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The measurement of gender wage discrimination: the distributional approach revisited

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Abstract This paper presents the advantages of taking into account the distribution of the individual wage gap when analyzing female wage discrimination. Several limitations of previous approaches such as the classic Oaxaca–Blinder and the recent distributive proposals using quantile regressions or counterfactual functions are thoroughly discussed. The methodology presented here relies on Jenkins' (J Econom 61:81–102, 1994) work and supports the use of poverty and deprivation literature techniques that are directly applicable to the measurement of discrimination. In an empirical illustration, we quantify the relevance of the glass ceiling and sticky floor phenomena in the Spanish labor market.

Keywords Wage discrimination · Distributive analysis · Economics of gender · Glass ceiling · Sticky floor

1 Introduction

The lower wages paid to female workers in comparison with males can easily be checked empirically in most labor markets. The fact that these wage differentials are not justified in terms of labor productivity is usually known as *gender wage discrimination*. In spite of the increasing interest on these matters, since the classical works of Oaxaca [32] and Blinder [6], not much is yet known about how this phenomenon affects the different subgroups of female workers.

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Recent lines of research aim to include distributional aspects in the study of wage discrimination in order to go beyond the mean when quantifying this phenomenon. Some of them propose the use of quantile regressions in the estimation of wage equations in order to increase the number of points in the earnings distribution at which the wage gap is evaluated (for example [16, 17] among others). Other proposals include a variety of techniques to estimate counterfactual earnings distribution functions in order to compare them with the original wage distribution and quantify the effects of wage differentials throughout the whole earnings range (see [7, 15]. Certainly both approaches allow us to obtain more information from the observed wage distributions than the classical methodology. Nevertheless, both approaches avoid considering the individual dimension of discrimination.

The analysis of individual discrimination experience was addressed by Jenkins [22], who underlined the need to consider the following two issues in any analysis of wage discrimination: (1) how to identify which individuals suffer discrimination and in what quantity; and (2) how to sum up the wage gaps using an index that verifies a set of desirable normative properties. He proposed to analyze the distribution of individual wage gaps using the theoretical advances in poverty research. Indeed, poverty and discrimination have strong similarities. More precisely, both imply some income or wage gap: either individual income does not provide a *minimum level of resources*, or similarly, the female wage is below what she would receive *if she was male but otherwise had identical attributes*. From this perspective, the wage gap reveals itself as genuinely individual, implying that its distribution should play a crucial role when quantifying the aggregate level of discrimination.

The classical methodology, widely used in empirical work, limits the analysis to the calculation of the *mean* wage gap. This implicitly means estimating all the individual wage gaps and aggregating them, attaching the same weight to each gap, independent of its relative relevance or value within the wage distribution. This does not seem a good idea from a distributive point of view, since the aggregation process includes value judgments in an obscure way. This process should be approached from a normative point of view, since it does not seem obvious whether or not the aggregate indicator should allocate identical weight to every individual, independently on the extent of her wage gap.

The aim of this paper is to bring this point, initially opened in the works by Jenkins [22] and Shorrocks [36], back into discussion and to propose a normative framework for the study of wage discrimination. This framework, based on the literature on poverty and deprivation, provides indicators that are explicit in incorporating the necessary judgments about how to aggregate wage gaps. The possibility of using an aggregate index derived from individual discrimination estimations allows for comparisons among countries, as well as among female groups, in a simple way. Our proposal presents advantages against the classical Oaxaca–Blinder decomposition, since aggregation is now obtained from an exhaustive distributive analysis that goes beyond the information summarized in the average wage gap. It also improves other distributive procedures that require simultaneous comparisons at all points of the wage distribution in which discrimination has been estimated since they lead to conclusive results only in extremely robust cases. Thus, for example, in quantifying the effectiveness of an anti-discriminatory policy measure, the quantile decomposition approach used so far to quantify discrimination allows one to identify the wage levels for which the measure has been effective and the wage levels for which it has not. However, it does not allow one to reach a general agreement about the goodness of such a measure if results of a different sign are found at different points of the wage distribution.

In any case, it is important to emphasize that the estimation of quantile regressions and our normative approach are complementary rather than mutually exclusive. In fact, this econometric technique provides a much more complete picture of the wage distribution than the classical OLS. For this reason, this technique seems more suitable than others within our normative framework. Thus, we propose to estimate individual wage gaps by using the aforementioned quantile technique, and, later, aggregate that information with discrimination indexes that satisfy good properties from a normative point of view. In some sense, our approach implies taking a step further in the distributive philosophy behind the quantile estimation of the wage gaps.

Our contribution in this paper is fourfold. First, we examine several limitations of recent distributional approaches for the analysis of wage discrimination. Second, we discuss the normative properties that any discrimination measure should satisfy when aggregating individual wage gaps, and we suggest a minimal set of them on which to reach a wide agreement. Next, we propose different discrimination measures, taken from the poverty literature, that are consistent with the above properties. These measures allow us to quantify both absolute and relative (with respect to different reference wages) discrimination. Third, unlike Jenkins [22], we propose the use of quantile regressions to identify and estimate individual wage gaps. It is shown that the estimation of wage equations by means of quantile regressions and the use of normative measures of discrimination *à la Jenkins* are not exclusive but complementary techniques. Fourth, we contrast the advantages of our approach by offering an empirical illustration. Using data from a Spanish survey on wages, we identify those subgroups of female workers who suffer highest discrimination levels. We show not only the existence of *glass ceilings* and *sticky floors* in the Spanish labor market, but we also quantify the contribution of each of them to total discrimination.[1] We should underline that, even if we recurrently refer to gender wage discrimination, the contributions of this paper are readily applicable to any other source of discrimination such as race, sexual orientation, nationality, age, religion, citizenship, etc.

The paper is organized as follows. Section 2 presents the classic approach to the measurement of discrimination and gives a sound justification of the importance of considering distributive aspects in discrimination measurement. In Section 3 we discuss the limitations of a variety of distributional techniques recently used in the measurement of wage discrimination. Section 4 presents our theoretical proposal detailing its main contributions. Section 5 provides two alternative empirical procedures to estimate individual wage gaps. In Section 6, we provide empirical evidence

[1] Usually, the literature has identified the existence of a *glass ceiling* when the gender pay gap is significantly larger at the top of the wage distribution. In contrast, Arulampalam et al. [3], after Booth et al. [8], identified a *sticky floor* when the gender wage gap is significantly larger at the bottom of the wage distribution.

on the advantages of our techniques by showing an empirical illustration based on a sample of Spanish wages micro-data. Finally, Section 7 concludes by presenting our main findings.

2 The relevance of the distributive approach in analysing wage discrimination

2.1 Wage discrimination: the identification problem

Usually, gender wage discrimination is identified as the difference in earnings between male and female workers who are otherwise identical in their attributes and thus in their expected productivity. In order to identify its presence and to measure its relevance, researchers have traditionally estimated wage equations conditional on a list of variables which, a priori, are potential determinants of the individual's salary. Thus, two separated *mincerian* log wage equations for males and females are commonly estimated:

$$\begin{aligned}\omega_{m_i} &= \ln\left(y_{m_i}\right) = Z'_{m_i}\beta_m + u_{m_i}\\ \omega_{f_i} &= \ln\left(y_{f_i}\right) = Z'_{f_i}\beta_f + u_{f_i}\end{aligned} \tag{1}$$

where m refers to males, f to females, y_i stands for the ith worker hourly wage, ω_i is the natural logarithm of y_i, Z'_i is the vector of characteristics, β are the characteristics' rates of return, and u_i is the corresponding error term.

2.2 Wage discrimination: the aggregation problem

Traditionally based on OLS estimations of these wage equations, discrimination has been evaluated in the mean distribution of the characteristics, and has thus quantified the wage discrimination suffered by the *mean* female worker when compared to the *mean* male worker. This is precisely the approach proposed by Oaxaca [32] and Blinder [6] in their seminal articles and which has been recurrently utilized in the literature ever since. In the original Oaxaca–Blinder decomposition, the mean observed wage gap is divided into two components: a first component, A, would quantify the labor market premium on the mean differences in characteristics between genders, while the second component, B, would show how different are the labor market rewards workers with a different gender evaluated at the mean female characteristics:[2]

$$\overline{\ln\left(y_m\right)} - \overline{\ln\left(y_f\right)} = \left(\overline{Z'_m} - \overline{Z'_f}\right)\hat{\beta}_m + \overline{Z'_f}\left(\hat{\beta}_m - \hat{\beta}_f\right) = \mathrm{A} + \mathrm{B}.$$

[2] Certainly, in empirical analysis, the B component could include factors other than discrimination if relevant characteristics are not controlled for when estimating wage equations. However, notice that this issue does not have its origin within the theoretical framework.

