

Furthermore, since the government incurs the administrative costs of tax collection as well as investments, it will need to generate a much higher social rate of return to break even. Optimal social welfare, therefore, requires three factors: (i) a tax system designed to be progressive; (ii) minimum administrative costs in collecting taxes; and (iii) efficient investments of tax revenues to maximize the social rates of return.

The progressivity index must increase when a tax is transferred from a richer individual to a poorer one, provided the transfer does not change the ranking. This requirement is satisfied by all the measures of tax progressivity discussed in the paper. However, an additional element is that the progressivity index must be more sensitive to such transfers if they take place among relatively poorer individuals. In the generalized Gini social welfare function, k is the parameter of inequality aversion. In Table 1, as k increases from 1 to 3, the progressivity index becomes more sensitive to tax transfers among poorer individuals. If a society's objective is to design a tax system that is more sensitive to tax transfers among those relatively poorer, then it should choose the progressivity index with a higher value of k. We note from Table 1 that as k rises, the social welfare losses fall for all nine countries, which implies that the social rate of return from their public investments needed for governments to break even also decreases. This observation has a similarity with the "leaky bucket experiment." As k rises, society tolerates more waste and inefficiency.[17]

Atkinson's social welfare functions also consider the different degrees of the aversion parameter, which makes it possible to assign higher weights to the transfers at the lower end of

[17] One of the referees pointed out this observation. See Okun (1975) for discussion on "leaky bucket experiments".

Table 2 Absolute Progressivity and Social Welfare Contributions of Taxation

Countries	Russia	Korea	UK	Israel	Germany	Finland	Canada	Australia	U.S.
Generalized Gini Social welfare k = 1									
Horizontal Inequity	−35	−40	−284	−286	−249	−96	−110	−118	−98
Proportional tax	−3796	−4382	9241	9789	−16,494	−15,076	−14,527	−14,770	−16,050
Progressivity	1746	1856	5191	5880	8808	7145	7737	9267	9898
Total welfare loss	−2084	−2566	4335	4194	−7935	−8027	−6900	−5620	−6250
Generalized Gini Social welfare k = 2									
Horizontal Inequity	−42	−46	−368	−404	−324	−108	−127	−128	−111
Proportional tax	−3796	−4382	9241	9789	−16,494	−15,076	−14,527	−14,770	−16,050
Progressivity	2346	2498	6744	7198	11,549	9524	10,221	11,601	12,337
Total welfare loss	−1492	−1930	2865	2994	−5269	−5660	−4433	−3296	−3824
Generalized Gini Social welfare k = 3									
Horizontal Inequity	−44	−47	−431	−501	−355	−112	−131	−121	−111
Proportional tax	−3796	−4382	9241	9789	−16,494	−15,076	−14,527	−14,770	−16,050
Progressivity	2661	2842	7461	7674	12,876	10,767	11,473	12,707	13,450
Total welfare loss	−1179	−1587	2211	2616	−3973	−4421	−3185	−2184	−2711
Bonferroni Social Welfare Function									
Horizontal Inequity	−38	−51	−651	1174	−370	−100	−112	−106	−95
Proportional tax	−3796	−4382	9241	9789	−16,494	−15,076	−14,527	−14,770	−16,050
Progressivity	2291	2438	6436	6630	11,077	9289	9861	11,157	11,880
Total welfare loss	−1543	−1995	3456	4333	−5788	−5886	−4778	−3719	−4265
Suits Social Welfare Function									
Horizontal Inequity	−84	−130	−294	−557	−908	−850	−921	−1522	−1280
Proportional tax	−3796	−4382	9241	9789	−16,494	−15,076	−14,527	−14,770	−16,050
Progressivity	1421	1710	4363	5677	8343	7123	7593	10,023	9877
Total welfare loss	−2458	−2802	5172	4668	−9059	−8803	−7856	−6268	−7453
Atkinson's Social Welfare Function: Inequality aversion parameter = 0.5									
Proportional tax	−3796	−4382	9241	9789	−16,494	−15,076	−14,527	−14,770	−16,050
Progressivity	615	949	2096	2858	3314	2488	3038	4118	4547
Total welfare loss	−3181	−3433	7146	6931	−13,180	−12,588	−11,489	−10,652	−11,504
Atkinson's Social Welfare Function: Inequality aversion parameter = 1.0									
Proportional tax	−3796	−4382	9241	9789	−16,494	−15,076	−14,527	−14,770	−16,050
Progressivity	1194	1583	3697	4666	6204	4787	5942	7299	8011
Total welfare loss	−2601	−2799	5544	5122	−10,290	−10,289	−8585	−7471	−8039
Atkinson's Social Welfare Function: Inequality aversion parameter = 1.5									
Proportional tax	−3796	−4382	9241	9789	−16,494	−15,076	−14,527	−14,770	−16,050
Progressivity	1847	2542	5529	6274	7903	5967	9258	10,103	11,650
Total welfare loss	−1949	−1840	3712	3515	−8591	−9109	−5269	−4667	−4400

Note: Results for 32 countries are available in the supplementary Excel file

Source: Authors' calculations

the income distribution. As expected, the conclusions emerging from them are similar to those from the generalized Gini social welfare functions.

