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Binary data, hierarchy of attributes, and multidimensional deprivation

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Abstract Empirical estimation of multidimensional deprivation measures has gained momentum in the last few years. Several existing measures assume that deprivation dimensions are cardinally measurable, when, in many instances, such data is not always available. In this paper, we propose a class of deprivation measures when the only information available is whether an individual is deprived in an attribute or not. The framework is then extended to a setting in which the multiple dimensions are grouped as basic attributes that are of fundamental importance for an individual's quality of life and non-basic attributes which are at a much lower level of importance. Empirical illustrations of the proposed measures are provided based on the estimation of multidimensional deprivation among children in Ethiopia, India, Peru and Vietnam.

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1 Introduction

In the last few years, assessment of multidimensional deprivation has emerged at the forefront of poverty research. Measurement of multidimensional poverty levels are a high-priority topic of research with enormous policy implications (Permanyer 2014). The importance of attributes of well-being, besides income, has long been recognized (e.g., Hicks and Streeten's 1979 basic needs approach, Sen's 1985 capabilities approach). Early attempts at measuring multidimensional deprivation date back to Townsend (1979) who provided a comprehensive assessment of poverty in the United Kingdom by compiling data on individual deprivations in multiple attributes besides income. Large datasets such as the Swedish Level of Living Survey (Erikson 1993), the ESRI studies on poverty in Ireland (Callan et al. 1999), and the SILC data in the European Union (Whelan et al. 2014) have facilitated the measurement of deprivation within countries. The United Nations Human Development Report (2010) for the first time estimated multidimensional poverty across a large number of developing countries. Statistical agencies in Mexico, Columbia, Bhutan and Philippines now publish annually estimates of multidimensional poverty at the national level. Despite the rapid pace at which the empirical literature is growing, the existing deprivation measures have not been quite amenable for estimation purposes.

In particular, we question two key assumptions made in the literature. The first assumption concerns measuring the extent of deprivation. It is usually assumed that each of the multiple attributes or dimensions under consideration is cardinally measurable along real intervals. However, in many instances, it is simply difficult to collect cardinal data and only ordinal data are available. For instance, in order to measure material deprivation or an asset index, typically the only data available is whether or not a household has a working toilet, a television set and so on. The assumption of cardinal measurement is demanding and unlikely to be satisfied for many crucial dimensions of deprivation. Ideally, a general theory of multidimensional deprivation should permit different (ordinal and cardinal) forms of measurement for different attributes. Such general theory will subsume, as special cases, the analytical framework based on the assumption of cardinal measurement for all attributes as well as the analytical framework, which we adopt here and which assumes binary, ordinal measurement for all attributes. In the absence of such general theory, however, it is still useful to explore the various special cases so as to gain further analytical insights, which may serve as building blocks for the construction of a general theory permitting measurements of different types for different attributes.[1]

A second, related assumption concerns aggregating the multiple deprivations. Again, it is typically assumed that an individual's deprivations along different dimensions are substitutable, so that, other things remaining the same, a small increase in an individual's deprivation along any dimension can be offset by a suitable simultaneous decrease in the individual's deprivation along any other dimension. Yet, it seems much more plausible to assume that some attributes are of fundamental importance for an individual's quality of life while there are other attributes which are at a much lower level of importance.[2] For instance, we may treat deprivation in certain attributes, such as food, as basic and more fundamental than deprivation in other attributes such as access to the Internet. Given a multiplicity

[1]There are relatively more examples in the literature where multidimensional measures use discrete data. For instance, see, Alkire and Foster (2011), Bossert et al. (2013), Lasso de la Vega (2010) but fewer examples of measures which use binary data (see Fusco and Dickes 2006 as an example).

[2]Cf. Maslow's (1943, 1954) theory of a hierarchy of human needs.

of attributes, policy makers often like to prioritize by focusing primarily on the removal of deprivation in terms of some of these attributes, which are considered to be basic, and relegating to a second place the objective of removing deprivation in terms of the other attributes, which are considered to be non-basic. Thus, we believe that both the assumptions are too strong and often difficult to fulfill when working with empirical data.[3]

In this paper, we propose a class of measures of multidimensional deprivation, which dispenses with these assumptions. We work with a coarse form of ordinal measurement of a person's deprivation, where, for every attribute, there are exactly two levels of deprivation: either the individual is deprived along that dimension (in which case her deprivation along the dimension takes the value 1) or she is not (in which case her deprivation along the dimension takes the value 0).[4] With this simple binary measure of an individual's deprivation in terms of each attribute, the informal basis of our group deprivation measures is given by an $n \times m$ matrix where n is the number of individuals in the group, m is the number of attributes, and, for every individual and every attribute, the corresponding entry in the matrix is either 0 or 1. The problem becomes one of aggregating such a deprivation matrix to reach a single index which reflects the overall deprivation of the group. Bossert et al. (2013) discussed a similar aggregation problem with a 0–1 dichotomous matrix. The main difference between their framework and ours is two-fold: they work in a framework with variable populations while our society is given and fixed, and we introduce a different notion of additive separability.

We consider a relatively simple structure where we have exactly two categories of attributes: basic attributes and non-basic attributes.[5] In assessing overall social deprivation, each basic attribute is assumed to have priority over the class of non-basic attributes. We formulate an intuitive notion of priority of basic attributes. Suppose the deprivation status, in terms of some basic attribute j, of an individual, i, changes from 1 (deprived) to 0 (non-deprived), when every other individual's deprivation status in terms of all attributes remains the same and individual i's deprivations in terms of all basic attributes other than j also remain the same, but individual i's deprivations in terms of non-basic attributes change in any way one likes. Then priority of basic attributes requires that the overall group deprivation must decrease compared with the initial situation. We explore the weights to be attached to the attributes and show that the class of such rules is in fact a subclass of the general class of measures formulated with binary data. Our notion of priority is closely related to the notion of "unrestricted hierarchy" proposed in Esposito and Chiappero-Martinetti (2010). They focus on prioritizing over two attributes while we develop the notion of priority for two classes of attributes: basic vs. non-basic attributes, and priority is given to the collection of basic attributes as a group over the collection of non-basic attributes as a group.