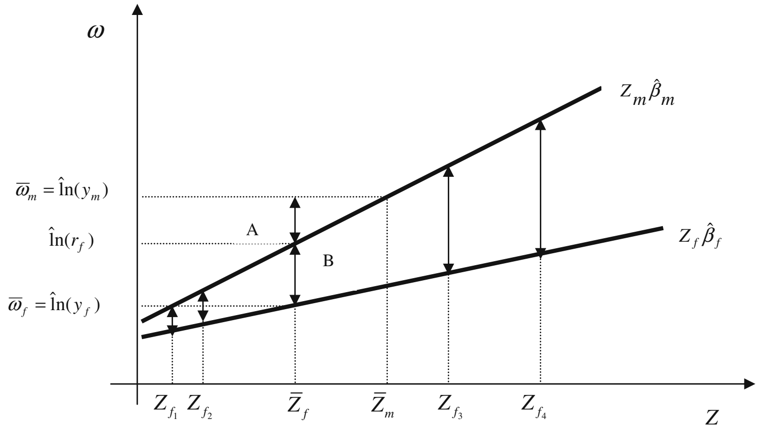


Fig. 1 Wage discrimination using OLS

Figure 1 shows, in the one-dimensional case, that the second component denotes the wage penalty the mean female worker faces given that she has a different remuneration of attributes compared to males.[3]

Even if seldom noted, it is easy to check that B is the mean of the individual differences between predicted male and female log wages estimated for each woman in the population. Thus,

$$B = \bar{Z}'_f\hat{\beta}_m - \bar{Z}'_f\hat{\beta}_f = \sum_i \left(Z_{f_i}\hat{\beta}_m - Z_{f_i}\hat{\beta}_f\right)/n$$

n being the total number of female workers.[4]

The choice of the male wage structure as the non-discriminatory reference is equivalent to considering discrimination as the disadvantage of any group with respect to the most favored group—this would not be true in the case of choosing some other reference. Whatever the non-discriminatory remuneration structure of reference, the use of the mean of the wage distribution is a large waste of information. In the first place, the mean does not allow for differences in the discriminatory experience at different points of the wage distribution. Furthermore, and most importantly, it implies assuming that to give the same weight to each different individual discrimination experience is a desirable way of aggregating wage gaps, independently of the actual degree of discrimination suffered by each individual. This all imposes, implicitly and in an obscure way, value judgments that are rather implausible from a normative point of view—since this implies, for example, that a hypothetical situation where a few individuals suffer high levels of discrimination would be equivalent

[3]In Fig. 1 the mean female and male worker is $\bar{Z}_f$ and $\bar{Z}_m$, respectively, and different individual female workers in the population are associated with Z_{f1}, Z_{f2}, Z_{f3} and Z_{f4}.

[4]Notice that if there are women enjoying negative wage gaps, $Z_{f_i}\hat{\beta}_m < Z_{f_i}\hat{\beta}_f$, positive and negative gaps would offset each other in B.

to another where many individuals experience low discrimination levels. There has been little discussion in the literature on the adequacy of these assumptions. Most probably this has been due to the attractive mathematical properties of the mean and also to the general lack of discussion of normative implications in discrimination measurement.

3 The limitations of recent distributive approaches

In recent years, a number of papers have introduced a variety of econometric techniques in order to incorporate distributive aspects in the comparative analysis of wage distribution. Since the Juhn et al. [26, 27] seminal papers, a large list of works have suggested that the market remuneration to individual endowments is not constant along the wage range.[5]

Within the studies that aim to measure gender wage discrimination some papers have also looked at distributional issues. Blau and Kahn [4, 5] explained the international differences in female wage gaps and their evolution in time using the methodology proposed by Juhn et al. [26]. This methodology allowed them to take into account the role played by the wage structure in the explanation of the gender wage gap. Fortin and Lemieux [15] analysed the wage gap along various years using *rank regressions* in order to estimate the probability that an individual receives a salary within a certain wage interval. And more recently, Bonjour and Gerfin [7] applied the methodology proposed by Donald et al. [13] to decompose the gender wage gap in Switzerland using wage distribution's flexible estimators based on duration models.

Further research on distributive aspects of the gender wage gap has used quantile regressions in order to decompose it at different points of the pay distribution. Examples of this are Reilly [33] and Newell and Reilly [31] in the analysis of ex-communist countries in transition; and García et al. [16], Gardeazábal and Ugidos [17], and De la Rica et al. [10] in their works for Spain. Finally, Albrecht et al. [1] in their study of the *glass-ceiling* in Sweden, Albrecht, Van Vuuren and Vroman [2] in their work for the Netherlands and Arulampalam et al. [3] exploring the gender pay gap over the European Union, apply techniques developed by Machado and Mata [29] where quantile regressions are used in order to estimate counterfactual density functions.

We claim that all these recent approaches to the analysis of discriminatory practices are a clear improvement on previous ones but present, nevertheless, some limitations in measuring discrimination from a distributive point of view.

[5] Buchinsky [9] presented empirical evidence on this using quantile regressions in the study of the evolution of wages in the US. Additionally, DiNardo et al. [12] quantified the effects generated by the change in the distribution of workers' characteristics on wage density using non-parametric regression techniques to estimate counterfactual wage distributions (which allows them to combine one period's population attributes with the returns structure of another). More recently, in their analysis of Portuguese wage inequality, Machado and Mata [28] used quantile regressions to model the conditional wage distribution on workers' characteristics allowing for the measurement of different returns for each attribute at different points of the wage range.

3.1 The comparison of conditional wage distributions: distributive aspects and misconceptions in measuring discrimination

In order to provide an illustration of the problems that arise when using counterfactual distribution functions in the estimation of wage discrimination, suppose that we know that the female wage distribution without discrimination is r_f and we compare it with the observed female wage distribution y_f. When moving from y_f to r_f, it will not come as a surprise that some female workers change their relative positions. This could imply that the earnings differentials between both distributions, evaluated at each quantile, would not show the true differences in discriminatory experiences of female workers. Let us show an example: Suppose that we depart from such a wage distribution as the density function $f(y_f)$ on the left hand side of Fig. 2 and, once we eliminate direct wage discrimination, the new density function moves uniformly to the right, $f(r_f)$. In this particular case, the distributive analysis using quantile differences would conclude that all female workers' deciles experience the same absolute level of discrimination, whatever their wage.

Nevertheless, this may not be necessarily true. It may be the case, as depicted in the graph, that all type A women, who initially earned y_A, earn r_A when the discriminatory component is eliminated. Additionally, a similar number of those female workers who were earning y_B could be experiencing a lower wage change once we eliminate discrimination and thus appear in r_B. The rest of type B women would reach the same wage level as females in A, the level $r'_B = r_A$. Obviously, the level of discrimination suffered by group A is much larger than that suffered by group B, but neither the study of the differences in the mean nor the comparison of quantile counterfactual distributions would detect it. In other words, when comparing density functions we are not only quantifying discrimination but also the re-orderings in the wage distribution. In this way, this measurement of discrimination is *contaminated* in the presence of mobility between quantiles.

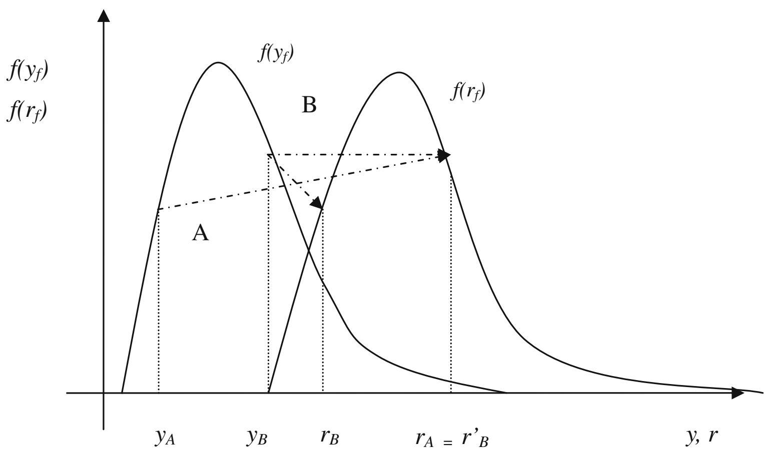


Fig. 2 Wage discrimination using counterfactual densities

The comparison of the means (variances, quantiles) of the actual and counterfactual wage distribution functions does not allow one to properly quantify the individual discriminatory experience. It is impossible to assure that a certain decile suffers more or less discrimination than another by comparing the wages that correspond to each decile in the actual and counterfactual distributions, since women who were placed in a decile in the former distribution may differ from those placed in the same decile in the latter. Nevertheless, various techniques in the literature on gender wage discrimination are based implicitly on the assumption that women maintain their initial ranking in the counterfactual distribution. Clearly, these papers should observe caution in the interpretation of some of their results.[6] The use of these techniques should remain within the interesting study of the distributive effects of discrimination. However, these effects should not be understood as levels of discrimination at different points of the wage distribution.[7]

3.2 The need for normative measures of wage discrimination

It is important to be aware that neither the methodologies based on conditional wage distribution functions nor those using quantile regressions consider the issue of how to weight the different levels of discrimination estimated throughout the wage range. Thus, implicitly, they avoid the construction of a single aggregated indicator. This decision may be argued as adequate in the aim of incorporating the least number of value judgments possible in the analysis. To provide measurements of discrimination at different quantiles without any aggregation criterion implies solving the judgments issue in a trivial way: no aggregation is undertaken and therefore no value judgments are incorporated. We should be aware, however, that this strategy makes it rather difficult to compare discrimination levels between distributions (apart from the trivial case in which a given wage distribution presents more discrimination in all estimated quantiles).

