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A reconsideration of the tradeoffs in the new human development index

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Abstract This paper demonstrates that the perfect tradeoff implication of the old Human Development Index can be avoided by employing a notion of imperfect tradeoff in the characterization of the Human Development Index. The resulting index is shown to be ordinally equivalent to the Chakravarty (Rev Dev Econ 7:99–114, 2003) generalized Human Development Index.

Keywords Human development index · Tradeoffs · Axioms · Characterization

One of the major objectives of the new Human Development Index (HDI) was to avoid perfect substitutability among different dimensions of well-being. The functional form that has been used as the new Human Development Index is the geometric mean of country level attainments in life expectancy, schooling and per capita income.

In a recent contribution, Ravallion [4] noted that the new version reduced the weight implicitly assigned to longevity in poor countries relative to rich ones substantially. It has also been observed that in the new form of the index unreasonably high valuation has been assigned to gains from extra schooling. In contrast, the weight assigned to income has been increased for most of the countries. Consequently, a poor country with a reduction in life expectancy and a very low increase in the rate of economic growth might see an improvement in its position in the human development scale. These all are consequences of the averaging principle employed in the construction of the new HDI.

In fact, the procedure 'of switching from the old to the new formula for the HDI is theoretically ambiguous' (Ravallion [4, p. 9]). One obvious way to avoid the perfect

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tradeoff (constant marginal rate of substitution) problem is to directly incorporate a notion of imperfect substitution in the derivation of HDI. This will enable us to choose the appropriate form of the index that possesses some desirable properties like ‘reflection of performance in any dimension’ by maintaining imperfect substitution. Thus, instead of looking at the tradeoff as an implication of the formulation, we directly use the tradeoff to derive the functional form.

1 Formal framework

Let us assume that there are k dimensions of well-being. Each dimension corresponds to a particular attribute or functioning. For instance, dimension 1 may represent housing; dimension 2 may indicate the level of literacy and so on.

Let x_i stand for the value of the i^{th} attribute for the population under consideration. It is assumed that x_i is bounded between m_i and M_i, where $0 < m_i \leq M_i < \infty$, that is, for each i, $0 < m_i \leq x_i \leq M_i < \infty$. We rule out the degenerate situation $m_i = M_i$, since in this case $x_i = m_i = M_i$. It is assumed that the indicator for the i^{th} attribute is of the form

$$a_i = \frac{x_i - m_i}{M_i - m_i}. \tag{1}$$

Since $x_i \in [m_i, M_i]$, $a_i \in [0, 1]$. Each a_i indicates the extent of attainment for a particular attribute i. To get an overall picture we have to consider all the attributes simultaneously. For this, we define the well-being index W of the society as a function of the single-dimensional indices. Note that this kind of assumption is quite common in economics. For instance, social utility is assumed to be a function of individual utilities. Since $a_i \in [0, 1]$, the real valued function W is defined on $[0,1]^k$. Formally, $W : [0, 1]^k \to R$, where $[0,1]^k$ is the $k-$fold Cartesian product of [0,1] and R is the real line. For $(a_1, a_2, \cdots, a_k) \in [0, 1]^k$, $W(a_1, a_2, \cdots, a_k)$ represents the level of well-being associated to $(a_1, a_2, \cdots, a_k)$.

We assume that W is continuous, increasing and strictly quasi-concave. We also assume that all partial derivatives of W exist. Given these assumptions, we now specify certain axioms for the function W.

Normalization (NOM) For any $z \in [0, 1]$, $W(z, z, \cdots, z) = z$.

Symmetry (SYM) For any $(a_1, a_2, \cdots, a_k) \in [0, 1]^k$, $W(a_1, a_2, \cdots, a_k)$ remains invariant under any permutation of $(a_1, a_2, \cdots, a_k)$.

Linear Homogeneity (LIH) For any $(a_1, a_2, \cdots, a_k) \in [0, 1]^k$, $W(ca_1, ca_2, \cdots, ca_k) = cW(a_1, a_2, \cdots, a_k)$, where $c > 0$ is any scalar.

