

50 is the new 30—long-run trends of schooling and retirement explained by human aging

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Abstract Workers in the US and other developed countries retire no later than a century ago and spend a significantly longer part of their life in school, implying that they stay less years in the work force. The facts of longer schooling and simultaneously shorter working life are seemingly hard to square with the rationality of the standard economic life cycle model. In this paper we propose a novel theory, based on health and aging, that explains these long-run trends. Workers optimally respond to a longer stay in a healthy state of high productivity by obtaining more education and supplying less labor. Better health increases productivity and amplifies the return on education. The health accelerator allows workers to finance educational efforts with less forgone labor supply than in the previous state of shorter healthy life expectancy. When both life-span and healthy life expectancy increase, the health effect is dominating and the working life gets shorter if the intertemporal elasticity of substitution for leisure is sufficiently small or the return on education is sufficiently large. We calibrate the model and show that it is able to predict the historical trends of schooling and retirement.

Keywords Healthy life expectancy · Longevity · Education · Retirement · Labor supply · Compression of morbidity

JEL Classification E20 · I25 · J22 · O10 · O40

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1 Introduction

In this paper we propose a novel life cycle theory of schooling, work, and retirement of aging individuals. The theory is useful to explain why younger generations spend more years in school but do not retire later, implying a shorter working life. This life cycle behavior is in particular intriguing because a popular explanation for a longer schooling period is based on increasing life expectancy, arguing that a longer expected lifespan allows to spend more time on education without reducing the working life, i.e. without reducing the return on education (Cervellati and Sunde 2005). Indeed, if a simple life cycle model (Ben-Porath 1967) is extended to allow for endogenous retirement, it predicts that increasing longevity induces more schooling and more or unchanged life time labor supply. Since life time labor supply is actually declining, it has been argued, that increasing life expectancy cannot explain the long-run trends of education, work, and retirement (Hazan 2009).

Figure 1 shows the long-run trends of life expectancy, education, age at retirement and expected years spent in retirement for cohorts of American men, born 1850–1970. For about a century, since 1870, education increased continuously, by about 8 years altogether. At the same time the age of retirement increased only slightly by about 2 years for cohorts born before 1940 and is predicted to decline by about 2 years for the later born cohorts, implying hardly any trend over the whole observation period (Lee 2001). During the same period of time, life expectancy at age 20 of American men rose from 43.7 years in 1850 to 57.7 years in 1990. These 14 more years were almost exclusively used to enjoy more leisure in retirement, as shown in the panel on the lower left-hand side of Fig. 1. Altogether this leaves us with the question why younger generations go longer to school when they do not want to work more.¹

In this paper we propose an answer by extending the life cycle model used by Hazan (2009) and taking into account that human capital consists not only of education, as assumed in the conventional model, but also of a health component. Specifically, we take into account that after a period in best health, productivity declines with age. Declining productivity motivates the individual to retire and provides a mechanism that complements and amplifies the usual leisure-driven motive for retirement. In order to explain the evolution of schooling and retirement behavior by trends of human aging we distinguish between life expectancy (lifespan) and healthy life expectancy. In Sect. 2 we document that healthy life expectancy increases one-to-one with life expectancy. Improving healthy life expectancy means that later born cohorts were able to work an extended period of their life in full health. For example, Costa (2000, 2002) finds that the share of elderly US American men (50–64 years old) having difficulties bending declined from 44 percent in 1900 to 8 % in 1994. The share of men having difficulties walking declined from 28 % in 1900 to 8 % in 1994. Similar trends are observed for a host of other health conditions potentially affecting labor productivity.

A longer stay in good health improves average labor productivity during working age at any level of education. Higher productivity makes more education desirable and the induced higher income leads to more demand for consumption goods and leisure. The increased demand for leisure causes the desired age of retirement to advance by less than education such that life time labor supply declines. As for the effects of increasing life expectancy, our model

¹ There is no contradiction between Lee's (2001) observation of a (mild) increase in age of retirement and the observed decline in labor force participation of the elderly (Costa 1998). The reason is that the labor force participation rate captures individuals who manage to survive until age 65; individuals who retired and died before the age of 65 do not count. During the period of observation improvements in health allowed more people to survive until the age of 65. This implies that the expected age of retirement increased while labor force participation declined.

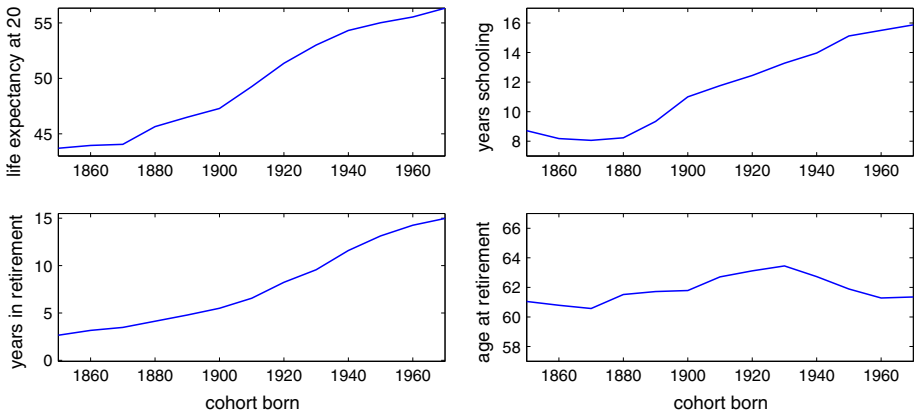


Fig. 1 Life expectancy at 20, schooling, and retirement: 1850–1970. Cohort data at 10-year intervals for US American males. Expected years in retirement is computed as life expectancy at 20 minus age at retirement. Life expectancy and age at retirement are from Lee (2001); years of schooling are from Hazan (2009)

predicts that both schooling and life time labor supply increase (as in Hazan 2009). However, since healthy life expectancy increases along with life expectancy, it can be that a longer life is observed along with a longer schooling period and a shorter working life. In the analytical part of the paper we prove that this outcome can be expected when the intertemporal elasticity of substitution for leisure is sufficiently small or the return to education is sufficiently large. In the numerical part of the paper we show that a calibrated version of the model explains the long-run trends of schooling, labor supply, and retirement reasonably well.

So far the literature has suggested other, potentially complementing, mechanisms to explain the long-run trends of schooling and labor supply. Cervellati and Sunde (2013) show that an increasing probability of survival during the working life may induce more schooling. Similar to our approach, improving survival during the working period increases average labor productivity. In contrast to our approach, it also increases years of consumption needs, which remain invariant to an improvement of health during the working period. More importantly, improving survival in middle age is bounded from above. The effect stressed by Cervellati and Sunde peters out when almost all individuals survive until retirement age and it continuously loses power as life expectancy of the middle aged improves. Our mechanism based on health improvements, is—at least in theory—unbounded and continues to work as long as healthy life expectancy raises in sync with life expectancy (Oeppen and Vaupel 2002), driven by continuing (medical) technological progress. Alternatively, Hansen and Lønstrup (2012) suggest a mechanism based on missing capital markets and accidental bequests. They consider a three period overlapping generations model in which individuals work during the education period as well as in old age. A higher probability to reach old age induces more savings and thus tends to lower consumption in middle age. To this individuals respond with more education in order to smooth consumption between the first and second period. Consumption smoothing, however, can motivate only a relatively small increase in the years of education, as the authors admit in the quantitative section of their analysis. Finally, Restuccia and Vandenbroucke (2013) propose non-homothetic preferences (subsistence needs) and the demand of education for pleasure as potential drivers of schooling and retirement behavior. The mechanism suggested in our paper—higher productivity through better health—was neglected by all three studies.

There exists also a related literature discussing the interaction of health, life expectancy, and labor supply independently from schooling behavior (Heijdra and Romp 2009; Bloom et al. 2014a; Kalemli-Ozcan and Weil 2010; d'Albis et al. 2012; Kuhn et al. 2015; Dalgaard and Strulik 2012, 2014). Recently, Heijdra and Reijnders (2012) have developed a general equilibrium model with which they predict that a longevity boost accompanied by reduced depreciation of human capital leads to more schooling, as in our paper, but also to less years spent in retirement and more life-time labor supply. The model is more detailed than ours but due to its complexity solved only numerically for a particular parametrization. Here, we follow a different strategy. Our model is decidedly simple such that we can formally prove under which conditions increasing life expectancy increases schooling and reduces life-time labor supply.