We argue here that in the distribution literature there are valuable options that incorporate judgments in a very reasonable way. The Lorenz dominance criterion aggregates income levels in order to compare different income distributions in terms of inequality under a minimum number of value judgments on which there has been an agreement.[8] This adds robustness, but incompleteness, to the orderings. In those cases in which the Lorenz criterion cannot order functions, complete inequality indices (Gini, Theil or Atkinson index) are unavoidable. These indices incorporate a larger number of value judgments than the Lorenz criterion but allow us to undertake slightly more delicate orderings. Often, the results offered by complete indices do not coincide, but differences between them are not at all random but consistent with

[6] Some works that suffer from this problem within the literature of gender wage discrimination are Albrecht et al. [1] and Bonjour and Gerfin [7].

[7] The decomposition of the gender wage gaps using quantile regressions does not suffer from this problem since the same women are considered when comparing the observed and the counterfactual distributions (see, for example, [17]).

[8] Basically resumed in two axioms: *symmetry* (or *anonymity*) and the *Pigou-Dalton Principle of Transfers*.

their particular normative properties. A deep analysis of these permits us the best comprehension of the analyzed phenomenon.

Jenkins' [22] approach advances in this direction and proposes discrimination measures that allow for the aggregation of wage gaps.[9] Our proposal extends his approach incorporating some improvements. We propose a normative framework in which to insert a discrimination measurement following the literature on deprivation.

4 Normative discrimination measures

So far we have shown that, firstly, when analyzing discrimination we should focus on the "experience of each individual." Given the bi-dimensional nature of this issue—which requires considering vector (y_{f_i}, r_{f_i}) for each individual—to quantify wage discrimination we should take into account the difference $(r_{f_i} - y_{f_i})$ for each i, rather than considering distributions r_f and y_f separately. Secondly, we need to aggregate these individual experiences. This implies taking value judgments into account, and these are, necessarily, of a subjective nature. Is this a problem? Not if we accept that discrimination is a *bad thing* in the same way that poverty or the duration of unemployment are. Hence the question is: what properties should a measure of discrimination satisfy? The literature on economic poverty has widely accepted a list of normative properties as satisfactory requirements for any poverty measure. We believe that these same properties are also adequate in the case of the study of wage discrimination. Let us discuss our proposal in detail.

4.1 Normative properties of discrimination indices

Consider two vectors of individual wage gaps, x_f and x'_f, where $x_f = (r_{f_1} - y_{f_1}, \ldots., r_{f_n} - y_{f_n})$, and $x'_f = (r'_{f_1} - y'_{f_1}, \ldots., r'_{f_s} - y'_{f_s})$, being y_{f_i} and r_{f_i} female wages with and without discrimination, and being n and s respectively the total number of female workers in each distribution. $d(x_f)$ represents the level of discrimination, which corresponds to distribution $\boldsymbol{x_f}$ for a given measure d. The minimal set of normative properties or axioms that $d(.)$ should satisfy are the following:

1. *Continuity axiom*. $d(\boldsymbol{x_f})$ must be a continuous function for any vector of wage gaps in its domain, $\boldsymbol{x_f}$.
2. *Focus axiom*. If we can obtain $\boldsymbol{x'_f}$ from $\boldsymbol{x_f}$ by rises in wages of non-discriminated women, then $d(\boldsymbol{x'_f}) = d(\boldsymbol{x_f})$.
3. *Symmetry* (or *anonymity*) *axiom*. If $\boldsymbol{x'_f}$ can be obtained from $\boldsymbol{x_f}$ by a finite sequence of permutations of individual wage gaps, then $d(\boldsymbol{x'_f}) = d(\boldsymbol{x_f})$.
4. *Replication invariance axiom*. If we can obtain $\boldsymbol{x'_f}$ from $\boldsymbol{x_f}$ by replications of the population, then $d(\boldsymbol{x'_f}) = d(\boldsymbol{x_f})$.

[9] A number of papers have used discrimination indices just as proposed by Jenkins [22]. We know of the empirical works of Denny et al. [11], Makepeace et al. [30], Gustafsson and Li [19], Ullibarri [37], and Hansen and Wahlberg [21].

5. *(Weak) monotonicity axiom*. If $\boldsymbol{x}'_f$ can be obtained from $\boldsymbol{x}_f$ by increasing the discrimination level of a woman, then $d(\boldsymbol{x}'_f) > d(\boldsymbol{x}_f)$.
6. (*Weak*) *transfer axiom*. If we can obtain $\boldsymbol{x}'_f$ from $\boldsymbol{x}_f$ by a sequence of "regressive transfers" between two discriminated female workers, so that the one with the highest discrimination suffers an increase in her wage gap equal to the decrease experienced by the other, then $d(\boldsymbol{x}'_f) > d(\boldsymbol{x}_f)$.

The *continuity axiom* is a reasonable property for any index in order to guarantee that small changes in wage gaps do not lead to large changes in discrimination levels. The *symmetry axiom* guarantees that the index does not favor any particular woman, and the *replication invariance axiom* is a technical property that allows for comparisons between distributions of different size. The two other final axioms lead to two basic properties. The *monotonicity axiom* refers to discrimination intensity, so that a worsening in the position of a discriminated woman yields a higher level of aggregate discrimination. The *transfer axiom* implies that a higher inequality level between discriminated women, in terms of their discrimination sharing, leads to an increase in the discrimination index. Thus, unlike the classic methodology where all women are attached the same weight regardless of their gap, the above axiom imposes that the more discrimination a woman suffers, the higher her contribution to the aggregate discrimination level.

Finally, the *focus axiom* requires the index to be dependent on the distribution of discriminated women while disregarding the wage level, but not the number, of the rest of the female workers. This does not mean that measures verifying this axiom are necessarily independent of the existence of women with wage advantages with respect to male workers,[10] but it does require that these salary advantages are not taken into account when measuring aggregate discrimination. We consider this axiom essential in order to properly aggregate individual discrimination. Suppose hypothetically that we find a labor market in which 40% of females suffer from a 100 € wage discrimination while another 40% earn a 100 € more than their equivalent males. An index that compensates these differences would measure the same discrimination in this labor market than in one in which all females earn exactly the same as identical males.[11] We consider these two situations as clearly different because we believe that discrimination is a form of individual (rather than group) deprivation, just like poverty or unemployment are. In all those cases, it is straightforward that an individual's deprivation situation cannot be counterbalanced by the lack of deprivation of others.[12]

[10]In fact, the share of these women over total female workers will be taken into account in all indices that verify *continuity*, *monotonicity* and *replication invariance axioms* (see Zheng [38] for the poverty case). Other things equal, the larger the share of discriminated women the larger discrimination will be.

[11]This is the case in the measurement of discrimination using the Oaxaca-Blinder decomposition.

[12]This is similar to considering that the existence of famous Gypsy musicians or African-American athletes should not offset the inferior economic position of most individuals from their ethnic or racial group.

The advantage of our approach is that it provides tools in order to aggregate individual discrimination using an index with reasonable normative properties. Further, it provides a framework to fully characterize discriminated individuals in a society given that it could be the case that some types of discrimination only appear in certain occupations or sectors and not in others. This definitely helps us in deepening the knowledge about discrimination in all possible settings.[13]

Accepting the axioms above, we will be able both to construct discrimination profiles by accumulating individual wage gaps and to develop some dominance criteria to rank wage distributions according to their discrimination level. Next we will be able to make a correspondence between these rankings and those obtained by using complete discrimination indices that also satisfy these properties. This is the case in the inequality and poverty fields, where there are valuable theorems that establish a relationship between the income distribution ranking obtained by "three 'I's of poverty" (TIP) or Lorenz's dominance criteria and those obtained by complete poverty and inequality indices compatible with those criteria. Thus, by using a minimal set of judgments, summarized in the above properties, we will be able to identify particular empirical cases where the discrimination distribution ranking is independent of the index chosen, since all indices yield the same result. This makes our analysis of discrimination significantly more robust.

This line of research was opened by Jenkins [22] when he used the Inverse Generalized Lorenz Curve (IGLC) in the discrimination field, and defined discrimination indices consistent with its dominance criterion.[14] Later, in the deprivation field, Shorrocks [36] generalized these relationships in the continuous case and summarized previous results obtained by different authors. In what follows, we extend this analysis and propose the use of discrimination curves and discrimination indices that will be defined so as to satisfy the above axioms.

4.2 Dominance relations between discrimination curves

Consider a vector of individual wage gaps, x_f, where $x_f = (x_{f_1}, ..., x_{f_n}) = (r_{f_1} - y_{f_1}, ..., r_{f_n} - y_{f_n})$, y_{f_i} and r_{f_i} being female wages with and without discrimination, respectively. Let us define $g(x_f)$ as the vector of individual wage discrimination, where each element, $g_i(x_f)$, is the maximum between x_{f_i} and zero:

$$g_i\left(x_f\right) = \max\left\{\left(r_{f\,i} - y_{f\,i}\right), 0\right\}$$

[13]Note that our approach allows also for the analysis and characterization of non-discriminated women. Alternatively, one could address the analysis of male discrimination using female wage structure as a reference although, presumably, it would be rather small given that only a minority of men, if any, would appear to be discriminated.