Strong Separability (STS) For all $k \geq 3$, $(a_1, a_2, \cdots, a_k) \in [0, 1]^k$ and for all pairs (i, j), $\frac{\partial W / \partial a_i}{\partial W / \partial a_j}$ is independent of $a_m, m \neq i \neq j$.

NOM demands that if indicator levels for different attributes take on the same value z, then W also takes on this common value. Thus, W is an average of individual indicator levels. SYM says that W does not change if we consider any reordering of the $a_i's$. Linear homogeneity means that an equi-proportionate increase in all the indicators will increase W equi-proportionately. Strong separability demands that the tradeoff, which is not necessarily constant, between any two indicators along a well-being contour will not depend on a third indictor. It may be noted that NOM, SYM and LIH are satisfied by both old and new HDIs.

The following theorem identifies the well-being index that satisfies axioms NOM, SYM, LIH and STS.

Theorem *The only well-being index satisfying axioms NOM, SYM, LIH and STS is given by*

$$W_\alpha = \left(\frac{1}{k}\sum_{i=1}^{k} a_i^\alpha\right)^{\frac{1}{\alpha}}, \tag{2}$$

where $0 < \alpha < 1$ *is a constant.*

Proof Following Kolm [3], we can say that W satisfying NOM, SYM and STS will be of the form

$$W(a_1, a_2, \cdots, a_k) = g^{-1}\left(\frac{1}{k}\sum_{i=1}^{k} g(a_i)\right). \tag{3}$$

By continuity and increasingness of W, g is also continuous and increasing. Using LIH in Eq. 3, we get

$$g^{-1}\left(\frac{1}{k}\sum_{i=1}^{k} g(ca_i)\right) = cg^{-1}\left(\frac{1}{k}\sum_{i=1}^{k} g(a_i)\right), \tag{4}$$

where $c > 0$ is any scalar. Equation 4 is a quasi-linear functional equation, of which the only continuous solution is

$$g(a_i) = A + B\frac{a_i^\alpha}{\alpha}, \tag{5}$$

where A, B and α are constants (see Aczel [1, p. 153]). Since a_i can take on the value 0, continuity of g demands that $\alpha > 0$. Increasingness of g requires that $B > 0$. By strict quasi-concavity of W, $0 < \alpha < 1$. Using g given by Eq. 5 in Eq. 4, we get the desired form of W. This completes the necessity part of the proof. The sufficiency is easy to verify. □

Note that the ordinal transformation $(W_\alpha)^\alpha$ of W_α given by Eq. 2 is the Chakravarty [2] generalized HDI defined as

$$HDI_\alpha^C = \frac{1}{k}\sum_{i=1}^{k} a_i^\alpha. \tag{6}$$

Since W_α and HDI_α^C are ordinally equivalent, ranking of different countries by these two indices will be the same. HDI_α^C is a decreasing function of α. For $\alpha = 1$, it coincides with HDI_o, the old HDI for k attributes. Because of additivity, it is possible to judge the performance in any dimension independently of other dimensions.

HDI_{α}^{C} takes on the value zero if and only if $a_i = 0$, that is, $x_i = m_i$ for all i. This clearly demonstrates that in HDI_{α}^{C} performance in any dimension is directly reflected.

If we assume that the domain of definition of a_i is (0,1], then in Eq. 5 we can also allow the possibility that $\alpha \leq 0$. For $\alpha = 0$, $g(a_i)$ becomes $g(a_i) = A + Blna_i$. The corresponding form of W is given by $\prod_{i=1}^{k} (a_i)^{\frac{1}{k}} = HDI_N$, the new HDI for k attributes. But if $x_i \to m_i$ for some i, then $HDI_N \to 0$ and this will happen irrespective of how large or small the values of other indicators are. This is an undesirable feature. In order to avoid this problem, we restrict attention on the general domain [0,1]. Our demonstration supports Ravallion's [4, p. 4] claim that 'the troubling tradeoffs found in the 2010 HDI could have been avoided to a large extent using an alternative aggregation function from the literature-indeed, a more general form of the old HDI, as proposed by Chakravarty [2].'

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