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Non-anonymous growth incidence curves, income mobility and social welfare dominance

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Abstract The distributional incidence of growth is generally analyzed by comparing the quantiles of the pre- and post-growth income distribution—e.g. the so-called Growth Incidence Curves. Such an approach based on an implicit re-ranking of individual incomes ignores income mobility by assuming that only post-growth income matters in social welfare. By contrast, this paper takes the view that "status quo matters" and that social welfare should logically be defined on both inital and terminal income. This leads to consider 'non-anonymous' Growth Incidence Curves that plot income growth rates against the various quantiles of the *initial* distribution. Dominance criteria that generalize those available for standard growth incidence curves are derived, which account for the inequality of individual income changes, conditional on initial income. An application to the cross-country distributional feature of global growth illustrates the analysis.

Keywords Growth · Pro-poor · Inequality · Income mobility · Dominance

1 Introduction

Growth incidence curves (GIC) are increasingly used to describe the distributional effects of growth. They simply plot the mean growth rate of real income in a population against income quantiles—see Fig. 1. A downward sloping GIC thus indicates that growth contributes to equalizing the distribution of income and vice-versa for an upward sloping curve. Of course, the shape of GICs may be very diverse. An important issue, therefore, is that of comparing different GICs. Under which

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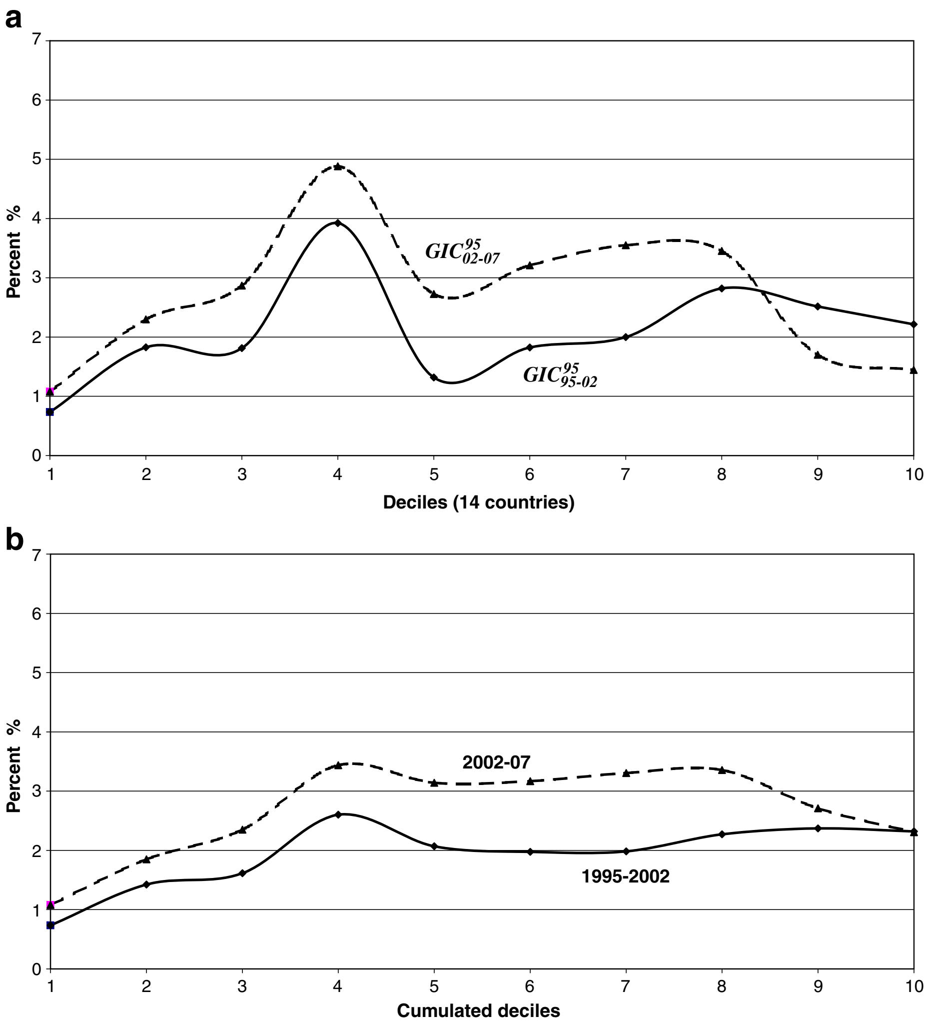


Fig. 1 **a** Growth incidence curve for global growth: average annual GDP per capita growth rates by decile, 1995–2002 vs. 2002–2007 (using 1995 as base year). **b** P-cumulative growth incidence curve for global growth: average annual GDP per capita growth rates for *p* poorest decile, 1995–2002 vs. 2002–2007

circumstances, is it possible to say that a growth episode or its GIC is "better" than another, and what is the meaning of such a statement?

Answers to that question have been provided by Ravallion and Chen [17] and Son [23]. They essentially rely on applying first-order and second-order dominance criteria—Atkinson [1]—to the terminal distribution of income. For instance, first order dominance implies that the GIC of a growth spell is everywhere above that of another. Second order dominance requires the mean growth rate of the *p* poorest in a growth episode—or the 'p-cumulative GIC'—be everywhere larger than in another.

These results are quite intuitive. Growth may be thought as a specific redistribution process, which can be analyzed with the standard tools of income redistribution analysis—i.e. tax-benefit incidence. In particular, GICs are very similar to tax progressivity charts whereas cumulative GICs bear some resemblance with "concentration curves".[1]

On second thought, there is a difference, though. It lies in the horizontal axis of GICs. In standard redistribution analysis, individuals are ranked according to their position in the *initial* distribution of income and the incidence curve shows how much their income is modified by redistribution. An important issue that arises in this context is that of the 're-ranking' of individuals by the redistribution system. Many results on the relative progressivity of a redistribution system vis-a-vis another are valid only if there is no re-ranking of individuals—see Lambert [15] for instance.

GICs compares the income of individuals who were not necessarily in the same initial position. The cumulative GIC shows the difference between the initial income of those individuals who are initially among the p poorest and the income of the p poorest individuals in the terminal distribution. They are not necessarily the same individuals. As redistribution analysis when it excludes re-ranking, GICs somehow ignore the issue of income mobility. Yet, GICs may have different shapes depending on whether mean growth rates are measured before or after the re-ranking of the population—see Fig. 3 below.

The present paper analyzes the distributional incidence of growth using the initial distribution as a reference. This leads to define 'non-anonymous' Growth Incidence Curves (na-GIC). Such an extension of the original analysis of the distributional features of growth logically leads to taking into account the full joint distribution of individual initial incomes and terminal incomes, or equivalently, initial income and income growth, or income change As with GICs, the goal is to define dominance criteria of a growth path over another. Reasonably enough, the comparison is restricted to growth paths with the same initial distribution, which we shall then call '*growth processes*' by analogy with the 'income processes' defined by Benabou and Ok [6].

Results similar to dominance criteria with GICs are obtained when social welfare functions are defined exclusively on final income. But things are different when it is recognized that "status quo matters", or, in other words, social welfare is defined on the bi-dimensional distribution of *both* initial and terminal incomes as assumed in part of the literature on income mobility—e.g. Atkinson [2], Chakravarty et al. [7], Dardanoni [8] or Gottshalk and Spolaore [12] among others. In that case, it is shown that the GIC dominance criteria has to be complemented by more restrictive criteria. In particular, social welfare dominance requires not only dominance of the cumulative na-GICs but also that of "inequality corrected cumulative na-GICs". Although a direct application of the sequential dominance criterion obtained by Atkinson and Bourguignon [4], these criteria do not seem to have ever been made explicit.[2]

Several authors already pointed to the difference that it makes to re-rank or not to re-rank income earners in drawing growth incidence curves and in comparing

[1] This analogy is also discussed in Jenkins and van Kerm [14].

[2] Fields et al. [10] refer explicitly to stochastic dominance of a distribution of income changes over another, but they do not relate it to initial incomes.

growth spells. In analyzing the pro-poorness of growth in different countries based on panel data on individual incomes Jenkins and van Kerm [14] and van Kerm [24] or Grimm [13] have shown how different conclusions were obtained when using standard GICs and what we call in this paper non-anonymous GICs. Although they discussed the issue of dominance of one growth episode over another based on non-anonymous GIC, however, they did not deal with the full implications of taking simultaneously into account both initial and terminal incomes in the evaluation of social welfare.

