Journal of Combinatorial Optimization (2020) 40:713–732
https://doi.org/10.1007/s10878-020-00611-2

Maximum weight induced matching in some subclasses of bipartite graphs

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Published online: 28 June 2020

Abstract

A subset $M \subseteq E$ of edges of a graph $G = (V, E)$ is called a *matching* in G if no two edges in M share a common vertex. A matching M in G is called an *induced matching* if $G[M]$, the subgraph of G induced by M, is the same as $G[S]$, the subgraph of G induced by $S = \{v \in V | \ v$ is incident on an edge of $M\}$. The Maximum Induced Matching problem is to find an induced matching of maximum cardinality. Given a graph G and a positive integer k, the Induced Matching Decision problem is to decide whether G has an induced matching of cardinality at least k. The Maximum Weight Induced Matching problem in a weighted graph $G = (V, E)$ in which the weight of each edge is a positive real number, is to find an induced matching such that the sum of the weights of its edges is maximum. It is known that the Induced Matching Decision problem and hence the Maximum Weight Induced Matching problem is known to be NP-complete for general graphs and bipartite graphs. In this paper, we strengthened this result by showing that the Induced Matching Decision problem is NP-complete for star-convex bipartite graphs, comb-convex bipartite graphs, and perfect elimination bipartite graphs, the subclasses of the class of bipartite graphs. On the positive side, we propose polynomial time algorithms for the Maximum Weight Induced Matching problem for circular-convex bipartite graphs and triad-convex bipartite graphs by making polynomial time reductions from the Maximum Weight Induced Matching problem in these graph classes to the Maximum Weight Induced Matching problem in convex bipartite graphs.

Keywords Matching · Induced matching · Bipartite graphs · Graph algorithm · NP-complete

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1 Introduction

Let $G = (V, E)$ be a graph. Let n and m denote the number of vertices and the number of edges of G, respectively. A set of edges $M \subseteq E$ is called a *matching* if no two edges of M are incident on a common vertex. Vertices incident to the edges of a matching M are called *saturated* by M. The Maximum Matching problem is to find a matching of maximum cardinality in a given graph. The Maximum Matching problem and its variations are extensively studied in literature. In this paper, we study an important variant of matchings called *induced matchings*. A matching M in G is called an *induced matching* if $G[M]$, the subgraph of G induced by M, is the same as $G[S]$, the subgraph of G induced by $S = \{v \in V | \, v$ is incident on an edge of $M\}$. A graph G with vertex set $V = \{a, b, c, d, g, h\}$ and edge set $E = \{ab, bc, cd, cg, gh, hd, ad, ac, bd, ch, gd\}$ is shown in Fig. 1. Let $M_1 = \{ab, gh\}$ and $M_2 = \{ab, cd, gh\}$. Note that M_1 is a matching as well as an induced matching in G, but M_2 is a matching but not an induced matching in G.

For a graph G, the Maximum Induced Matching problem is to find an induced matching of maximum cardinality in G. The maximum induced matching problem and its decision version are defined as follows:

Maximum Induced Matching problem (MIMP)

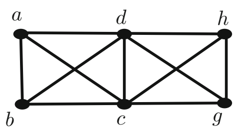
Instance	A graph $G = (V, E)$.
Solution	An induced matching M in G.
Measure	Cardinality of the set M.

Induced Matching Decision problem (IMDP)

Instance	A graph $G = (V, E)$ and a positive integer $k \leq |V|$.
Question	Does there exist an induced matching M in G such that $|M| \geq k$?

The Maximum Induced Matching problem was introduced by Stockmeyer and Vazirani as "Risk-free Marriage problem" in 1982 (Stockmeyer and Vazirani 1982). The Induced Matching Decision problem is NP-complete for general graphs (Stockmeyer and Vazirani 1982), and remains so even for bipartite graphs (Cameron 1989) and k-regular graphs for $k \geq 4$ (Kobler and Rotics 2003; Zito 1999) (see Duckworth et al. 2005 for a survey). The Induced Matching Decision problem also remains NP-complete for bipartite graphs with maximum degree 3, and C_4-free bipartite graphs, which are two special subclasses of bipartite graphs (Lozin 2002). On the other hand, the Maximum Induced Matching problem is polynomial time solvable for many graph classes, for example, chordal graphs (Cameron 1989), chordal bipartite graphs (Cameron et al. 2003), trapezoid graphs, interval-dimension graphs and cocomparability graphs (Golumbic and Lewenstein 2000) etc.

Fig. 1 Graph G



Recently, Klemz and Rote studied the weighted version of the Maximum Induced Matching problem (Klemz and Rote 2017). The Maximum Weight Induced Matching problem is defined as follows:
Maximum Weight Induced Matching problem (MWIMP)

Instance	A graph $G = (V, E)$ with positive real weights $w(e)$ for each $e \in E$.
Solution	An induced matching M in G.
Measure	Weight of M, that is $\sum_{e \in M} w(e)$.

In this paper, we study the Maximum weight Induced Matching problem for some subclasses of bipartite graphs: perfect elimination bipartite graphs, comb-convex bipartite graphs, star-convex bipartite graphs, circular-convex bipartite graphs, and triad-convex bipartite graphs. The class of circular-convex bipartite graphs was introduced by Liang and Blum (1995) and has been studied recently by researchers (see Liu 2014; Liu et al. 2015, 2014; Pandey and Panda 2019). The triad-convex bipartite graphs, star-convex bipartite graphs, and comb-convex bipartite graphs are studied in Chen et al. (2016), Jiang et al. (2013), Liu et al. (2015), Song et al. (2012), Wang et al. (2014). The main contributions of the paper are summarized below.

1. We show that the Induced Matching Decision problem is NP-complete for star-convex bipartite graphs, comb-convex bipartite graphs, and perfect elimination bipartite graphs.
2. We propose an $O(m^2)$ time algorithm to solve the Maximum Weight Induced Matching problem in circular-convex bipartite graphs.
3. We propose an $O(mn^6)$ time algorithm to solve the Maximum Weight Induced Matching problem in triad-convex bipartite graphs.

Our algorithms for the Maximum Weight Induced Matching problem in circular-convex bipartite graphs and triad-convex bipartite graphs are based on a polynomial time reduction for the Maximum Weight Induced Matching problem from these graph classes to convex bipartite graphs. The following result is already known for the Maximum Weight Induced Matching problem in convex bipartite graphs.

Theorem 1 (Klemz and Rote 2017) *The* Maximum Weight Induced Matching *problem can be solved in $O(n + m)$ time in convex bipartite graphs.*

A preliminary version of this paper for unweighted graphs appeared in Pandey et al. (2017).

2 Preliminaries

We consider only simple, connected and undirected graphs. In a graph $G = (V, E)$, the sets $N_G(v) = \{u \in V(G) \mid uv \in E\}$ and $N_G[v] = N_G(v) \cup \{v\}$ denote the *open neighborhood* and *closed neighborhood* of a vertex v, respectively. For a vertex v, the degree of v is the cardinality of open neighborhood of v, and is denoted by $d_G(v)$. A vertex v is called a pendant vertex if $d_G(v) = 1$. For a set $S \subseteq V$ of the graph

$G = (V, E)$, the subgraph of G *induced* by S is defined as $G[S] = (S, E_S)$, where $E_S = \{xy \in E | x, y \in S\}$. For a set $E' \subseteq E$ of the graph $G = (V, E)$, the subgraph of G *induced* by E' is defined as $G[E'] = (V_{E'}, E')$, where $V_{E'} = \{x \in V | x$ is incident on an edge of $E'\}$. A graph G is said to be *chordal* if every cycle in G of length at least four has a *chord*, that is, an edge joining two non-consecutive vertices of the cycle. A graph $G = (V, E)$ is said to be *bipartite* if V can be partitioned into two disjoint sets X and Y such that every edge of G joins a vertex in X to a vertex in Y, and such a partition (X, Y) of V is called a *bipartition*. A bipartite graph with bipartition (X, Y) of V is denoted by $G = (X, Y, E)$. A bipartite graph G is said to be *chordal bipartite* if every cycle of length at least 6 has a chord. A weighted graph is a graph $G = (V, E)$ together with a weight function $w : E \rightarrow R^+$ from the edge set to the set of positive real numbers.