A large empirical literature has investigated the association between health and education. A major problem, hampering identification of the impact of longevity on education is reverse causality. Several mechanisms are conceivably generating a positive impact of education on longevity (Grossman 2006) and there exists a rich microeconomic literature suggesting that indeed a part of the observable positive correlation is explained by a causal impact of education on health and longevity (see Cutler et al. 2011 for a survey of the literature). Acemoglu and Johnson (2006, 2007) suggested to exploit the third stage of the epidemiological transition, i.e. the decline of communicable diseases in the 1940s, as an exogenous health shock in order to uncover the causal effect of life expectancy at birth on education and GDP per capita. Their non-finding was subsequently challenged by several follow-up studies and extensions, which, however were mainly concerned with GDP or GDP growth (Aghion et al. 2011; Cervellati and Sunde 2011; Bloom et al. 2014b). The focus on communicable diseases with their large impact on child mortality appears to be not optimally suited to uncover the nexus between adult longevity and education as it is stated by the Ben-Porath model. Indeed, the conventional overlapping generation model with endogenous fertility and child mortality predicts that improving child mortality should leave net fertility and thus education provided by the parents unchanged (Galor 2011). A more appropriate test of the Ben-Porath model and of our suggested extension of it is provided by two recent studies. Oster et al. (2013) investigate the indication of Huntington disease and Hansen and Strulik (2015) investigate the cardiovascular revolution of the 1970s. Both studies find a significant impact of improving adult health and longevity on education.

Reestablishing a positive role of life expectancy for education is important because a host of economic theories uses this mechanism in order to explain the onset of industrialization and the take-off to growth as well as the existence of contemporaneous cross-country differences in economic development (Kalemli-Ozcan et al. 2000; Boucekinne et al. 2002, 2003; Cervellati and Sunde 2005, 2015; Soares 2005; Strulik and Werner 2014). Our study corroborates the longevity-education channel, albeit with a refinement of the mechanism: It suggests that it is healthy life expectancy rather than life expectancy as such that promotes education and economic growth.

The paper is organized as follows. The next section establishes the nexus between life expectancy and healthy life expectancy and surveys the literature on the impact of declining adult health on productivity. Section 3 sets up the life cycle model and discusses its mechanism. Section 4 presents the solution and discusses the comparative statics. Section 5 generalizes the basic model, calibrates it with US data, feeds in the historical time series of life expectancy at 20 for the US, and confronts the model predictions with the historical series presented in Fig. 1. Section 6 concludes. Proofs of the propositions, longer derivations of equations, as well as further extensions of the basic model can be found in the Appendix.

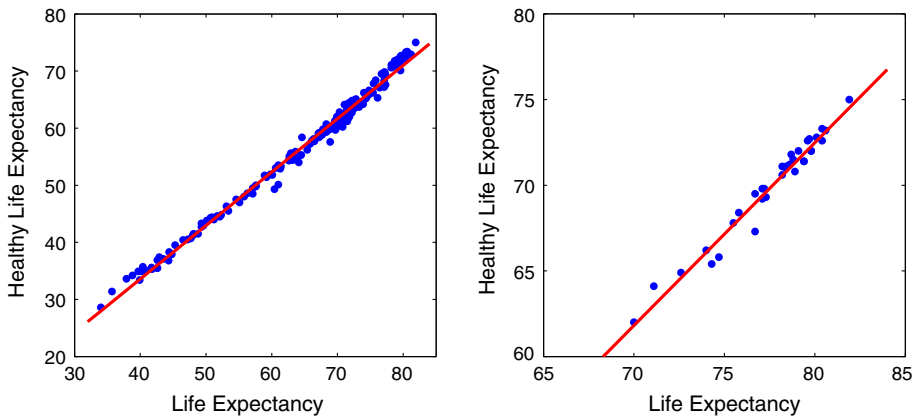


Fig. 2 Life expectancy versus healthy life expectancy across countries. Data for 191 countries from WHO (2012) for the year 2002. *Left*: all countries (regression line: $HALE = 0.94 LE - 3.8$). *Right*: OECD countries (regression line: $HALE = 1.07 LE - 12.8$)

2 Human aging and productivity

The WHO defines healthy life expectancy (HALE) as the average number of years that a person can expect to live in “full health” (WHO 2012). Over the last century healthy life expectancy increased about one-to-one with life expectancy. This means that members of later born cohorts can not only expect to live longer but also to live a longer part of their life in full health. Figure 2 shows for the year 2002 life expectancy and healthy life expectancy at birth across countries, according to WHO (2012). The panel on the left hand side considers all 191 countries. The association appears to be almost linear and the slope is close to unity. Taking into account that the full sample contains countries at all levels of economic development, at different stages of the epidemiological transition, and with distinct age distributions and disease environments, it is amazing that there are no clear outliers discernable and that the linear association seems to hold everywhere. As a second exercise we look at OECD countries, i.e. rich countries, which can be considered to be in the fourth phase of the epidemiological transition. The panel on the right hand side shows that a similar linear association holds whereby the slope of the regression line is now mildly larger than unity.

The cross-section analysis, however, is not optimal for our purpose since we aim to apply our theory to the long-run historical evolution of life expectancy, education, and labor supply (Fig. 1). Ideally, to corroborate our theory, we would need long-run data on the evolution of healthy life expectancy within the U.S. Given that the idea of healthy life expectancy is relatively new, there is, unfortunately, not sufficiently many data available for a thorough time series analysis.

A second best alternative is to look at medium-run time differences across countries. Figure 3 shows the results of this exercise, using data from Salomon et al. (2013). This study provides estimates of life expectancy and healthy life expectancy for males and females in 1990 and 2010. Again, we obtain an almost linear relationship suggesting that within countries male healthy life expectancy increases by 0.84 years for any 1 year increase of life expectancy. The association is somewhat smaller for females. At the aggregate (world) level Salomon et al. report also the increase of healthy life expectancy per age group. Using this data one can show that all age groups above age 5 and below age 50 gained on average

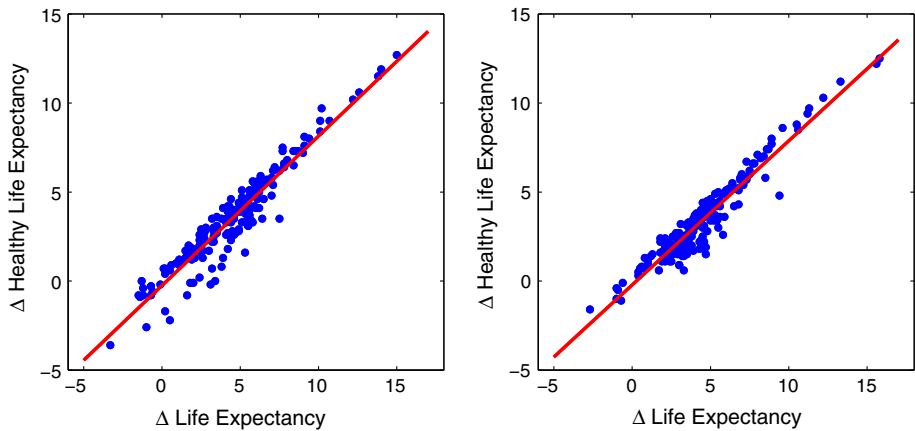


Fig. 3 Change in life expectancy and healthy life expectancy 1990–2010. Data for 187 countries from Salomon et al. (2013) *Left*: males (regression line: $\Delta\text{HALE}=0.84 \text{ LE}-0.25$). *Right*: females (regression line: $\Delta\text{HALE}=0.81 \text{ LE}-0.22$)

about 2 more years of healthy life from 1990 to 2010. Above age 50 the gains deteriorate and converge to zero. This observation indicates that the extension of a healthy adult life was mainly experienced by workers in their prime working age.