The paper is organized as follows. The next section recalls the basic results about GIC dominance relying on simple empirical examples based on the cross-country distribution of global growth in different periods. Section 3 presents sufficient and necessary conditions for the dominance of one growth process over another for general classes of social welfare functions defined on initial income and income change and for a given initial distribution of income. These conditions imply in turn dominance criteria for na-GICs and for an extension of na-GIC that takes into account the inequality of income changes conditionally on initial income. Links with GIC dominance are also discussed. The analysis is conducted with continuous distributions and is shown to combine standard one-dimensional dominance results in the terminal income space and income-mobility specific criteria. Section 4 illustrates those various dominance conditions with a comparison of the cross-country structure of global economic growth in two recent periods using the 1995 distribution of world income as common initial reference. The concluding section summarizes the various results in the paper.

2 Alternative representations of the distributional incidence of growth

Let the initial distribution of income in the economy being studied be described by its cumulative density function (cdf) $F(y)$, with support $(0, a)$. Growth takes place over some time period. Its distributional impact may be described by the conditional distribution function, $\widetilde{\Phi}(z \mid y)$ of terminal incomes, z, conditionally on initial incomes y. Finally, let $\Phi(z)$ be the marginal distribution of income at terminal time implied by $\widetilde{\Phi}(\,)$. We are interested in comparing two growth paths described by the transition functions $\widetilde{\Phi}(\,)$ and $\widetilde{\Phi}^*(\,)$ and their corresponding terminal distributions $\Phi(z)$ and $\Phi^*(z)$. Unless specified otherwise, we shall assume that the two growth paths have the same initial cdf, $F(y)$.

Growth incidence curves are defined on the initial and terminal distributions of income. Define the 'quantile function', $y_F(p)$ as the inverse of the cumulative density $F()$:

$$y_F(p) \iff p = F(y)$$

and similarly for the terminal cdf $\Phi()$ or $\Phi^*()$. The Growth Incidence Curve corresponding to the growth path with terminal cdf Φ may then be defined as [17]:

$$g_{\Phi F}(p) = \frac{y_\Phi(p)}{y_F(p)} - 1$$

It simply shows the rate of growth of the p^{th} quantile of the distribution. The distributional impact of growth is thus represented through the inverse of the cumulative density functions rather than those functions themselves.

An obvious property of the GIC is First Order Dominance. Assume that the terminal distribution $\Phi()$ first order dominates (FOD) that of the alternative growth path $\Phi^*(\,)$. In other words, the (additive) social welfare associated with $\Phi()$ is larger or equal to that associated with $\Phi^*(\,)$ for all individual income utility functions that are increasing. It is well known that this is equivalent to $\Phi(z) \leq \Phi^*(z)$ for all z. This in turn implies that the GIC associated with the first growth path must be everywhere above that corresponding to the second one, as long, of course, as initial distributions are the same for the two paths. Formally:

$$\Phi() \;\; FOD \;\; \Phi^*() \iff \Phi(z) \leq \Phi^*(z) \, \forall z \iff g_{\Phi F}(p) \geq g_{\Phi^* F}(p) \;\; \forall p$$

Second order social welfare dominance (SOD) refers to individual utility functions that are not only increasing but also concave in income. It is equivalent to the integral of the cumulative density function $\Phi()$ being no larger than the integral of $\Phi^*(\,)$. In turn, we know this is equivalent to the 'generalized Lorenz curve' being no smaller with $\Phi()$ than with $\Phi^*()$ for all p. Using quantile functions, the Generalized Lorenz curve can be expressed as:

$$\widetilde{L}_{\Phi}(p) = \int_0^p y_{\Phi}(q)dq$$

Using the GIC, this can be rewritten as:

$$\widetilde{L}_{\Phi}(p) = \int_0^p \left[g_{\Phi F}(q) + 1\right] y_F(q)dq = \int_0^p g_{\Phi F}(q) y_F(q)dq + \widetilde{L}_F(p)$$

Comparing social welfare associated with two growth paths Φ and Φ^*, we thus have:

$$\Phi() \;\; SOD \;\; \Phi^*() \iff \int_0^p x_{\Phi}(q)dq \geq \int_0^p x_{\Phi^*}(q)dq \quad \text{for all} \quad p$$

where $x_{\Phi}(q)$ is the absolute—rather than relative—change in quantile income q over the growth path Φ, and similarly for Φ^*. Equivalently, this relationship may also be expressed in terms of the 'p-cumulative GIC', $G_{\Phi F}()$ defined by:

$$G_{\Phi F}(p) = \frac{\int_0^p g_{\Phi}(q) y_F(q)dq}{\int_0^p y_F(q)dq}$$

Second order dominance then requires that $G_{\Phi F}(p) \geq G_{\Phi^* F}(p)$ for all p–see Son [23].

Empirical illustrations in this paper are based on the global distribution of income per capita, with equal weight given to all countries.[3] Income is assimilated to GDP per capita. Data are drawn from the World Development Indicators [25]. Figure 1a shows the GIC curve of the global economy for the 1995–2002 period, which may be denoted GIC^{95}_{95-02}. To illustrate the comparison of growth paths discussed above and to stick to the case of identical initial distributions, Fig. 1 also shows the GIC curve

[3] This is what Milanovic [16] called 'inter-country' rather than 'global' distribution. Here we shall use the two terms interchangeably. The relevance of this application is discussed below.

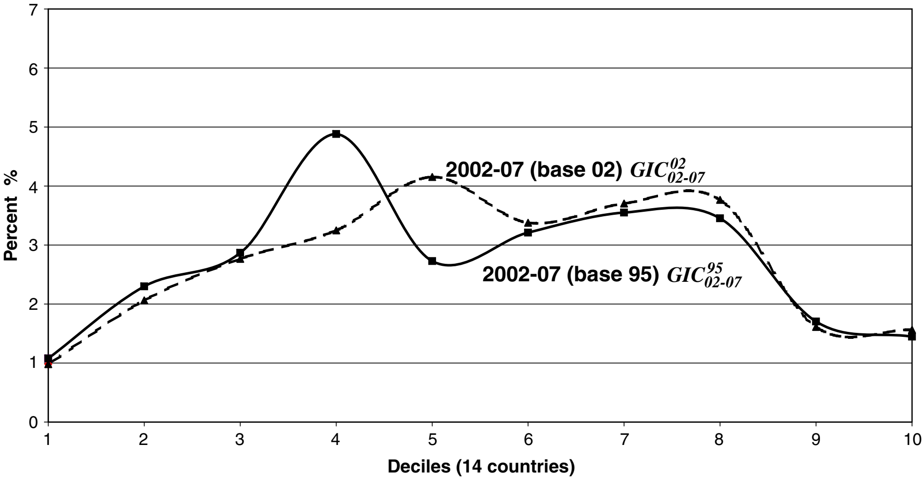


Fig. 2 Global growth incidence curves: changing the base year for 2002/2007 growth from 2002 to 1995 (annual rates)

that would have been obtained for the same 1995–02 period had country growth rates over that period been those observed between 2002 and 2007. This curve is denoted GIC^{95}_{02-07}.[4] It can be seen that there is no first order dominance because the mean income of the two upper deciles did not grow as fast in 2002–2007 as in 1995–2002. Figure 1b shows the p-cumulative growth incidence curves for the two periods. It can be checked that second order dominance holds. In other words, social welfare would have been higher in 2002 with the 2002–2007 growth rates than with the actual 1995–2002 growth rates for all social welfare functions based on increasing and concave country income utility functions.

In the preceding example, it is important to stress that the comparison of two growth spells in the global economy is done using the same base year. Of course, it would be possible to compare GIC^{95}_{95-02} on the one hand and GIC^{02}_{02-07}, using 2002 as initial year, on the other hand. As a matter of fact the latter may be considered as the 'true' GIC for the 2002–2007 period. Two remarks are in order, however. First, the shape of GIC^{02}_{02-07} and GIC^{95}_{02-07} is not the same—as can be seen on Fig. 2. Second, it must be clear that the FOD and SOD dominance results apply only when the initial distributions of the two growth paths being compared are the same. One can always compare GIC^{95}_{95-02} and GIC^{02}_{02-07} but the comparison would have little meaning in terms of social welfare.