Finally, we provide some examples of indices belonging to the proposed class of deprivation measures.We use data from the Young Lives, an international study on childhood poverty, conducted by the University of Oxford. The study conducted household surveys among poor communities in four developing countries: Ethiopia, India, Peru and Vietnam. We compile data which is binary in nature; for example, whether a child is underweight or

[3]Both the assumptions are implicitly made in several existing measures; see for example Bourguignon and Chakravarty (2003), Chakravarty and Silber (2007), Duclos et al. (2006), Dutta et al. (2003) and Tsui (2002).

[4]We consider binary variables which are ordinal. For examples of binary variables which can be nominal, cardinal or ratio-scale, see Alkire et al. (2015).

[5]The analysis can be extended to a setting with more than two tiers in our hierarchical structure for the attributes, though we shall not undertake this exercise here.

stunted, whether a child can read or write, and whether the household has access to electricity, sanitation and drinking water. The extent of deprivation among the poor in these countries differs significantly. For instance, 65 % of the surveyed children in India lacked access to sanitation facilities whereas in Peru only 9 % of children were deprived of sanitation facilities. In Ethiopia, 85 % children were illiterate, compared to only 17 % in Vietnam. Thus, it is not obvious which of the four countries had the highest incidence of deprivation in a multidimensional setting. We estimate multidimensional deprivation measures using alternative functional forms and rank order countries according to the extent of deprivation. We use different classification of basic and non-basic attributes by varying the weighting structure of attributes. Overall, we find that Ethiopia and India had substantially higher multidimensional deprivation among children relative to Vietnam and Peru.

The plan of the paper is as follows. In Section 2, we introduce some basic notation, in Section 3, we formulate several axioms and in Section 4 we use the axioms to characterize the main class of group deprivation measures. In Section 5, we extend our analysis by introducing a hierarchy of attributes. Empirical illustrations of the proposed measures of group deprivation are given in Section 6. We conclude in Section 7. The proofs of all the results are given in the Electronic Supplementary Material to the paper.

2 Basic notation

Let $N = \{1, \cdots, n\}$ be a given finite set of individuals with $n \geq 2$ and let $F = \{f_1, \cdots, f_m\}$ be a finite set of attributes with $m \geq 2$; we shall refer to N as the group or society under consideration. Let $M = \{1, \cdots, m\}$. Let $\mathcal{D}$ be the class of all $n \times m$ matrices, D, such that each entry in D is either 0 or 1. The elements of $\mathcal{D}$ will be denoted by $C = (c_{ij})$, $D = (d_{ij})$, etc., and will be called deprivation matrices. For all $D \in \mathcal{D}$, all $i \in N$ and all $j \in M$, the entry d_{ij} in the deprivation matrix D will be interpreted as i's level of deprivation in terms of attribute f_j: if $d_{ij} = 0$, then i is not deprived in terms of attribute f_j in the matrix D; on the other hand, if $d_{ij} = 1$, then i is deprived in terms of attribute f_j in deprivation matrix D. For each $i \in N$, let $d_{i\bullet} = (d_{i1}, \cdots, d_{im})$ denote the deprivation status of individual i, in D, along the m dimensions. Similarly, for each $j \in M$, let $d_{\bullet j} = (d_{1j}, \cdots, d_{nj})$ denote the vector of the n individuals' deprivation levels, in D, in terms of the given attribute f_j.

A group deprivation measure (or a deprivation measure for short) is a function h from $\mathcal{D}$ to the closed interval [0, 1]. For all $C, D \in \mathcal{D}$, $h(C) \geq h(D)$ is interpreted as the degree of deprivation that the society has under C is at least as high as the degree of deprivation under D, $h(C) > h(D)$ and $h(C) = h(D)$ being interpreted in a corresponding fashion.

Our central concern is the class of all group deprivation measures h, such that:

(1) for some increasing function g: $[0, 1] \to [0, 1]$ with $g(0) = 0$ and $g(1) = 1$, and some positive constants $\omega_1, \cdots, \omega_m$ with $\omega_1 + \cdots + \omega_m = 1$, we have $[h(C) = \frac{1}{n} \sum_{i \in N} g(\sum_{j=1}^{m} \omega_j c_{ij})$, for all $C = (c_{ij}) \in \mathcal{D}]$.

Let H denote the class of group of all group deprivation measures h which satisfy (1). For all $h \in H$, let E_h be the set of all $(g, \omega_1, ..., \omega_m)$, such that g is an increasing function from [0, 1] to [0, 1] with $g(0) = 0$ and $g(1) = 1$; $\omega_1, \cdots, \omega_m$ are positive constants with $\omega_1 + \cdots + \omega_m = 1$; and $[h(C) = \frac{1}{n} \sum_{i \in N} g(\sum_{j=1}^{m} \omega_j c_{ij})$ for all $C = (c_{ij}) \in \mathcal{D}]$.