Let $G = (X, Y, E)$ be a bipartite graph with $|X| = n_1$ and $|Y| = n_2$. G is called a *convex bipartite graph* if there exists a linear ordering $<$ on X, say $x_1 < x_2 < \cdots < x_{n_1}$, such that for every vertex y in Y, $N_G(y) = \{x_i, x_{i+1}, \ldots, x_j\}$ for $1 \le i \le j \le n_1$, that is, vertices in $N_G(y)$ are *consecutive* in the linear ordering $<$ on X. A set of consecutive vertices in the linear ordering $<$ on X is called an *interval*. G is called a *circular-convex bipartite graph* if there exists a circular ordering $\prec$ on X, say $x_1 \prec x_2 \prec \cdots \prec x_{n_1} \prec x_{(n_1+1)} = x_1$, such that for every vertex y in Y, either $N_G(y) = \{x_i, x_{i+1}, \ldots, x_j\}$ or $N_G(y) = \{x_j, x_{j+1}, \ldots, x_{n_1}, x_1, \ldots, x_i\}$ for $1 \le i \le j \le n_1$, that is, vertices in $N_G(y)$ are *consecutive* in the circular ordering $\prec$ on X. A set of consecutive vertices in the clock-wise direction in the circular ordering $\prec$ on X is called a *circular arc* and the first vertex and the last vertex in the circular arc are called the *left end point* and the *right end point* of the circular arc, respectively.

A tree with exactly one non-pendant vertex is a *star*. A *comb* is a graph obtained by attaching a pendant vertex (tooth) to every vertex of a path (backbone). A bipartite graph $G = (X, Y, E)$ is called a *tree-convex bipartite graph*, if a tree $T = (X, E^X)$ can be defined such that for every vertex y in Y, the neighborhood of y induces a subtree of T. Tree-convex bipartite graphs are recognizable in linear time, and the associated tree T can also be constructed in linear-time (Bao and Zhang 2012). A tree-convex bipartite graph with a corresponding tree is shown in Fig. 2. For T a star, G is called a *star-convex bipartite graph*. For T a *triad*, that is, three paths with a common end-vertex, G is called a *triad-convex bipartite graph*. For T a comb, G is called a *comb-convex bipartite graph*. If T is a path, then G is called a *convex bipartite graph*. Note that both the definitions of convex bipartite graphs are equivalent.

For a bipartite graph $G = (X, Y, E)$, an edge $uv \in E$ is a *bisimplicial edge* if $N_G(u) \cup N_G(v)$ induces a complete bipartite subgraph in G. Let $(e_1, e_2, \ldots, e_k)$ be an ordering of pairwise non-adjacent edges (no two edges have a common end vertex) of G (not necessarily all edges of E). Let S_i be the set of endpoints of edges $e_1, e_2, \ldots, e_i$ and let $S_0 = \emptyset$. Ordering $(e_1, e_2, \ldots, e_k)$ is a *perfect edge elimination ordering* for G if $G[(X \cup Y) \setminus S_k]$ has no edge and each edge e_i is bisimplicial in the remaining induced subgraph $G[(X \cup Y) \setminus S_{i-1}]$. A graph G is called a *perfect elimination bipartite graph* if G admits a perfect edge elimination ordering. The class of perfect elimination bipartite graphs was introduced by Golumbic and Gauss (1978). The hierarchial relationship between subclasses of bipartite graphs is shown in Fig. 3.

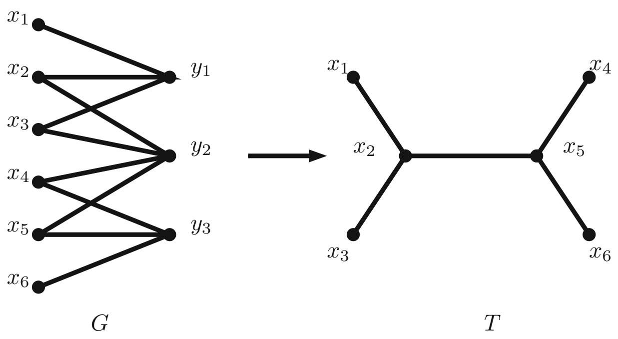


Fig. 2 A tree-convex bipartite graph G with a corresponding tree T

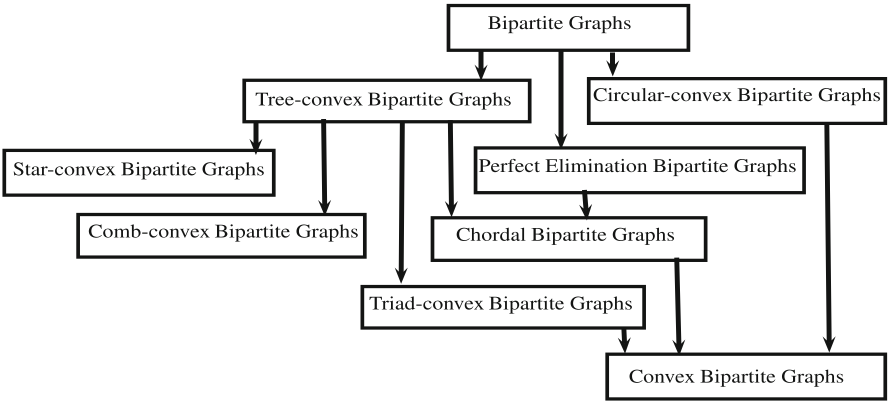


Fig. 3 The hierarchical relationship between subclasses of bipartite graphs

3 NP-completeness results

In this section, we study the NP-completeness of the INDUCED MATCHING DECISION problem. The INDUCED MATCHING DECISION problem is NP-complete for bipartite graphs. We strengthen the complexity result of the INDUCED MATCHING DECISION problem, by showing that it remains NP-complete for star-convex bipartite graphs, comb-convex bipartite graphs, and perfect elimination bipartite graphs, three important subclasses of the class of bipartite graphs.

3.1 Star-convex bipartite graphs

In this subsection, we prove the hardness result for the INDUCED MATCHING DECISION problem in star-convex bipartite graphs. Recall that a bipartite graph $G = (X, Y, E)$ is called a *star-convex bipartite graph*, if a star $T = (X, E^X)$ can be defined on partition X such that for every vertex y in Y, the neighborhood of y induces a connected subgraph of T. A star-convex bipartite graph with a corresponding star is shown in Fig. 4.

