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A modified firefly algorithm based on light intensity difference

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Abstract Firefly algorithm (FA) is a swarm-intelligence-based, meta-heuristic algorithm and has been widely applied since its establishment in 2009. In this paper, a modified FA based on light intensity difference (LFA) is proposed. The light intensity of a firefly is determined by the landscape of the objective function in FA. The modifications are established in consideration of the variation trend of light intensity differences. As the light intensity differences vary with movements of fireflies, the parameter settings could be adjusted pertinently and self-adaptively at any moment for different problems. The applications to numeric experiments show that, LFA is well adaptive and efficient for different problems, and can make a trade-off between global exploration and local exploitation so as to decrease the risk of premature convergence effectively.

Keywords Firefly algorithm · Swarm intelligence · Optimization · Light intensity difference

1 Introduction

Nowadays, problems in many fields are highly non-linear and usually have multiple local optimum. To cope with these problems, global optimization algorithms based on swarm intelligence are widely used. Firefly algorithm (FA), one of the recent swarm-intelligence-based global optimization methods, is developed by Yang (2010a), Yang (2010b, 2009) in 2009. It is a kind of stochastic, nature-inspired, meta-heuristic algorithm that can be applied for solving the hardest optimization problems (also NP-hard

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problems) (Fister et al. 2013). The performance study of FA shows that FA is provided with higher efficiency and better accuracy comparing with other algorithms, such as genetic algorithm, in solving non-linear benchmark functions (Yang 2010b; Senthilnath et al. 2011; Bhushan and Pillai 2013; Vashistha et al. 2013; Arora and Singh 2013; Basu and Mahanti 2011). Nowadays, FA and its variants have been adopted for solving many optimization and engineering problems. Farahani et al. (2011) and Nasiri and Meybodi (2012) have studied FA for its applications to continuous optimization problems in dynamic environments. Gandomi et al. (2011), Miguel et al. (2013) and Younes et al. (2014) adopted FA to solve multi-objective continuous/discrete optimization problems. Azad and Azad (2011), Talatahari et al. (2014) and Kamarian et al. (2014) carried out the optimum design of structures with both sizing and geometry design variables based on Hassanzadeh et al. (2011), Horng (2012), Agarwal et al. (2014) segmented and compressed images by using FA in image processing. Besides, Sayadi et al. (2010), Jati (2011) and Marichelvam et al. (2014) used FA to deal with many NP-hard problems, such as travelling salesman problem.

The basic FA has some potential drawbacks such as premature convergence when dealing with multimodal problems with many local peaks and valleys. What's more, it will converge quite slowly while solving optimization problems with large search domain because of the adoption of absolute distance between fireflies. To improve the performance of FA, it is an effective approach to enhance the global exploration and mutation by modifications of parameter settings. And as far as now, there are three methodologies to implement the modifications of parameter settings. First, modify the parameters by optical choice. Yang (2009) has suggested the variation range of light absorption coefficient and randomization coefficient. Arora and Singh (2013) and Mo et al. (2013) studied the optical choice range of each parameter for different cases in numeric experiments. Second, tune the parameters referring to some laws. Bidar and Rashidy (2013) proposed a new method in which fuzzy controller used as parameter controller in FA with the aim of gaining balance between exploration and exploitation. Yang and He (2013), Olamaei et al. (2013) and Gandomi et al. (2013) generated the randomization coefficient using a function to the geometrical annealing schedule. Third, change the structure of some elements. Łukasik and Żak (2009) proposed a novel formulation of light absorption coefficient based on the maximum distance in search domain. Yang (2010b), Gandomi et al. (2011), Yang and Deb (2009) and Farahani et al. (2011) has studied to generate the vector of random numbers with uniform distribution, Gaussian distribution and Levy flights. However, the method of parameters' optical choice can heighten the performance of FA limitedly due to the fixed alterations doesn't improve the conformation of FA essentially. And the other two methodologies can enhance the performance of FA effectively, yet they are not targeted and self-adaptive for various problems. That means, they may be quite helpful in some problems, but is of poor effect in others. Besides, modifications suggested in some literatures are unilateral or segmental. Some modifications can only affect the mutation of FA, and other improvements would influence the global exploration partially. Hence, there are still many problems we need to work on.