Suppose the group deprivation measure h satisfies (1). Then the weighted average, $\sum_{j=1}^{m} \omega_j c_{ij}$ figuring in (1) may be thought of as individual i's overall "nominal" deprivation when i has the deprivation vector $c_{i\bullet}$, such "nominal" deprivation being the multidimensional counterpart of the notion of an individual's normalized "shortfall" from the poverty benchmark, which is used in the literature on the measurement of income poverty. Thus, though for every individual attribute, there are only two levels of deprivation, 0 and 1, the overall nominal deprivation of an individual can have many different levels and hence the notion of the "depth" of an individual's overall nominal deprivation is non-trivial. The expression $g(\sum_{j=1}^{m} \omega_j c_{ij})$ figuring in (1) has the obvious interpretation as individual i's overall "real" deprivation when individual i has the deprivation vector $c_{i\cdot}$.

In the following section, we shall provide axiomatic characterization of the class, H, of group deprivation measures and discuss the formal structures of several subclasses of H. One of these subclasses is the class, H', of all group deprivation measures h, such that:

(2) for some positive constants $\omega_1, \cdots, \omega_m$ with $\omega_1 + \cdots + \omega_m = 1$, we have $[h(C) = \frac{1}{n} \sum_{i \in N} \sum_{j=1}^{m} \omega_j c_{ij}$, for all $C = (c_{ij}) \in \mathcal{D}]$.

It is clear that $H' \subseteq H$ and that, when the weights $\omega_1, \cdots, \omega_m$ for the different attributes are such that $[h(C) = \frac{1}{n} \sum_{i \in N} \sum_{j=1}^{m} \omega_j c_{ij})$, for all $C = (c_{ij}) \in \mathcal{D}]$, we have a very specific individual deprivation function g under which the overall nominal deprivation of an individual coincides with her overall "real" deprivation so that the deprivation of the group or the society is simply the average of the overall nominal deprivations of all individuals. Such a group deprivation measure can be viewed as the multidimensional counterpart of the average normalized shortfall familiar in the literature on the measurement of income poverty.

For any $C = (c_{ij}),\ D = (d_{ij}) \in \mathcal{D}$, any $p \in N$ and any $k \in M$, we say that, (i) C and D are (pk)-variant if $c_{pk} \neq d_{pk}$ and $c_{ij} = d_{ij}$ for all $ij \neq pk$; that is, C and D are identical except the pk-th elements, and (ii) C and D are (pk)-invariant if $c_{pk} = d_{pk}$; that is, C and D have the same pk -th elements. A deprivation matrix $D = (d_{ij}) \in \mathcal{D}$ is said to be a simple deprivation matrix if, for some $p \in N$, $d_{i\bullet}$ is the zero vector for all $i \in N \backslash \{p\}$; that is, D is such that there are at least $n - 1$ individuals each of whom is not deprived in terms of any attribute.

3 A class of deprivation measures using binary data

In this section, we axiomatically characterize the class H of group deprivation measures. We introduce the following properties that are to be imposed on a measure h.

Normalization For all $D = (d_{ij})$ and for all $\delta \in \{0, 1\}$, if $[d_{ij} = \delta$ for all $i \in N$ and all $j \in M]$, then $h(D) = \delta$.

Anonymity Let σ be a bijection from N to N. Then, for all $C, D \in \mathcal{D}$, if $c_{i\bullet} = d_{\sigma(i)\bullet}$ for all $i \in N$, then $h(C) = h(D)$.

Monotonicity For all $C = (c_{ij}),\ D = (d_{ij})$, if $(c_{ij} \geq d_{ij}$ for all $i \in N$ and all $j \in M)$ and $C \neq D$, then $h(C) > h(D)$.

Independence For all $C, D, C', D' \in \mathcal{D}$, and for all $k \in N$, if $[(c_{i\bullet} = d_{i\bullet}$ and $c'_{i\bullet} = d'_{i\bullet}$ for all $i \in N \backslash \{k\}$ and $(c_{k\bullet} = c'_{k\bullet}, d_{k\bullet} = d'_{k\bullet})]$, then $h(C) - h(D) = h(C') - h(D')$.

Additivity For each $j \in M$, there exists a function g_j such that, for all simple deprivation matrices, $C, D \in \mathcal{D}$, and for all $p \in N$, $h(C) \geq h(D) \Leftrightarrow g_1(c_{p_1}) + ... g_m(c_{p_m}) \geq g_1(d_{p_1}) + ... + g_m(d_{p_m})$.

Normalization is straightforward: if no one in the group N is deprived along any dimension, then the overall deprivation index for N is 0, and if everyone in N is deprived along every dimension, then the overall deprivation index for N is 1. Anonymity requires that the interchange of any two rows of a deprivation matrix does not affect the overall deprivation. It essentially says that the name of an individual has no significance in measuring overall deprivation of the society. Anonymity is also called Symmetry in the literature. Monotonicity requires that, if every individual under C is as at least deprived as under D and some individual under C is deprived while the same individual is non-deprived under D, then the overall deprivation level under C is higher than that under D. These three properties are fairly standard in the literature on the multi-dimensional approach to deprivation, see, among others, Bourguignon and Chakravarty (2003), and Tsui (2002).

Independence requires that the overall deprivation measure is separable with respect to individuals' deprivations: if two deprivation matrices differ only with respect to a single individual's deprivations along the m dimensions, then the difference between the overall deprivations under the two deprivation matrices are independent of all other individuals' deprivations. The intuitive idea underlying Independence is identical to that of the well-known axiom of Sub-group Decomposability proposed in the literature on poverty and deprivation. See, for example, Tsui (2002) and Bourguignon and Chakravarty (2003). Formally, Independence is weaker than the property of Sub-group Decomposability.