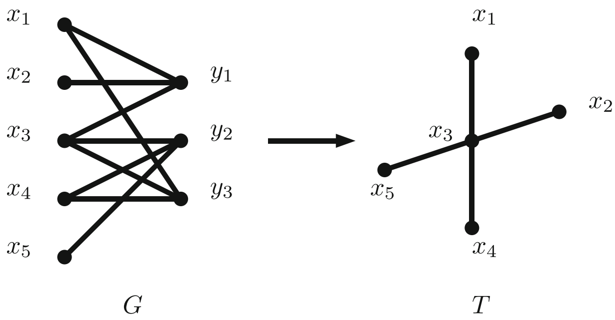


Fig. 4 A star-convex bipartite graph G with a corresponding star T

The following necessary and sufficient condition for a bipartite graph to be a star-convex bipartite graph will be useful in the polynomial time reduction.

Lemma 1 (Pandey and Panda 2019) *A bipartite graph $G = (X, Y, E)$ is a star-convex bipartite graph if and only if there exists a vertex x in X such that every vertex y in Y is either a pendant vertex or is adjacent to x.*

Theorem 2 *The* Induced Matching Decision *problem is NP-complete for star-convex bipartite graphs.*

Proof Clearly, the Induced Matching Decision problem is in NP for star-convex bipartite graphs. To show the NP-completeness, we give a polynomial time reduction from the Induced Matching Decision problem for bipartite graphs, which is already known to be NP-complete (Cameron 1989).

Given a bipartite graph $G = (X, Y, E)$, we construct a star-convex bipartite graph $H = (X_H, Y_H, E_H)$ in the following way: $X_H = X \cup \{u\}$, $Y_H = Y$, and $E_H = E \cup \{uy \mid y \in Y_H\}$. The construction of H from G is shown in Fig. 5. By Lemma 1, it is clear that the constructed graph H is a star-convex bipartite graph (as every vertex in Y_H is adjacent to the vertex $u \in X_H$). Now, the following claim is sufficient to complete the proof of the theorem. □

Claim *G has an induced matching of size at least k if and only if H has an induced matching of size at least k.*

Proof Let M be an induced matching in G and $|M| \geq k$. Then M is also an induced matching in H (As, in the construction of H from G, we have not added any edge whose both endpoints are the vertices of G). Hence H contains an induced matching of size at least k.

Conversely, let M' be an induced matching in H, and $|M'| \geq k$. If M' saturates u, that is, M' contains an edge, say $e = uy_i$, whose one endpoint is u, then $|M'| = 1$. If any other edge $x_r y_s \in M'$, then M' will not be an induced matching in H (as $uy_s \in E(H)$). In this case, G contains an induced matching with at least one edge. Otherwise, if M' does not contain any edge whose one of the endpoints is u, then $M' \subseteq E(G)$. Also, since M' is an induced matching in H and G is a subgraph of H,

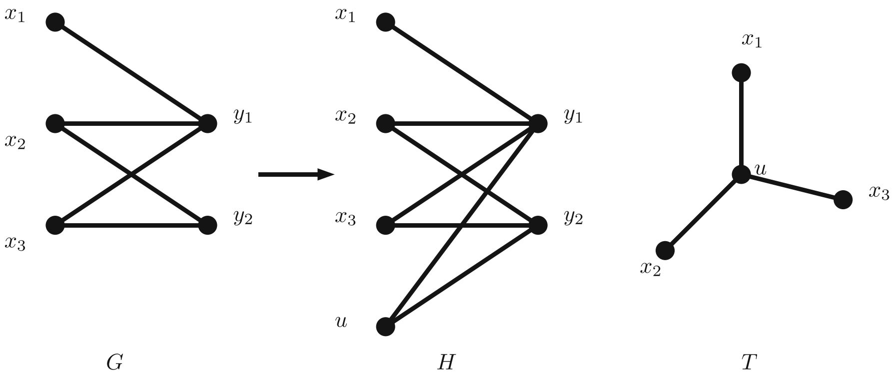


Fig. 5 An illustration of the construction of H from G

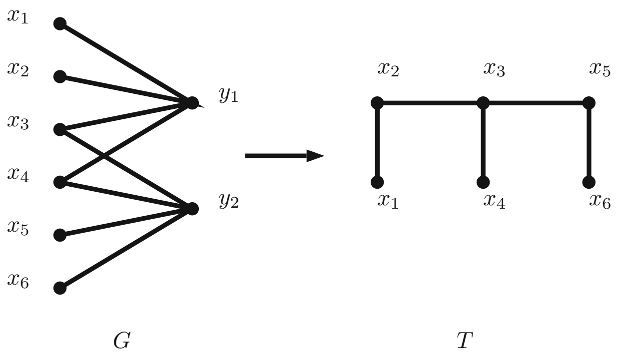


Fig. 6 A comb-convex bipartite graph G with a corresponding comb T

M' is also an induced matching in G. Hence G contains an induced matching of size at least k. □

Hence, the theorem is proved. □

3.2 Comb-convex bipartite graphs

In this subsection, we show that the Induced Matching Decision problem remains NP-complete for comb-convex bipartite graphs, which is a subclass of tree-convex bipartite graphs. Recall that a bipartite graph $G = (X, Y, E)$ is called a *comb-convex bipartite graph*, if a comb $T = (X, E^X)$ can be defined on partition X such that for every vertex y in Y, the neighborhood of y induces a connected subgraph of T. A comb-convex bipartite graph with a corresponding comb is shown in Fig. 6.

Theorem 3 *The* Induced Matching Decision *problem is NP-complete for comb-convex bipartite graphs.*

Proof Clearly, the Induced Matching Decision problem is in NP for comb-convex bipartite graphs. To show the NP-completeness, we give a polynomial time reduc-

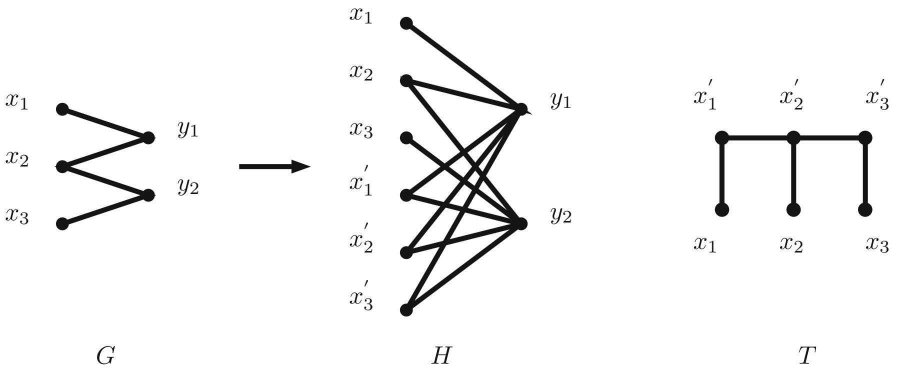


Fig. 7 An illustration of the construction of H from G

tion from the Induced Matching Decision problem for bipartite graphs, which is already known to be NP-complete (Cameron 1989).

Given a bipartite graph $G = (X, Y, E)$, we construct a comb-convex bipartite graph $H = (X_H, Y_H, E_H)$ in the following way: $X_H = X \cup X'$, where X' contains a copy of x for each $x \in X$, $Y_H = Y$ and $E_H = E \cup \{x'y \mid x' \in X' \text{ and } y \in Y\}$. Note that H can be constructed from G in polynomial time. The construction of H from G is shown in Fig. 7. It is easy to see that H is a comb convex bipartite graph if X' is taken as the backbone and X is taken as the teeth of the comb. □

Claim *G has an induced matching of size at least k if and only if H has an induced matching of size at least k, where $k > 1$.*

Proof Let M be an induced matching in G of size at least k. Then, M is also an induced matching in H. Hence, H has an induced matching of size at least k.