An ambiguity in interpreting GICs comes from the fact that the quantiles on the horizontal axis do not comprise the same statistical units. For instance, the 1995–2002 GIC in Fig. 1a compares the mean income of the various deciles of the 1995 and 2002 distributions but the country composition of these deciles has changed during that time interval. GICs thus are '*anonymous*' in the sense that the composition of the various quantiles of the distribution does not matter. For instance, Uganda and

[4]In that comparison, the initial distribution is indeed the same—i.e. the 1995 distribution- for the two growth paths being compared as specified above.

Tanzania moved out of the first decile whereas Eritrea and Madagascar moved in. *If one is interested in whether global growth has been pro-poor between 1995 and 2002 there does not seem to be any good reason for ignoring what happened to countries that grew fast enough to move out of the bottom deciles.*

An alternative to the GIC approach to the distributional features of growth consists of *keeping the ranking of statistical units constant*, whereas comparing the initial and terminal quantile functions $y_F(p)$ and $y_\Phi(p)$, as done before, is equivalent to re-ranking them. The no-reranking approach to the distributional incidence of growth then associates to every quantile in the initial income distribution $y_F(p)$ the terminal incomes $z(p)$ of individual units in that quantile. The corresponding Growth Incidence Curve becomes '*non-anonymous*' since it is now possible to put a name on each point of the curve. Formally, non-anonymous Growth Incidence Curves (na-GIC)[5] can be defined by:

$$\widetilde{g}(p) = \frac{\int_o^a z d\widetilde{\Phi}(z \mid y_F(p))}{y_F(p)} - 1$$

In other words, the na-GIC associates to every quantile in the initial distribution the mean income growth of all individual units in that quantile. Likewise, the cumulative na-GIC curve may be defined as:

$$\widetilde{G}(p) = \frac{\int_0^p \widetilde{g}(q) y_F(q) dq}{\int_0^p y_F(q) dq}$$

Figure 3a plots the na-GIC for the 1995–2002 global growth spell and compares it to the original GIC. The discrepancy between both curves is striking. The same is true of the cumulative na-GIC and original GICs in Fig. 3b. The difference is such that comparing two different growth paths might well lead to different conclusions about their distributional impact depending on whether one does or does not re-rank the statistical units between the initial and the terminal year. The main differences between the GIC and the na-GIC in Fig. 3a comes from the fact that, between 1995 and 2002, India and Bosnia moved up from the third to the fourth decile of the distribution. The fast growth of these two countries is explicitly taken into account in the 3rd decile of the na-GIC whereas it is somewhat hidden in the change in the composition of the 3rd and 4th decile in the standard GIC.[6]

It remains now to see whether the simple dominance criteria in comparing the standard GICs or cumulative GICs have some counterpart with na-GICs and whether they make any difference when comparing different growth paths.

[5] Jenkins and van Kerm [14] and van Kerm [24] refer to the same curves as "mobility profiles".

[6] A paradoxical situation is that described in Robilliard et al. [18] about the simulation of the distributional effects of the 1997 crisis in Indonesia. The standard GIC curve suggests the crisis has been strongly regressive with poor people more severely affected than others, whereas the na-GIC for the same simulation points to the crisis being 'progressive', initially poor people having done on average better than the others.

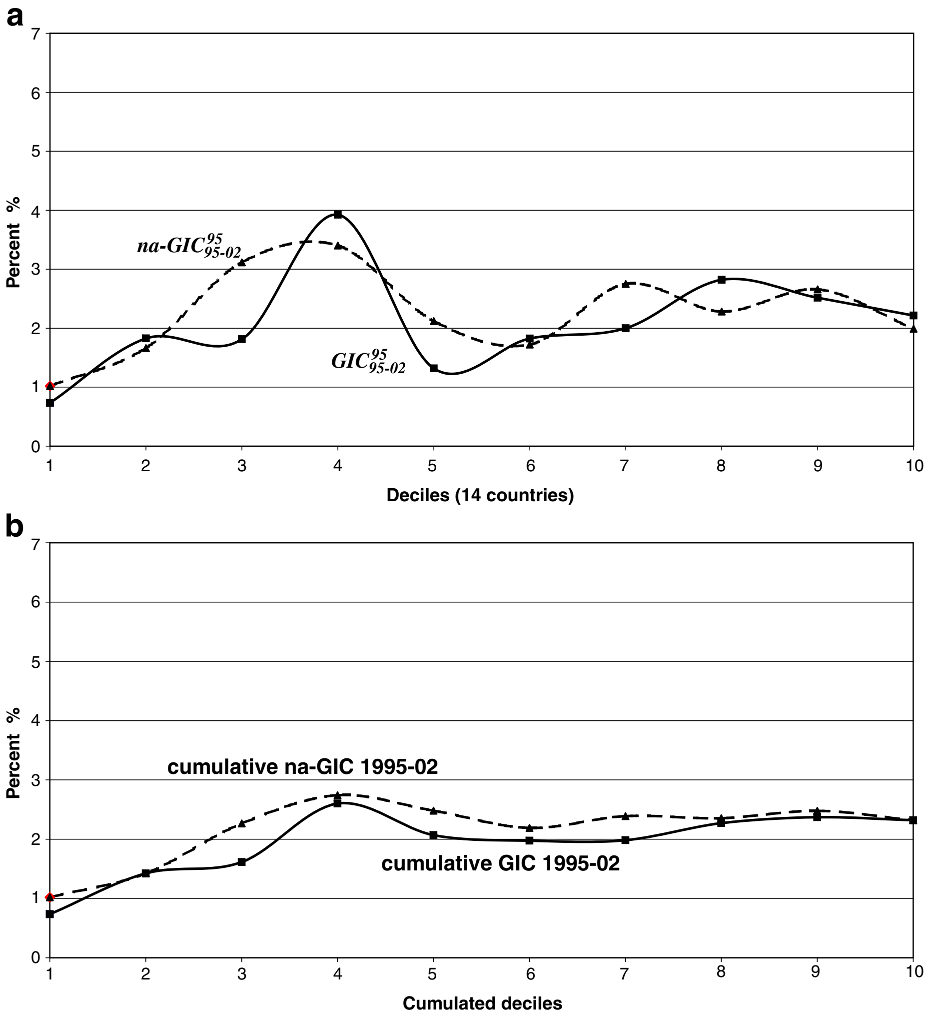


Fig. 3 **a** Anonymous (standard) vs. non-anonymous growth incidence curves: global growth, 1995–2002, annual rates. **b** Anonymous (standard) vs. non-anonymous p-cumulative growth incidence curves: global growth, 1995–2002, annual rates

3 The bi-dimensional approach: dominance criteria when simultaneously accounting for initial and terminal incomes

As the preceding example shows an important issue in evaluating the distributional impact of growth is whether the analysis must refer to the initial or the terminal situation of individuals or countries. If we are interested in pro-poor growth, should pro-poorness be measured comparing the poor in the initial and terminal distributions or should those individuals or countries that escaped poverty be taken into account?

Interestingly enough, this issue is related to the length of the growth spell that is analyzed. In the preceding example, India was in the bottom three deciles of the global distribution in 1995. It is not anymore in 2002. Therefore, it is not included in the calculation of the mean growth rate of the bottom three deciles with the GIC approach. But, of course, it would have been included if the analysis had focused on a much shorter time period, say 1995-96. In other words, GICs are modified when applying the same growth rates over a variable number of years. This would not be the case with na-GICs.[7] This difference may not matter if the goal of the analysis is only to *describe* distributional changes between the initial and terminal points. It may not be so if it is to evaluate social welfare along alternative growth paths.

This section considers the general case where social welfare depends on both initial and terminal incomes, or equivalently, initial income and income change. The case where social welfare depends solely on terminal income, which corresponds to the GIC approach, is a particular case of this general specification.