The purpose of this paper is to present a modified FA based on the light intensity difference (LFA). The light intensity is determined by the landscape of the objective function in FA. In consideration of the variation trend of light intensity differences through the whole optimization process, the modifications are established. As the light intensity differences vary with the movements of fireflies, the values of parameters could be changed pertinently and self-adaptively at any moment for any problems. Moreover, the experimental study on LFA shows that the LFA is well adaptive and efficient for different problems, and its capacity of global optimization and mutation is enhanced so as to decrease the risk of premature convergence. The structure of this paper is as follows: Sect. 2 discusses the fundamentals of the FA. The bionic principle of FA is stated firstly, and the algorithmic structure is then presented. Section 3 expounds the modifications of FA based on the light intensity difference. The mechanism of movements of fireflies which is related to the light intensity in the FA is discussed. Based on various definitions of light intensity differences, the modifications are then presented. And in Sect. 4, the modified FA is applied to six benchmark functions, and the results are presented following from that.

2 Fundamentals of firefly algorithm

2.1 Bionic principle of FA

Fireflies are characterized by their flashing light produced by biochemical process bioluminescence (Fister et al. 2013). Such flashing light may serve as the primary courtship signals for mating, or to warn off potential predators. And as they are living in harmony, the flashing light behavior is also used to communicate among group members and make their collective decisions in order to achieve the overall goals. Simulating the social behavior of fireflies and the phenomenon of bioluminescent communication, the FA is developed (Mo et al. 2013).

In order to formulate the FA, there are three theoretical rules as following which must be obeyed (Yang 2009): (1) All fireflies are unsex so that one firefly will be attracted to other fireflies regardless of their sex. The brightness of a firefly is determined by the landscape of the objective function, the better position has the higher brightness; (2) Attractiveness is proportional to their brightness, thus for any two fireflies, the less brighter one will be attracted to the brighter one and the attractiveness decrease as their distance increases; (3) If there is no brighter one than a particular firefly, it will move randomly.

In FA, it use the point of the search space to simulate individual firefly in the nature, the process of search and optimization is simulated as the process of attraction and moving of a firefly, the measurement of advantages and disadvantages of individual locations based on objective function, and a firefly move to another which is located a better position in the neighborhood structure so as to evolution its position (Mo et al. 2013). Therefore, there are two important issues: the brightness and attractiveness. Brightness reflects the advantages and disadvantages of the location of the firefly, and determines the direction of movement. Attractiveness decides the moving distance. As the brightness and attractiveness are constantly updated, we can achieve the objective optimization gradually.

2.2 Descriptions of FA

For designing the brightness and attractiveness properly, two important issues need to be defined: the variation of light intensity and the formulation of attractiveness.

In the basic FA, the light intensity I of a firefly representing the solution x is proportional to the value of objective function $I(x) \propto f(x)$, and decreases with the increase in the square of the distance r^2 because of absorption in the light propagation media. Therefore, as the distance from the light source increases, the light absorption causes that light becomes weaker and weaker. And the light intensity $I(r)$ can be given by:

$$I(r) = I_0 \times e^{-\gamma r^2} \tag{1}$$

where I_0 denotes the light intensity of the source, and γ is the light absorption coefficient of the propagation media.

The attractiveness β of a firefly is proportional to its light intensity $I(r)$. Therefore, the attractiveness β can be defined in a similar pattern as Eq. (1):

$$\beta = \beta_0 \times e^{-\gamma r^2} \tag{2}$$

where β_0 is the attractiveness at $r = 0$. In some way, the light intensity I and attractiveness β are synonymous for each firefly. While the intensity is referred to as an absolute measure of flash by the firefly, the attractiveness is a relative measure of its light that should be seen and judged by other fireflies.

The distance between any two fireflies x_i and x_j is expressed as the Euclidean distance by the basic FA, as follows:

$$r_{ij} = \left\| x_i - x_j \right\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \tag{3}$$

where d denotes the dimensionality of the problem, and $x_{i,k}$ is the kth component of the firefly x_i.

While the firefly x_i is attracted to another more attractive firefly x_j, the movement of the ith firefly is determined by the following equation:

$$x_i^{t+1} = x_i^t + \beta_0 e^{-\gamma r_{ij}^2} \times (x_j^t - x_i^t) + \alpha \times (rand - 1/2) \tag{4}$$

where t is the iteration number, α is the randomization parameter deciding the size of random walk, and *rand* is a random number generator uniformly distributed in [0, 1]. For most cases, we can take $\beta_0 = 1$, $\gamma = 1$, and $\alpha \in [0, 1]$. In addition, if the scales vary significantly in different dimensions, it is better to replace α by αS_k where the parameters $S_k (k = 1, \cdots, d)$ are the actual scales of the d-dimensions problem.