Conversely, assume that M is an induced matching in H of size at least k, $k > 1$. Observe that if a vertex $x' \in X'$ is saturated by M, that is $x'y \in M$ for some $y \in Y$, then M does not contain any other edge of H. Since x' is adjacent to every vertex $y \in Y$, for any edge $x_i y_i$, where $x_i \in X_H$ and $y_i \in Y$, $x'y_i$ is also an edge in H. Hence, the edge $x_i y_i$ can not be present in M and $|M|$ should be 1.

As $|M| \geq k > 1$, any vertex $x' \in X'$ is not saturated by M, that is, M does not have any edge from E'. In this case, M is also an induced matching in G, and $|M| \geq k$. □

Hence, the theorem is proved. □

3.3 Perfect elimination bipartite graphs

In this subsection, we prove the hardness result for the Induced Matching Decision problem in perfect elimination bipartite graphs. Since the class of perfect elimination bipartite graphs is a subclass of bipartite graphs, and a superclass of chordal bipartite graphs, our result reduces the complexity gap between bipartite graphs and chordal bipartite graphs.

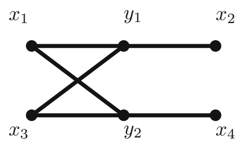


Fig. 8 A perfect elimination bipartite graph

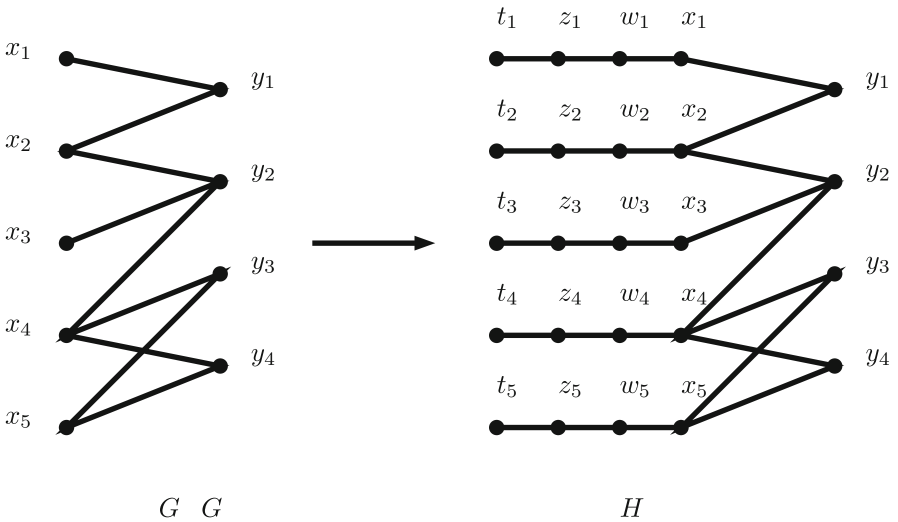


Fig. 9 An illustration of the construction of H from G

A perfect elimination bipartite graph with a perfect edge elimination ordering $\sigma = (x_4y_2, x_2y_1)$ is shown in Fig. 8.

Theorem 4 *The* INDUCED MATCHING DECISION *problem is NP-complete for perfect elimination bipartite graphs.*

Proof Clearly, the INDUCED MATCHING DECISION problem is in NP for perfect elimination bipartite graphs. To show the NP-completeness, we give a polynomial time reduction from the INDUCED MATCHING DECISION problem for bipartite graphs, which is already known to be NP-complete (Cameron 1989).

Given a bipartite graph $G = (X, Y, E)$ where $X = \{x_1, x_2, \ldots, x_{n_1}\}$ and $Y = \{y_1, y_2, \ldots, y_{n_2}\}$, we construct a bipartite graph $H = (X_H, Y_H, E_H)$ in the following way: For each $x_i \in X$, add a path $P_i = x_i, w_i, z_i, t_i$ of length 3. Formally $X_H = X \cup \{z_i \mid 1 \leq i \leq n_1\}$, $Y_H = Y \cup \{w_i, t_i \mid 1 \leq i \leq n_1\}$, and $E_H = E \cup \{x_iw_i, w_iz_i, z_it_i \mid 1 \leq i \leq n_1\}$. □

Figure 9 illustrates the construction of H from G. Clearly H is a perfect elimination bipartite graph and $(z_1t_1, z_2t_2, \ldots, z_{n_1}t_{n_1}, x_1w_1, x_2w_2, \ldots, x_{n_1}w_{n_1})$ is a perfect edge elimination ordering for H. Now, the following claim is sufficient to complete the proof of the theorem.

Claim *G has an induced matching of size at least k if and only if H has an induced matching of size at least $k + n_1$.*

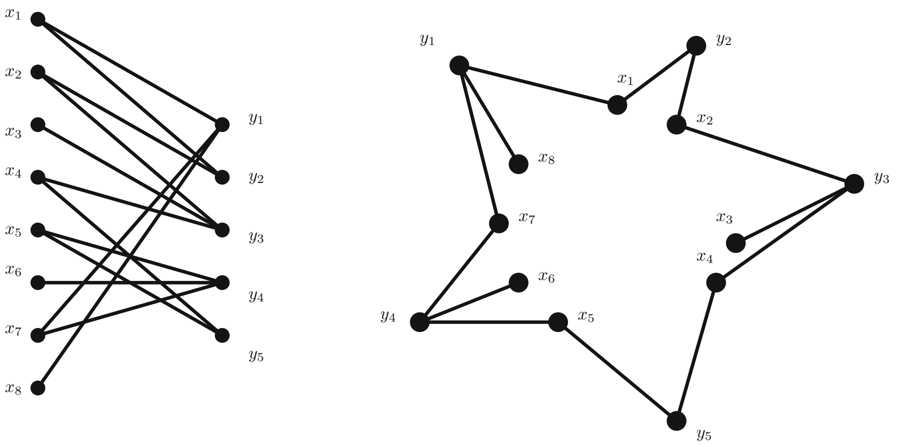


Fig. 10 A circular-convex bipartite graph

Proof Let M be an induced matching in G, and $|M| \geq k$. Then $M' = M \cup \{z_i t_i \mid 1 \leq i \leq n_1\}$ is an induced matching in H, and $|M'| \geq k + n_1$. Hence H has an induced matching of size at least $k + n_1$.

Conversely, let M' be an induced matching in H, and $|M'| \geq k+n_1$. Then for each i, $1 \leq i \leq n_1$, $|M' \cap \{x_i w_i, w_i z_i, z_i t_i \mid 1 \leq i \leq n_1\}| \leq 1$, that is, at most one edge from the set $\{x_i w_i, w_i z_i, z_i t_i\}$ belongs to M'. Define $S = \{x_i w_i, w_i z_i, z_i t_i \mid 1 \leq i \leq n_1\}$. Also define $M = M' \setminus S$. Then $|M| \geq k$. Since $M \subseteq M'$, M is also an induced matching in H. Also, G is a subgraph of H, and $M \subseteq E_G$. Hence M is also an induced matching in G. Hence G contains an induced matching of size at least k. □

Hence, the theorem is proved. □

4 Circular-convex bipartite graph

In this section, we propose a polynomial time algorithm to compute a maximum weight induced matching in a weighted circular-convex bipartite graph. Recall that a circular-convex bipartite graph is a bipartite graph that exhibits a circular ordering $\prec$ on X, say $x_1 \prec x_2 \prec \cdots \prec x_{n_1} \prec x_{(n_1+1)} = x_1$, such that for every vertex y in Y, either $N_G(y) = \{x_i, x_{i+1}, \ldots, x_j\}$ or $N_G(y) = \{x_j, x_{j+1}, \ldots, x_{n_1}, x_1, \ldots, x_i\}$ for $1 \leq i \leq j \leq n_1$, that is, vertices in $N_G(y)$ are *consecutive* in the circular ordering $\prec$ on X. A circular-convex bipartite graph with a corresponding circular ordering $x_1 \prec x_2 \prec \cdots \prec x_8 \prec x_1$ is shown in Fig. 10.