3.1 Decomposing the difference between two growth paths into initial and transitional distribution difference

Now that the analytical framework is explicitly bi-dimensional, it is possible to deal rigorously with the issue of comparing growth paths with different initial distributions. Consider then two growth paths described by the joint density functions of initial and terminal incomes: $\omega(y, z)$ and $\omega^*(y, z)$. One way or another, what matters is the difference between these two functions. With the same notations as above this can be written as:

$$\Delta\omega(y, z) = \omega(y, z) - \omega^*(y, z) = f(y).\widetilde{\Phi}(z \mid y) - f^*(y)\widetilde{\Phi}^*(z \mid y)$$

Then, a natural decomposition of that difference is:

$$\Delta\omega(y, z) = f(y).\Delta\widetilde{\Phi}(z \mid y) + \widetilde{\Phi}^*(z \mid y)\Delta f(y) \quad (1)$$

where $\Delta\widetilde{\Phi}(z \mid y) = \widetilde{\Phi}(z \mid y) - \widetilde{\Phi}^*(z \mid y)$ and $\Delta f(y) = f(y) - f^*(y)$.

At this stage, it is convenient to refer to $\Delta\omega(y, z)$ as the difference in 'growth paths' and to $\Delta\widetilde{\Phi}(z \mid y)$ as the difference in 'growth processes'. The idea here is that a growth path is the combination of a growth process—essentially a transitional or conditional density function—with an initial distribution. The first part of Eq. 1 is the contribution to the difference in growth paths of the 'growth processes', i.e. densities of terminal income conditional on initial income. The second part is the contribution of the difference in initial distributions for a given growth process. To the extent that growth processes may be considered independently from initial distributions, then it is the first part of Eq. 1 that matters. Hence the focus on growth paths with identical initial distributions in the preceding section, or the imposition of a given initial distribution for the comparison of two growth processes.

This choice of focusing on growth paths with identical initial distributions is important for social welfare comparisons. Considering simultaneously initial and terminal incomes within a social welfare dominance context can be done using the

[7] Actually this would be the case only if annual growth rates are small enough so that they can simply be added over time.

general bi-dimensional framework proposed by Atkinson and Bourguignon [3]. But, if the point is to compare growth paths with the same initial distribution, then simpler and more powerful criteria derived in Atkinson and Bourguignon [4] can be used. The latter referred to the comparison of two income distributions among households with different needs, the distribution of needs itself being constant. In the present context, 'needs' is simply replaced by 'initial income'.

In what follows, we briefly recall the results obtained in this latter paper, show how they apply to the present case and derive simple necessary conditions for dominance of one growth process over another that seem of practical use. Before doing so, however, some remarks are in order about the concept of social welfare dominance in two dimensions and the shape of social welfare functions to be used.

3.2 Defining welfare dominance among growth paths

Formally, the problem is to compare two growth processes given by the conditional distributions $\widetilde{\Phi}(z \mid y)$ and $\widetilde{\Phi}^*(z \mid y)$ where z stand for terminal income and y for initial income. The initial distribution of income is given by the cumulative density function $F(y)$. Atkinson and Bourguignon [3] suggested that such comparison could be made by considering social welfare functions where individual utilities would depend on both initial and terminal income. This idea was analyzed further by Atkinson [2] when analyzing mobility—assuming identical marginal distribution of terminal incomes too—and at a later stage by Dardanoni [8] and Gottshalk and Spolaore [12] among others. Other authors suggested to define social welfare on permanent incomes—see for instance Shorrocks [20] or Chakravarty et al. [7].[8] The same perspective is adopted in what follows, except for the fact that, for analytical convenience, individual utility functions are specified as functions of initial income, y, and *change in income* , $x \ (= z - y)$ with support $[-a, a]$.

Let $\Psi(x \mid y)$ and $\Psi^*(x \mid y)$ be the corresponding conditional distributions over the growth processes being compared and

$$\Delta\Psi(x \mid y) = \Psi(x \mid y) - \Psi^*(x \mid y)$$

the difference between them. Denoting the utility an individual draws from his/her own growth process by $u(y, x)$, the difference in overall social welfare between the two growth paths is defined as:

$$\Delta W = \int_0^a \int_{-a}^a u(y, x) d\Delta\Psi(x \mid y) dF(y) \tag{2}$$

or, using quantile functions for the common marginal distribution $F()$:

$$\Delta W = \int_0^1 \int_{-a}^a u\left[y(p), x\right] d\Delta\Psi(x \mid y(p)) dp \tag{3}$$

Welfare dominance of growth process $\widetilde{\Phi}$ over $\widetilde{\Phi}^*$ holds in the sense of a family of utility functions, V, if $\Delta W \geq 0$ for all utility functions $u(\ )$ belonging to family V. Formally:

$$\widetilde{\Phi} \succsim_V \widetilde{\Phi}^* \iff \Delta W \geq 0 \ \forall u \in V$$

[8] See the survey by Fields and Ok [9] and also the recent paper by Fields [11].

An interesting property readily apparent in Eqs. 2 and 3 is that the dominance of a growth process over another in general depends on the initial distribution of income. It is thus possible that the growth process $\widetilde{\Phi}$ dominates the process $\widetilde{\Phi}^*$ for a given initial distribution $F()$ but not for another.

3.3 Social welfare functions

Families of social welfare functions may be defined by restrictions imposed on the marginal utility of income change, x. In effect, this is equivalent to imposing restrictions on the marginal utility of terminal income. Three families of functions will be considered depending on the number of restrictions imposed on the marginal utility of income change.

In the first family, V_1, utility functions are only required to have positive marginal utilities of income change:

$$V_1 = \{u;\ u_x \geq 0\}.$$

Note that no restriction is imposed on the marginal utility of initial income. This is because the two growth paths being compared have the same distribution of initial income so that no comparison has to be performed in that dimension. This family of functions will be said to define '*first order' dominance*' comparison criteria among growth processes.

A more restrictive family of social welfare functions requires the marginal utility of income change to decline with initial income. In other words, a drop in income is more painful the poorer people initially are and likewise an increase in income brings more additional utility the poorer they are.

$$V_2 = \left\{u;\ u_x \geq 0,\ u_{xy} \leq 0\right\}$$

As it is close to the principle of declining marginal utility of income in the unidimensional case, this family of bi-dimensional utility functions will be said to define 'second order dominance' criteria in the space of growth processes.

'Third order dominance' will be obtained by imposing restrictions on the sign and slope of the second derivative of utility with respect to income changes. In this respect, a sensible family of social welfare functions is defined by:

$$V_3 = \left\{u;\ u_x \geq 0,\ u_{xx} \leq 0, u_{xy} \leq 0, u_{yxx} \geq 0\right\}$$

The marginal utility of income change is thus required to decline with the income change itself, a rather standard requirement, but this decline is supposed to be slower when initial income rises. The general intuition behind this is the same as for V_2. It is essentially that utility depends less and less on income change as initial income rises. The marginal utility of a given income change is lower for higher initial incomes in V_2 (second cross derivative u_{xy}) and it declines more slowly with income change in V_2 (third order cross derivative u_{yxx}).

It must be noted that both families V_2 and V_3 include utility functions that depend solely on terminal income, that is functions of the form:

$$u(y, x) = h(y + x)$$

V_2 requires functions $h()$ to be increasing and concave, whereas V_3 requires in addition third derivatives to be positive. In that sense, the family of functions V_3 truly refers to third order dominance relationships. Practically, dominance criteria based on V_2 will necessarily be consistent with the standard unidimensional dominance analysis where utility is assumed to depend solely on terminal income. Criteria based on V_3 will be less demanding.[9] It can also be seen that both V_2 and V_3 are consistent with the well-known Pigou-Dalton principle, but, of course add further restrictions to social welfare.

A much more general family of utility functions that belong to V_2 and V_3 is given by the following CES-like family of functions:

$$u(y, x) = \left[ay^{\alpha} + b\,(y + x)^{\alpha}\right]^{\beta} \quad \text{with } 0 \leq \beta \leq \alpha \leq 1 \text{ and } a, b \geq 0. \tag{4}$$

Somehow, this functional form generalizes the standard permanent income hypothesis. In that expression, initial and terminal incomes are indeed substitutes but not necessarily perfect substitutes (case of $\alpha = 1$).

3.4 Dominance criteria

We now state the dominance criteria corresponding to the three preceding families of social welfare functions.