A longer stay in a healthy state increases labor productivity by reducing the force of aging through better maintenance and repair of the human body. From this viewpoint, the concept of healthy life expectancy as defined by the WHO is not ideal to corroborate our theory below. The WHO's conceptualization of "full health" does, for example, not consider the aging-driven loss of muscle force and cognitive abilities. Here, we will use a slightly different conceptualization of "full health" and define as healthy life expectancy the age at which labor productivity starts to decline due to deteriorating bodily function.

Most of our cognitive skills and motor skills start deteriorating around the age of 30 or even earlier (Nair 2005). Physiologically the phenomenon is explained by an increasing loss of redundancy in the human body and therewith deteriorating reliability, which eventually ends in death (see Arking 2006, on human aging; Gavrilov and Gavrilova 1991, on reliability of the human body). Deteriorating muscle strength and reliability cause gross motor skills to decline with age. Costa (2000, 2002) provides evidence that the deterioration of the human body slowed down since the beginning of the 20th century such that later born cohorts of elderly US American men (50–64 years old) experience less impairments of bodily function. For a host of health conditions, Costa shows that all indicators improved, most of them quite strongly, some (e.g. joint problems, back problems, heart and circulatory conditions) improved at an annual rate of 0.7% or higher. Since adult life expectancy in the US improved over the same period by about 0.3 percent annually (at age 20 from 42 years in 1900 to 56 years in 2000), these health conditions improved at a rate more than twice as fast as the improvement of life expectancy.

In principle, the force of aging acts on cognitive abilities as well (Skirbekk 2004). But there are also important differences. In particular, so called *crystallized abilities*, i.e. the abilities to use knowledge and experience, remain relatively stable until most of adulthood and start declining after the age of 60, or even later. As a result some measures of psychological competence, like verbal skills and inductive reasoning start declining "only" around age 50, as shown in Fig. 4, taken from Schaie (1994).

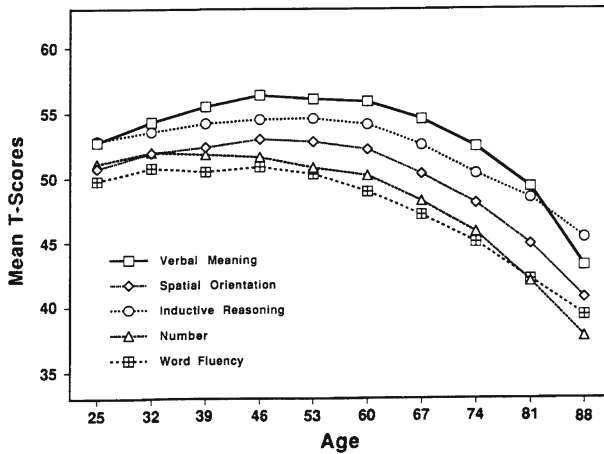


Fig. 4 Human aging: crystallized abilities. Source [Schaie \(1994\)](#)

These observations suggest two different channels through which the years in full health improved for later born cohorts of workers. The first channel operates through improving health due to better nutrition and medical technological progress. Over the last centuries the state of health of workers improved together with the long-run trend towards better nutrition, in particular during childhood, and its impact on productivity through alleviated body size and cognitive ability ([Fogel 1994](#); [Case and Paxson 2008](#); [Case 2010](#); [Floud et al. 2011](#); [Strauss and Thomas 1998](#)).² Medical technological progress is perhaps even more important for the health of older workers. Progress in medical science increased the possibilities of maintenance and repair of the human body (e.g. [Porter 2001](#)) and thus allowed for a longer stay at full health in a learned occupation. For example, a century ago a chronic knee or hip injury would have severely reduced the productivity of a farmer or a mason. Today medical technological progress allows for a knee or hip replacement, keeping the worker – after operation and rehab – at full health and productivity. Medical technological progress improves healthy aging such that the (repaired) body starts declining later in life.

The second channel operates through structural change and its impact on the employment structure of the economy. In some occupations (woodcutter, farmer) the health needed to perform at highest productivity declines earlier than in others (salesman, lawyer). This means that the sectoral structure of the modern economy requires less gross motor skills (farm and construction work) and less fine motor skills (manufacturing) but more crystallized abilities (the service sector). The average worker was more likely to be a farmer or industrial worker hundred years ago and is more likely to be a salesperson or consultant today. Since crystallized abilities decline later in life, the average worker can expect to stay a longer period of the working life at highest productivity. In other words, when technological progress

² There is also a direct link from better health in childhood to performance at school ([Bleakley 2007](#); [Bharadwaj et al. 2013](#)) and to parental investment in education ([Hazan and Zoabi 2006](#)) and thus to adult health and productivity. [Andersen et al. \(2015\)](#) find a strong impact of eye disease on contemporaneous income across countries and argue that it results partly from the delay of the fertility transition caused by the strong exposure to UV light in some countries and the reduced return on human capital due to the entailed high prevalence of eye disease. The trade-off between fertility and education is not explored in the present paper but it is potentially amplifying the positive association between life-expectancy, healthy life expectancy, and productivity.

moves workers from physically demanding jobs to less demanding jobs (requiring crystalized abilities) it increases healthy life expectancy of the representative individual in the economy.³

The aging-driven decline of bodily function maps into an (occupation-specific) decline of productivity, which maps into a decline of earnings (Skirbekk 2004). This mapping is not necessarily one-to-one. The age-earnings curve may decline at a later age than the age productivity curve. Lazear (1981) and Akerlof and Katz (1989) rationalize age earning profiles below productivity at young age and above productivity at old age with the motive to reduce shirking and increase effort. In the OECD, in 1995, age-earning profiles reached their peak for 45–54 age group in 17 out of 19 countries (earlier in the UK and Czech Republic; OECD 1998). Kotlikoff and Gokhale (1992) show that earnings of US American salesmen closely follow productivity, declining at about age 50, while earnings of office clerks and managers decline significantly later than productivity, around about age 57. In our model we ignore the potential deviation of wage and productivity profile and focus instead on the change of the onset of decline over time. In support of our theory, Burtless (2013, Fig. 10) observes that the age-earning profile of US American workers shifted out over time. In 1985 the decline of earnings started at age 50 whereas in 2010 it started at age 60.

The concept of aging-driven declining productivity relates our paper to an earlier literature based on the Ben-Porath model, which studied the role of depreciation of human capital for labor supply at the intensive and extensive margin. Heckman (1976) investigates how depreciation of human capital affects investment in education, labor supply, and the age-earnings profile. Blinder and Weiss (1976) derive an endogenous foundation of the life cycle consisting of schooling, on the job education, work, and retirement. Mincer (1974) introduces into the Ben-Porath model the assumption that the return to education on the job declines linearly with age and derives the “Mincer equation”. The impact of increasing life expectancy on education and labor supply, however, remained uninvestigated in this literature.

3 The model

The starting point is the simple Ben-Porath (1967) model as re-introduced by Hazan (2009). Individuals expect to live for T years. Given this constraint they choose to attend school for s years (before work) and choose to retire from work after R years. In contrast to the earlier literature we take into account that human capital consists of two components, education and health. A person with s years education and health $a(t)$ supplies human capital $h(s, t)$ which equals individual productivity.

As in the standard model, attending school for s years produces education-specific human capital $e^{\theta(s)}$. Additionally, we take into account that health and productivity erode with age such that human capital of a person with s years of education and age t is given by $h(s, t) = e^{\theta(s)} \cdot a(t)$. Specifically, we assume that life consists of a period of best health of length τ , after which health deteriorates with age. In order to obtain an explicit solution of the life cycle problem we assume that health deteriorates linearly, i.e.

$$a(t) = \begin{cases} 1 & \text{for } t \leq \tau \\ 1 - \frac{t-\tau}{\lambda-\tau} & \text{for } \tau < t < \lambda \\ 0 & \text{for } t \geq \lambda. \end{cases} \quad (1)$$

³ Recent research by Hamermesh (2013) suggests that this is even true for economists. Hamermesh observes a trend towards higher average age of authors publishing in the top five journals and explains this by a trend that made the profession less like pure mathematics (requiring mostly fluid abilities, which decline early in life) and more like a humanistic field (requiring mostly crystallized abilities, which are more persistent).