Additivity requires that, for simple deprivation matrices, a deprivation measure is additively separable among the attributes. A similar but different property has been proposed in Bossert et al. (2013). Additivity captures the contribution of an attribute to an individual's overall deprivation and thus to the overall deprivation of the society. Viewed this way, Additivity is appealing from a policy point of view: it would enable researchers and policy makers to identify the attributes that contribute most to the overall deprivation. It may be noted that, additivity among the attributes is often associated with a kind of separability/independence among the attributes. In the context of finite domains like ours, a property of separability among the attributes is not sufficient for cases involving more than four attributes (see, for example, Kraft et al. 1959, and Fishburn 1996). For the purpose of simplicity and convenience, here we invoke a full-fledged version of additivity. For a more primitive treatment of the relevant properties of additivity in our context of finite domains to ensure additive separability among the attributes, see Dhongde et al. (2015) where they use a conventional separability property to deal with the case involving four or less attributes, and introduce a separability property along the idea of Kraft et al. (1959) to derive the additivity property used in the current paper for cases with more than four attributes.

4 Characterizing group deprivation measures

To begin with, we present the following result which shows that the overall deprivation of the society is the sum of individuals' overall deprivations along the m dimensions if certain axioms are imposed on the overall deprivation of the society. The proofs of all the propositions are given in the Electronic Supplementary Material.

Proposition 1 *A deprivation measure h satisfies Normalization, Monotonicity, Anonymity and Independence if and only if there exists an increasing function $\varphi : \{0, 1\}^m \to [0, 1]$ with $\varphi(0, \cdots, 0) = 0$, $\varphi(1, \cdots, 1) = 1$, such that, for all $C = (c_{ij}) \in \mathcal{D}$, $h(C) = \frac{1}{n} \sum_{i \in N} \varphi(c_{i\bullet})$.*

Therefore, the combination of Normalization, Anonymity, Monotonicity and Independence to be imposed on a deprivation measure h implies that h is additive across individuals. Our next result characterizes H.

Proposition 2 *A group deprivation measure h belongs to H if and only if it satisfies Normalization, Monotonicity, Anonymity, Independence and Additivity.*

5 Introducing a hierarchy of attributes

The basic class of deprivation measures that we have focused on is given by $h(C) = \frac{1}{n}\sum_{i\in N} g(\sum_{j\in M}\omega_j c_{ij})$, where $h(C)$ is the index of group deprivation when the deprivation matrix is C , $g(\sum_{j\in M}\omega_j c_{ij})$ is the overall deprivation of individual i with deprivation vector $c_{i\bullet}$, and $\omega_1, ..., \omega_m$ are the weights for the different attributes. $\sum_{j\in M}\omega_j c_{ij}$ can be interpreted as the overall "nominal deprivation" of individual i; $\sum_{j\in M}\omega_j c_{ij}$ is thus the counterpart of the notion of an individual's normalized shortfall from the poverty benchmark, which figures in the literature on the measurement of income poverty. When we start with binary (0–1) deprivation data for each individual and each attribute, $\sum_{j\in M}\omega_j c_{ij}$ provides a plausible cardinal measure of the depth of overall nominal deprivation of individual i. In this section, we explore the structure of several subclasses of H by introducing a distinction between basic and non-basic attributes.

5.1 A characterization of H'

We first provide a characterization of the class H', i.e., the class of all deprivation measures h which satisfy (2). To do this, we introduce a new property.

Strong additivity For all $C, D, C', D' \in \mathcal{D}$, all $p \in N$, and all $q \in M$, if $c_{pq} = d_{pq} = 1$, $c'_{pq} = d'_{pq} = 0$, C and C' are (pq)-variant, and D and D' are (pq)-variant, then $h(C) \geq h(D) \Leftrightarrow h(C') \geq h(D')$.

Strong Additivity, which was introduced in Pattanaik et al. (2012), imposes restrictions on how the comparison of two deprivation matrices, C and D, changes when the same individual along a given dimension under C and D switches from the same status of being deprived (resp. being non-deprived) to the same status of being non-deprived (resp. being deprived).

Proposition 3 *Let h be a group deprivation measure. $h \in H'$ if and only if h satisfies Normalization, Monotonicity, Anonymity, Independence and Strong Additivity.*

The class of measures characterized in Proposition 3 also has been obtained in Bossert et al. (2013) in a different setting where they deal with a richer domain by including variable societies.

5.2 Basic dimensions and priority of basic dimensions

In addition to H', we now consider subclasses of H which are based on a simple two-fold distinction between what may be called "basic" attributes and "non-basic" attributes. In the introductory section, we sketched the intuition of a simple framework where the policy maker distinguishes between basic and non-basic attributes and gives priority to each basic attribute over the entire group of non-basic attributes. We now formally introduce this notion of priority and study the implications for group deprivation measures in H. Let F_B denote the set of dimensions that are regarded as basic, and let F_{NB} denote the set of non-basic dimensions. We assume $F_B \neq \emptyset$. Let M_B denote the set of all $j \in M$, such that $f_j \in F_B$, and let M_{NB} denote $M \backslash M_B$. m_B and m_{NB} denote the cardinalities of M_B and M_{NB}, respectively.

We introduce a property of group deprivation measures, which embodies the notion of priority of basic attributes. What this property requires is that, if the deprivation status of an individual, i, along a basic dimension changes from non-deprived to deprived while her deprivation status remains unchanged along every other basic dimension and every other individual's deprivation vector remains the same, then, irrespective of any changes in i 's deprivation status along non-basic dimensions, the overall group deprivation must increase. Formally,

Priority of basic attributes (PBA) For all $C, D \in \mathcal{D}$, all $i \in N$, all $j \in M_B$, if [$c_{k\bullet} = d_{k\bullet}$ for all $k \in N \setminus \{i\}$], [$c_{ij} = 1$, ($c_{ij'} = 0$ for all $j' \in M_{NB}$)] and [$d_{ij} = 0$, ($d_{ij'} = 1$ for all $j' \in M_{NB}$), ($d_{ip} = c_{ip}$ for all $p \in M_B \setminus \{j\}$)], then $h(C) > h(D)$.