3.4.1 First-order dominance

Given the absence of any restriction on the way utility depends on initial income, the family of social welfare functions V_1 simply leads to the standard unidimensional first-order criterion for every value of initial income:

$$\widetilde{\Phi} \succsim_{V_1} \widetilde{\Phi}^* \iff \Delta W_u \geq 0 \;\; \forall u \in V_1 \iff \Delta\Psi(x \mid y_F(p)) \leq 0 \;\; \forall x, p. \tag{5}$$

Note that this criterion implies that the mean income change must be higher for $\widetilde{\Phi}$ than $\widetilde{\Phi}^*$ for all p.

3.4.2 Second-order dominance

Applying directly the results in Atkinson and Bourguignon [4] with a continuous rather than a discrete specification for the dimension with the same marginal distribution leads to the following dominance condition:

$$\widetilde{\Phi} \succsim_{V_2} \widetilde{\Phi}^* \iff \Delta W_u \geq 0 \;\; \forall u \in V_2 \iff \tag{6}$$
$$\Delta\Theta(x, p) = \frac{1}{p}\int_0^p \Delta\Psi(x \mid y_F(q))dq \leq 0 \;\; \forall x, p.$$

In that expression, the function $\Theta(x, p)$ is simply the cdf of the income change x for the p smallest initial incomes, or the p-cumulative cdf of the income change. This is the straight generalization of the first order dominance results in a single dimension to two dimensions. Yet, the 'second-order' dominance label may seem somewhat misleading here since the preceding criteria still refer to the cdf of income

[9] In the uni-dimensional case, restrictions on the third derivatives of the utility functions are equivalent to the 'transfer sensitivity' axiom used by Shorrocks and Foster [22] to generalize standard Lorenz dominance to cases where the Lorenz curves cross each other once from above.

changes. The difference with first-order dominance, however, is that the cdfs of income changes are now integrated with respect to initial income. It is in that sense that this is a 'second-order dominance' criterion.

It can be seen that second-order dominance is less demanding that first-order dominance since it is now possible not to have dominance in the sense of Eq. 5 for some values of initial income but to have it in the sense of Eq. 6 when integrating along the initial income dimension.

3.4.3 Third-order dominance

The second-order dominance criterion (Eq. 6) is still very demanding since it requires the p-cumulative cdf of income changes of one growth path to be everywhere below that of another. Intuitively, this property must certainly hold for low initial income levels and low income changes, but not so much for high levels of initial income or income changes. The third-order dominance criterion based on the family V_3 of welfare functions weakens that requirement by allowing p-cumulative cdf of the growth paths being compared to cross each other.

$$\widetilde{\Phi} \succsim_{V_3} \widetilde{\Phi}^* \iff \Delta W_u \geq 0 \;\; \forall u \in V_3 \iff \Delta H(x,p) = \int_{-a}^{x} \Delta\Theta(z,p)dz \leq 0 \;\; \forall x \in [-a,a] \;\; \textit{and} \;\; \forall p \in [0,1] \tag{7}$$

In this expression $H(x, p)$ is the integral of the cumulative function $\Theta(x, p)$ with respect to income change. It is well-known since Atkinson [1] and Shorrocks [21] that the condition $\Delta H(x, p) \leq 0$ is strictly equivalent to Generalized Lorenz dominance. In the present case, given the fact that income changes may take negative values the concept of '*incomplete mean change*' is preferred to the generalized Lorenz curve. The following equivalence can easily be shown:

$$\Delta H(x,p) = \int_{-a}^{x} \Delta\Theta(z,p)dz \leq 0 \, \forall x \in [-a,a] \;\; \textit{and} \;\; \forall p \in [0,1] \iff X(q,p) \geq X^*(q,p) \, \forall p,q \in [0,1] \tag{8}$$

where $X(q, p)$ (resp $X^*(q, p)$) is the mean income gain among the p poorest individuals in the initial distribution and, among them, the q poorest in terms of income change. X and X^* are referred to as the 'p-cumulative *incomplete mean income change*'. The word 'incomplete' refers here to the fact that only a fraction q among the p poorest are taken into account. Formally, it is given by:

$$X(q,p) = \frac{1}{q}\int_{-a}^{Z(q,p)} z\Theta_z(z,p)dz \;\; \text{with} \;\; \Theta\left[Z(q,p),p\right] = q$$

and equivalently for X^*, where $\Theta_z(z, p)$ is the density function of the distribution of income changes for the p poorest in terms of initial income.

3.5 Some necessary conditions for dominance

Both Eqs. 6 and 7 criteria compare surfaces or manifolds, associated with growth processes. Dominance is achieved when one surface is everywhere above or below another. These surfaces may not be very convenient to perform actual comparisons of growth paths, but it is always possible to restrict the comparison to a few values of p, q or x as done below in Fig. 5.

If this is found to be too cumbersome, it is still possible to focus on a few indirect indicators of the relative position of these surfaces. Some interesting necessary conditions for dominance of one growth process over another can be derived in this way.

First, notice that by integrating below the $\Theta(x, p)$ surface, second-order dominance requires that the p-cumulative mean income changes be higher for the dominating growth process for all values of p. This is also a requirement for third order dominance when considering the intersection of the $X(q, p)$ surface with the $q = 1$ plane:

$$\widetilde{\Phi} \succeq_{V_2 \text{ or } V_3} \widetilde{\Phi}^* \implies X(1, p) \geq X^*(1, p)\ \forall p \in [0, 1] \qquad (9)$$

This condition has a direct relationship with the na-GICs associated with $\widetilde{\Phi}$ and $\widetilde{\Phi}^*$as will be seen below.

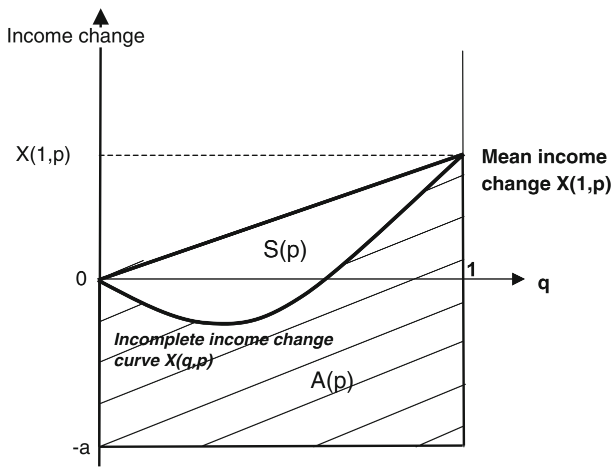
Second, integrating below the p-cumulative incomplete mean income change curves, $X(q, p)$ with respect to q provides another necessary condition for third order dominance. Figure 4 shows the p-cumulative incomplete income change curve, $X(q, p)$, for a given value of p. Integrating below this curve from the lower bound of income changes $-a$, one obtains an area with size $A(p)$. The same could be done for the growth process $\widetilde{\Phi}^*$ leading to an area of size $A^*(p)$. If the growth process $\widetilde{\Phi}$ dominates $\widetilde{\Phi}^*$ in V_3 , then Eq. 8 implies that $A(p) \geq A^*(p)$ for all p.

Another way of expressing this necessary condition for third order dominance of $\widetilde{\Phi}$ over $\widetilde{\Phi}^*$ is as follows. In Fig. 4 it can be seen that :

$$A(p) = X(1, p)/2 + a - S(p) \qquad (10)$$

where $S(p)$ is the area between the bisector and the p-cumulative incomplete income change curve. Clearly, $S(p)$ depends on the degree of inequality of the distribution of income changes. If they were all equal then the incomplete income change curve

Fig. 4 P-cumulative incomplete mean income changes $X(p, q)$ for given p



would simply be the bisector and $S(p)$ would be 0. By analogy with the Gini coefficient, one can thus define an inequality index for the distribution of x as:

$$\Gamma(p) = 2.S(p)/X(1, p) \tag{11}$$

Combining Eqs. 10 and 11 the necessary condition $A(p) \geq A^*(p)$ for the third order dominance of $\widetilde{\Phi}$ over $\widetilde{\Phi}^*$ becomes:

$$X(1, p)\left[1 - \Gamma(p)\right] \geq X^*(1, p)\left[1 - \Gamma^*(p)\right] \quad \forall p \in [0, 1] \tag{12}$$

According to Eq. 12 dominance of $\widetilde{\Phi}$ over $\widetilde{\Phi}^*$requires not only that the p-cumulative mean income changes be higher for the dominating growth process but that this property still holds when mean income changes have been *corrected* by a term that takes into account the inequality in the distribution of income changes. One can thus refer to Eq. 12 as a condition on '*inequality-corrected p-cumulative mean income changes*'.