In this paper, we have also computed tax progressivity measures using the Bonferroni and Suits social welfare functions. Table 1 reveals that for the Australian tax system, the Bonferroni social welfare function results in a welfare loss of 25 cents, while the Suits social welfare function yields a much larger welfare loss of 42 cents. The welfare gain due to the progressivity of taxation is also much higher for the Bonferroni social welfare function at 25 cents compared with 18 cents when the Suits social welfare function is used. Although different social welfare functions result in different magnitudes of tax progressivity measures and welfare losses, the overall conclusions emerging from our empirical results are not much different.

10.2 Absolute Measures of Tax Progressivity

Table 2 presents the absolute measures of tax progressivity and welfare losses due to taxation based on 2011 PPP international dollars. Taking Canada as an example, tax progressivity derived from the Gini social welfare function contributes to an increase in social welfare equivalent to $7,737 per person. The Canadian tax system, nevertheless, results in a total welfare loss of $6,900 per person. The Canadian government mobilizes tax revenues worth $14,527 per person. To break even, the government needs to generate social welfare of an amount equivalent to more than $6,900 per person. Otherwise, the Canadian society would suffer a net social welfare loss, in which case the government should not be imposing a tax on the population.

10.3 Rank Correlation Analysis

This paper presents eight alternative social welfare functions from which we obtained eight alternative measures of tax progressivity. All progressivity measures satisfy the fundamental axiom of equity in taxation. The absolute measures satisfy the fundamental axiom that the rich should pay more tax than the poor. Similarly, the relative measures satisfy a stronger axiom that the rich should pay tax at a higher rate. Empirical calculations of the tax progressivity

Table 3 Spearman Rank Correlation for 32 Countries

		Tax	Generalized Gini			Bonferroni	Suits	Atkinson with aversion		
	Income	Rate	$k=1$	$k=2$	$k=3$			0.5	1.0	1.5
Income	1.00	0.68*								
Tax rate	0.68*	1.00								
Progressivity measures										
Generalized Gini $k=1$	−0.32	−0.24	1							
Generalized Gini $k=2$	−0.31	−0.27	0.96*	1						
Generalized Gini $k=3$	−0.25	−0.25	0.88*	0.97*	1					
Bonferroni	−0.32	−0.24	0.94*	0.97*	0.96*	1				
Suits	−0.27	−0.24	0.98*	0.91*	0.81*	0.90*	1			
Atkinson aversion = 0.5	−0.41	−0.44	0.79	0.69*	0.58*	0.67*	0.83*	1		
Atkinson aversion = 1	−0.39	−0.38	0.90*	0.83*	0.75*	0.81*	0.91*	0.94*	1	
Atkinson aversion = 1.5	−0.38	−0.47	0.68*	0.66*	0.61*	0.62*	0.67*	0.72*	0.81*	1
Redistribution of tax										
Generalized Gini $k=1$	−0.52*	0.72*	1							
Generalized Gini $k=2$	−0.50*	0.72*	0.98*	1						
Generalized Gini $k=3$	−0.47*	0.70*	0.95*	0.99*	1					
Bonferroni	−0.36	0.62*	0.89*	0.94*	0.96*	1				
Suits	−0.56*	0.70*	0.98*	0.95*	0.92*	0.84*	1			
Atkinson aversion = 0.5	−0.58*	0.71*	0.91*	0.86*	0.82*	0.68*	0.92*	1		
Atkinson aversion = 1	−0.57*	0.71*	0.95*	0.94*	0.92*	0.80*	0.95*	0.95*	1	
Atkinson aversion = 1.5	−0.39	−0.43	0.64*	0.64*	0.62*	0.53*	0.62*	0.73*	0.70*	1

Note: * indicates statistical significance at 1%

Source: Authors' calculations

show different magnitudes of the change in progressivity. A question that accordingly arises is whether the various measures derived from different social welfare functions result in significantly different rankings of countries.

Formby et al. (1981) presented an empirical study using the U.S. income tax system for the period 1962–1976 and concluded that the Kakwani and Suits measures of progressivity displayed opposite rankings in three out of fourteen years. If this conclusion is generally correct, then it is essential to know which social welfare functions we should use in analyzing progressivity in taxation. We address this issue using the rank-correlation method.