The movements of fireflies consist of three terms: the current position of ith firefly, the movement towards to another brighter firefly affected by attraction β, and a random walk constituted by a randomization parameter α and the random generated number *rand*. Therefore, the parameter γ, which mainly determines the attraction β, has a crucial impact on the convergence speed which affects the global exploration of FA, and the parameter α determine the size of random walk which affects the mutation and local exploitation of FA. Hence, lots of studies or modifications are focus on the settings of these three parameters so as to make a balanced trade-off between local exploitation and global exploration for different problems. Schematically, the basic FA can be summarized as the pseudo code in pseudo **Code 1**.

Code 1. Pseudo code of the base Firefly Algorithm

Objective function $f(\mathbf{x}), \mathbf{x} = (x_1, \cdots, x_d)^T$

Generate initial population of fireflies $\mathbf{x}_i \, (i = 1, \cdots, n)$

Light intensity I_i at $\mathbf{x}_i$ is determined by $f(\mathbf{x}_i)$

Define light absorption coefficient γ

While (t<MaxGeneration)

  For i=1:n (all N fireflies)

    For j=1:i (all N fireflies)

      If ($I_j > I_i$), Move firefly i towards j in all dimensions(Apply Eq.**(4)**); ***End if***

      Attractiveness varies with distance r via exp$[-\gamma r^2]$

      Evaluate new solutions and update light intensity

    End for j

  End for i

  Rank the fireflies and find the current best

End while

Post process results and visualization

3 Modified FA based on light intensity difference

In this section, a new modified FA based on the light intensity difference is proposed to change the values of parameters pertinently and self-adaptively at any moment for any problems. The definitions of light intensity difference in different situations are expounded. And the modifications are established with the purpose of enhancing the global exploration and mutation of FA so as to reduce the risk of premature convergence.

3.1 Definitions of light intensity differences

According to Eq. (4) and the pseudo **Code 1**, the direction of movement is determined by the comparison of light intensities of any two fireflies in the basic FA, and the displacement of movement depends on the attractiveness function. Since the light absorption coefficient and the initial attractiveness are generally constant, the attractiveness is only decided by the distance between fireflies. Therefore, the displacement of movement only depends on the distance between fireflies. And that means, light intensities are only used to determine the direction of movement, but not to affect the displacement of movement for different fireflies. However, for any two fireflies with distance r, firefly i both move towards firefly j while they are in different cases of $I_j > I_i$ and $I_j \gg I_i$, and they will possess same displacements on account of the same distance r. However, the corresponding solution of firefly j is obviously better than the corresponding solution of firefly i with different degrees. Accordingly, the movements should be of different displacements. Thinking about this reason, we will make use of the light intensity to determine the displacement of movement according to different situations. Therefore, some modifications for FA are proposed based on light intensity difference which can change the values of parameters pertinently and self-adaptively at any moment for any problems.

So as to measure the light intensity differences in different situations, some definitions of light intensity differences are presented firstly.

Definition 1 The light intensity difference between fireflies in the tth iteration cycle is

$$\Delta I_{ij}^t = I_j^t - I_i^t \tag{5}$$

where $I_j^t > I_i^t$, $i, j \in [1, N]$, $t \in [1, T]$, N is the population size, T is the total iteration generations.

Definition 2 The max light intensity difference in the tth iteration cycle is

$$\Delta I_{\max}^t = \max(I^t) - \min(I^t) \tag{6}$$

where I^t are the light intensities of all fireflies in the tth iteration cycle.

Definition 3 The global light intensity difference in the last t iteration cycles is

$$\Delta I_{\max} = \max(I) - \min(I) \tag{7}$$

where I are all the light intensities of fireflies emerged in the last t iteration cycles.

For any two fireflies with a certain distance r, firefly i need move quickly to firefly j while the value of ΔI_{ij}^t is large, and it implies that there are multiple optimum if ΔI_{ij}^t is small. Similarly, for fireflies with a changeless light intensity difference ΔI_{ij}^t, there are multiple optimum if the distance r is large, and the two fireflies are in the vicinity of the same optima while the distance r is quite small.