[14]This curve represents the per capita cumulative sum of wage gaps, on absolute values, for each cumulative proportion of women, once they have been ranked from higher to lower absolute wage gap. Note that Jenkins [22], when defining the IGLC on absolute values of $\boldsymbol{x_f}$, does not impose the *focus axiom*. However, as it has been shown, it seems reasonable to redefine the variable, the dominance criterion and the indices he proposes taking that axiom into account.

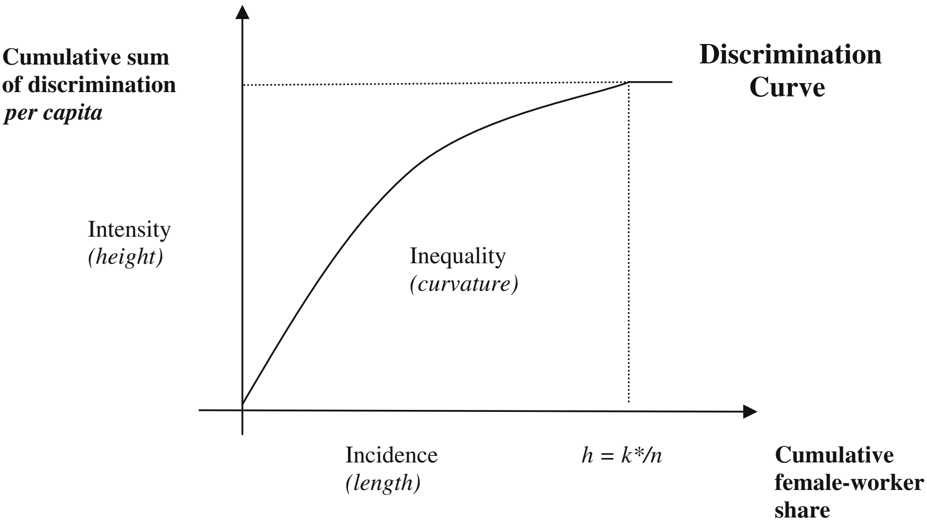


Fig. 3 Discrimination curve

The discrimination curve represents for each $0 \leq p \leq 1$ the sum of the first $100^{*}p$ percent of $g_i(x_f)$ values divided by the total number of female workers, n, once these have been ranked from a higher to a lower wage discrimination level. Hence, $g(x_f) = (g_1, g_2, \ldots, g_n)$ satisfies that $g_1 \geq g_2 \geq \ldots \geq g_n$, and for each value of $p = k/n$ the curve can be written as:

$$D_p\left(g\left(x_f\right)\right) = \sum_{i=1}^{k} \frac{g_i\left(x_f\right)}{n} \tag{2}$$

where k is any integer number such that $k \leq n$.[15] $D(g)$ accumulates individual discrimination levels, from higher to lower discrimination, divided by n. As shown in Fig. 3,[16] $D(g)$ is a positive, increasing and concave function; where $D_0(g) = 0$, $D_1(g) = \bar{g}$, and takes a constant value when we consider the last discriminated woman, k^*. The shape of the above curve provides us with useful information. First, it shows the incidence of discrimination so that to identify the proportion of discriminated women, we only need to know the percentile where the curve becomes a horizontal line, $h = k^*/n$. Second, it informs us about its intensity, since the height of the curve is the accumulated discrimination averaged by the number of female workers. Third, it also shows the inequality aspect of the discrimination distribution by the degree of concavity of the curve before point h.

[15]The Discrimination Curve is the IGLC defined for $g(x_f)$ rather than for absolute values of wage gaps, $|x_f|$, as in Jenkins [22]. The latter implies, counter-intuitively, considering positive and negative wage gaps as equivalent.

[16]This is an adaptation of Fig. 1 in Jenkins and Lambert [24], where the properties of the TIP curves are shown to measure aggregate poverty.

Definition of dominance in discrimination Given two wage discrimination distributions, g^1 and g^2, we would say that:

$$g^1 \text{ dominates } g^2 \text{ in a discriminatory sense if}$$
$$g^1 \neq g^2 \text{ and } D_p\left(g^1\right) \leq D_p\left(g^2\right) \text{ for any } p \in [0,\ 1].$$

It is straightforward then to show that this dominance criterion is closely linked to the six properties mentioned above. Thus, we can establish a relationship between dominance in the discriminatory sense and the set of aggregate indices, $d^*(x_f)$, that satisfy in $g(x_f)$ the *continuity, focus*, *monotonicity*, *symmetry*, *transfer* and *replication invariance axioms*.

Theorem[17] *For any pair of wage discrimination distributions, g^1 and g^2, it follows that,*

$$g^1 \text{ dominates } g^2 \text{ in a discriminatory sense}$$
$$\Leftrightarrow$$
$$d\left(x_f^1\right) < d\left(x_f^2\right) \text{ for any } d(\cdot) \in d^*$$

Hence, a higher discrimination curve leads, unambiguously, to a higher discrimination level for an extensive set of discrimination indices.[18]

4.3 Complete indices consistent with dominance discrimination

Since the dominance criterion is not always able to give us conclusive results in empirical applications (the estimated discrimination curves can cross) it is interesting to explore some of the indices belonging to d^*. We are interested in those that satisfy both the normative axioms above and any other property that may be of special interest for empirical analysis, such as *decomposability*.

Additive decomposability Consider a partition within $\boldsymbol{x_f}$, where $n_1 + n_2 + \ldots + n_J = n$ are the sizes of J subpopulations, $\boldsymbol{x_f^{(1)}}, \boldsymbol{x_f^{(2)}}, \ldots, \boldsymbol{x_f^{(J)}}$. A discrimination index d is said to be additively decomposable if:

$$d\left(x_f\right) = \sum_{j=1}^{J} \left(\frac{n_j}{n}\right) d\left(x_f^{(j)}\right).$$

[17]This result was first shown in Shorrocks [35], where it was used to study the duration of unemployment, and in Jenkins and Lambert [23] in the poverty field. This work established the basis for later results on TIP curves [24, 25]. The continuous case is shown in Shorrocks [36]. Jenkins [22] first used this approach in the wage discrimination field, where he defined wage discrimination as the difference, in absolute terms, between the wages estimated with and without discrimination.

[18]Notice that this theoretical result also makes it possible to quantify the differences in discrimination between two wage distributions without using complete indices. This stems from Theorems 4 and 5 in Jenkins and Lambert [25].

This property suggests that it may be desirable to decompose overall discrimination as the weighted sum of subpopulation discrimination levels. However, this is not a widely accepted criterion in the poverty field if, for example, we consider that the poverty level in a group cannot be independent of that in other groups. Despite this serious criticism, the above property is clearly very helpful in most empirical applications, since it allows us to measure the contribution of each population group, j, to the total level of detected discrimination, $\left(n_j/n\right) d\left(x_f^{(j)}\right)$. This means that we can study discrimination for different female characteristics. Thus not only can we classify women by earnings (as in the quantile estimations mentioned above) but also by any other variables, such as education level, age, or geographical location.

Jenkins [22] proposed the use of different *families* of aggregate discrimination indices. If they were conveniently defined over $\boldsymbol{x_f}$, instead of $|r_f - y_f|$ as he initially proposed, the main difference of Jenkins' approach with respect to our proposal would be the *transfer axiom*. Jenkins shows a preference for the use of indices that do not satisfy this axiom.[19] In fact, the family of decomposable indices that he uses in his empirical analysis, J_α, is a concave function that depends on the relative individual discrimination level (with respect to the average wage). The concavity of this index means that given a constant aggregate wage gap, the more discrimination is focused on fewer women, the lower the discrimination level will be. It follows that evenness in the distribution of discrimination will increase the value of the index. This is inconsistent with what is generally assumed in other forms of relative deprivation like poverty, where increasing the level of deprivation of a more deprived person should have a larger impact on the aggregate level of deprivation than when the increase affects a less deprived person. In our view, that should be the case for discrimination too and thus we propose the use of indices that satisfy the *transfer axiom*.

Taking into account all the above, we consider that it is not necessary to define new discrimination indices, as Jenkins suggests, but only to make good use of those with the best normative properties within the poverty literature. Therefore, if we adapt the family indices proposed by Foster et al. [14] to measure [absolute] discrimination, we can write a discrimination index such that:

$$d_\alpha x_f = \left(\frac{1}{n}\right) \sum_{i=1}^{k^*} \left(x_{f_i}\right)^\alpha, \quad \alpha \geq 0 \tag{3}$$

where k^* denotes again the number of discriminated female workers and α is the discrimination aversion parameter. For the special case $\alpha = 0$ the index is a headcount measure of the incidence of discrimination among women, h, and for $\alpha = 1$ it accounts for the average level of discrimination per woman. Further, it is well

[19]Even though he offers theoretical results for both cases depending on the sign and value of a parameter.

known that for values of α strictly higher than 1 these indices satisfy our normative requirements, $d_\alpha \in d^*$, and are additively decomposable.[20]

4.4 Absolute versus relative discrimination

An additional issue in the measurement of discrimination is whether to use a relative rather than an absolute approach. In order to do this we need to define new indices, dr_α, which would be a function of the wage gap vector normalized with respect to some average wage, for example the mean female wage without discrimination, $\bar{r}_f$:[21]

$$dr_\alpha \left(x_f/\bar{r}_f\right) = \left(\frac{1}{n}\right) \sum_{i=1}^{k^*} \left(x_{f\,i}/\bar{r}_f\right)^\alpha$$