Table 3 presents the empirical results on the Spearman rank correlation that indicates changes in the rankings of 32 countries by various progressivity measures. As shown in the table, the estimates of the rank correlation are all positive and statistically significant at the 1% level of significance. We, therefore, conclude that there is a significant monotonic relationship among the measures of progressivity derived from different social welfare functions. These observations are inconsistent with Formby, Seaks, and Smith's conclusion that the measures of progressivity are fundamentally different and can move in the opposite direction. One possible explanation for such divergence is that, as pointed out earlier, Suits measure cannot be directly assigned a social welfare interpretation unless it is normalized by $(1 + G_x)$. Unfortunately, Formby et al. (1981) did not perform this normalization.

Various measures of redistribution of taxation are also derived from different social welfare functions. The empirical results in Table 3 also reveal that there is a significant monotonic relationship among various measures of the redistribution effects of taxation. The rankings of countries by the progressivity and redistribution effects of tax, therefore, do not change significantly. This paper also pointed out earlier that for a given degree of progressivity, the average tax rate contributes to a reduction in post-tax income inequality—that is, it increases the redistribution of taxation. Our rank-correlation analysis is consistent with this theory. The rank correlations presented in Table 3 also reveal that a tax rate has no significant relationship among various measures of progressivity. Still, as expected, there is a significant relationship between the tax rate and the redistribution of taxation.

11 Conclusions

The design of public policies and programs of tax progressivity should account for assessments of the extent to which tax systems lead to social welfare gains or losses. The literature on taxation, however, has yet to explore the social welfare implications of the measures of progressivity. This paper undertakes a pioneering effort in this field by developing a social welfare function framework for deriving the measures of tax progressivity and exploring their social welfare implications.

We propose a social welfare function framework, from which we derive a general progressivity index. We use this general progressivity index to obtain progressivity indices from particular social welfare functions. Using this methodology, the paper introduces eight alternative measures of tax progressivity from eight different social welfare functions.

This paper provides a generalization of the Kakwani index from which the Kakwani index is obtained as a particular case of the generalized progressivity measures. This general class of progressivity measures relies on parameter k. The parameter k is similar to Atkinson's inequality aversion parameter that assigns a higher weight to the poorer populations in the income distribution. The paper also derives new progressivity measures from Atkinson's class of social welfare functions.

We also propose a modification of the Suits measure of progressivity. While this measure is widely used to analyze equity in taxation, it needs to be appropriately normalized so that it can be assigned a social welfare interpretation. This paper also develops a new progressivity measure based on the Bonferroni social welfare function.

Using the methodology developed in this paper, we compare tax progressivity using the income distribution data from 32 developed countries, selected based on the availability of comparable tax data and household surveys conducted around 2013. The relative measures of progressivity are interpreted as the welfare gains (losses) of tax progressivity (regressivity) when society pays an average one dollar of tax. Taking Australia as an example, the value of the Kakwani index of tax progressivity for the Australian tax system is calculated at 0.22. This finding implies that the Australian tax system is progressive. For every dollar in tax the Australian government mobilizes, the tax system contributes to a social welfare gain of 22 cents. If the Australian tax system were proportional, there would be a loss of social welfare equal to 59 cents, resulting in a net loss of social welfare of 37 cents. The paper finds that the Australian tax system also incurs a welfare loss of 0.01 cents due to horizontal inequity; hence, the tax system's contribution to the total social welfare loss is 38 cents. The Australian government needs to generate a social rate of return of 38% from its public investments funded by the revenue collected from taxes.

The paper concludes that optimizing social welfare requires three factors: a progressive tax system, minimum administrative costs for collecting taxes, and efficient investments of tax revenues to maximize the social rate of return.

Although different social welfare functions result in varying magnitudes of tax progressivity measures and welfare losses, the overall conclusions are by and large similar, and the overall results are pretty robust regardless of the social welfare function used.

Considering various kinds of distortions that lead to lower efficiency in the tax system, the optimum taxation literature has attempted to determine how progressive a tax system should be. Covering both the issues of efficiency and equity, it deals with determining the optimal tax structure that is associated with maximizing social welfare. This paper shows that the welfare loss of taxation can be substantial and may get worse if the tax system is not efficient. The efficiency in taxation is beyond the scope of this paper. However, we have emphasized the importance of efficient and equitable investments of tax revenues by governments so that society does not suffer a welfare loss from taxation. Thus, maximizing social rates of return from government investments should be an essential component of any debate on taxation policy.

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Appendix

Effect of rank change due to taxation on social welfare

In Section 3, we discussed the general classes of additive separable and rank-order social welfare functions. We demonstrate in this Appendix that for these classes of social welfare functions, the change in ranking between the pre- and post-tax incomes will always result in a loss of social welfare.