As is shown in many optimization processes, light intensity differences of fireflies are generally large at the beginning, hence the inferior fireflies should move towards the superior fireflies with larger spans and randomizations which will speed up the convergence and increase the diversification. And in the last phase fireflies are concentrated near optimum and the light intensity differences are always quite small, so the movement of fireflies should be with smaller spans and randomizations which would increase the capacity of exploitation and suppress the oscillation near the optimum. However, how much should the light intensity difference be that we can say it is big or small? In order to measure the values of the light intensity differences, we will define the ratios of light intensity difference in different cases.

Definition 4 The ratio of light intensity difference between fireflies in the tth iteration cycle is

$$\xi^t = \Delta I_{ij}^t / \Delta I_{\max}^t \tag{8}$$

Hence, while ξ^t is comparatively big at the beginning of optimization, the values of attractiveness function β and randomization coefficient α also should be big; while ξ^t is enough small in the last phase, the values of attractiveness β and randomization coefficient α should turn to small. But, the ratio of light intensity difference ξ^t will change with the iteration number t, so it is proper to choose a ratio of global light intensity difference as the measure criterion.

Definition 5 The ratio of global light intensity difference in the last t iteration cycles is

$$\xi = \Delta I_{ij}^t / \Delta I_{\max} \tag{9}$$

As the global light intensity difference $\Delta I_{\max}$ always changes not big enough, so it is more effective to measure the corresponding solutions by the ratio of global light intensity difference. That means, the larger the value of ξ is, the more inferior the corresponding solutions are; and the smaller the value of ξ is, the more superior the corresponding solutions are. Moreover, the corresponding solutions are in the vicinity of global optimum while the value of ξ is approximately equal to zero.

3.2 The proposed modifications

Based on the definitions of light intensity difference, the modifications for FA are established as follows. The modifications about the light absorption coefficient and initial attractiveness are with the purpose of enhancing the global exploration, and the modification about randomization coefficient could increase the mutation so as to decrease the risk of premature convergence.

3.2.1 Light absorption coefficient γ

As is known, the displacement of movement is mainly determined by the attractiveness β and the parameter γ has a crucial impact on the attractiveness and convergence speed. As the value of light absorption coefficient γ is always constant, the attractiveness β decreases with the increase in the square of the distance r^2 according to Eq. (2). Thinking about the characteristic of exponential term of the attractiveness function, it implies that, if the span of search domain S is rather big, it will decrease the convergence speed and solving efficiency of the FA. Therefore, some modifications are required to make.

Combined with the achievements in literatures, we can let

$$\gamma = \gamma_0 / r_{\max}^2 \tag{10}$$

where $r_{\max} = \max d(x_i, x_j), \forall x_i, x_j \in S$ is the max distance in search domain S, γ_0 is the initial light absorption constant. Based on this modification, the attractiveness will be a second order function of non-dimensional distance which is the ratio of distance r and max distance $r_{\max}$ referred to Eq. (2). Thus, FA can have a suitable convergence speed for any problems.

On the other hand, a characteristic distance over which the attractiveness changes significantly from β_0 to $\beta_0 e^{-1}$ is defined as

$$\Gamma = 1/\sqrt{\gamma} \tag{11}$$

Mathematically, the characteristic distance Γ controls the average distance of a group of fireflies that can be seen by adjacent groups and affects the global exploration of FA. Therefore, the characteristic distance of the modified FA in this paper is

$$\Gamma = r_{\max} / \sqrt{\gamma_0} \tag{12}$$

As a result, the global exploration of the modified FA would be strong enough if $\gamma_0 \leq 4$, and the local exploitation of the modified FA shall be better when the value of γ_0 is fairly large. Hence, it should determine the value of γ_0 based on practical situations in applications.

3.2.2 Initial attractiveness β_0

As is shown in Eq. (2), the value of the attractiveness β is affected by the initial attractiveness β_0 which determines its amplitude and has a crucial effect on the convergence speed of FA. Thinking about the common pattern of optimization process, we can let

$$\beta_0 = \begin{cases} \xi (\xi > \eta_1) \\ \eta_1 (\xi \leq \eta_1) \end{cases} \tag{13}$$

As $\xi \in [0, 1]$, so the value of β_0 is changed in the domain of $\left[\eta_1, 1\right]$. At the same time, thinking about that η_1 with a very small value may lead to a slow convergence speed in the last phase, we can let the value of η_1 is 0.3.