Another interesting possibility consists in normalizing each female wage gap individually by dividing it by her earnings without discrimination,

$$v_{f_i} = x_{f_i}/r_{f_i}$$

This implies that the critical point is no longer the average wage but, instead, the highest discrimination level that each woman could suffer:[22]

$$dr_\alpha \left(v_{f_i}\right) = \left(\frac{1}{n}\right) \sum_{i=1}^{k^*} \left(v_{f_i}\right)^\alpha \tag{4}$$

In order to guarantee that these indices satisfy the same properties as $d_\alpha\left(x_f\right)$, we need to redefine the discrimination curves on the normalized wage discrimination vector, $\mathrm{D}\left(\Gamma\left(x_f/\bar{r}_f\right)\right)$ or $\mathrm{D}(\Gamma(v_f))$, where

$$\Gamma_i\left(\frac{x_f}{\bar{r}_f}\right) = \max\left\{\left(\frac{r_{f_i} - y_{f_i}}{\bar{r}_f}\right),\ 0\right\} \quad \text{and} \quad \Gamma_i\left(v_f\right) = \max\left\{\left(\frac{r_{f_i} - y_{f_i}}{r_{f_i}}\right),\ 0\right\},$$

[20] It would also be interesting to measure discrimination adapting our approach to the use of different poverty indices that satisfy other normative properties such as those proposed by Sen [34] or Hagenaars [20]. The latter would allow us to measure discrimination as the social welfare loss it causes.

[21] Another possibility would be to use the mean observed wage, $\bar{y}_f$.

[22] The role played by r_{f_i} in this kind of normalization is similar to that of the poverty line in the deprivation literature. Hence, by dividing the individual wage gap by r_{f_i}, we do something similar to what is done in the poverty literature when constructing relative poverty gaps by using individual poverty lines for each household (depending on its size, composition, location,...).

reformulating the dominance criterion and the theorem in a consistent way.[23] Hence, the normalised discrimination curve, D(Γ), which maintains the same graphic characteristics than D(g), can be written as:

$$D_p(\Gamma) = \sum\nolimits_{i=1}^{k} \frac{\Gamma_i}{n} \tag{5}$$

once the vector Γ has been ranked from higher to lower relative wage discrimination: $\Gamma_1 \geq \Gamma_2 \geq \ldots \geq \Gamma_n$.

Definition of dominance in normalised discrimination Given two normalised discrimination vectors, Γ^1 and Γ^2, we say that:

$$\Gamma^1 \text{ dominates } \Gamma^2 \text{ in a discriminatory sense if}$$
$$\Gamma^1 \neq \Gamma^2 \text{ and } D_p\left(\Gamma^1\right) \leq D_p\left(\Gamma^2\right) \text{ for any } p \in [0, 1]$$

The dominance theorem for the relative case could be stated as follows:

Theorem (relative case) *For any pair of normalised wage discrimination distributions,* Γ^1 *and* Γ^2*, it follows that,*

$$\Gamma^1 \text{ dominates } \Gamma^2 \text{ in a discriminatory sense}$$
$$\Leftrightarrow$$
$$dr\left(x_f/\bar{r}_f\right)^1 < dr\left(x_f/\bar{r}_f\right)^2 \text{ for any } dr(\cdot) \in dr^*$$
$$\left[dr\left(v_f^1\right) < dr\left(v_f^2\right) \text{ for any } dr(\cdot) \in dr^*\right]$$

being $dr^*(\cdot)$ *the discrimination indices set which satisfies the aforementioned axioms in* $\Gamma\left(x_f/\bar{r}_f\right)$ $\left[\text{or } \Gamma\left(v_f\right)\right]$.

5 Estimating individual wage gaps

In order to implement the above analysis we need complete information either on x_{f_i} or v_{f_i} (depending on whether we are interested in analyzing absolute or relative discrimination). However, notice that our theoretical contribution is completely independent on the way this information is obtained. The measures we propose depend on the quality of the information on individual discrimination just like income inequality measures depend on the information on equivalent individual income. Indeed, the way in which individual discrimination or income are obtained is definitely an empirical issue.[24]

[23] This point was missed by Jenkins [22], and implies an inconsistency in his Results 1 and 2 when relating them to J_α and R_v indices.

[24] One could rightly argue here that determining individual income and individual discrimination involves a different type of difficulty. However, both of them are just imperfect measures of ideal concepts.

A plausible way of obtaining x_{f_i} and v_{f_i} is by estimating y_{f_i} and r_{f_i}. As far as we know, only Jenkins' [22] proposes such a procedure. In doing so, he uses OLS estimations of *mincerian* wage equations for men and women (see Eq. 1). Following this approach, it is possible to predict both the estimated wage of a female worker, $\hat{y}_{f_i}$, and her potential wage if her attributes were remunerated as if she was male, $\hat{r}_{f_i}$:

$$\hat{y}_{f_i} = \exp\left(Z'_{f_i}\hat{\beta}_f + \hat{\sigma}_f^2/2\right)$$
$$\hat{r}_{f_i} = \exp\left(Z'_{f_i}\hat{\beta}_m + \hat{\sigma}_f^2/2\right) \tag{6}$$

where $\hat{\sigma}_f^2$ is the estimated variance of u_f.[25] The conditional wage gap $(\hat{r}_{fi} - \hat{y}_{fi})$ reflects the estimated wage discrimination experienced by a female worker i, being $(\hat{r}_f - \hat{y}_f)$ the distribution of the estimated discrimination in the female workers group.[26]

Alternatively we propose to estimate individual wage gaps by quantile regressions. When log wage equations are estimated by quantile regressions, $\exp(Z'_{f_i}\hat{\beta}_f^q)$ represents the conditional quantile q of female wage distribution y_f:

$$\hat{y}_{f_i}^q = \exp\left(Z'_{f_i}\hat{\beta}_f^q\right)$$
$$\hat{r}_{f_i}^q = \exp\left(Z'_{f_i}\hat{\beta}_m^q\right). \tag{7}$$

Since we can estimate Eq. 7 in several quantiles, $\{q_1, q_2, \ldots, q_Q\}$, it is not obvious how to obtain the conditional wage gap for each female worker. For illustrative purposes, we proceed as follows. First, we attach to each working woman, i, the quantile, $q = q_i^*$, whose associated female conditional quantile function allows us minimizing her individual wage residual, i.e. $q_i^* \in \{q_1, q_2, ..., q_Q\}$ solves:

$$\min\left(y_{f_i} - \hat{y}_{f_i}^q\right).$$

Thus, the estimated wage for woman i will be: $\hat{y}_{f_i}^{q_i^*} = \exp\left(Z'_{fi}\hat{\beta}_f^{q_i^*}\right)$. And second, for this woman, i, $\hat{r}_{f_i}^q$ is computed using the same conditional quantile, q_i^*, in the *male* wage structure, i.e., $\hat{r}_{f_i}^{q_i^*} = \exp\left(Z'_{fi}\hat{\beta}_m^{q_i^*}\right)$. In this way, what we are actually doing for

[25] $\text{Exp}(Z'_{fi}\hat{\beta}_f + \hat{\sigma}_f^2/2)$ is the expected value of the log-normal variable, y_f, conditional to Z_{fi} in the OLS regression. Note that in Jenkins [22], there is a mistake because $\left(\hat{\sigma}_f^2/2\right)$ was dropped out in the above expression. In the second equation we could substitute $\left(\hat{\sigma}_f^2/2\right)$ by $(\hat{\sigma}_m^2/2)$, and use the male variance of residuals. We have checked that in doing so, our empirical results would not change.

[26] We are assuming that individual women's residual wages are unaffected by discrimination, thus the characteristics included in Z explain the full phenomenon. Therefore, two women with identical observed characteristics will present the same level of estimated discrimination. Obviously, as in Oaxaca-Blinder decomposition, any misspecification of the model drives to measurement errors in our discrimination level.

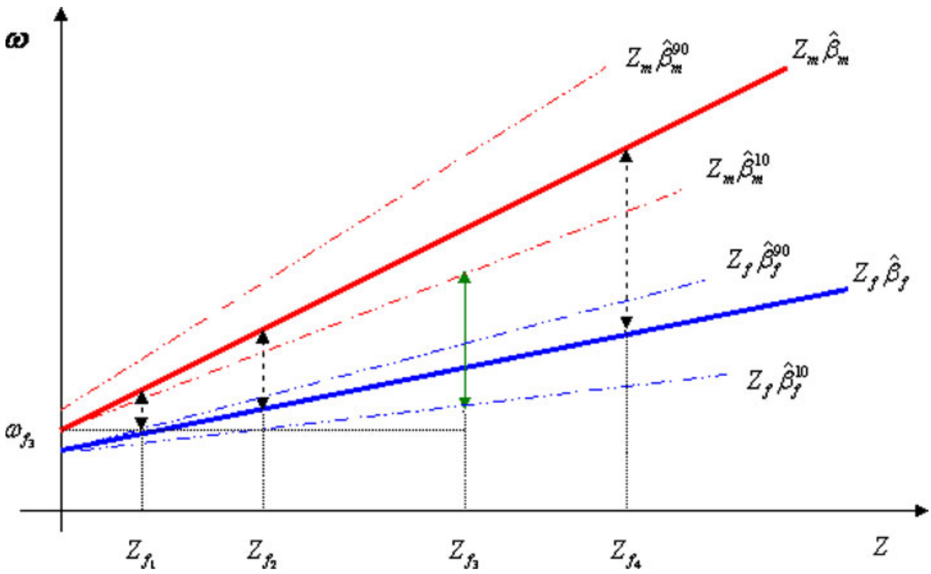


Fig. 4 Wage discrimination using OLS and quantile regressions

each woman is selecting her predicted wage, $\hat{y}_{f_i}^{q_i^*}$, as the closest to her actual wage, y_{f_i}, and comparing it with a male wage, $\hat{r}_{f_i}^{q_i^*}$, estimated for a hypothetical man with her characteristics and situated in the same relative ranking within the conditional male wage distribution.