A problem with this inequality corrected mean income change is that it may be negative. Indeed, it can be seen on Fig. 4 that the area $S(p)$ may be larger than $X(1, p)/2$, which implies that the inequality coefficient $\Gamma(p)$ may be larger than unity. This is essentially due to the fact that inequality is defined here on a variable that can take negative values. However, to the extent that what matters is the comparison between two growth processes, the sign of inequality corrected mean income changes may not be that important.

A simple way of eliminating the inconvenience of an inequality index being possibly negative would be to replace $\Gamma(p)$ by

$$\widetilde{\Gamma}(p) = 2.S(p)/[X(1, p) + a]$$

which belongs to [0, 1].[10] The dominance condition 12 would then rewrite:

$$[X(1, p) + a].\left[1 - \widetilde{\Gamma}(p)\right] \geq [X^*(1, p) + a].\left[1 - \widetilde{\Gamma}^*(p)\right]$$

This is indeed possible but arbitrary because of the choice of the income upper bound a. It is also somewhat artificial. The dominance condition actually writes:

$$X(1, p) - 2.S(p) \geq X^*(1, p) - 2.S^*(p)$$

and Eq. 12 is simply a way of expressing the criterion on both sides of that inequality according to the conventional aggregate welfare definition suggested by Sen [19], i.e. "mean income * (1 - Gini)". Here the mean income is replaced by the mean income change and the Gini coefficient by another inequality index.

3.6 Dominance criteria and growth incidence curves

We now can get back to the Growth Incidence Curves and examine the implications of the preceding social welfare criteria. Equivalence between the dominance criteria or some of the preceding necessary conditions and na-GIC curves is rather straightforward.

[10] I thank a refereee for this suggestion.

First, first-order dominance implies dominance of na-GIC curves. Denote $\overline{x}(p)$ and $\overline{x}^*(p)$ the income change means of the two growth paths for the p^{th} quantile of the initial income distribution. The na-GIC curves are then given respectively by $\overline{x}(p)/y(p)$ and $\overline{x}^*(p)/y(p)$. As first-order dominance (Eq. 5) requires the conditional cdf of income change to be lower for the dominating growth process, this implies the mean income change to be larger on that path for all p:

$$\widetilde{\Phi} \succsim_{V_1} \widetilde{\Phi}^* \iff \Delta\Psi(x \mid y_F(p)) \leq 0 \quad \forall x, p \implies$$
$$\widetilde{g}(p) = \overline{x}(p)/y(p) \geq \widetilde{g}^*(p) = \overline{x}^*(p)/y(p) \; \forall p.$$

Second, both second-order and third-order dominance imply p-cumulative na-GIC dominance. This is the necessary condition 9 seen above. To extend that condition to p-cumulative na-GIC curves, it is necessary to switch from income changes to growth rates. This can be done by dividing the p-cumulative mean income change $X(1, p)$ by the p-cumulative mean initial income, $Y(p)$ defined by:

$$Y(p) = \int_0^p y_F(q)dq.$$

The following implication then holds:

$$\widetilde{\Phi} \succeq_{V_2 \text{ or } V_3} \widetilde{\Phi}^* \implies X(1, p) \geq X^*(1, p) \; \forall p \in [0, 1] \implies$$
$$\widetilde{G}(p) = X(1, p)/Y(p) \geq \widetilde{G}^*(p) = X^*(1, p)/Y(p).$$

Third, third-order dominance leads to an extension of p-cumulative na-GIC curves that takes into account the inequality in the distribution of income changes. These inequality-corrected p-cumulative na-GIC curves, $\overline{G}(p)$, are defined by:

$$\overline{G}(p) = \widetilde{G}(p)\left[1 - \Gamma(p)\right]$$

and the following implication holds:

$$\widetilde{\Phi} \succeq_{V_3} \widetilde{\Phi}^* \implies \overline{G}(p) \geq \overline{G}^*(p).$$

Conceptually, this extension of na-GIc curves is important because it introduces a notion of 'income mobility' or 'horizontal inequality' into the description of the distributional features of growth. Standard GICs compare the distribution of terminal income and thus refer implicitly to the change in vertical inequality associated with growth. Na-GICs account for income mobility but focus on mean income changes conditionally on initial income. Inequality-corrected na-GICs introduce horizontal inequality into the description of growth. Two growth paths may have the same p-cumulative na-GIC curves and quite different inequality-corrected p-cumulative na-GIC curves, indicating that there is more disparity in individual growth rates for given initial income in one path. That path could not dominate the other in the sense of V_3.

A last point to be scrutinized is the relationship between the dominance criteria stated above and standard GICs. It has been seen that all families of social welfare functions defined on initial income and income change included social welfare functions defined on terminal income only. It should thus be the case that dominance criteria established in this section imply the same unidimensional dominance criteria which lie behind standard GICs.

This is easily checked. Consider for instance the partial first-order dominance criterion (Eq. 5). The cumulative density function of terminal income, $\Phi()$, associated with the conditional cdf of income changes $\Psi(x \mid y)$ is given by:

$$\Phi(z) = \int_0^1 \Psi(z - y(p) \mid y(p))dp.$$

It follows that the first-order dominance criterion (Eq. 5) implies first order dominance of the distribution of terminal income on growth path $\widetilde{\Phi}$ over path $\widetilde{\Phi}^*$. In turn this implies GIC dominance, i.e. $g_{\Phi F}(p) \geq g_{\Phi^* F}(p)$. The same result can be proven for second-order dominance using the cumulative GIC curves. The equivalence is slightly more intricate for third-order dominance since it should rely on unidimensional welfare comparisons where the third derivative of utility functions is negative, a condition that is not usually used in the context of GICs.[11]

4 An empirical illustration with the global distribution of income

To illustrate the preceding criteria, we use again the change in the global distribution of income by country, comparing the 1995–2002 period with what would have been the growth path if growth rates had been during that period what they have actually been between 2002 and 2007. Thus, the comparison between the two growth paths is made using the same initial income distribution as a reference, in accordance with a previous argument in this paper.

One may wonder whether this is the best example for applying the analytical tools developed in this paper. GICs generally refer to national growth experiences, the statistical unit being the citizen and the data base a panel of individual incomes, or possibly the results of the simulation of the effects of some policy reform on the whole population. These were indeed the kind of examples used by Grimm [13] and Jenkins and van Kerm [14] when introducing non-anonymity in the analysis of the distributional effects of growth. Formally, however, there is no reason why global growth could not be studied with the same tools as national growth. There are policies that do affect the global structure of growth (development aid, trade, migration, ...) and the focus on globalization issues in various policy circles requires analytical instruments to form social judgements on the structure of global growth.

The restrictive aspect of the example used in this paper is that global growth is reduced to its pure cross-country component, all countries being given the same weight as in the 'convergence literature' of the late 1990s—see for instance Barro and Sala-i-Martin [5]. Two remarks are in order here. First, it would have been possible to introduce population weights in the analysis, and also to take into account the distribution of income within countries. This has not been done for the sake of simplicity and because the focus of the paper is more analytical than empirical. The numerical example that follows is mostly illustrative. Second, the convergence

[11] Following Shorrocks and Foster [22], it is to be expected that third-order dominance would imply that cumulative GIC curves for two growth may cross each other -unlike second-order dominance—but only once and from above for the dominating growth path.

analysis of global growth is essentially positive, whereas the analysis in this paper is purely normative. The convergence literature asks whether the inter-country distribution of income tends to equalize or to polarize. The question asked in this paper is whether global growth in one period may be considered as socially 'better' than global growth in another period, or by extension whether some policy reform affecting global growth is better than another. These are very different questions.