So, if the ratio of light intensity difference is generally large at the beginning of optimization process, large values of initial attractiveness β_0 will increase the convergence speed and the possibility to find diverse optimum. With the iterations carrying on, the ratio of light intensity difference and the displacements of movements will decrease. While the value of β_0 equals to η_1 with no decay in the last phase, fireflies are all in the vicinity of optimum, and small displacements of movements which is affected by a small value of β_0 will lead fireflies to move towards to optimum slowly with little oscillations.

3.2.3 Randomization coefficient α

As the randomization coefficient α essentially control the randomness of the FA, a large value of the randomization coefficient α will encourage fireflies to explore unknown regions and avoid the FA trapping in local optimum. At the same time, a small value will let fireflies focusing on local exploitation with little oscillations. Similarly, we can let

$$\alpha = \alpha_0 \times 0.02 r_{\max}, \alpha_0 = \begin{cases} \xi & (\xi > \eta_2) \\ \eta_2 & (\xi \leq \eta_2) \end{cases} \tag{14}$$

where α_0 is the randomization ratio, the value of η_2 may be arranged to 0.1. The constant 0.02 comes from the fact that random walks requires a number of steps to reach the optimum for fireflies while balancing the local exploitation without jumping too far in a few steps.

Based on this modification, the random walks of FA will be fairly big at the beginning of optimization process. Hence, the mutation of FA will be enhanced so as to avoid premature convergence. And in the last phase, the random walks will be small enough to suppress the oscillation and enhance the local exploitation.

4 Experiments and results

In this section, numeric experiments are designed to study the performance of LFA and verify the availability of the modifications. Some benchmark functions are tested in experiments, and simulation results of LFA are compared with other algorithms. Finally, some important conclusions have been got.

4.1 Numeric experiments

Performance of LFA is tested on six typical benchmark functions (Table 1) which have been extensively used in literatures (Yang 2009; Bhushan and Pillai 2013; Arora and Singh 2013; Mo et al. 2013; Farahani et al. 2011). The six benchmark functions could be subdivide in two types. The previous three functions, which are highly nonlinear with lots of local optimum, are adopted to verify the global exploration and mutation of different algorithms. And remaining three functions are used to verify the local exploitation and convergence speed of algorithms. The function, the equation, the admissible range of the variable x_i, and the global optima are summarized in Table 1. Each experiment was run with uniform random initial values of x in range $[x_{\min}, x_{\max}]$ indicated in Table 1. During the optimization process the fireflies were not allowed to fly outside the region defined by $[x_{\min}, x_{\max}]$. For simplicity, the simulations of the six

Table 1 Informations of benchmark functions

Function	Equation	Range x_i	Global optima
Rastrigin	$f_1(x) = \sum_{i=1}^{D} \left[x_i^2 - 10\cos(2\pi x_i) + 10\right]$	[−5.12, 5.12]	0
Ackley	$f_2(x) = 20 - 20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)\right) + e$	[−10, 10]	0
Schaffer	$f_3(x) = \frac{\sin^2\sqrt{\sum_{i=1}^{D} x_i^2} - 0.5}{\left[1 + 0.001(\sum_{i=1}^{D} x_i^2)\right]^2} + 0.5$	[−50, 50]	0
Rosenbrock	$f_4(x) = \sum_{i=1}^{D-1}\left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right]$	[−2, 2]	0
Schwefel	$f_5(x) = \sum_{i=1}^{D} \lvert x_i\rvert + \prod_{i=1}^{D} \lvert x_i\rvert$	[−10, 10]	0
Sphere	$f_6(x) = \sum_{i=1}^{D} x_i^2$	[−50, 50]	0

benchmark functions are all with 2 dimensions. Because all of the six test functions have the unique global optima 0 at [0,0], we can view the convergence process of algorithms intuitively and effectively only through the convergence curves of function fitness.