Figure 4 shows in the one-dimensional case individual (log) wage gaps using OLS and quantile regressions. In the former case, all individual wage gaps are estimated by using the same OLS $\hat{\beta}$'s. Thus, for any woman, *i*, having characteristic Z_{f_i}, the conditional wage gap is the difference between $Z_{f_i}\hat{\beta}_m$ and $Z_{f_i}\hat{\beta}_f$, independently of her actual wage (see Z_{f_1}, Z_{f_2}, Z_{f_4}). Quantile regressions are also shown in Fig. 4 (for simplicity we only represent the conditional quantile functions of men and women corresponding to the 10th and 90th quantiles). Assume, for example, that female worker Z_{f_3} has associated the 10th quantile according to their actual (log) wage, ω_{f_3} (i.e. $q_3^* = 10$). To obtain the quantile wage gap for this woman we must compare her estimated wage in quantile 10 $\left(Z_{f_3}\hat{\beta}_f^{10}\right)$ with the wage that a man with the same characteristic as hers would get in the corresponding 10th quantile of the conditional wage distribution of men $\left(Z_{f_3}\hat{\beta}_m^{10}\right)$.[27]

[27]This is an ad hoc choice that might be forcing the interpretation of this type of estimates. However, it seems reasonable to measure individual discrimination comparing women and men with the same characteristics and at the same position in their corresponding conditional wage distributions since it could reduce the effect of unobservable characteristics in the estimation of individual wage gaps.

6 An empirical illustration: the case of Spain

In this section, we show the advantages of our approach identifying those female workers who suffer the highest discrimination levels in the Spanish labor market. Thus, we will compare aggregate discrimination levels estimated by OLS and quantile regressions (QR) for males and females using a sample of private sector employees.[28] The variable to be explained is the logarithm of hourly wage, and explanatory variables are those usually included in the related literature and available in the database: tenure, experience, education, region, type of contract, occupation, firm size, type of collective agreement, firm-ownership and type of reference market (international, national or local).[29] Coefficients are reported in Table 2 in the Appendix. Once we check that wage regressions results are roughly consistent with those in other previous empirical analyses, we construct wage distributions for working women, estimated with and without discrimination. These estimates are denoted respectively by $\hat{y}_f$ and $\hat{r}_f$ in the OLS case, and $\hat{y}_f^q$ and $\hat{r}_f^q$ in the quantile case (see expressions 6 and 7).[30]

The non-parametric *kernel* wage densities are depicted in Fig. 5a and b. Observed wages result in a more accurate fit using QR, especially evident in the lower tail, and thus showing a greater dispersion in QR than in OLS. Furthermore, as it is shown in Table 3 in the Appendix, QR also presents a greater dispersion of wage gap density estimations in the absolute case, even if not so much in the relative case when each individual wage gap is normalized by $\hat{r}_{f_i}$ and $\hat{r}_{f_i}^q$.

Descriptive statistics for wages and conditional wage gaps estimated with both models are also reported in Table 3 in the Appendix. An interesting result is that the average absolute wage gap, 315.1 pesetas in OLS and 319.5 pesetas in QR, represents around 27% of the observed average female wage (1,188 pesetas), both in OLS and QR estimations. This turns out to be relatively high compared to estimations on female wage discrimination for other developed countries in the literature.[31]

[28] Data come from the *Encuesta de Estructura Salarial* (Survey of Wage Structure) undertaken by the *Instituto Nacional de Estadística* (INE) in 1995. This survey covers employees in firms with ten or more workers and does not include any wage information for employees in Agriculture, Public Administration, Health Services or Education. Those individuals who did not work the entire month or who worked part-time were removed from the sample. The final number of observations for analysis are 27,085 women and 100,208 men.

[29] It was not possible, however, to control for other relevant workers' personal characteristics such as marital status or the presence of children in the household. Furthermore, this database only contains working women and men. For this reason, we do not control for *selection bias*, which implies that our results must be interpreted with caution when interested in conclusions for the whole population.

[30] We compute quantile regressions in ten different points of the distribution (exactly at the middle quantile within each decile: i.e. 5th, 15th, 25th,...., 95th). For simplicity, from now on we will employ the term *discrimination* although we are conscious that our estimations may be including other dimensions in which the wages of women and men differ.

[31] However, we should be cautious in making comparisons when studies follow different methodological approaches. Besides, notice that this survey does not include any Public Administration employees whose wages would presumably reduce discrimination in the Spanish case.

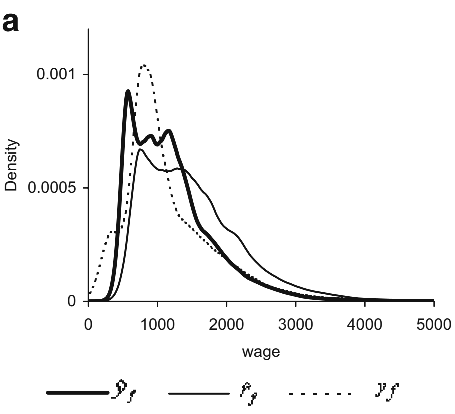


Fig. 5 **a** Observed and predicted wage with and without discrimination (OLS). **b** Observed and predicted wage with and without discrimination (QR)

In order to compare discrimination levels captured by both procedures from a normative and distribute point of view, absolute and normalized discrimination curves are depicted in Fig. 6a and b, (following the expressions 2 and 5, respectively). Both figures show that OLS gender gap distribution dominates QR in discrimination. Thus, our second result is that, in the Spanish case, QR discrimination is always larger than OLS for all discrimination indices fulfilling the axioms proposed (in both absolute and relative cases). This result may be the consequence of the better fit of the QR wage estimations.

With the purpose of deepening the distributive analysis, we divide female workers in deciles defined by their observed wages and calculate absolute and relative

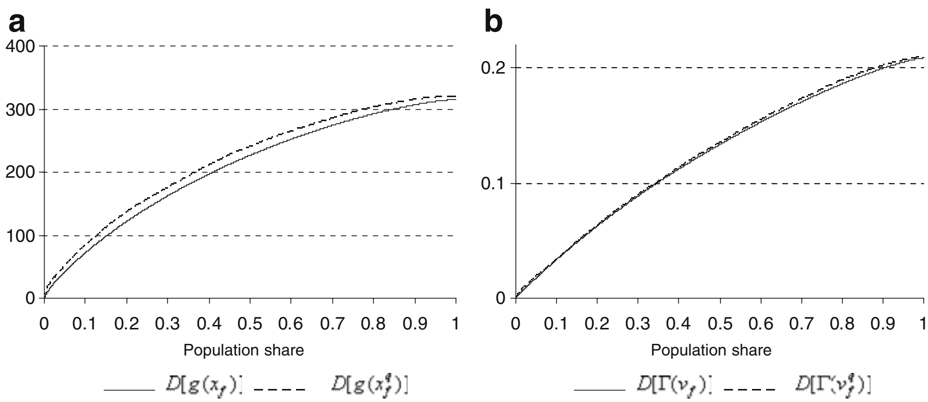


Fig. 6 **a** Absolute discrimination curves. **b** Normalized discrimination curves

discrimination curves separately for each group, using the above individual wage gaps estimated over the total population of male and female workers. From Fig. 7a and b, it is clear that absolute discrimination increases as wages grow in both OLS and QR estimations. If we drew the curves for relative discrimination by deciles, however, we would see that they show an ambiguous pattern due to the appearance of some crosses. In any case, we can assert, differently from the absolute case, that relative discrimination is larger at the bottom than at the top of the female wage distribution. This is a third finding of our analysis and would be consistent with the existence of a *sticky floor* phenomenon in the Spanish labor market.