Our algorithm is based on the reduction from circular-convex bipartite graphs to convex bipartite graphs. Below we first give a construction of a convex bipartite graph from a given circular-convex bipartite graph.

Construction 1 Let $G = (X, Y, E)$ be a weighted circular-convex bipartite graph with positive edge weights $w(e)$ for each $e \in E$. Let $|X| = n_1$ and $|Y| = n_2$. Let $e = x_i y_j \in E$. Without loss of generality, we can always assume a circular ordering

$\prec$ on X, say $x_1 \prec x_2 \prec \cdots \prec x_{n_1} \prec x_{(n_1+1)} = x_1$, such that for every vertex y in Y, $N_G(y)$ is a circular arc. Now, construct the graph $G_e = (X_e, Y_e, E_e)$ as follows: $X_e = X \backslash N_G(y_j)$, $Y_e = Y \backslash N_G(x_i)$, and $E_e = \{xy \in E \mid x \in X_e, y \in Y_e\}$.

Lemma 2 *G_e is a convex bipartite graph.*

Proof Let $G' = (X', Y', E')$ be the graph constructed from G by removing x_i and all its neighbors. Now, we define a linear ordering $<$ on the vertices of X' as follows, $x_{i+1} < x_{i+2} < \cdots < x_{n_1} < x_1 < \cdots < x_{i-1}$. Since we have removed all the neighbors of x_i, none of the vertex in Y' is adjacent to both x_{i-1} and x_{i+1}. Therefore for every vertex $y \in Y'$, $N_{G'}(y)$ is an interval on X'. Hence G' is a convex bipartite graph. Also, observe that G_e is a subgraph of G'. Since, G' is a convex bipartite graph, G_e is also a convex bipartite graph. $\square$

Lemma 3 *Let M be a maximum weight induced matching in G and $e \in M$. Let M_e be a maximum weight induced matching in G_e. Then $M' = M_e \cup \{e\}$ is also a weighted induced matching in G and $w(M) = w(M')$. In other words, M' is a maximum weight induced matching in G.*

Proof Note that G_e is constructed from G by removing some of its vertices. Hence every weighted induced matching in G_e is also a weighted induced matching in G. Therefore M_e is also a weighted induced matching in G. Also, there is no edge in G, which joins an endpoint of e and an endpoint of some edge of G_e, which implies that there is no edge in G which joins an endpoint of e and an endpoint of some edge in M_e. Hence $M' = M_e \cup \{e\}$ is also a weighted induced matching in G. Since M is a maximum weight induced matching in G, $w(M) \geq w(M')$. To complete the proof, we need to show that $w(M) \leq w(M')$.

Let x_i, y_j are the end points of the edge e, and $S = N_G(x_i) \cup N_G(y_j)$. Then $M \backslash \{e\}$ does not contain any edge incident on any vertex in S, otherwise M is not a weighted induced matching in G. Hence $M \backslash \{e\}$ is a weighted induced matching in $G[(X \cup Y) \backslash (N_G(x_i) \cup N_G(y_j))]$, which is exactly the graph G_e. Since M_e is a maximum weight induce matching in G_e, $w(M_e) \geq w(M) - w(e)$. Therefore $w(M) \leq w(M_e) + w(e) = w(M')$.

Hence $w(M) = w(M')$, and M' is also a maximum weight induced matching in G. $\square$

The detailed algorithm for finding a maximum weight induced matching in a weighted circular-convex bipartite graph G is given in Fig. 11.

The following theorem directly follows from the Lemma 3 and the algorithm WEIGHTED- INDUCED- M- CIRCULAR- CONVEX.

Theorem 5 *A maximum weight induced matching in a weighted circular-convex bipartite graph can be computed in $O(m^2)$ time.*

Proof Since for each edge $e \in E$, we are computing a maximum weight induced matching in graph G_e. The time taken by algorithm is atmost $O(m \times g(n))$, where $g(n)$ is the time complexity of computing a maximum weight induced matching in a convex bipartite graph. By Theorem 1, it is clear that $g(n) = O(m)$. So, the overall time complexity of WEIGHTED- INDUCED- M- CIRCULAR- CONVEX is at most $O(m^2)$. $\square$

Algorithm 1: WEIGHTED-INDUCED-M-CIRCULAR-CONVEX(G)

Input: A weighted circular-convex bipartite graph $G = (X, Y, E)$ with edge weights $w : E \to \mathbb{R}^+$, where $X = \{x_1, x_2, \ldots, x_{n_1}\}$ and $Y = \{y_1, y_2, \ldots, y_{n_2}\}$.
Output: A maximum weight induced matching M^* in G.
$M = \emptyset$, $M^* = \emptyset$;
foreach $e \in E$ **do**
  Construct G_e using Construction 1;
  Find a maximum weight induced matching M in G_e;
  Update $M = M \cup \{e\}$;
  if $w(M^*) < w(M)$ **then**
    $M^* = M$;
return M^*.

Fig. 11 Algorithm to compute a maximum weight induced matching in a circular-convex bipartite graph

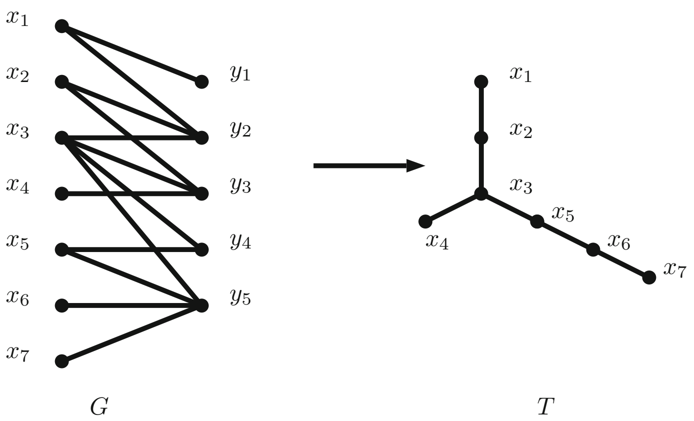


Fig. 12 A triad-convex bipartite graph G with a corresponding triad T

5 Triad-convex bipartite graph

In this section, we propose a polynomial time algorithm to compute a maximum weight induced matching in a weighted triad-convex bipartite graph.

Let $G = (X, Y, E)$ be a weighted triad-convex bipartite graph with positive edge weights $w(e)$ for each $e \in E$ and $T = (X, E^X)$ be a triad defined on X, such that for every vertex $y \in Y$, $T[N_G(y)]$ is a connected subgraph of T. Let $X = \{x_0\} \cup X_1 \cup X_2 \cup X_3$ be such that for each i, $1 \leq i \leq 3$, $X_i = \{x_{i,1}, x_{i,2}, \ldots, x_{i,n_i}\}$. Also suppose that $x_{i,0} = x_0$ for all i, $1 \leq i \leq 3$. For each i, $1 \leq i \leq 3$, let $P_i = x_0, x_{i,1}, x_{i,2}, \ldots, x_{i,n_i}$ be a path in $T = (X, E^X)$. Note that x_0 is a common vertex in all the three paths P_1, P_2, and P_3. A triad-convex bipartite graph with a corresponding triad is shown in Fig. 12.