The genetic algorithm (GA) (Goldberg 1989) and particle swarm optimization (PSO) (Kennedy and Eberhart 1995; Kennedy et al. 2001), two typical bio-inspired meta-heuristic algorithms, are widely used in performance researches (Yang 2010b; Yang 2009; Gandomi et al. 2011; Yang and Deb 2009; Mohammadi et al. 2013). Hence, the GA, PSO and the basic FA are adopted to be the contrast. The population sizes of groups are 20 all the time, and other parameter settings of the four algorithms are illustrated as follows:

(1) GA: mutation probability $p_m = 0.05$, crossover probability $p_c = 0.8$, gene length $l = 10$;
(2) PSO: weight constant $w = 1$, learning parameters $\alpha = \beta = 2$;
(3) FA: $\beta_0 = \gamma = 1, \alpha = 0.02$;
(4) LFA: $\gamma_0 = 4, \eta_1 = 0.3, \eta_2 = 0.1$.

4.2 Results and discussions

When the initial positions were determined randomly, the numeric experiments can be carried out by compiling MATLAB programs. The results of numeric experiments are shown in Table 2, and the function fitness which are varied with iteration numbers are illustrated in Figs. 1, 2, 3, 4, 5, and 6. The fitness in figures means the transient

Table 2 Comparison of results of different algorithms

Algorithm	Optimal values					
	Rastrigin	Ackley	Schaffer	Rosenbrock	Schwefel	Sphere
GA	0.089	0.0442	0.037377	1.5344e−4	0.0196	4.777e−3
PSO	8.45e−4	0.2506	0.078663	1.4423e−4	0.01416	8.069e−3
FA	8.158e−5	0.0018	0.22769	8.331e−8	0.0953	0.62305
LFA	1.79e−5	3.404e−4	3.1229e−6	1.502e−9	1.73077e−4	5.0906e−8

values of objective functions during the optimization process. Besides, the ranges of vertical axis are trimmed in order to exhibit the comparison of convergence curves more clearly.

There are many criteria in literatures for evaluating the performance of the algorithms. Here, the successful optimization is defined as error of optimal values are not big than 0.001. As is shown in Table 2, LFA has achieved all the six global optimum successfully in the light of this criteria. It implies that LFA has better performance of global exploration and mutation which lead to get rid of trapping into local optimum and approach the global optima finally. Moreover, LFA also has better performance of local exploitation, which means better computational accuracy, than other algorithms. In addition, the basic FA has terrible results for the functions of Schaffer and Sphere owing to their large search domain.

For the previous three functions which are of many peaks and valleys, the goal of numeric experiments is to verify the LFA's capacity of global exploration and mutation. The comparisons of convergence curves are shown in Figs. 1, 2, and 3. In general, LFA converges quickly to the global optima without trapping into local optimum for all the three functions, which demonstrates the good capacity of global exploration and mutation to avoid premature convergence. And, the basic FA have a better performance of optimization efficiency than GA and PSO for functions of Rastrigin and Ackley as is shown in Figs. 1 and 2. In addition, because of a large search domain in Schaffer Function, both of the attractiveness and random walks of FA appears to be so small that the convergence curve of FA seems to be changeless as is illustrated in Fig. 3.

For the rest three functions which are of only one optima, the goal of numeric experiments is mainly to test the local exploitation and convergence speed of LFA. The comparisons of function fitness are illustrated in Figs. 4, 5, and 6. Due to a small search domain for the Rosenbrock Function, all the four algorithms find the optimal solution successfully in the given iteration generation, yet LFA is more efficient and more precise referred to Fig. 4 and Table 2. Similar to the functions of Schwefel and Sphere, LFA approach the global optima rapidly which reveals its advantage of local exploitation and convergence speed. And owing to the large span of search domain which lead to small value of attractiveness, FA converges so slowly that it couldn't reach the global optima in the given generations. As is implied in Figs. 5 and 6, the larger the span of search domain, the slower the convergence speed of FA.

In summary, the results of numeric experiments have demonstrated the advantage of LFA. Based on the proposed modifications, LFA is not only well adaptive, but also