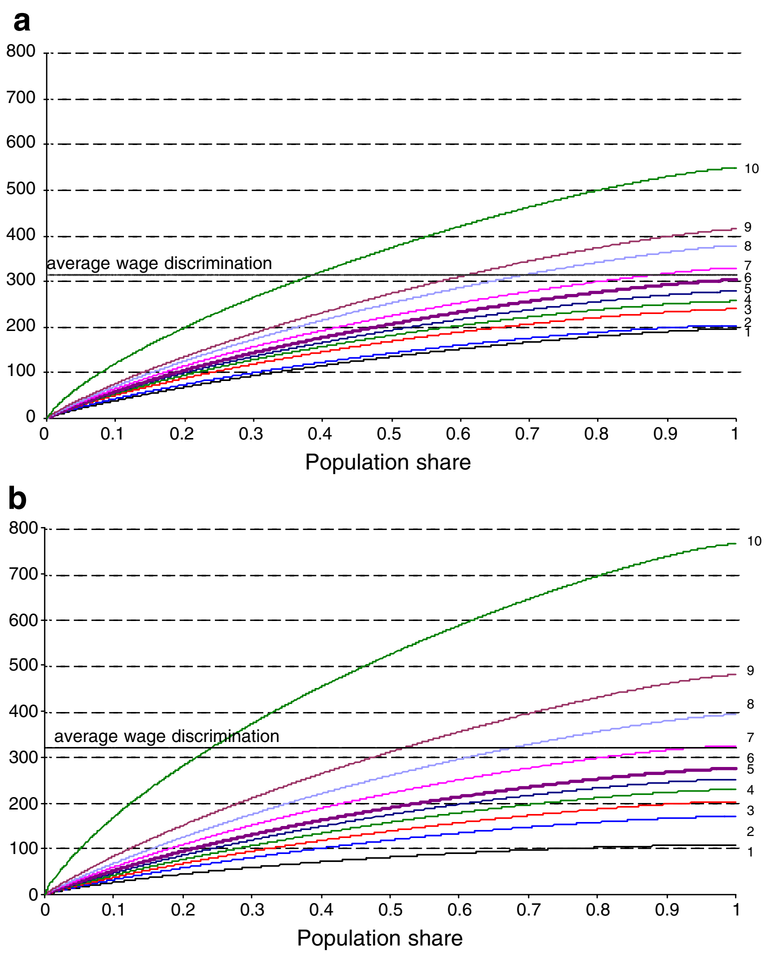
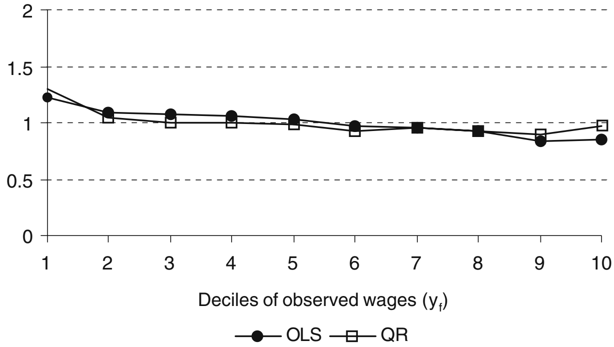


Fig. 7 **a** Absolute discrimination curves by deciles OLS. **b** Absolute discrimination curves by deciles QR

Fig. 8 Relative discrimination by deciles ($dr\ \alpha = 2$) (normalized using average discrimination)

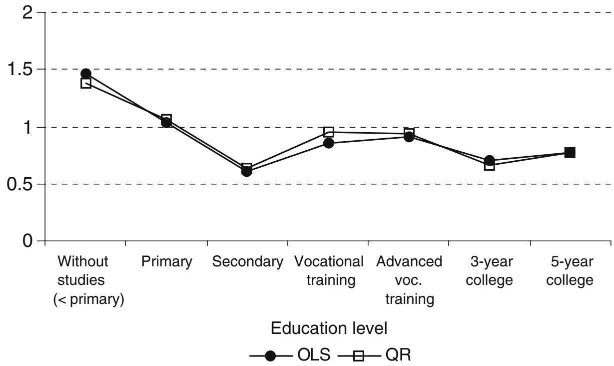


Aiming for a more explicit result on the ranking of relative discrimination by deciles, we propose the use of additively decomposable indices of relative discrimination. This strategy clearly offers less robust results but provides us with some evidence for intermediate deciles whose discrimination curves cross. Figure 8 displays, for each decile, the ratio of the within-group discrimination against the average discrimination using index $dr_{\alpha=2}$ (see expression 4). A value above (below) one indicates that that decile has a discrimination level larger (smaller) than the average. Both OLS and QR estimations show very similar patterns. We observe that females in the first decile experience the largest relative discrimination. As we move along the wage distribution, discrimination decreases slightly (with the exception of the last decile). Similarly, when separating females by their education levels, relative discrimination is much higher than average for those without studies and lower for those with higher studies (see Fig. 9). However, in this second case the evolution of discrimination along the educational career decreases but without a clear pattern.

Trying to move onwards in this analysis, we follow the strategy in De la Rica et al. [10] and break the sample into those females holding a university degree and the rest. Figure 10a and b present relative discrimination results by deciles for these two groups, here again we use the overall population average discrimination as a reference.[32] The results for females without university studies (the largest group) resemble the slightly decreasing pattern of total female workers (see Fig. 8), now including the last decile too. In contrast, among females with a university degree, relative discrimination has a considerably different pattern: it surprisingly increases

[32]Note that deciles are constructed for each sub-population. Table 4 in the Appendix shows the demographic weight of each group in the overall population deciles.

Fig. 9 Relative discrimination by education (dr $\alpha = 2$) (normalized using average discrimination)



with the wage level. This increase is even sharper for the last decile when using QR.[33] Thus, among the more skilled women, it is the group of top-wage female earners that face the largest relative discrimination level. This interesting result indicates the existence of a *glass ceiling* for some female employees.[34]

Therefore, while there seems to be a *sticky floor* for low educated women in the Spanish labor market, for the highest ed ucational group there also seems to be a *glass ceiling*. De la Rica et al. [10] obtained a similar result by decomposing the gender wage gap in different percentiles using quantile regressions. The advantage of our approach compared to theirs is that our approach allows us to quantify and compare both phenomena. We do this by calculating the contribution of each group's relative discrimination to the whole relative discrimination level and present our results in Table 1. In the first column, we include the demographic weight of each subgroup of female workers, $(n_j/n)^*100$, in the second and fifth columns we present their relative discrimination levels, $dr_{\alpha=2}\left(x_f^{(j)}\right)$, and finally we detail their contribution to the whole relative discrimination level, $\left(\frac{n_j}{n}\right) dr_{\alpha=2}\left(x_f^{(j)}\right)$, both in absolute and percentage values (for OLS and quantile regressions).

In both cases, it can be seen that the highest educated women with the highest salaries bear much more relative discrimination than the highest educated women with the lowest salaries: 0.061 and 0.031 in quantile regressions, respectively. However, their contribution to overall discrimination only represents three decimal points over their demographic weight: 1.4 in comparison with 1.1%. In contrast, for female workers with the lowest wages and educational attainments, these percentages are 11.9 and 8.9, respectively. This means that although the *glass ceiling* phenomenon

[33]Note, however, that its highest value is lower than that experienced by low-wage women without a university degree.

[34]It is relevant to emphasize that this result is not only associated with the index dr_2, but it can be obtained with other discrimination indices due to dominance by deciles in their respective normalized curves.

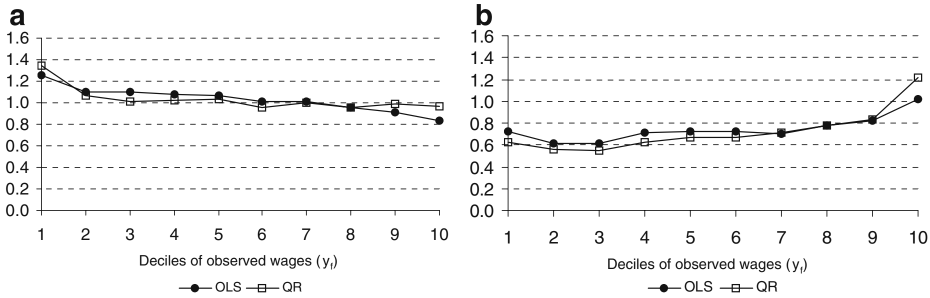


Fig. 10 **a** Relative discrimination by deciles (drα = 2). Females with a non-university degree (normalized using overall average discrimination). **b** Relative discrimination by deciles (drα = 2) Females with a university degree (normalized using overall average discrimination)

Table 1 Relative discrimination by education groups

Groups	Population	OLS			Quantile regressions		
		Within-group discrimination	Contribution to overall discrimination		Within-group discrimination	Contribution to overall discrimination	
	% of all women	$dr^j_{\alpha=2}$	Absolute	%	$dr^j_{\alpha=2}$	Absolute	%
Non-university degree	88.6	0.050	0.044	91.5	0.051	0.045	91.8
By deciles							
1	8.9	0.061	0.0054	11.2	0.067	0.0059	11.9
2	8.9	0.053	0.0047	9.7	0.053	0.0047	9.4
3	8.9	0.054	0.0047	9.8	0.050	0.0045	9.0
4	8.9	0.052	0.0046	9.6	0.051	0.0045	9.0
5	8.9	0.052	0.0046	9.5	0.051	0.0045	89.2
6	8.9	0.049	0.0043	8.9	0.047	0.0042	8.5
7	8.9	0.049	0.0044	9.0	0.050	0.0044	8.9
8	8.9	0.046	0.0041	8.5	0.048	0.0042	8.5
9	8.9	0.044	0.0039	8.1	0.049	0.0043	8.8
10	8.9	0.040	0.0036	7.3	0.048	0.0043	8.6
University degree	11.4	0.036	0.004	8.5	0.036	0.004	8.2
By deciles							
1	1.1	0.035	0.0004	0.8	0.031	0.0004	0.7
2	1.1	0.030	0.0003	0.7	0.028	0.0003	0.6
3	1.1	0.030	0.0003	0.7	0.027	0.0003	0.6
4	1.1	0.035	0.0004	0.8	0.031	0.0004	0.7
5	1.1	0.035	0.0004	0.8	0.033	0.0004	0.8
6	1.1	0.035	0.0004	0.8	0.033	0.0004	0.8
7	1.1	0.034	0.0004	0.8	0.035	0.0004	0.8
8	1.1	0.038	0.0004	0.9	0.039	0.0004	0.9
9	1.1	0.040	0.0005	0.9	0.041	0.0005	1.0
10	1.1	0.050	0.0006	1.2	0.061	0.0007	1.4
All women	100	0.049	0.049	100	0.050	0.050	100

has a qualitative relevance, it is of a relatively small importance if we compare it with the *sticky floor* phenomenon.[35]

7 Conclusions

In this paper we have detailed the advantages of analyzing wage discrimination from a distributive point of view, considering each individual discriminatory experience. Our theoretical contributions are two: First, we underline the imprecise measurement of discrimination comparing counterfactual functions. Second, and most importantly, we propose a new normative framework for the study of wage discrimination based on the poverty and deprivation literature. For the latter we provide a variety of improvements to Jenkins' [22] approach to the aggregation of individual discriminatory experiences by adding to its consistency and normative power.