Let $G = (X, Y, E)$ be a weighted triad-convex bipartite graph with x_0 as the common vertex of the three paths of a triad. Note that a maximum weight induced matching of G may saturate x_0 or may not saturate x_0. Among all the weighted induced matchings of G not saturating x_0, let M_1 be a maximum weight induced matching. Similarly, among all the weighted induced matchings of G saturating x_0, let M_2 be a maximum weight induced matching. If weight of M_1 is at least the weight of M_2, then M_1 is a maximum weight induced matching of G. Otherwise, M_2 is a maximum weight induced matching of G. So, it is enough to find a maximum weight induced matching of G saturating x_0 and a maximum weight induced matching of G not saturating x_0. We first find a maximum weight induced matching M of G not saturating x_0. We first show that in this case, M can saturate at most 3 neighbors of x_0. This allows us to consider only four cases: M does not saturate any neighbor of x_0, M saturates exactly one neighbor of x_0, M saturates exactly two neighbors of x_0 and M saturates exactly three neighbors of x_0. In each of these cases, we construct a convex bipartite graph G' from G and we compute a maximum weight induced matching of G' using the known algorithm given in Klemz and Rote (2017). We then use this matching to find a maximum weight induced matching of G. Similarly, we solve the case when M saturates x_0. So, in each of these cases, we first construct a convex bipartite graph G' from G and we find a maximum weight induced matching M' of G' using the known algorithm given in Klemz and Rote (2017). We then construct a maximum weight induced matching of G by suitably adding some edges which depends on the corresponding case. The details are given below through a series of lemmas.

Lemma 4 *Let M be a maximum weight induced matching in G. Let M does not saturate x_0, but M saturates some of the neighbors of x_0. Then M saturates at most 3 neighbors of x_0.*

Proof We prove it by contradiction. Suppose M saturates 4 neighbors of x_0, say y_{j_1}, y_{j_2}, y_{j_3}, and y_{j_4}. Also suppose that $\{x_{j_1}y_{j_1}, x_{j_2}y_{j_2}, x_{j_3}y_{j_3}, x_{j_4}y_{j_4}\} \subseteq M$. Then at least two vertices of the set $\{x_{j_1}, x_{j_2}, x_{j_3}, x_{j_4}\}$ belong to the same path P_i for some i, $1 \leq i \leq 3$ (see Fig. 13). Without loss of generality, we may assume that $x_{j_1}, x_{j_2} \in P_1$. Also assume that the distance between x_0 and x_{j_1} is less than the distance between x_0 and x_{j_2} in path P_1. Notice that y_{j_2} is adjacent to x_0 as well as x_{j_2}. Also, by the definition of triad-convex bipartite graph, $T[N_G(y_{j_2})]$ is a subtree of T. Hence y_{j_2} must be adjacent to x_{j_1}. But by the definition of induced matching, if $x_{j_1}y_{j_1}$ and $x_{j_2}y_{j_2}$ are edges in an induced matching, then $x_{j_1}y_{j_2} \notin E$. So, we arrive at a contradiction. □

Now let M be a maximum weight induced matching in G, then one of the following possibilities must occur:

(a) M does not saturate any vertex in $N_G[x_0]$.
(b) x_0 is saturated by M.
(c) M does not saturate x_0 but M saturates at most 3 neighbors of x_0. In this case, again three possibilities arise.

- M saturates exactly one neighbor of x_0.
- M saturates exactly two neighbors of x_0.
- M saturates exactly three neighbors of x_0.

Now we discuss in detail that how to find a maximum weight induced matching M in G in each of the above cases:

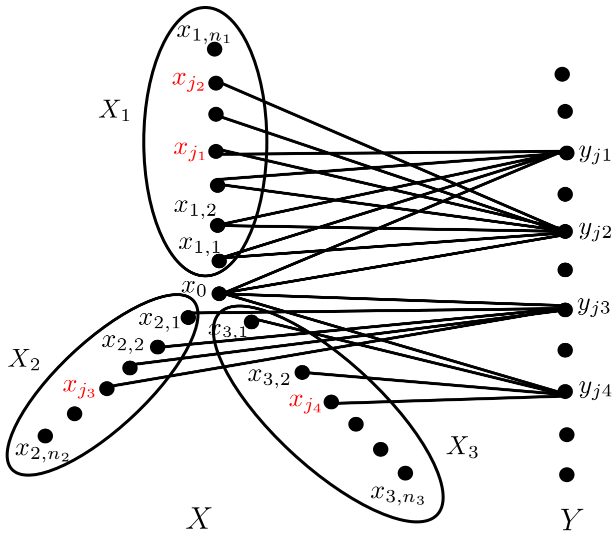


Fig. 13 A triad-convex bipartite graph

Case 1 M does not saturate any vertex in $N_G[x_0]$.
We construct a weighted convex bipartite graph $G_0 = (X_0, Y_0, E_0)$ in the following way:

Construction 2 $G_0 = G[V \backslash N_G[x_0]]$.

Lemma 5 *G_0 is a convex bipartite graph.*

Proof G_0 is constructed by removing the vertex x_0 and all its neighbors from G. Now, we define a linear ordering $<$ on the vertices of X_0 as follows, $x_{1,1} < x_{1,2} < \cdots < x_{1,n_1} < x_{2,1} < x_{2,2} < \cdots < x_{2,n_2} < x_{3,1} < x_{3,2} < \cdots < x_{3,n_3}$. Then for every vertex $y \in Y_0$, $N_{G_0}(y)$ is an interval on X_0. Hence G_0 is a convex bipartite graph. □

Lemma 6 *Let M_0 be a maximum weight induced matching in G_0. Then M_0 is a weighted induced matching in G. Moreover, if there exists a maximum weight induced matching M in G which does not saturate any vertex in $N_G[x_0]$ then $w(M) = w(M_0)$. In other words, M_0 is a maximum weight induced matching in G.*

Proof Note that G_0 is a subgraph of G, and there does not exist any edge $e \in E(G) \backslash E(G_0)$ such that both the end points of e are in G_0. Hence every induced matching in G_0 is also an induced matching in G. So, M_0 is an induced matching in G.

Now, if M is a maximum weight induced matching in G and M does not saturate any vertex in $N_G[x_0]$, then M is also a maximum weight induced matching in the graph obtained from G by removing x_0 and all its neighbors, which is exactly G_0. Hence $w(M) = w(M_0)$, and M_0 is also a maximum weight induced matching in G. □

Case 2 x_0 is saturated by M, that is $x_0 y \in M$ for some $y \in N_G(x_0)$.

We construct a weighted convex bipartite graph G_0^y in the following way:

Construction 3 $G_0^y = G[V \backslash (N_G(x_0) \cup N_G(y))]$.

Lemma 7 *G_0^y is a weighted convex bipartite graph.*

Proof Since G_0^y is a subgraph of G_0, and G_0 is a convex bipartite graph (by Lemma 5), G_0^y is also a convex bipartite graph. □

Lemma 8 *Let M_0 be a maximum weight induced matching in G_0^y. Then $M_0 \cup \{x_0 y\}$ is a weighted induced matching in G. Moreover, if there exists a maximum weight induced matching M in G containing the edge $x_0 y$ then $w(M) = w(M_0) + w(x_0 y)$.*

Proof Clearly M_0 is a induced matching in G, as G_0^y is a subgraph of G, and no edge $e \in E(G) \backslash E(G_0^y)$ contains both end points in G_0^y. Also, notice that the distance of any vertex, say v saturated by M_0 is at least 2 from x_0 as well as y in G. Hence $M_0 \cup \{x_0 y\}$ is also a weighted induced matching in G.