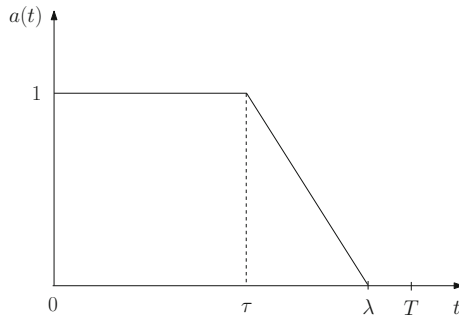


Fig. 5 Life cycle with aging

The parameter τ is conceptualized as healthy life expectancy. Figure 5 illustrates the implied health over the life cycle. After τ years, which are spent on schooling and work, productivity and health begin to deteriorate and at age λ productivity has declined to zero. The person dies at age T . The figure has been drawn such that the person continues to live for a while after reaching zero productivity, i.e. $T > \lambda$.⁴ As a consequence of increasing τ , a person spends a longer time of his or her life in a healthy condition after which health deteriorates faster until death. Taken by itself this effect describes a compression of morbidity (Fries 1980). An increase of T , holding τ constant, could then be conceptualized as an expansion of morbidity. The earlier literature (e.g. Hazan 2009) can be understood as a special case for which $\tau = \lambda = T$, that is the case in which the person stays in perfect health until death. Here we discuss not only how individuals respond to an improvement in life expectancy T , as in the related literature, but also how they respond to an improvement of healthy life expectancy τ .

Assumption (1), although stylized, captures an important insight from gerontology, namely that the rate of health deficit accumulation is increasing in the stock of health deficits or, in other words, that older people age faster (Gavrilov and Gavrilova 1991; Mitnitski et al. 2002; Arking 2006). This can be seen by noticing that, according to (1), the rate of loss of health (depreciation of human capital) is given by $\dot{h}/h = \dot{a}/a = -[1/(\lambda - \tau)]/a$ for $\tau < t < \lambda$. In absolute terms this rate is the higher the lower the state of health $a(t)$ is, i.e. the older the individual is. Notice that an increase of healthy life expectancy, conceptualized as an increase in τ implies, ceteris paribus, a *faster* decline of productivity (loss of human capital) after age τ . This effect is known in the literature as compression of morbidity (Fries 1980). The standard human capital model, based on a constant rate of human capital depreciation, in contrast, predicts the opposite. There, the loss of human capital at age t , given a constant rate of depreciation δ , is $\dot{h}(t) = -\delta h(t)$. This assumption (counterfactually) implies that the loss of human capital $\dot{h}(t)$ is large when $h(t)$ is large, i.e. when people are young and that the loss is small when people are old.⁵

The standard Mincer (1974) equation contains two further terms, experience and “experience squared”, which we ignore subsequently. We neglect the linear experience term in

⁴ The assumption that $T > \lambda$ is plausible but not necessary for our results, which hold for $\lambda \rightarrow T$ as well. Moreover, health could decrease non-linearly without affecting our results. The linearity assumption is made to obtain a closed-form solution. The crucial assumption is that there exists a point in life after which health and productivity decline, which allows us to disentangle the effects of increasing life expectancy and increasing healthy life expectancy.

⁵ In Appendix F we compare the treatment of depreciation in the standard human capital model vs. our approach in more detail.

order to get an analytically tractable solution. In the Appendix we show robustness of our quantitative results with respect to a “tent-shaped” age-productivity pattern. This procedure acknowledges that productivity increases through experience in early work-life, a la Mincer, before it declines in later work life because of deteriorating health. The robustness of results is not surprising because the crucial mechanism of healthy life expectancy does not affect the early work period but the later period, in which health deteriorates and in which a retirement decision is made. The estimated Mincer equation usually obtains a negative coefficient for “experience squared”, generating declining productivity at older ages. Since it is hard to believe that productivity is decreasing because of “too much experience”, we are convinced that the squared term is meant to measure the effect of increasing age on health and productivity. The effect of deteriorating health is captured explicitly by Eq. (1) in our model.

Individuals maximize lifetime utility consisting of the discounted stream of instantaneous utility $u(c(t))$ derived from consumption $c(t)$ at any age t plus utility derived from leisure. We assume, as in the related literature, that labor is indivisible. Individuals decide whether to work or not. Since individual productivity is continuously decreasing during the second period of the working life, individuals start working after a period of schooling s and then retire at age R . Notice that rational individuals retire before or at age λ , i.e. before or at the time when their productivity vanishes to zero, $R \leq \lambda$.

Summarizing, life time utility is given by

$$\max_{s,R} V = \int_0^T e^{-\rho t} u(c) dt + v(\lambda - R), \quad (2)$$

in which ρ denotes the time preference rate and v is a utility function for leisure. Additionally, individuals may experience utility from leisure after their health induced productivity deteriorated to zero. Since they would never consider retiring in this last period of their life, the experienced utility (which may be positive, zero, or even negative due to bad health) is not relevant for the solution of the optimization problem. Without loss of generality we normalize units such that a unit of human capital gets a unit wage (net of labor supply costs). The lifetime budget constraint requires that discounted lifetime earnings equal discounted lifetime consumption, i.e.

$$\int_s^R e^{-rt} h(s, t) dt = \int_0^T e^{-rt} c(t) dt. \quad (3)$$

Individuals maximize (2) subject to (1) and (3). In order to simplify the problem, we follow Hazan (2009) and assume that $r = \rho$ such that individuals prefer a smooth consumption profile over life. We furthermore assume a logarithmic form for utility experienced from consumption, $u(c) = \log(c)$, a constant return to schooling, $\theta(s) = \theta \cdot s$, and an isoelastic form for utility experienced from leisure, $v(\lambda - R) = \omega(\lambda - R)^{1-\eta}/(1-\eta)$, $\eta > 0$. The term $1/\eta$ is the intertemporal elasticity of substitution for leisure. Furthermore, we normalize the interest rate to zero.

While these assumptions reduce the generality of our results, they are also quite powerful in that they allow for a neat explicit solution of the life-cycle problem with aging. This in turn, allows us to formally prove our results. We are thus convinced that the end justifies the means, in particular since the related literature on aging, schooling, and labor supply is usually confined to numerical simulations. In Sect. 5 we present a generalized version of the model with positive interest rates. This provides more realism and a better approximation of the historical data but leaves the main mechanism unaffected. Further extensions of the model are discussed in the Appendix.

With the assumptions made, problem (1)–(3) can be restated as

$$\max_{s \geq 0, R \geq \tau} T \log \left\{ e^{\theta s} (\tau - s) + e^{\theta s} \frac{(2\lambda - R - \tau)}{2(\lambda - \tau)} (R - \tau) \right\} + \omega \frac{(\lambda - R)^{1-\eta}}{1 - \eta}, \quad \omega \geq 0, \eta \geq 0. \tag{4}$$

The first term in curly parenthesis is the income earned during the first period of the working life, which is of length $(\tau - s)$. The second term is the income earned during the second period of the working life, which is of length $(R - \tau)$.

The first order condition for optimal schooling is given by

$$\theta = \frac{2(\lambda - \tau)}{2(\tau - s)(\lambda - \tau) + (2\lambda - R - \tau)(R - \tau)} \equiv g(\tau, \lambda, s, R). \tag{5}$$

The left hand side of (5) is the marginal gain in utility per time unit elicited by an extra time unit of education (the return on education). The right hand side of (5) is the marginal cost in terms of utility per time unit and per unit of education, that is $g \equiv -\partial u(\tilde{c})/\partial s$ with $\tilde{c} = c/\exp(\theta s)$. As in by the earlier literature (e.g. Hazan 2009), the education decision does not directly depend on life-span T . Here, it depends on the length of the healthy working life τ . A longer healthy life increases human capital per unit of education and reduces the marginal opportunity cost of education. Formally, $\partial g/\partial \tau = -2(2\lambda - R - \tau)(R - \tau)/N^2 < 0$, in which N is the numerator of g .

Thus, an increase in healthy life expectancy reduces the right hand side of (5) while the left hand side remains unchanged. In order to balance the equation, s and R respond such that the right hand side rises again to its original value. The marginal cost of education rises if the education period s is prolonged or the age of retirement R is reduced. Formally, $\partial g/\partial s = 4(\lambda - \tau)^2/N^2 > 0$, and $\partial g/\partial R = -4(\lambda - R)(\lambda - \tau)/N^2 < 0$. Intuitively, both responses reduce the length of the working life and thus income per unit of education. They thus increase the opportunity cost of education. The fact that a longer healthy life increases income per unit of human capital has not been considered so far in the literature on life expectancy and labor supply. A longer healthy life makes more education attractive and entails an income effect that makes earlier retirement attractive.