The empirical illustration using Spanish data allows us to analyze the differences and similarities between OLS and quantile regressions. We should emphasize three basic results. First, for the case of Spain, quantile regressions reveal a significantly higher level of aggregate discrimination compared to that detected using classical estimation techniques. This result seems to be the consequence of the better fit of the QR wage estimations, especially evident in the lower tail. For this reason, this technique appears as more suitable than others within our normative framework. Nevertheless, more empirical evidence for other countries would be useful to find out the strength and robustness of this finding. Second, in spite of the previous result, OLS and QR methods raise roughly similar discrimination patterns throughout the wage range. Finally, it seems clear that absolute discrimination increases with observed wages. However, conclusions are not so straightforward in the relative case. On the one hand, women with very low wages register significantly higher relative discrimination levels than the rest. On the other hand, those females who hold a University degree are a particular case: Those who are earning the highest salaries bear relative discrimination levels around the total wage distribution average, but much larger than all other female workers holding a university degree. All this suggests the existence of both *sticky floors* and *glass ceilings* in the Spanish labor market. The former has the highest quantitative relevance while the latter has a more qualitative significance.

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[35] Given that female activity rate in low-income households is remarkably low in Spain (see [18], our intuition is that it is likely that the *sticky floor* phenomenon is more affected by the lack of control for selection bias compared to the *glass ceiling* phenomenon. However, the direction of the effect of this on wage discrimination is not clear.

Appendix

Table 2 OLS and Quantile regressions estimates for hourly wage in logarithms

	Females						Males					
	OLS	QR at percentiles					OLS	QR at percentiles				
		5	25	45	75	95		5	25	45	75	95
Tenure	0.040	0.054	0.036	0.029	0.024	0.017	0.028	0.041	0.025	0.021	0.015	0.011
$Tenure^2$	−0.001	−0.001	−0.001	−0.001	0.000	0.000	−0.001	−0.001	0.000	0.000	0.000	0.000
Experience	0.024	0.014	0.017	0.020	0.024	0.028	0.032	0.025	0.027	0.029	0.033	0.037
$Experience^2$	−0.0003	−0.0002	−0.0002	−0.0003	−0.0003	−0.0003	−0.0004	−0.0004	−0.0004	−0.0004	−0.0004	−0.0005
Education [reference: without studies or less than primary]												
Primary	0.065	0.014[a]	0.047	0.044	0.065	0.089	0.046	0.041	0.024	0.041	0.054	0.072
Secondary	0.275	0.185	0.225	0.236	0.282	0.353	0.234	0.182	0.181	0.220	0.254	0.324
Vocational training	0.143	0.078	0.109	0.121	0.137	0.145	0.135	0.125	0.108	0.133	0.150	0.159
Advanced voc. training	0.234	0.171	0.196	0.197	0.225	0.322	0.234	0.206	0.209	0.238	0.261	0.280
3-year college	0.380	0.241	0.310	0.357	0.414	0.443	0.379	0.302	0.332	0.361	0.382	0.430
5-year college	0.570	0.343	0.474	0.523	0.625	0.703	0.582	0.439	0.503	0.561	0.610	0.679
Type of contract [reference: fixed term contact]												
Indefinite contract	0.257	0.710	0.408	0.206	0.122	0.154	0.286	0.793	0.405	0.200	0.160	0.169
Occupation [reference: non-qualified workers (9)]												
Managers	0.664	0.456	0.624	0.658	0.738	0.883	0.742	0.509	0.651	0.732	0.849	0.991
Professionals	0.540	0.503	0.553	0.523	0.516	0.616	0.495	0.432	0.487	0.488	0.512	0.614
Technicians	0.430	0.380	0.406	0.404	0.431	0.520	0.364	0.271	0.316	0.349	0.414	0.522
Clerks	0.219	0.250	0.228	0.206	0.210	0.257	0.191	0.184	0.168	0.183	0.208	0.267
Qualified (services)	0.149	0.184	0.172	0.144	0.112	0.111	0.063	0.095	0.070	0.058	0.049	0.122
Qualified (industry)	0.045	0.046[a]	0.019[a]	0.018[a]	0.045	0.079	0.138	0.160	0.134	0.124	0.125	0.167
Operators	0.017[a]	0.005[a]	−0.003[a]	−0.011[a]	0.025	0.060	0.128	0.131	0.123	0.123	0.130	0.151

Size of the firm [reference: 10–19 workers]												
20–49 workers	0.010[a]	0.012[a]	0.008[a]	0.019	0.022	0.041	0.063	0.059	0.046	0.056	0.085	0.092
50–99 workers	0.044	0.030[a]	0.019[a]	0.061	0.084	0.106	0.136	0.111	0.131	0.137	0.156	0.158
100–199 workers	0.116	0.074	0.100	0.128	0.135	0.176	0.179	0.152	0.191	0.189	0.195	0.196
>200 workers	0.165	0.139	0.160	0.197	0.216	0.256	0.276	0.281	0.302	0.286	0.289	0.262
Type of labor agreement [reference: firm labor agreement]												
National labor agreement	−0.072	−0.050	−0.104	−0.109	−0.105	−0.037	−0.066	−0.074	−0.087	−0.088	−0.074	−0.049
Sector or provincial agreement	−0.096	−0.063	−0.103	−0.122	−0.127	−0.071	−0.067	−0.055	−0.088	−0.094	−0.086	−0.061
Type of sector [reference: private sector]												
Public sector	0.140	0.243	0.03[a]	0.076	0.210	0.144	0.027	0.167	0.061	0.049	−0.019[a]	−0.061
Market [reference: foreign market]												
Local–regional market	−0.057	−0.116	−0.066	−0.049	−0.046	−0.034	−0.016	−0.015[a]	−0.019	−0.006[a]	−0.007[a]	−0.007[a]
National market	−0.012[a]	−0.030[a]	−0.014[a]	0.002[a]	0.003[a]	0.011[a]	0.018	−0.023	0.007[a]	0.017	0.030	0.060
Constant	5.938	5.101	5.783	6.110	6.330	6.421	6.009	5.073	5.804	6.147	6.379	6.580
R^2 or Pseudo-R^2	0.59	0.45	0.35	0.37	0.43	0.43	0.62	0.46	0.38	0.40	0.42	0.44
Observations	27,085						100,208					

Quantile regressions were performed also at percentiles 15, 35, 55, 65 and 85, not displayed for simplicity

[a]Coefficient is not significant at 10%. Coefficient for regions omitted. OLS variances computed using White estimator

Table 3 Summary statistics: average and inequality

	Average	Theil (0)	Theil (1)	Theil (2)	Gini
Wages					
Observed					
y_f	1,188	0.182	0.175	0.210	0.320
Predicted by OLS					
$\hat{y}_f$	1,204	0.116	0.116	0.128	0.269
$\hat{r}_f$	1,519	0.111	0.110	0.122	0.262
Predicted by QR					
$\hat{y}_f^q$	1,177	0.166	0.160	0185	0.308
$\hat{r}_f^q$	1,496	0.167	0.163	0.193	0.310
Conditional wage gaps					
Predicted by OLS					
Absolute: $\hat{r}_f - \hat{y}_f$	315.1	0.176	0.163	0.185	0315
Absolute: $(\hat{r}_f - \hat{y}_f)/\hat{r}_f$	0.208	0.070	0.061	0.059	0.196
Predicted by QR					
Absolute: $\hat{r}_f^q - \hat{y}_f^q$	319.5	0.276	0.248	0.312	0.383
Absolute: $(\hat{r}_f^q - \hat{y}_f^q)/\hat{r}_f^q$	0.209	0.087	0.071	0.069	0.209

Average values in pesetas

Table 4 Women with and without a university degree

Decile of y_f	Without a university degree	With a university degree
1	94.4	5.6
2	96.1	3.9
3	96.4	3.6
4	95.8	4.2
5	93.8	6.2
6	93.4	6.6
7	89.3	10.7
8	85.9	14.1
9	80.0	20.0
10	60.7	39.3
Overall population	88.6	11.4

Percentage by decile of observed wage (y_f)

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