Next, if M is a maximum weight induced matching in G and $x_0 y \in M$. Then $M \backslash \{x_0 y\}$ must be maximum weight induced matching in $G[V \backslash (N_G(x_0) \cup N_G(y))]$, which is exactly G_0^y. Hence $w(M_0) = w(M) - w(x_0 y)$, that is, $w(M) = w(M_0) + w(x_0 y)$. Hence $M_0 \cup \{x_0 y\}$ is also a maximum weight induced matching in G. □

Case 3 M does not saturate x_0 but M saturates at most 3 neighbors of x_0.
Again the following three cases arise:
Subcase 3.1 M saturates exactly one neighbor y_i of x_0, that is, $x y_i \in M$ for some $x \in N_G(y_i) \backslash \{x_0\}$.
We construct a weighted convex bipartite graph $G_x^{y_i}$ in the following way:

Construction 4 First remove all the neighbors of x_0 other than y_i from G. Let us call the resultant graph G'. Now define $G_x^{y_i} = G'[V(G') \backslash (N_{G'}(x) \cup N_{G'}(y_i))]$.

Lemma 9 *$G_x^{y_i}$ is a weighted convex bipartite graph.*

Proof Since $G_x^{y_i}$ is a subgraph of G_0, and G_0 is a convex bipartite graph (by Lemma 5), $G_x^{y_i}$ is also a convex bipartite graph. □

Lemma 10 *Let M_i be a maximum weight induced matching in $G_x^{y_i}$. Then $M_i \cup \{x y_i\}$ is a weighted induced matching in G. Moreover, if there exists a maximum weight induced matching M in G such that M does not saturate x_0, and M saturates exactly one neighbor y_i of x_0, and $x y_i \in M$, then $w(M) = w(M_i) + w(x y_i)$. In otherwords, $M_i \cup \{x y_i\}$ is a maximum weight induced matching in G.*

Proof Clearly, M_i is also a weighted induced matching in G. Also every vertex of $G_x^{y_i}$ is at distance at least two from x as well as y_i. Hence every vertex which is saturated by M_i is at distance at least two from both x and y_i. Hence $M_i \cup \{x y_i\}$ is a weighted induced matching in G.

Now, if M is a maximum weight induced matching in G, and M does not saturate x_0, and M saturates exactly one neighbor y_i of x_0, and $x y_i \in M$, then M is also a maximum weight induced matching in $G' = G[V \backslash (N_G(x_0) \backslash \{y_i\})]$. Also, $M \backslash \{x y_i\}$

is a maximum weight induced matching in $G'[V(G')\backslash(N_{G'}(x) \cup N_{G'}(y_i))]$, which is exactly $G_x^{y_i}$. Hence $w(M_i) = w(M) - w(xy_i)$, that is, $w(M) = w(M_i) + w(xy_i)$. Therefore, $M_i \cup \{xy_i\}$ is a maximum weight induced matching in G. □

Subcase 3.2 M saturates exactly two neighbors y_i, y_j of x_0, that is, $x_r y_i, x_s y_j \in M$ where $x_r \in N_G(y_i)\backslash N_G(y_j)$, and $x_s \in N_G(y_j)\backslash N_G(y_i)$.
We construct a weighted convex bipartite graph $G_{x_r x_s}^{y_i y_j}$ in the following way:

Construction 5 First remove all the neighbors of x_0 other than y_i, y_j from G. Let us call the resultant graph G'. Now define $G_{x_r x_s}^{y_i y_j} = G'[V(G')\backslash(N_{G'}(x_r) \cup N_{G'}(x_s) \cup N_{G'}(y_i) \cup N_{G'}(y_j))]$.

Lemma 11 $G_{x_r x_s}^{y_i y_j}$ *is a weighted convex bipartite graph.*

Proof Since $G_{x_r x_s}^{y_i y_j}$ is a subgraph of G_0, and G_0 is a convex bipartite graph (by Lemma 5), $G_{x_r x_s}^{y_i y_j}$ is also a convex bipartite graph. □

Lemma 12 *Let M_{ij} be a maximum weight induced matching in $G_{x_r x_s}^{y_i y_j}$. Then $M_{ij} \cup \{x_r y_i, x_s y_j\}$ is a weighted induced matching in G. Moreover, if there exists a maximum weight induced matching M in G such that M does not saturate x_0, and M saturates exactly two neighbors y_i, y_j of x_0, and $x_r y_i, x_s y_j \in M$, then $w(M) = w(M_{ij}) + w(x_r y_i) + w(x_s y_j)$. In other words, $M_{ij} \cup \{x_r y_i, x_s y_j\}$ is a maximum weight induced matching in G.*

Proof Since $G_{x_r x_s}^{y_i y_j}$ is constructed from G by removing some of its vertices, M_{ij} is also a weighted induced matching in G. Observe that there is no edge in G joining the two edges $x_r y_i$ and $x_s y_j$. Also, in graph G every vertex saturated by M_{ij} is at distance at least 2 from every vertex in $\{x_r, x_s, y_i, y_j\}$. Hence $M_{ij} \cup \{x_r y_i, x_s y_j\}$ is a weighted induced matching in G.

Now, if M is a maximum weight induced matching in G, and M does not saturate x_0, and M saturates exactly two neighbors y_i, y_j of x_0, and $x_r y_i, x_s y_j \in M$, then M is also a maximum weight induced matching in $G' = G[V\backslash(N_G(x_0)\backslash\{y_i, y_j\})]$. Also, $M\backslash\{x_r y_i, x_s y_j\}$ is a maximum weight induced matching in $G'[V(G')\backslash(N_{G'}(x_r) \cup N_{G'}(x_s) \cup N_{G'}(y_i) \cup N_{G'}(y_j))]$, which is exactly $G_{x_r x_s}^{y_i y_j}$. Hence $w(M_{ij}) = w(M) - w(x_r y_i) - w(x_s y_j)$, that is, $w(M) = w(M_{ij}) + w(x_r y_i) + w(x_s y_j)$. Therefore, $M_{ij} \cup \{x_r y_i, x_s y_j\}$ is a maximum weight induced matching in G. □

Subcase 3.3 M saturates exactly three neighbors y_i, y_j, y_k of x_0, that is, $x_r y_i, x_s y_j, x_t y_k \in M$ where $x_r \in N_G(y_i)\backslash(N_G(y_j) \cup N_G(y_k))$, and $x_s \in N_G(y_j)\backslash(N_G(y_i) \cup N_G(y_k))$, and and $x_t \in N_G(y_k)\backslash(N_G(y_i) \cup N_G(y_j))$.
We construct a weighted convex bipartite graph $G_{x_r x_s x_t}^{y_i y_j y_k}$ in the following way:

Construction 6 First remove all the neighbors of x_0 other than y_i, y_j, y_k from G. Let us call the resultant graph G'. Now define $G_{x_r x_s x_t}^{y_i y_j y_k} = G'[V(G')\backslash(N_{G'}(x_r) \cup N_{G'}(x_s) \cup N_{G'}(x_t) \cup N_{G'}(y_i) \cup N_{G'}(y_j) \cup N_{G'}(y_k))]$.