Additive social welfare functions

Atkinson's class of additive social welfare functions for the pre-tax income x is presented in (1). Suppose $y(x) = x - T(x)$ is the post-tax income of a person with pre-tax income x, then the social welfare of $y(x)$ can be written as

$$u\left(W_y\right) = \int_0^\infty u[y(x)]f^*(y)dy \tag{65}$$

where $f^*(y)$ is the density function of the post-tax income y. We now introduce the idea of pseudo-social welfare function of $y(x)$ as

$$u\left(\widehat{W}_y\right) = \int_0^\infty u[y(x)]f(x)dx \tag{66}$$

which uses the density function of pre-tax income x. If $y'(x) = [1 - T'(x)] > 0$ for all x, it means that the marginal tax rate $T'(x) < 1$ for all x. This requirement implies that $y(x)$ increases monotonically with x. Using the basic statistics of monotonic transformation, the density function of $y(x)$ is then given by

$$f^*(y) = \frac{f(x)}{\left[1-T'(x)\right]} \tag{67}$$

which immediately implies that $f^*(y)dx = f(x)dx$. Thus, from (A.1.1) and (A.1.2), we obtain $W_y = \widehat{W}_y$, meaning that the social welfare of $y(x)$ will be the same as the pseudo-social welfare of $y(x)$.

The monotonicity property will not hold if $T'(x) > 1$ for some x, implying that taxpayers pay a tax of more than one dollar for every additional dollar they earn. This situation also means that ranking between the pre- and post-tax incomes will change.

Suppose we can partition income into a finite number of k intervals such that $y(x)$ is strictly monotonic and differentiable on each partition, then $f^*(y)$ is given by

$$f^*(y) = \sum_{j=1}^{k} \frac{f\left(x_j\right)}{\left[1-T'\left(x_j\right)\right.} \tag{68}$$

If $T'(x_j) < 1$ for all j, then $f^*(y)dy = f(x)dx$. Suppose $T'(x_j) > 1$ for some j. In this case, $1 - T'(x_j) < 0$, which from (A.1.4) would imply $f^*(y)dy < f(x)dx$. Accordingly from (A.1.1) and (A.1.2), we immediately obtain $u(W_y) < u\left(\widehat{W}_y\right)$, which, on using $u'(x) > 0$ for all x, leads to $W_y < \widehat{W}_y$. Therefore, we arrive at the following theorem.

Theorem A.1.

For a general class of Atkinson's social welfare functions, if there is a change in ranking between the pre- and post-tax incomes, then there will always be a loss of social welfare given by $H = W_y - \widehat{W}_y < 0$.

Rank-order social welfare functions

A general form of rank-order social welfare function is presented in (3.2). Given this, the social welfare of $y(x) = x - T(x)$ is obtained as

$$W_y = \int_0^\infty y(x) v\left(F^*(y)\right) f^*(y) dy \tag{69}$$

which on using (3.3) is expressed as

$$W_y = \text{Covariance } \left[y(x), v\left(F^*(y)\right)\right]. \tag{70}$$

We obtain the pseudo-social welfare function of $y(x)$ using the weights of the pre-tax income, therefore, is given by

$$\widehat{W}_y = \int_0^\infty y(x) v(F(x)) f(x) dx \tag{71}$$

which can be written as

$$\widehat{W}_y = \text{Covariance } [y(x), v(F(x))]. \tag{72}$$

Given a fixed number of persons in the population, the variance of $F(x)$ and $F^*(y)$ will be the same, then (A.2.2) and (A.2.4) will yield

$$\frac{\widehat{W}_y}{W_y} = \frac{R[y(x), v(F(x))]}{R\left[y(x), v\left(F^*(y)\right)\right]}. \tag{73}$$

$R(a, b)$ stands for the coefficient of correlation between a and b. Note that $F(x)$ is the cumulative proportion of individuals when they are arranged in ascending order of their pre-tax income, while $F^*(y)$ is the cumulative proportion of individuals when their post-tax income arranges them. The difference between $F(x)$ and $F^*(y)$ will, therefore, be due to the difference in rankings between x and y. Thus, if $F(x)$ and $F^*(y)$ are replaced by rankings of x and y, the correlation coefficient in (A.2.5) will not change. Since $v(F(x))$ and $v(F^*(y))$ are monotonically decreasing functions $F(x)$ and $F^*(y)$, respectively, equation (A.2.5) becomes

$$\frac{\widehat{W}_y}{W_y} = \frac{R[y(x), r(-x)]}{R[y(x), r(-y)]} \tag{74}$$

If $y'(x) > 0$, x and y will have the same rank, then from (A.2.6), $W_y = \widehat{W}_y$. But if $y'(x) < 0$, then x and y will have a different rank, in which case $W_y < \widehat{W}_y$. This thus leads to the following theorem.

Theorem A.2.

For a general class of rank-order social welfare functions, if there is a change in ranking between the pre- and post-tax incomes, there will always be a loss of social welfare given by $H = W_y < \widehat{W}_y < 0$.

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