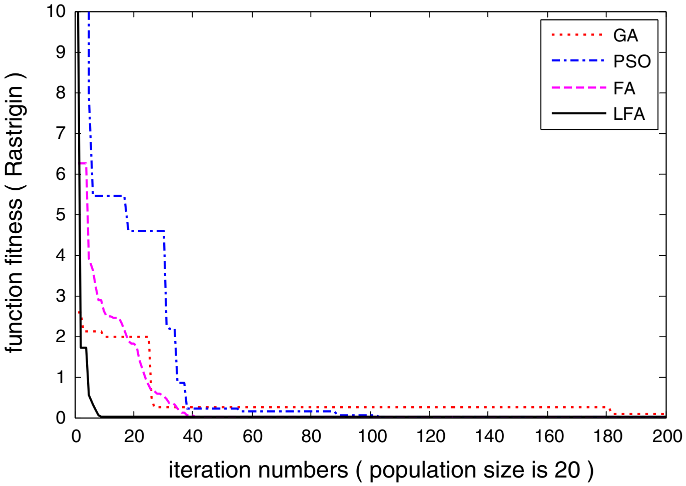

Fig. 1 Comparison of function fitness of Rastrigin function

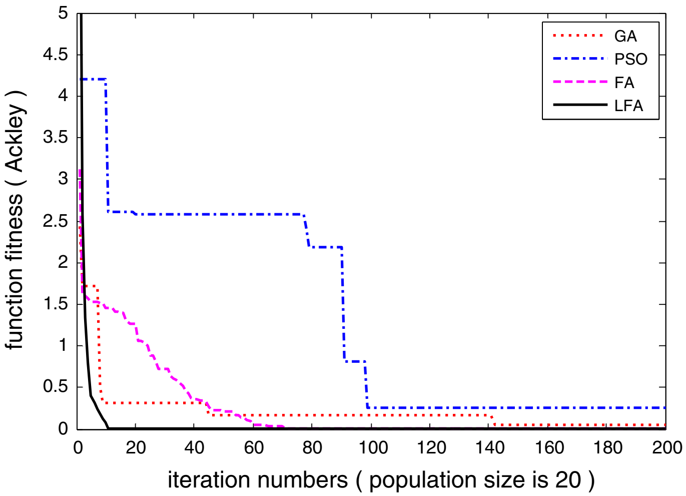

Fig. 2 Comparison of function fitness of Ackley function

of stronger ability of global exploration and local exploitation than other algorithms. After improving, the attractiveness is a function of non-dimensional distance which is the ratio of absolute distance between fireflies and max distance of search domain, and the random walk is also related to the max distance of search domain. As a result, the convergence speed is upgraded and the risk of premature is reduced effectively, especially for multimodal problems with large search domain. Hence, the modified FA could have a suitable ability of global exploration and mutation for any optimization problems. At the same time, parameter settings of initial attractiveness and randomization ratio are proposed to depend on the ratio of global light intensity difference. And as the light intensity differences vary with the movements of fireflies, their values can