The first order condition for the optimal retirement age is given by (6),

$$\frac{\omega(\lambda - R)^{-\eta}}{T} = \frac{2(\lambda - R)}{2(\tau - s)(\lambda - \tau) + (2\lambda - R - \tau)(R - \tau)} \equiv \tilde{g}(\tau, \lambda, s, R). \tag{6}$$

The left hand side of (6) is the utility gain from an extra unit of time spent in retirement. The right hand side is the marginal cost, in terms of utility, of retiring a time unit earlier. Take the derivatives with respect to τ , s , and R to observe $\partial \tilde{g}/\partial \tau = 4(\lambda - R)(\tau - s)/N^2 > 0$, $\partial \tilde{g}/\partial s = 4(\lambda - R)(\lambda - \tau)/N^2 > 0$, and $\partial \tilde{g}/\partial R = -(4(\lambda - R)^2 + 2N)/N^2 < 0$, in which N is the numerator of \tilde{g} . A longer healthy life increases income per unit of human capital. This means that the marginal cost of retirement increases, which moves the right hand side of (6) upwards. In order to balance the equation, s has to go down and R has to go up such that the length of the working life increases and the marginal cost of retirement declines.

The partial response to increasing τ through the retirement condition thus tends to compensate the responses through the schooling condition. Which effect dominates cannot be said without explicitly solving model. Generally, in order to observe more schooling and less labor supply as a response to a longer healthy life the effect of healthy life time on schooling has to be stronger than on retirement. Below we demonstrate that this is indeed the case for ordinary logarithmic and iso-elastic utility functions, given that a mild condition on the elasticity of labor supply or the return on education is fulfilled.

With respect to a longer life T , the optimal responses of schooling and life time labor supply are familiar from Hazan (2009). A longer life T reduces the left hand side in (6). This means that R has to go up or s has to go down. A higher retirement age, however, elicits more schooling according to (5). This means that the dominant effect is later retirement and thus life time labor supply tends to increase in response to a longer life.

4 The solution

We focus on the interior solution for education. If the return on education θ were too low or if the length of life T were too short, a non-negativity constraint on s applies and education assumes a corner solution. Here, however, we are interested in the interaction of education and lifetime labor supply and assume henceforth that parameter values are such that the interior solution applies. After some algebra we obtain from (5) and (6) an explicit solution for the optimal years of schooling s , the age at retirement R , and life time labor supply L , $L \equiv R - s$.

$$R = \lambda - \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{1}{1+\eta}} \tag{7}$$

$$s = \frac{\lambda + \tau}{2} - \frac{1}{\theta} - \frac{1}{2(\lambda - \tau)} \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{2}{1+\eta}} \tag{8}$$

$$L = \frac{1}{\theta} + \frac{1}{2(\lambda - \tau)} \left[\lambda - \tau - \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{1}{1+\eta}} \right]^2 \tag{9}$$

The following propositions are proved in the Appendix.

Proposition 1 (Life Expectancy) *An increase in life expectancy leads to more education, later retirement and more life time labor supply .*

Proposition 2 (Healthy Life Expectancy) *An increase in healthy life expectancy τ leads to more education, later retirement, and less life time labor supply*

Proposition 3 (Equal Increase of τ and T) *An equal increase of life expectancy and healthy life expectancy (i.e. $dT = d\tau$) leads to more education and later retirement. It leads to less lifetime labor supply for*

$$\eta \geq 1 + \frac{2(\lambda - \tau)}{T} \tag{10}$$

or, if $\eta < 1 + 2(\lambda - \tau)/T$, for

$$\theta > \frac{(\lambda - \tau)\omega}{T} \left[\frac{2(\lambda - \tau) + T(1 - \eta)}{T(1 + \eta)(\lambda - \tau)} \right]^{1+\eta} \tag{11}$$

If we just focus on lifespan T there is nothing new here. Proposition 1 confirms the familiar result: With rising lifespan, individuals want to educate more and work longer. Since life time labor supply actually declined, one may conclude from this result that increasing life-expectancy cannot explain the historical schooling trends (Hazan 2009).

The picture changes once we focus on healthy life-expectancy τ . With improving healthy life-expectancy individuals want to obtain more education and to reduce lifetime labor supply (Proposition 2). A longer healthy life increases labor productivity and induces more schooling.

Higher income allows more consumption, the marginal utility from consumption declines and leisure becomes relatively more precious. Consequently the individual reduces labor supply. We explained the mechanism in detail when we discussed the first order conditions.

Actually, as established in Sect. 2, life expectancy and healthy life expectancy increase simultaneously and the association is basically linear with a slope close to unity. It is thus interesting to explore the prediction for labor supply elicited by an equal increase of T and τ . Proposition 3 shows two alternative conditions under which the effect of healthy life expectancy is dominating and increasing life expectancy is observed along with more schooling and less lifetime labor supply. Condition (10) requires that the intertemporal elasticity of substitution for leisure $1/\eta$ (i.e. the labor supply elasticity at the extensive margin) is sufficiently small. As can be seen from (7) and (8), a large value of η guarantees that retirement and schooling respond only weakly to changing T and that retirement responds weakly to changing τ . In this case the positive effect on schooling from increasing τ , i.e. from better health and productivity, is dominating. With a large response of schooling and a small response of retirement, life time labor supply declines.⁶

Condition (10) is straightforward, general, and also likely to hold. To see this, notice that it requires $\eta > 1$ for $\tau \rightarrow \lambda$. For, say $\tau = 50$, $\lambda = 70$, and $T = 90$, it requires $\eta > 1.5$. When we assume that the labor supply elasticity at the extensive and intensive margin are similar and that individuals work 1/3 of their daily time, then a Frisch elasticity of labor supply of 0.25, as recently estimated by Chetty et al. (2011), would suggest $\eta = 8$. Even for a labor supply elasticity twice as large, implying $\eta = 4$, condition (10) is met.

If (10) does not hold, condition (11) ensures a negative response of life time labor supply. It requires that the return to schooling θ is sufficiently large compared to the weight of leisure in utility ω . As can be seen from (7) and (8), a small value of ω means that retirement and schooling respond only weakly to changing T and that retirement responds weakly to changing τ . This means that the positive effect on schooling from increasing τ is dominating and this effect is large if the return to education is large. Summarizing, a relatively low ω and high θ ensure that the response of schooling to the positive impulse from better health and productivity dominates such that life time labor supply declines.

Finally we address robustness of our main result when healthy life expectancy increases by less than life expectancy. We know that the result of declining life time labor supply holds according to Proposition 3 for an equal increase and that it fails to hold according to Proposition 1 when life expectancy improves but healthy life expectancy stays constant. This means that there exists a critical factor of proportionality of life expectancy and healthy life expectancy below which our result breaks down. Generally, suppose τ and λ are both increasing linearly in T , such that $\tau := c_\tau + \alpha T$ and $\lambda := c_\lambda + \beta T$, in which c_τ and c_λ are constants and α and β are factors of proportionality (for Proposition 3, $\alpha = 1$ and $\beta = 0$). Then, life time labor declines as a response of increasing longevity T if

$$\frac{dL}{dT} = \frac{\partial L}{\partial T} + \alpha \frac{\partial L}{\partial \tau} + \beta \frac{\partial L}{\partial \lambda} < 0 \Leftrightarrow \alpha - \beta > \frac{\partial L / \partial T}{\partial L / \partial \lambda} = \frac{2}{T \left((1 + \eta) \left(\frac{(\lambda - \tau)\omega}{T\theta} \right)^{-\frac{1}{1+\eta}} + \frac{\eta - 1}{\lambda - \tau} \right)}. \tag{12}$$

Since the right hand side of (12) is strictly positive, our main result breaks down when $\alpha = \beta$, i.e. when there is no compression in morbidity. From here we have to proceed

⁶ Better health also increases the hours worked per time increment because of reduced sickness absence. In our model a reduction of sickness absence would be captured by an increase of supply of human capital per time increment, i.e. an upward shift of the $a(t)$ -curve. In the Appendix we show that such a shift leaves all results unaffected.