The global growth na-GIC for the two growth processes are shown in Fig. 5. Despite the fact that the overall average growth rate has been larger in 2002–2007, there is no first order dominance in the sense of Eq. 5, since the two curves in Fig. 5a cross each other. In other words, there is no Pareto-superiority. If growth rates had

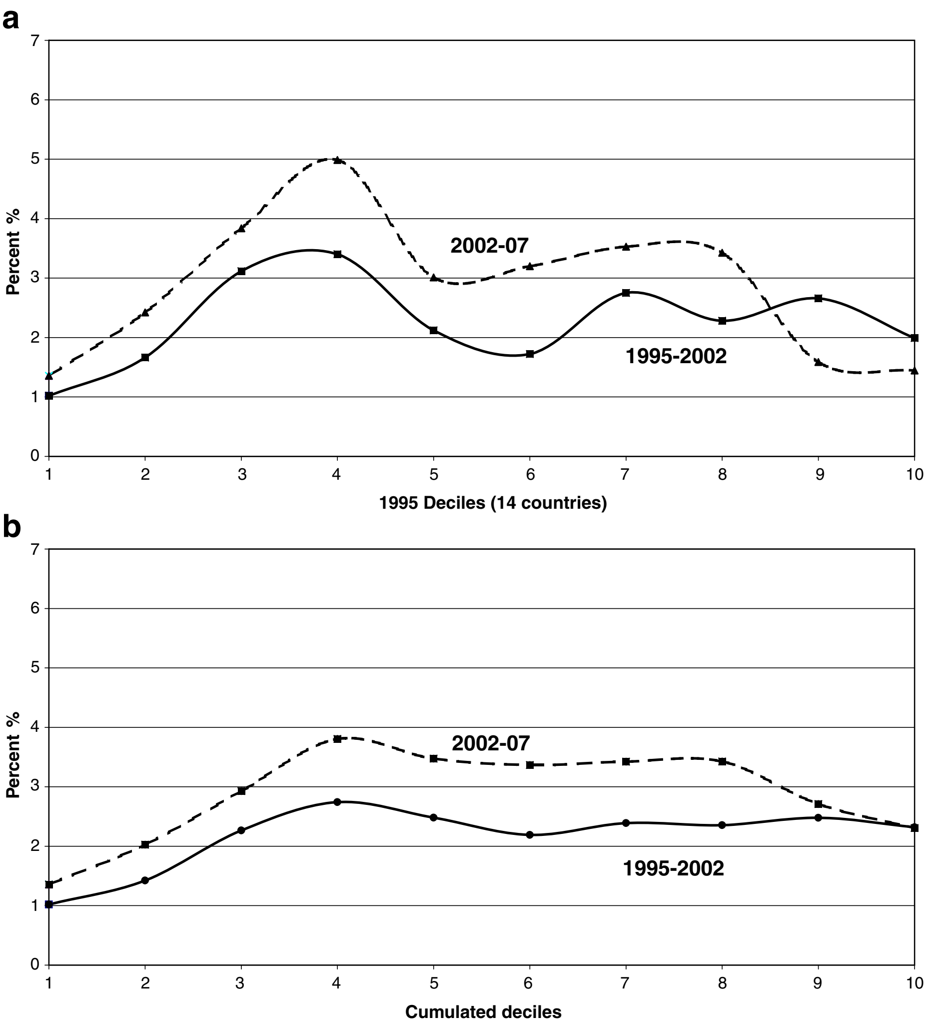


Fig. 5 **a** Non-anonymous growth incidence curve for global growth: average annual growth rates by 1995 decile, 1995–2002 vs. 2002–2007. **b** Non-anonymous p-cumulative growth incidence curve for global growth: average annual growth rates for *p* poorest 1995 deciles, 1995–2002 vs. 2002–2007

been in 1995–2002 those that were observed in 2002–2007, countries in the lower part of the distribution in 1995 would have been better off but countries at the very top would have been worse off. On the contrary, second order dominance holds, in the sense of Eq. 6 (Fig. 5b). The inferior growth performance of initially rich countries thus is compensated, in welfare terms, by the better performance of poorest countries.

Moving now to second-order dominance in the sense of (7), Fig. 6 shows the projections of the incomplete income gain surface, $X(q, p)$, on the q plane for selected values of p. For the ease of comparison with other charts, incomplete income gains are normalized by the mean 1995 income of the p deciles. So, the ordinate of the various curves at $q = 100\%$ for the various values of p corresponds to the mean growth rate shown on the p-cumulative na-GICs in Fig. 5b. Figure 6 suggests that second-order dominance holds. Indeed, the incomplete income gain curves $X(q, p)$ are everywhere higher for 2002–2007 than for 1995–2002, for all values of p being considered.

It logically follows from this that dominance must also hold for inequality-corrected p-cumulative na-GICs. Those curves are shown in Fig. 7 and this is indeed the case. Note on this figure that the inequality-corrected p-cumulative na-GIC is negative for the first decile of the distribution in 1995–2002. The inequality coefficient, $\Gamma(.1)$, turns out to be negative on that growth path, indicating a high degree of inequality of income changes among the 10 per cent poorest countries according to 1995 incomes.

This example shows that the dominance criteria derived in the preceding section are not so restrictive as to prevent full dominance when comparing alternative growth paths for the global economy. Alternatively, one may wonder whether the record mean growth rate observed during 2002–2007 is not sufficient for dominance to hold whatever the country structure of growth behind that mean. The simple simulation reported in Fig. 8 shows that this is not the case.

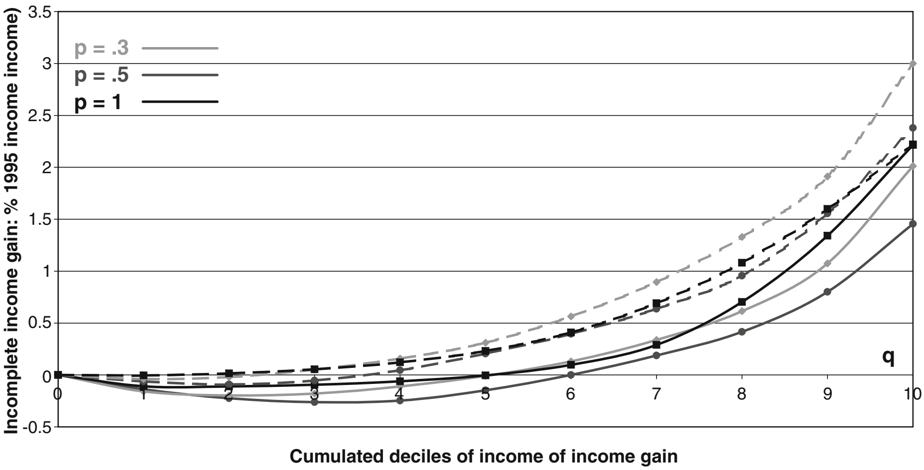


Fig. 6 Projections of the p-cumulative incomplete income gain curve, $X(q, p)$, on the q plane ($p = .3, .5, 1$): 1995–2002 (*solid lines*); 2002–2007 (*dotted lines*) (income gains expressed as percentage of p-cumulative 1995 income)

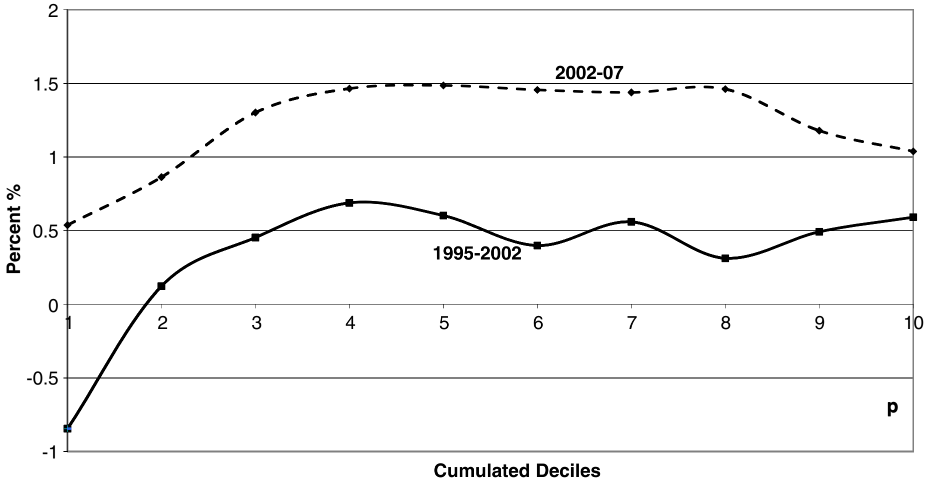


Fig. 7 Inequality corrected non-anonymous p-cumulative growth incidence curve for global growth: 1995–2002 vs. 2002–2007