Our next proposition clarifies the implications of PBA for group deprivation measures in H.

Proposition 4 *Let $h \in H$. Then h satisfies PBA if and only if, for all $(g, \omega_1, ..., \omega_m) \in E_h$,* $\min\{\omega_j : j \in M_B\} > \sum_{j' \in M_{NB}} \omega_{j'}$.

Since g has the obvious interpretation as a function which specifies the overall deprivation of an individual given her vector of deprivations in terms of the different attributes, the restriction on the weights, $\omega_1, \cdots, \omega_m$,which go with g, in Proposition 4 amounts to evaluating an individual's overall deprivation in a lexicographic fashion: for all $c_{i\bullet}, d_{i\bullet}$, $g(c_{i\bullet}) \geq g(d_{i\bullet}) \Leftrightarrow (\sum_{j \in M_B} \omega_j c_{ij}, \sum_{j \in M_{NB}} \omega_j c_{ij}) \geq_{lex} (\sum_{j \in M_B} \omega_j d_{ij}, \sum_{j \in M_{NB}} \omega_j d_{ij})$, where $\geq_{lex}$ is the standard lexicographic relation defined over $[0,\infty)^2$. To see this, let $c_{i\bullet}, d_{i\bullet}$ be such that, for some $j' \in M_B$, $c_{ij'} = 1$ and $c_{ij} = 0$ for all $j \in M \setminus \{j'\}$, and ($d_{ij} = 0$ for all $j \in M_B$ and $d_{ij} = 1$ for all $j \in M_{NB}$). Then, $g(c_{i\bullet}) = g(\omega_{j'})$ and $g(d_{i\bullet}) = g(\sum_{j \in M_{NB}} \omega_j)$. Note that, from the above results, we have $\omega_{j'} > \sum_{j \in M_{NB}} \omega_j$ implying that $g(c_{i\bullet}) > g(d_{i\bullet})$. On the other hand, $(\sum_{j \in M_B} \omega_j c_{ij}, \sum_{j \in M_{NB}} \omega_j c_{ij}) = (\omega_{j'}, 0)$, and $(\sum_{j \in M_B} \omega_j d_{ij}, \sum_{j \in M_{NB}} \omega_j d_{ij}) = (0, \sum_{j \in M_{NB}} \omega_j)$. Then, $(\omega_{j'}, 0) >_{lex} (0, \sum_{j \in M_{NB}} \omega_j)$.

5.3 Deprivation-decreasing switch and the function g

Proposition 2 provides characterizations of H, and, by definition, for all $h \in H$ and all $(g, \omega_1, \cdots, \omega_m) \in E_h$, g is an increasing function of the weighted sum of an individual's deprivations along the different dimensions. From Proposition 2, we do not know much about the function g beyond the fact that it is increasing. If, however, one has further specific intuition about how the overall deprivation of the society should respond to certain changes in individual deprivations, then the function g can be further restricted.

Assume that the society consists of two individuals and that there are three attributes. Suppose $h \in H$ and $(g, \omega_1, \cdots, \omega_m) \in E_h$. Consider the following deprivation matrices:

$$C = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Then, $h(C) = \frac{1}{2}[g(\omega_1+\omega_2+\omega_3)+g(0)] = \frac{1}{2}g(\omega_1+\omega_2+\omega_3)$ and $h(D) = \frac{1}{2}[g(\omega_1+\omega_3)+g(\omega_2)]$. Note that, in C, individual 1 is deprived along each dimension, while individual 2 is non-deprived in each dimension. Intuitively, individual 1 is (unambiguously) more deprived overall than individual 2 in C. Suppose attribute f_2 is the least important among the three attributes, that is, $w_2 < w_1$ and $w_2 < w_3$. In that case, individual 1 is more deprived overall than individual 2 in D also. Suppose, starting with C, the deprivation matrix of the society changes to D. Then, this transition from C to D can be viewed as a "transfer" of 1's deprivation in terms of f_2 to individual 2, the deprivation of both individuals in terms of every other attribute, as well as the ranking of the two individuals in terms of overall deprivation, remaining the same. One may feel that, in this case, the change reduces the overall deprivation of the society, i.e., $h(C) = \frac{1}{2}g(\omega_1+\omega_2+\omega_3) > h(D) = \frac{1}{2}[g(\omega_1+\omega_3)+g(\omega_2)]$. In general, consider the following axiom for group deprivation measures in H.

Deprivation-decreasing switch Let $h \in H$. Then, for all $(g, \omega_1, \cdots, \omega_m) \in E_h$, for all $C, D \in \mathcal{D}$, and for all $i, i' \in N$ and all $j \in M$, if $[(c_{k\bullet} = d_{k\bullet}$ for all $k \in N\backslash\{i, i'\}]$, $c_{i\bullet}$ and $d_{i\bullet}$ are identical except that $c_{ij} = 1$ and $d_{ij} = 0$, $c_{i'\bullet}$ and $d_{i'\bullet}$ are identical except that $c_{i'j} = 0$ and $d_{i'j} = 1$, and $(g(c_{i\bullet}) > g(c_{i'\bullet})$ and $g(d_{i\bullet}) \geq (g(d_{i'\bullet})]$, then $h(C) > h(D)$.