Table 1 Compression of morbidity and life time labor supply

The table shows for alternative parametrizations of the model the critical factor of proportionality (in percent) between life expectancy and healthy life expectancy above which increasing life expectancy is observed together with increasing schooling and reduced life time labor supply

	$T = 60$	$T = 70$	$T = 80$
$\theta = 0.075$	0.481	0.405	0.350
$\theta = 0.1$	0.466	0.393	0.339
$\theta = 0.125$	0.455	0.383	0.331
$\omega = 0.1$	0.374	0.315	0.272
$\omega = 1$	0.481	0.405	0.350
$\omega = 10$	0.618	0.521	0.450
$\eta = 2$	2.516	2.051	1.718
$\eta = 8$	0.481	0.405	0.350
$\eta = 10$	0.374	0.316	0.273

numerically by computing the critical $\alpha - \beta$ for which the left hand side and the right hand side of (12) are equal. Table 1 shows the critical $\alpha - \beta$ for the parameters of our benchmark calibration ($c_\lambda = 51$; $c_\tau = -5$; $\omega = 1$; $\theta = 0.075$; $\eta = 8$; see Sect. 4 for details). The table shows the critical value by which α has to exceed β . Assuming $\beta = 0$, it shows the critical factor of proportionality between life expectancy and health life expectancy. In other words, it shows how strong the compression of morbidity has to be for life time labor supply to decline. Table 1 shows this value for alternative assumptions of longevity T , return to schooling θ , weight of leisure in utility ω , and the inverse of the extensive labor supply elasticity η . The main takeaway from the numerical analysis is that the critical $\alpha - \beta$ is small, suggesting that a mild compression of morbidity along with increasing longevity is sufficient to produce our main result.⁷

5 Extensions and calibration

This section generalizes the model and confronts the predictions made by a calibrated version with the historical time series shown in the introduction. For this purpose we drop the assumption of zero interest rates.⁸ Assuming $r > 0$ and keeping all other assumptions from the basic model, problem (1)–(3) becomes

$$\max_{s,R} U = \int_0^T e^{-rt} \log(c) dt + \omega \frac{(\lambda - R)^{1-\eta}}{1 - \eta}$$

with

$$c = \frac{e^{\theta s} (e^{r(T-\tau)} + e^{r(T-R)} (-1 + r(\lambda - R)) + e^{r(T-s)} r(\tau - \lambda))}{(e^{rT} - 1) r(\tau - \lambda)}$$

⁷ For the benchmark calibration of the model we assume a mild increase of λ together with T . The implied β is given by $[\lambda(1970) - \lambda(1850)]/[T(1970) - T(1850)] = 0.119$. Thus $\alpha > 0.119 + 0.03 = 0.22$ is sufficient for life time labor supply to decline. For the benchmark calibration we use $\alpha = 0.84$, based upon the estimate provided with Fig. 3.

⁸ Keeping a zero interest rate does not much affect the predicted trends but it implies a too high level of education for all cohorts. The reason is that the opportunity costs of education would be too low when consumption during the schooling period is financed at a zero interest rate.

Table 2 Parameter values for benchmark case

θ	η	ω	r
0.074	8	1	0.07

From here we proceed numerically with a calibration for the U.S. Following Mehra and Prescott (1985) we set $r = 0.07$, representing the average real return on the stock market for (most of) the last century. We set $\eta = 8$, a value that corresponds to a labor supply elasticity of 0.25, as recently estimated by Chetty et al. (2011) for labor supply at the extensive margin. We feed into the model the historical time series for life expectancy at 20 from Lee (2001) based on cohort data. The parameters of the calibration are summarized in Table 2.

Healthy life expectancy is a relatively new concept such that there are no long-run time series that we could use for the calibration. As an approximation we take the association between life expectancy and healthy life expectancy obtained by medium run time-differences for men estimated from data by Salomon et al. (2013) and depicted in Fig. 3, i.e. we assume that an additional year of life expectancy leads to 0.84 additional healthy years. We assume that education begins at age five. Let k denote the index number of the birth year of a cohort starting 1850 ($k = 1$) and ending 1970 ($k = 13$) and let $T_5(k)$ denote life expectancy at age 20 plus 15 years (approximating life expectancy at 5).

According to our theory, healthy life expectancy τ is defined as the age at which individual productivity starts declining. In order to go from the estimated medium run differences according to Salomon et al. (2013) to τ we adjust the “origin” and keep the slope. Specifically we compute $\tau_5(k) = 0.84 \cdot T_5(k) - 5$. This means that healthy life expectancy (at age 5) improves from 44.3 to 54.9 years (i.e. healthy life expectancy at birth improves from 49.3 to 59.9 years) when life expectancy (at 5) improves from 58.7 to 71.3 years (i.e. life expectancy at birth improves from 63.7 to 76.3 years). For the end of the observation period it is thus predicted that labor income starts declining at about age 60, an outcome which is approximately consistent with the actual behavior of wages in the US (Burtless 2013). We also assume that λ_5 improves mildly from 57.5 to 59 (i.e. λ_0 improves from 62.5 to 64) during the observation period. Overall this means a strong compression of morbidity; $(\lambda - \tau)$ improves from 13 to 4.0 years during the period of observation.

Finally we search for values of θ and ω that produce a reasonable prediction of the historical trends of schooling and retirement. Figure 6 shows results for $\omega = 1$ and $\theta = 0.074$, represented by red (dashed) lines. Solid (blue) lines re-iterate the historical US time series from the introduction. Results are relatively insensitive with respect to the choice of ω . A tenfold increase or decrease of ω shifts the age-at-retirement series by about half a year. Results are much more sensitive to the choice of the return to schooling θ . It is thus reassuring that our numerical estimate of θ is not outlandish in comparison with the econometric estimates. A value of 0.074 lies within the confidence bands of most of the OLS and IV estimates of the studies surveyed by Card (1999).⁹

The upper left panel in Fig. 5 shows the evolution of life expectancy that we feed into the model. The upper right panel shows the actual and predicted evolution of schooling. The model gets the trend right but, naturally, fails to predict the downturn of schooling during the civil war years. The middle left panel shows the evolution of retirement. The model cannot capture the mildly hump shaped trajectory of retirement but gets the long-run trend right by

⁹ In the Appendix we extend the model by a constant trend of wage growth (due to technological progress). In this case the best fit of the data is obtained for $\theta = 0.0715$, a value slightly smaller than in the benchmark case.

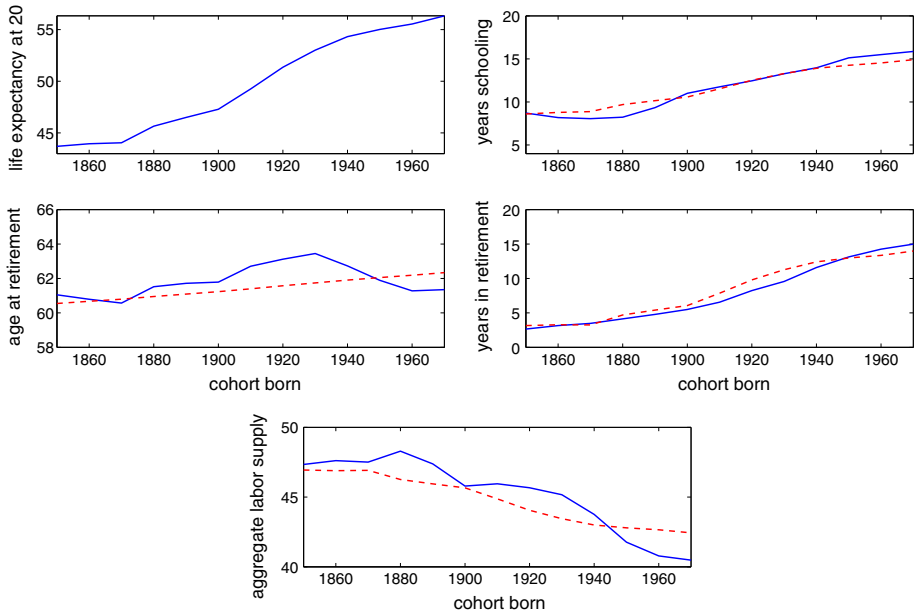


Fig. 6 Data and model prediction: 1850–1970. *Solid lines*: data from Fig. 1. *Dashed lines*: model prediction

predicting an almost invariant age of retirement. Together with the predicted trajectory for schooling this means that the model gets the evolution of years in retirement about right, as shown in the middle right panel. In line with the data the model predicts that years in retirement increased by about 11 years during the observation period. The bottom panel shows the predicted years of labor supply ($L = R - s$). The model gets the trend about right for the cohorts born 1850 to 1940 and underestimates the decline of labor supply for the cohorts born 1950 to 1970 by about 2 years, indicating that other forces like social welfare policy contributed further to the decline of labor supply for later born cohorts.