Lemma 13 $G_{x_r x_s x_t}^{y_i y_j y_k}$ *is a weighted convex bipartite graph.*

Algorithm 2: WEIGHTED-INDUCED-M-TRIAD-CONVEX(G)

Input: A weighted triad-convex bipartite graph $G = (X, Y, E)$ with a triad $T = (X, E^X)$, where $X = \{x_0\} \cup X_1 \cup X_2 \cup X_3$, and positive edge weights $w(e)$ for each $e \in E$.
$|X_1| = n_1$, $|X_2| = n_2$, $|X_3| = n_3$.
Output: A maximum weight induced matching M^* in G.

```
M = ∅, M* = ∅;
Construct G_0 using Construction 2;
Find a maximum weight induced matching M in G_0;
M* = M;
foreach y ∈ N_G(x_0) do
    Construct G_0^y using Construction 3;
    Find a maximum weight induced matching M in G_0^y;
    Update M = M ∪ {x_0 y};
    if w(M*) < w(M) then
        M* = M;
foreach y_i ∈ N_G(x_0) do
    foreach x ∈ N_G(y_i) \ {x_0} do
        Construct G_x^{y_i} using Construction 4;
        Find a maximum weight induced matching M in G_x^{y_i};
        Update M = M ∪ {x y_i};
        if w(M*) < w(M) then
            M* = M;
foreach y_i, y_j ∈ N_G(x_0) such that N_G(y_i) \ N_G(y_j) ≠ ∅ and N_G(y_j) \ N_G(y_i) ≠ ∅ do
    foreach x_r ∈ N_G(y_i) \ N_G(y_j) do
        foreach x_s ∈ N_G(y_j) \ N_G(y_i) do
            Construct G_{x_r x_s}^{y_i y_j} using Construction 5;
            Find a maximum weight induced matching M in G_{x_r x_s}^{y_i y_j};
            Update M = M ∪ {x_r y_i, x_s y_j};
            if w(M*) < w(M) then
                M* = M;
foreach y_i, y_j, y_k ∈ N_G(x_0) such that N_G(y_i) \ (N_G(y_j) ∪ N_G(y_k)) ≠ ∅ and
  N_G(y_j) \ (N_G(y_i) ∪ N_G(y_k)) ≠ ∅ and N_G(y_k) \ (N_G(y_i) ∪ N_G(y_j)) ≠ ∅ do
    foreach x_r ∈ N_G(y_i) \ (N_G(y_j) ∪ N_G(y_k)) do
        foreach x_s ∈ N_G(y_j) \ (N_G(y_i) ∪ N_G(y_k)) do
            foreach x_t ∈ N_G(y_k) \ (N_G(y_i) ∪ N_G(y_j)) do
                Construct G_{x_r x_s x_t}^{y_i y_j y_k} using Construction 6;
                Find a maximum weight induced matching M in G_{x_r x_s x_t}^{y_i y_j y_k};
                Update M = M ∪ {x_r y_i, x_s y_j, x_t y_k};
                if w(M*) < w(M) then
                    M* = M;
return M*.
```

Fig. 14 Algorithm to compute a maximum weight induced matching in a triad-convex bipartite graph

Proof Since $G^{y_i y_j y_k}_{x_r x_s x_t}$ is a subgraph of G_0, and G_0 is a convex bipartite graph (by Lemma 5), $G^{y_i y_j y_k}_{x_r x_s x_t}$ is also a convex bipartite graph. □

Lemma 14 *Let M_{ijk} be a maximum weight induced matching in $G^{y_i y_j y_k}_{x_r x_s x_t}$. Then $M_{ijk} \cup \{x_r y_i, x_s y_j, x_t y_k\}$ is a weighted induced matching in G. Moreover, if there exists a maximum weight induced matching M in G such that M does not saturate x_0, and M saturates exactly three neighbors y_i, y_j, y_k of x_0, and $\{x_r y_i, x_s y_j, x_t y_k\} \subseteq M$, then $w(M) = w(M_{ijk}) + w(x_r y_i) + w(x_s y_j) + w(x_t y_k)$. In other words, $M_{ijk} \cup \{x_r y_i, x_s y_j, x_t y_k\}$ is a maximum weight induced matching in G.*

Proof Since $G^{y_i y_j y_k}_{x_r x_s x_t}$ is constructed from G by removing some of its vertices, M_{ijk} is also a weighted induced matching in G. Observe that there is no edge in G joining any two edges in the set $\{x_r y_i, x_s y_j, x_t y_k\}$. Also, in graph G every vertex saturated by M_{ijk} is at distance at least 2 from every vertex in $\{x_r, x_s, x_t, y_i, y_j, y_k\}$. Hence $M_{ijk} \cup \{x_r y_i, x_s y_j, x_t y_k\}$ is a weighted induced matching in G.

Now, if M is a maximum weight induced matching in G, and M does not saturate x_0, and M saturates exactly three neighbors y_i, y_j, y_k of x_0, and $x_r y_i, x_s y_j, x_t y_k \in M$, then M is also a maximum weight induced matching in $G' = G[V \backslash (N_G(x_0) \backslash \{y_i, y_j, y_k\})]$. Also, $M \backslash \{x_r y_i, x_s y_j, x_t y_k\}$ is a maximum weight induced matching in $G'[V(G') \backslash (N_{G'}(x_r) \cup N_{G'}(x_s) \cup N_{G'}(x_t) \cup N_{G'}(y_i) \cup N_{G'}(y_j) \cup N_{G'}(y_k))]$, which is exactly $G^{y_i y_j y_k}_{x_r x_s x_t}$. Hence $w(M_{ijk}) = w(M) - w(x_r y_i) - w(x_s y_j) - w(x_t y_k)$, that is, $w(M) = w(M_{ijk}) + w(x_r y_i) + w(x_s y_j) + w(x_t y_k)$. Therefore, $M_{ijk} \cup \{x_r y_i, x_s y_j, x_t y_k\}$ is a maximum weight induced matching in G. $\square$

Based on the above discussion, the detailed algorithm for finding a maximum weight induced matching in a weighted triad-convex bipartite graph G is given in Fig. 14.

The following theorem directly follows from Lemmas 6, 8, 10, 12, 14 and the algorithm WEIGHTED- INDUCED- M- TRIAD- CONVEX.

Theorem 6 *A maximum weight induced matching in a weighted triad-convex bipartite graph can be computed in $O(mn^6)$ time.*

Proof The time taken by algorithm is atmost $O(\Delta^6 \times g(n))$, where $g(n)$ is the time complexity of computing a maximum weight induced matching in a convex bipartite graph. By Theorem 1, it is clear that $g(n) = O(m)$. So, the overall time complexity of WEIGHTED- INDUCED- M- TRIAD- CONVEX is at most $O(mn^6)$. $\square$

6 Conclusion

In this paper, we prove the NP-completeness result for the INDUCED MATCHING DECISION problem in the following subclasses of bipartite graphs: star-convex bipartite graphs, comb-convex bipartite graphs and perfect elimination bipartite graphs. On the positive side, we propose an $O(m^2)$-time algorithm for the MAXIMUM WEIGHT INDUCED MATCHING problem in circular-convex bipartite graphs. We also propose an $O(mn^6)$-time algorithm for the MAXIMUM WEIGHT INDUCED MATCHING problem in triad-convex bipartite graphs. Further, it will be interesting to study algorithms with better time complexity for the MAXIMUM WEIGHT INDUCED MATCHING problem in these graph classes.

Acknowledgements The first author thanks the SERB, Department of Science and Technology for their support vide Diary No. SERB/F/12949/2018-2019. The third author wants to thank the Department of Science and Technology (INSPIRE) for their support.

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