The intuition underlying the axiom, deprivation-decreasing switch, is as follows. Suppose we start with a situation where, individual i is deprived in terms of f_j, but individual i' is not, and, further, i's overall deprivation is higher than that of i'. Now suppose the two individuals switch their deprivation statuses in terms of f_j so that, after the switch, i becomes non-deprived in terms of f_j and i' becomes deprived in terms of f_j, but there is no change in the deprivation status of either i or i' in terms of any attribute other than f_j and there is no change in the deprivation status of any individual other than i and i' in terms of any attribute. Further, suppose that, even after the change, i's overall deprivation is at least as great as that of i'. Then what the axiom of deprivation-decreasing switch stipulates is that the overall deprivation of the society decreases as a result of the change. This intuition is similar to that of "prioritarianism" where the most deprived individuals are given some priority.[6]

The following result shows the implication of Deprivation-Decreasing Switch for group deprivation measures in H; we omit the proof of the result which is fairly straightforward. Before stating the result formally, we introduce a notation. For a given vector $\omega = (\omega_1, \cdots, \omega_m) \in (0, 1)^m$ with $\sum_{j \in M} \omega_j = 1$, let $R_\omega = \{t \in [0, 1] : t = \sum_{j \in M} \omega_j x_{\omega j}$ for some $x_{\omega j} \in \{0, 1\}^m\}$.

Proposition 5 *Let $h \in H$. h satisfies Deprivation-Decreasing Switch if and only if, for all $(g, \omega_1, \cdots, \omega_m) \in E_h$, g has the following property:*

(d-convexity): for all $\alpha, \beta \in R_\omega$ and all $\gamma > 0$ with $\alpha - \gamma \in R_\omega$ and $\beta + \gamma \in R_\omega$,
$[\alpha - \gamma \geq \beta + \gamma] \Rightarrow [g(\alpha) - g(\alpha - \gamma) > g(\beta + \gamma) - g(\beta)]$.

[6] See, for example, Bosmans et al. (2013), and Parfit (1997). It may be noted that the underlying idea of Deprivation-Decreasing Switch is similar to some relevant properties such as Correlation Increasing Majorization or Non-decreasing Poverty under Correlation Increasing Switches considered in the literature; see, for example, Bourguignon and Chakravarty (2009), and Tsui (1999). The intuition underlying deprivation-decreasing switch may depend on whether attributes are *substitutes* or *complements*, and this intuition may not be entirely compelling when some attributes are "complements" of each other. For a discussion, see Atkinson (2003), Bosmans et al. (2015), Bourguignon and Chakravarty (2003, 2009), and Pattanaik et al. (2012).

It may be noted that, if g is increasing and convex, then g satisfies d-convexity. For example, a power function $g(t) = t^{\alpha}$, with $\alpha > 1$, satisfies d-convexity.

6 An empirical illustration

In this section, we provide an empirical illustration of deprivation indices that belong to the class of deprivation measures characterized in the previous sections. Using data from the study, Young Lives, conducted by the Department of International Development at the University of Oxford, we study multi-dimensional deprivation of children in four countries.Young Lives is a longitudinal study which has traced about 12,000 children over the last 15 years. Data are available for three rounds, namely, 2002 (Round 1), 2006 (Round 2), and 2009 (Round 3). The study follows two cohorts of children-the older cohort of children was born in 1994–1995, and the younger cohort of children was born in 2001-2002. Data are collected in Ethiopia, India, Peru and Vietnam, representing the four major regions (Africa, South Asia, Latin America and East Asia) of the developing world. In each country, twenty sites predominantly located in poor areas, are selected to reflect heterogeneity of location, ethnicity and religion in country populations.[7] Household surveys of children and their primary caregivers comprise of questions focused on the causes and consequences of childhood poverty. The sample is designed to include a high proportion of poor children, but also includes other children. The sample comprises of approximately 2000 children from the younger age-group, and approximately 1000 children from the older age-group in each country. We compile data on the younger cohort using the 2009 round, so that the children in our sample are between 7 and 9 years old.

6.1 Deprivation attributes

Deprivation among children is measured in terms of seven attributes: (1) weight; (2) height; (3) drinking water; (4) sanitation; (5) literacy; (6) electricity; and (7) consumer durables. Data available on most of the attributes are of binary form. The Young Lives Study classifies a child is classified as underweight if the child's weight-for-age z-score is more than two standard deviations below the World Health Organization's (WHO) prescribed benchmark. Similarly, a child is classified as stunted if the height-for-age z-score is more than two standard deviations below the WHO benchmark. Access to safe drinking water is determined by the source of drinking water (e.g. bore well, tube well, piped, water tank), data is collected on whether a household has a flush or a septic toilet and whether a household has electricity or not. Literacy is defined as the ability of a child to read and write without difficulty. Each child is tested by the field worker. A child is deprived in this attribute if the child cannot read anything and cannot write anything. A child who reads letters or words or writes with difficulty is not considered illiterate. Access to consumer durables includes a radio, television, bicycle, motorbike, automobile, landline phone, and mobile phone.[8] A child is deprived in terms of access to consumer durables if he/she has access to less than three consumer durables in the relevant list. For each of the seven attributes, a child's deprivation

[7]The sampling method is not designed to be nationally representative of children the right age. More information on the dataset is available at http://www.younglives.org.uk/.

[8]In addition to the common items listed, the surveys include refrigerator and fan for India and Vietnam, sofa and bedstead for Ethiopia, and refrigerator, iron, blender, and stove for Peru.

takes exactly one of the two values: 1 (deprived) and 0 (not deprived). Table 1 provides a summary of the indicators and the benchmarks used.