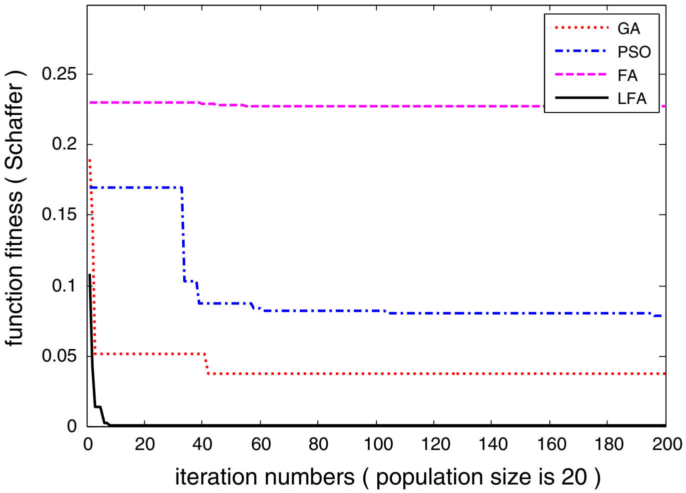

Fig. 3 Comparison of function fitness of Schaffer function

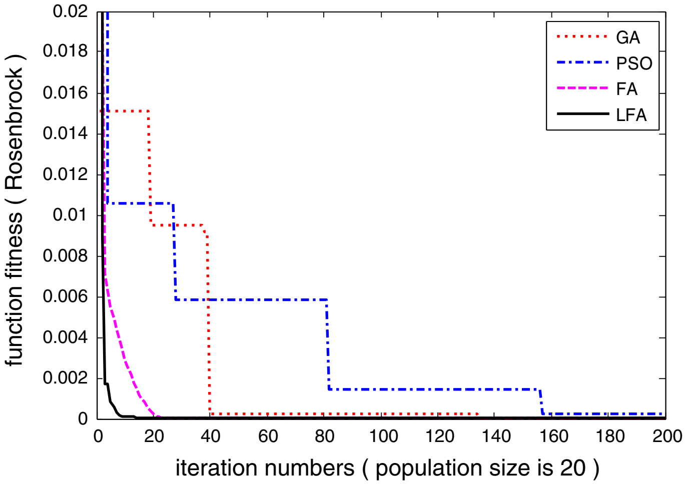

Fig. 4 Comparison of function fitness of Rosenbrock function

be adjusted pertinently and self-adaptively at any moment for different problems. As a result, the capability of global exploration and mutation for LFA will be reinforced at the beginning, while the ability of local exploitation will be enhanced in the last phase. Additionally, we can adjust the performance of LFA and make a trade-off between local exploitation and global exploration by tuning the parameter settings involved in the modifications mentioned above.

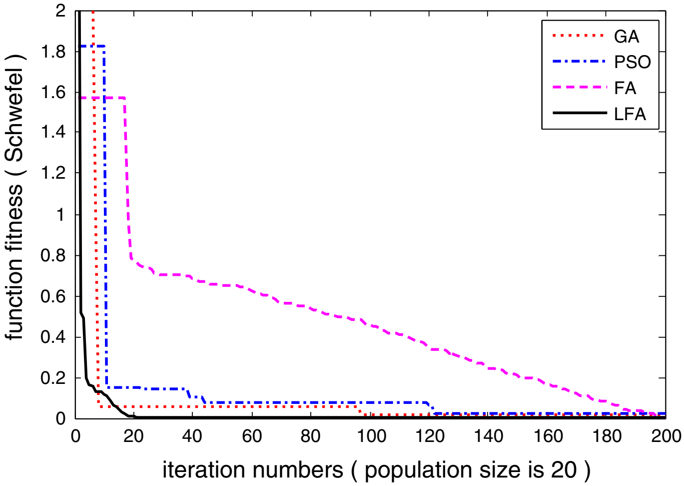

Fig. 5 Comparison of function fitness of Schwefel's problem

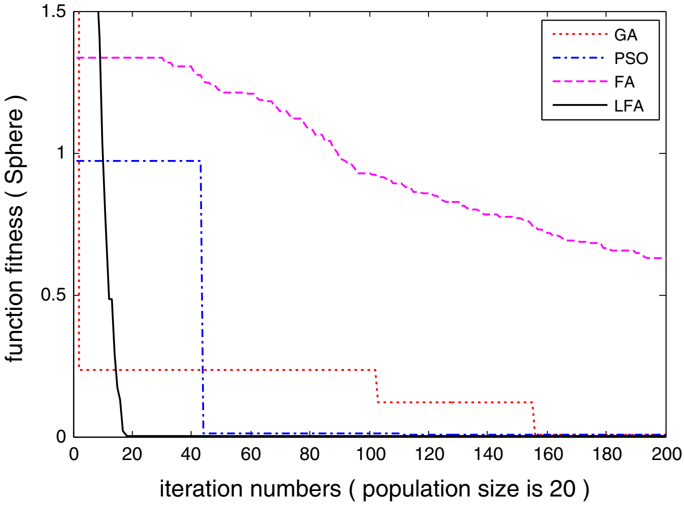

Fig. 6 Comparison of function fitness of Sphere function

5 Conclusions

This paper proposed a modified FA based on the light intensity difference (LFA). In FA, the light intensity is determined by the landscape of the objective function. After analyzing the movement mechanism of fireflies, we find that the light intensity can not only decide the direction of movement, but also be used to determine the displacement of movement according to practical situations. Therefore, the modifications, which can alter the values of parameters pertinently and self-adaptively at any moment for different problems, are established in consideration of the variation trend of light intensity differences. The structure improvements about the light absorption coefficient and randomization coefficient are suggested to avoid converging slowly and enhance the mutation for problems with large search domains. At the same time, initial attractiveness and randomization ratio are depending on light intensity differences. Thus, their values will decrease gradually and self-adaptively in pace with light intensity differences of fireflies trend to zero by the whole optimization process. As a result, the capability of global exploration and mutation for LFA will be reinforced at the beginning, while the ability of local exploitation is enhanced in the last phase. In order to effectively verify the performance of LFA, we make a carefully comparison between GA, PSO, the basic FA and LFA. The investigated results show that, when comparing with GA, PSO and the basic FA, LFA is well adaptive and efficient, and it also can perform much better in terms of global optimization and mutation so as to decrease the risk of premature convergence.

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