We next provide a brief sensitivity analysis of the results. Table 3 shows the increases of the years spent in school and in retirement between the years 1850 and 1970 for alternative assumption about the degree of relative risk aversion σ , the return to schooling θ , the interest rate r , and the elasticity of extensive labor supply, captured by η . The predicted increase of schooling over time is remarkably robust against alternative numerical specifications of the model. The same is true for the predicted years in retirement. Table 3 also documents that our results are not driven by the simplifying assumption of logarithmic utility from consumption. For that purpose we assumed a general iso-elastic utility function $u(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$ and alternative values of σ . With rising σ the predicted increase in schooling gets somewhat lower and the estimate for the rise in years in retirement changes insignificantly. Overall, the largest changes vis a vis the benchmark case are obtained for predicted years in retirement for alternative labor supply elasticities (η). At the extreme, when η is 1/4th of its calibrated value, the 12 years more in life expectancy trigger “only” 9 more years in retirement. The model loses its quantitative precision but the main qualitative results continue to hold. Individuals are still predicted to increase schooling and to reduce life time labor supply in response to improving life expectancy. In the Appendix we show the robustness of these results for a tent-shaped age-productivity profile with an initial phase in working age of increasing productivity through increasing experience.

Table 3 Increase of years of schooling and years in retirement 1840–1970

σ	θ			r			η		
	0.075	0.1	0.125	0.03	0.05	0.07	2	8	10
Schooling:									
0.75	6.31	6.38	6.41	6.16	6.26	6.31	5.8	6.31	6.34
1	6.31	6.38	6.39	6.15	6.25	6.31	5.9	6.31	6.34
1.5	6.31	6.36	6.36	6.15	6.25	6.31	5.78	6.31	6.34
3	6.31	6.31	6.22	6.09	6.22	6.31	5.75	6.31	6.34
Retirement:									
0.75	10.84	10.89	10.91	10.94	10.91	10.84	8.96	10.84	10.92
1	10.84	10.89	10.91	10.94	10.91	10.84	8.96	10.84	10.92
1.5	10.84	10.9	10.91	10.95	10.92	10.84	8.98	10.84	10.92
3	10.85	10.92	10.95	10.97	10.93	10.85	9.01	10.85	10.92

The table shows the increase in years of schooling and years in retirement predicted by the model for alternative interest rates r , alternative returns on schooling θ , alternative degrees of relative risk aversion σ and alternative elasticities η . For $\sigma = 1$ we set $u(c) = \log(c)$, for $\sigma \neq 1$ we set $u(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$. All other parameter values as for Fig. 6

6 Conclusion

In this paper we proposed a theory based on human aging that helps to understand the long-run trends of life expectancy, schooling and retirement. The expectation of more healthy years, or more generally, less early declining labor productivity, induces more schooling. More education and better health increase lifetime income and consumption and induce a higher demand for leisure, i.e. a longer stay in retirement. We have also shown that the calibrated model generates quantitatively reasonable long-run trends of schooling and retirement behavior.

In order to illustrate one particular mechanism we have chosen a simple partial equilibrium model that extends [Ben-Porath \(1967\)](#) and [Hazan \(2009\)](#) by one aspect, human aging understood as declining labor productivity. We have thus neglected other potentially important drivers of schooling and retirement behavior. We have, for example, ignored that individuals may die while still in the workforce, as suggested by [Cervellati and Sunde \(2013\)](#), that preferences may be non-homothetic, that individuals demand education for pleasure, as suggested by [Restuccia and Vandenbroucke \(2013\)](#), and that individuals may actively invest in their health to raise productivity and to live longer, as suggested by [Dalgaard and Strulik \(2012, 2014\)](#). Furthermore the public economics literature emphasizes fiscal incentives as an important driver of retirement (e.g. [Gruber and Wise 1998](#)) and the growth and human capital literature emphasizes technological change and globalization as important drivers of demand for an educated workforce (e.g. [Galor and Weil 2000](#); [Galor and Mountford 2008](#)). Combining the mechanism explored in this paper with these complementing channels would further improve the understanding of long-run trends of schooling and retirement.

In the paper we focussed on historical trends but it may also be interesting to ponder the future predicted by the model. Unlike the mechanism based on increasing survival probability to retirement age ([Cervellati and Sunde 2013](#)), which is naturally bounded from above, the mechanism proposed by our model is, in principle, unbounded. At least in theory it is conceivable that life expectancy continues to increase and that healthy life expectancy

increases “in sync”. So far, there is little evidence for declining trend growth of life expectancy (Oeppen and Vaupel 2002; Strulik and Vollmer 2013). Manton et al. (2006) estimate that active (healthy) life expectancy at age 65 will increase by 5.2 years from 2015 to 2080 while life expectancy is predicted to increase by 4.7 years, implying a disproportionate increase of healthy life time. Other scholars are more pessimistic. Olshansky (2005) argue that the obesity epidemic as a relative recent phenomenon was not fully captured in the historical trends and that thus historical trends are not informative about future developments. Taking current obesity trends into account would substantially reduce the positive trends of health and life expectancy. On the other hand, optimistic scholars envision a future of “negligible senescence” (De Grey and Rae 2007; Kurzweil and Grossman 2010), i.e. a world in which medical technological progress allows to repair all health deficits and to maintain the human body in perfect health. In the context of the present model this would be captured by both τ and T approaching infinity, implying convergence towards a state of “perpetual youth”, a concept well known to economists.

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Appendix A: derivation of the optimal solution

The interior solution of the optimization problem of the household is derived from the first order conditions (5) and (6), i.e.

$$0 \stackrel{!}{=} \theta + \frac{2(\lambda - \tau)}{R^2 - 2\lambda R + 2s(\lambda - \tau) + \tau^2}, \tag{13}$$

$$0 \stackrel{!}{=} \frac{2T(R - \lambda)}{R^2 - 2R\lambda + 2s(\lambda - \tau) + \tau^2} - (\lambda - R)^{-\eta}\omega. \tag{14}$$

Rearranging Eq. (13) we obtain

$$\frac{2}{R^2 - 2R\lambda + 2s(\lambda - \tau) + \tau^2} = -\frac{\theta}{\lambda - \tau}.$$

Inserting this into Eq. (14) we arrive at

$$\omega(\lambda - R)^{-\eta} = -\frac{T\theta(R - \lambda)}{\lambda - \tau} \Leftrightarrow R = \lambda - \left(\frac{(\lambda - \tau)\omega}{T\theta}\right)^{\frac{1}{1+\eta}}. \tag{15}$$

Optimal schooling follows from inserting the optimal retirement age R from (15) into Eq. (14), i.e.

$$s = \frac{\lambda + \tau}{2} - \frac{1}{\theta} - \frac{\left(\frac{(\lambda - \tau)\omega}{T\theta}\right)^{\frac{2}{1+\eta}}}{2(\lambda - \tau)}. \tag{16}$$

Therefore, optimal labor supply is, given by $L = R - s$, is

$$L = \frac{1}{\theta} + \frac{1}{2(\lambda - \tau)} \left(\tau - \lambda + \left(\frac{(\lambda - \tau)\omega}{T\theta}\right)^{\frac{1}{1+\eta}} \right)^2. \tag{17}$$