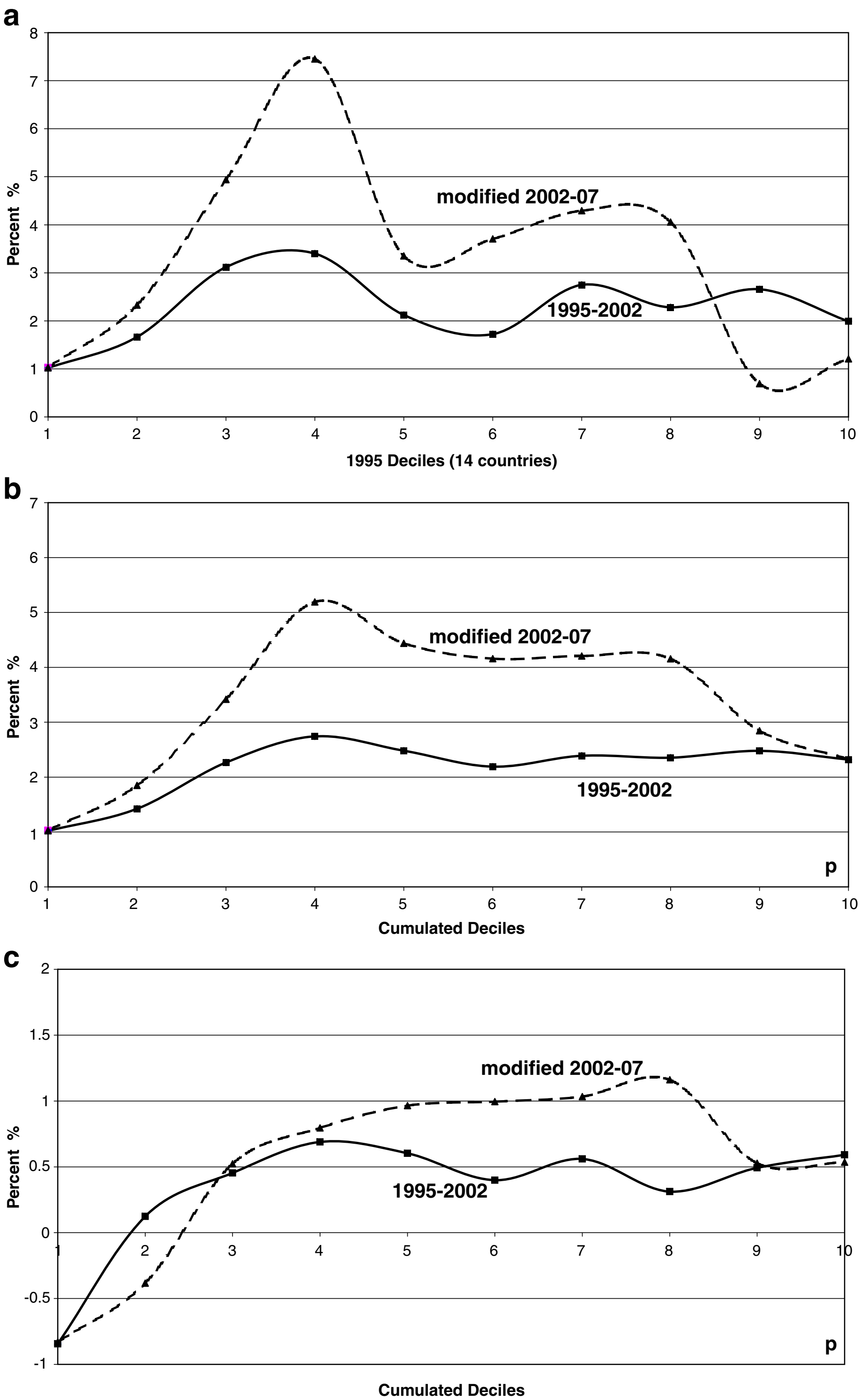
The simulation consists of keeping the overall mean growth rate of the 2002–2007 period constant while increasing the dispersion of growth rates across countries. Growth rates are then artificially modified applying the following mean preserving spread rule:

$$g_i \ \to \overline{g} + S(g_i - \overline{g}) + u_i$$

where g_i is the growth rate of country i, $\overline{g}$ the overall growth rate of the global economy, S a scale factor arbitrarily set to 1.8 and u_i a corrective term ensuring that full first-order dominance holds for the poorest countries while mean growth is preserved. This transformation is equivalent to increasing the inequality of income changes or *growth-related income mobility*. In other words, countries which initially (i.e. 1995) were close to each other in terms of income find themselves more distant, and presumably at more distant ranks of the global income distribution in the terminal year of the period considered.

Figure 8b shows that the simulated 2002–2007 growth path still dominates 1995–2002 when considering p-cumulative na-GICs. However, Fig. 8c shows that dominance does not hold anymore when considering the inequality-corrected p-cumulative na-GICs. It follows that there cannot be dominance in terms of the incomplete income change (Eq. 12) or the social welfare criterion (Eq. 7).

Fig. 8 **a** Non-anonymous growth incidence curve for global growth: average annual growth rates by 1995 decile: 1995–2002 vs. modified 2002–2007. **b** Non-anonymous p-cumulative growth incidence curve for global growth: average annual growth rates for *p* poorest 1995 deciles, 1995–2002 vs. modified 2002–2007. **c** Inequality corrected non-anonymous p-cumulative growth incidence curve for global growth: 1995–2002 vs. modified 2002–2007


a
modified 2002-07
1995-2002
Percent %
1995 Deciles (14 countries)
b
modified 2002-07
1995-2002
Percent %
p
Cumulated Deciles
c
modified 2002-07
1995-2002
Percent %
p
Cumulated Deciles

This example illustrates the different meaning of the p-cumulative and the inequality corrected p-cumulative na-GICs. Even though these are only necessary conditions for dominance of a growth process over another, whether one is satisfied and the other is not gives some indication on the way the structure of growth affects overall dominance. If Eq. 9 holds and not Eq. 12 then the reason for no dominance is likely to come from more inequality in growth rates for initially close observations. In the opposite case, no dominance is more likely to be due to a lower overall mean growth rate. But, of course, a complete diagnosis can only be obtained by considering the whole p-cumulative incomplete income gain curves, $X(q, p)$.

5 Conclusion

This paper extended the concept of Growth Incidence Curve to that of non-anonymous Growth Incidence Curves where growth is evaluated for the various quantiles of the initial distribution of income without any re-ranking. This simple extension of the original growth incidence framework leads to considering simultaneously initial and terminal incomes in the evaluation growth, or equivalently to explicitly introducing income mobility into the description of the distributional features of growth.

The main contribution of this paper is to provide a rigorous bi-dimensional framework for the social welfare evaluation of growth, under the assumption that both terminal and initial incomes enter individual welfare. Bi-dimensional social welfare dominance criteria obtained in previous work have been adapted to this particular case and some interesting necessary conditions have been derived that compare growth processes on the basis of different definitions of non-anonymous growth incidence curves. Of special relevance is the 'inequality-corrected cumulative non-anonymous growth incidence curve' where the mean income growth of cumulative quantiles of the initial distribution are scaled down by a factor that depends negatively on the degree of inequality of income changes within these quantiles. This simple dominance criterion thus takes into account changes in vertical inequality but also horizontal inequality, that is differences among people who are initially in the same situation.

Applying such criteria to evaluate growth clearly requires the availability of panel data on individual incomes. The empirical application in this paper relies on a particular panel which is the cross-country distribution of GDP per capita. It helped illustrating the various concepts being discussed, but it may be considered as too specific, especially in view of the fact that population weights were simply ignored. Other authors have worked on panel data of individual incomes in particular countries at various points of time and it might be interesting to study the properties of the instruments proposed in the present paper in this kind of framework—an important difficulty being the way to deal with entries in and exits from the panel.

Another field of application is the modeling of tax-benefit reforms. Typically, the distributional aspects of these reforms are analyzed through 'micro-simulation' techniques which simulate the income of every individual in a data base if a given reform were to be implemented. Comparing reforms with the help of the tools developed in this paper which allow combining vertical and horizontal inequality concerns seems promising.

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