Table 2 lists the percent of children deprived in each attribute in the four countries. About 45 % children in India were underweight, 35 % in Ethiopia and only 5 % in Peru. Roughly 20 % children in each country were stunted. Few children in India (3 %) had no access to drinking water or electricity. But a majority of children in Vietnam (81 %) and in Ethiopia (50 %) lived in households with no drinking water or electricity. Only 9 % children in Peru lived in households with no sanitation compared to 65 % children in India, where according to the 2011 census nearly half of the population had no access to a latrine. Percent of children who had difficulty in reading and writing was high in most countries, with Ethiopia having highest rate (85 %), and Vietnam with the least rate (17 %). Relatively few children in Vietnam (9 %) but many children in Ethiopia (54 %) lived in households with two or less consumer durables. Given the significant variability in the incidence of deprivation in each attribute, we are interested in finding the extent of multidimensional deprivation in each country.

6.2 Illustrations of deprivation measures

Using our analytical framework, we measure and compare the overall deprivation of children in each country (rank 1 denotes the highest level of deprivation, rank 2 denotes the second highest level of deprivation, and so on). We have $n = 2000$, number of children in each country, and $m = 7$ attributes. Our basic result for the calculation of overall deprivation of a group is given by $h(C) = \frac{1}{n} \sum_{i \in N} g(\sum_{j=1}^{m} \omega_j c_{ij})$ for all $C \in \mathcal{D}$ where $c_{ij} = 1$, if the i-th individual is deprived in attribute f_j and $c_{ij} = 0$ otherwise, w_j is the weight attached to attribute f_j and g specifies individual's overall deprivation function. We shall consider two different forms for the function g, namely: $g\ (t) = t$ and $g\ (t) = t^2$.

Earlier we suggested the interpretation of $\sum_{j \in M} \omega_j c_{ij}$ as individual i's overall nominal deprivation and the interpretation of $g(\sum_{j=1}^{m} w_j c_{ij})$ as individual i's overall real deprivation. Given these interpretations, $g(t) = t$ will amount to eliminating the distinction between an individual's nominal deprivation and her real deprivation. Then in this case, the overall deprivation, $\sum_{i \in N} g(\sum_{j=1}^{m} w_j c_{ij})$, of the society will be the counterpart of the "average of normalized shortfalls" in the literature on the measurement of a group's income poverty. Similarly, given our interpretation of $\sum_{j=1}^{m} w_j c_{ij}$ as individual i's overall nominal deprivation, overall social deprivation for the case where $g(t) = t^2$ will be analogous to the "mean of squared normalized shortfalls" in the literature on the measurement of a groups income poverty.

Table 1 Attributes indicating deprivation among children

Attributes	A child is deprived if
Weight	Underweight, i.e., the weight-for-age z score < -2 std. dev.
Height	Stunted, i.e., the height-for-age z score < -2 std. dev.
Drinking water	No access to tap drinking water
Sanitation	No access to flush/septic toilet
Electricity	No electricity in the house
Literacy	Unable to read or write without problems
Consumer Durables	Household has less than 3 consumer durables

Table 2 Percent of deprived children in each country

Attributes	Ethiopia	India	Peru	Vietnam
Weight	35	45	5	25
Height	21	29	20	20
Drinking water	51	03	20	81
Sanitation	43	65	09	38
Electricity	52	03	14	03
Literacy	85	63	40	17
Consumer Durables	54	41	23	09

Source: The Young Lives Dataset, 2009

6.2.1 Substitutability between attributes

We start by making no distinction between basic attributes and non-basic attributes. Then the deprivation measure $h(C)$ with $g(t) = t$ illustrates the class of measures which satisfy Normalization, Monotonicity, Anonymity, Independence and Strong Additivity (Proposition 3) and the deprivation measure $h(C)$ with $g(t) = t^2$ illustrates the class of measures which satisfy Normalization, Monotonicity, Anonymity, Independence, Additivity and Deprivation-decreasing switch (Proposition 2).

Table 3 contains estimates of the deprivation measures and the resulting country ranking. Suppose all attributes are weighed equally, i.e., $w_1 = \ldots = w_7 = \frac{1}{7}$. In this case (Case 0), when we use the function $g(t) = t$, the overall deprivation index $h(C)$ gives the

Table 3 Deprivation indices with substitutability between attributes

Case	Attributes (weights)		Countries	$g(t) = t$	Rank	$g(t) = t^2$	Rank
0	Weight	Electricity	Ethiopia	0.485	1	0.294	1
	Height	Literacy	India	0.359	2	0.179	2
	Drinking water	Durables	Vietnam	0.275	3	0.121	3
	Sanitation	(1/7)	Peru	0.191	4	0.080	4
	(1/7)						
I	Weight	Drinking water	Ethiopia	0.440	1	0.256	1
	Height	Sanitation	India	0.362	2	0.194	2
	(2/9)	Electricity	Vietnam	0.263	3	0.122	3
		Literacy	Peru	0.178	4	0.072	4
		Durables					
		(1/9)					
II	Weight	Electricity	Ethiopia	0.478	1	0.281	1
	Height	Durables	India	0.382	2	0.201	2
	Drinking water	(1/12)	Vietnam	0.311	3	0.149	3
	Sanitation		Peru	0.190	4	0.077	4
	Literacy						
	(2/12)						