If the Hessian of U is negative definite at the critical point (s, R) , then this point is a local maximum. The Hessian of U is given by

$$H_U(s, R) := \begin{pmatrix} \frac{\partial^2 U}{\partial s^2} & \frac{\partial^2 U}{\partial s \partial R} \\ \frac{\partial^2 U}{\partial R \partial s} & \frac{\partial^2 U}{\partial R^2} \end{pmatrix} =: \begin{pmatrix} H_{1,1} & H_{1,2} \\ H_{2,1} & H_{2,2} \end{pmatrix}, \tag{18}$$

where

$$\begin{aligned} H_{1,1} &= \frac{2T(-R^2 + 2R\lambda + 2(s - \lambda)\lambda - 2s\tau + \tau^2)}{(R^2 - 2R\lambda + 2s(\lambda - \tau) + \tau^2)^2} - \eta\omega(\lambda - R)^{-1-\eta}, \\ H_{1,2} &= -\frac{4T(R - \lambda)(\lambda - \tau)}{(R^2 - 2R\lambda + 2s(\lambda - \tau) + \tau^2)^2}, \\ H_{2,1} &= H_{1,2}, \\ H_{2,2} &= -\frac{4T(\lambda - \tau)^2}{(R^2 - 2R\lambda + 2s(\lambda - \tau) + \tau^2)^2}. \end{aligned}$$

The two principal minors of H evaluated in the critical point are

$$H_{U,1} = -\eta \frac{T\theta}{(\lambda - \tau)} - \frac{T\theta \left(\lambda - \tau + \theta \left(\frac{(\lambda - \tau)\omega}{T\theta} \right)^{\frac{2}{1+\eta}} \right)}{(\lambda - \tau)^2} < 0, \tag{19}$$

$$H_{U,2} = \det H(R, s) = \frac{T^2\theta^3}{\lambda - \tau} (1 + \eta) > 0. \tag{20}$$

Hence, the Hessian is negative definite and the critical point is a maximum.

Appendix B: proofs of the propositions

Proof of Proposition 1 The partial derivatives of R, s and L , c.f. (7) – (9), with respect to T are

$$\begin{aligned} \frac{\partial s}{\partial T} &= \frac{\omega \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{1-\eta}{1+\eta}}}{T^2(1 + \eta)\theta} > 0, \\ \frac{\partial R}{\partial T} &= \frac{\left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{1}{1+\eta}}}{T(1 + \eta)} > 0, \\ \frac{\partial L}{\partial T} &= -\frac{\omega \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{-\eta}{1+\eta}}}{T^2(1 + \eta)\theta} \left(-\lambda + \tau + \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{1}{1+\eta}} \right). \end{aligned}$$

We have $\partial L / \partial T > 0$ because

$$\frac{\omega \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{-\eta}{1+\eta}}}{T^2(1 + \eta)\theta} \geq 0 \text{ and } \left(-\lambda + \tau + \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{1}{1+\eta}} \right) = \tau - R \leq 0.$$

□

Proof of Proposition 2 The partial derivative of s , c.f. (8), with respect to τ is

$$\frac{\partial s}{\partial \tau} = \frac{1}{2} \left(1 + \frac{(1 - \eta) \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{2}{1+\eta}}}{(1 + \eta)(\lambda - \tau)^2} \right).$$

For $\eta \leq 1$ it is easy to see that $\frac{\partial s}{\partial \tau} > 0$. If $\eta > 1$, it holds that

$$1 + \frac{(1 - \eta) \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{2}{1+\eta}}}{(1 + \eta)(\lambda - \tau)^2} > 0 \Leftrightarrow (1 + \eta)(\lambda - \tau)^2 > (\eta - 1) \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{2}{1+\eta}}$$

This inequality holds because

$$(\eta - 1) \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{2}{1+\eta}} \leq (\eta - 1)(\lambda - \tau)^2 \stackrel{!}{<} (1 + \eta)(\lambda - \tau)^2 \Leftrightarrow \eta - 1 < 1 + \eta.$$

The partial derivatives of R and L , c.f. (7) and (9), with respect to τ are

$$\begin{aligned} \frac{\partial R}{\partial \tau} &= \frac{\left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{1}{1+\eta}}}{(1 + \eta)(\lambda - \tau)} > 0, \\ \frac{\partial L}{\partial \tau} &= \frac{\left(-\lambda + \tau + \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{1}{1+\eta}} \right) \left((1 + \eta)(\lambda - \tau) + (\eta - 1) \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{1}{1+\eta}} \right)}{2(1 + \eta)(\lambda - \tau)^2}. \end{aligned}$$

It holds

$$\begin{aligned} \frac{\partial L}{\partial \tau} < 0 &\Leftrightarrow \left((1 + \eta)(\lambda - \tau) + (\eta - 1) \left(\frac{(\lambda - \tau)\omega}{\theta T} \right)^{\frac{1}{1+\eta}} \right) > 0 \\ &\Leftrightarrow (1 + \eta)(\lambda - \tau) + (\eta - 1)(\lambda - R) > 0 \\ &\Leftrightarrow \frac{\lambda - \tau}{\lambda - R} > \frac{1 - \eta}{1 + \eta}, \end{aligned}$$

which holds $\forall \eta \geq 0$. □

Proof of Proposition 3 An equal increase in τ and T is captured by the directional derivative $\partial L / \partial T + \partial L / \partial \tau$, i.e.

$$\frac{\partial L}{\partial T} + \frac{\partial L}{\partial \tau} = \frac{\left(-\lambda + \tau + \left(\frac{(\lambda - \tau)\omega}{T\theta} \right)^{\frac{1}{1+\eta}} \right)}{2T^2(1 + \eta)\theta(\lambda - \tau)^2} \cdot \Lambda \tag{21}$$

with

$$\Lambda := -\omega \left(\frac{(\lambda - \tau)\omega}{T\theta} \right)^{\frac{-\eta}{1+\eta}} 2(\lambda - \tau)^2 + T^2\theta \left((1 + \eta)(\lambda - \tau) + (\eta - 1) \left(\frac{(\lambda - \tau)\omega}{T\theta} \right)^{\frac{1}{1+\eta}} \right).$$

For the numerator of the first term of Eq. (21) it holds that

$$\left(-\lambda + \tau + \left(\frac{(\lambda - \tau)\omega}{T\theta} \right)^{\frac{1}{1+\eta}} \right) = \tau - R < 0.$$

The denominator is greater than 0. Therefore, the problem simplifies to

$$\begin{aligned}
 & -\omega \left(\frac{(\lambda - \tau)\omega}{T\theta} \right)^{\frac{-\eta}{1+\eta}} 2(\lambda - \tau)^2 + T^2\theta \left((1 + \eta)(\lambda - \tau) + (\eta - 1) \left(\frac{(\lambda - \tau)\omega}{T\theta} \right)^{\frac{1}{1+\eta}} \right) \stackrel{!}{>} 0 \\
 & \Leftrightarrow \frac{\omega}{T^2\theta} \left(\frac{(\lambda - \tau)\omega}{T\theta} \right)^{\frac{-\eta}{1+\eta}} 2(\lambda - \tau)^2 - (\eta - 1) \left(\frac{(\lambda - \tau)\omega}{T\theta} \right)^{\frac{1}{1+\eta}} < (1 + \eta)(\lambda - \tau) \\
 & \Leftrightarrow \left(\frac{(\lambda - \tau)\omega}{T\theta} \right)^{\frac{1}{1+\eta}} \left[\frac{2(\lambda - \tau)^2\omega}{T^2\theta} \frac{T^2\theta}{(\lambda - \tau)\omega} - (\eta - 1) \right] < (1 + \eta)(\lambda - \tau) \\
 & \Leftrightarrow \left(\frac{(\lambda - \tau)\omega}{T} \right)^{\frac{1}{1+\eta}} \left[\frac{2(\lambda - \tau) + T(1 - \eta)}{T(1 + \eta)(\lambda - \tau)} \right] < \theta^{\frac{1}{1+\eta}},
 \end{aligned}$$

which is always fulfilled for $\eta \geq \frac{2(\lambda - \tau)}{T} + 1$. If $\eta < \frac{2(\lambda - \tau)}{T} + 1$, the last inequality becomes

$$\theta > \frac{(\lambda - \tau)\omega}{T} \left[\frac{2(\lambda - \tau) + T(1 - \eta)}{T(1 + \eta)(\lambda - \tau)} \right]^{1+\eta}.$$

□

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