Source: Authors' calculations

sum of deprivations $\left(\sum_{i\in N}\sum_{j=1}^{m} c_{ij}\right)$ as a proportion of the maximum deprivations possible present in a country $(n \times m)$. Ethiopia has the highest deprivation level, followed by India, Vietnam and Peru has the lowest deprivation level. Now suppose we assign different weights to attributes. Note that the weights are assigned such that they do not impose any lexicographic ordering among the attributes, i.e. the weights do not satisfy the conditions specified in Section 5. In Case I, being underweight or stunted is given greater weight whereas in Case II all attributes except access to electricity and durables are given greater weights. The deprivation index in each country varies as we alternate the weights on the attributes but there is no change in the ranking of the countries. This ranking is in line with the ranking of these four countries based on other published indices, such as the Human Development Index (HDI), the Multidimensional Poverty Index (MPI) (the ranking switches between Peru and Vietnam) and the Child Development Index (CDI).[9]

6.2.2 *Priority of basic attributes*

We now make a distinction between basic attributes and non-basic attributes. The deprivation measure $h(C)$ with $g(t) = t$ illustrates the class of measures in Proposition 3 and $h(C)$ with $g(t) = t^2$ in Proposition 5. Both indices also satisfy the property of priority of basic attributes (PBA).

In Table 4 we revisit Case I and Case II from Table 3. Unlike in Table 3, however the weights chosen in Table 4 are such that they ensure a lexicographic priority of basic attributes over non-basic attributes. For instance in Case I, we treat the anthropometric attributes as basic and all the other attributes as non-basic. The weights are chosen as follows: suppose w_1, w_2 are weights attached to weight-for-age and height-for-age respectively, then $\min\{w_1, w_2\} > (w_3+w_4+w_5+w_6+w_7)$, $w_j > 0$ for all j and $\sum_{j\in M}\omega_j = 1$. A lexicographic priority implies the following. Suppose a child's deprivation status for a basic attribute, say, weight-for-age, changes from normal to underweight, while her deprivation status remains unchanged for the other basic attribute, namely, height-for-age. Then, no favorable changes of her deprivation status in any of the non-basic attributes can possibly offset the unfavorable change in her status with respect to weight. On comparing the rankings in Tables 3 and 4, we find that in Case I, there is a switch in ranking between Ethiopia and India. Recall that in Case I, the weight of each of the basic attribute in Table 3 is about 0.22, whereas in Table 4 the weight of each basic attribute is increased to 0.35. Hence the extent of multidimensional deprivation India which had the highest proportion of children who are underweight and stunted, exceeds that in Ethiopia. In Case II, the ranking is preserved in both the tables.

In Case I and II in Table 4, each of the basic attributes have equal weights. However lexicographic orderings can take many forms depending on the weights assigned. As an illustration, in Case III, we assign greater weights to the anthropometric attributes within the basic attributes. Compared to Case II, we find that there is a change in the ranking between Ethiopia and India. Deprivation index for India increases in value as we place greater weight on the anthropometric attributes. Of course, we could repeat the calculations for other identifications of basic and non-basic attributes.

[9]The HDI in 2009 is: Ethiopia: 0.324, India: 0.512, Vietnam: 0.566, Peru: 0.718 (UNDP 2010). The MPI in 2010 is: Ethiopia: 0.582, India: 0.296, Vietnam: 0.075, Peru: 0.0854 (Alkire and Santos 2010). The CDI between 2000-2006: Ethiopia: 36.43, India: 26.62, Vietnam: 11.9, Peru: 6.2 (Save the Children UK 2008). A higher HDI value denotes greater development whereas higher values of MPI and CDI denote greater deprivation.

Table 4 Deprivation indices with non-substitutability between basic and non-basic attributes

Case	Basic (weights)	Non-Basic (weight)	Countries	$g(t) = t$	Rank	$g(t) = t^2$	Rank
I	Weight	Drinking water	Ethiopia	0.365	2	0.223	2
	Height	Sanitation	India	0.367	1	0.235	1
	(6/17)	Electricity	Vietnam	0.244	3	0.140	3
		Literacy (1/17)	Peru	0.155	4	0.071	4
II	Weight	Electricity	Ethiopia	0.475	1	0.278	1
	Height	Durables	India	0.391	2	0.211	2
	Drinking water	(1/17)	Vietnam	0.325	3	0.162	3
	Sanitation		Peru	0.190	4	0.077	4
	Literacy (3/17)						
III	Weight	Electricity	Ethiopia	0.355	2	0.220	2
	Height	Durables	India	0.376	1	0.251	1
	(3/8)	(1/32)	Vietnam	0.255	3	0.153	3
	Drinking water		Peru	0.152	4	0.073	4
	Sanitation Literacy (1/16)						

Source: Authors' calculations

7 Conclusions

In this paper, we explored the structure of several classes of group deprivation measures, using an analytical framework which required, in a sense, minimal information on deprivation dimensions. The proposed measures were based on data where each individual's deprivation in terms of any given attribute is assumed to take one of two values, 0 and 1. Furthermore, intrinsically and for policy purposes there is a need to rank order the multiple attributes. The proposed framework was flexible to allow for such hierarchy of attributes. We introduced a simple distinction between basic and non-basic attributes and showed that when weights attached to non-basic attributes are sufficiently small, the individual deprivation function takes a lexicographic form with priority being given to the basic attributes. Finally, we provided an empirical illustration by measuring group deprivation measures for a sample of children in four countries. In general, it was evident that the extent of multidimensional deprivation among children was higher in India and Ethiopia, lower in Vietnam, and least in Peru. The literature so far has largely assumed that attributes are cardinally measurable; in this paper, we proposed a class of indices suitable for attributes which are binary, ordinally measurable. However, in the real world, most datasets have a combination of both cardinal as well as ordinal measures. We hope that the framework proposed will be extended in the future to cover the more general case where some attributes are binary, ordinally measured and other attributes are cardinally measured.

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