ORIGINAL PAPER

Efficacy of Two Mathematics Interventions for Enhancing Fluency with Elementary Students

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Published online: 12 November 2010 - Springer Science+Business Media, LLC 2010

Abstract An alternating treatments design was used to evaluate two curriculumbased mathematics interventions designed to enhance fluency with three elementary school students. Results indicate that both the Math to Mastery (MTM) intervention and the Cover, Copy, Compare (CCC) intervention were effective at increasing mathematics fluency, as measured by digits correct per min, for all students. However, MTM was more effective than CCC for two of the three students. Followup data taken 6 and 18 days following termination of the intervention phase indicated that all participants achieving mastery performance maintained mastery levels across both interventions.

Keywords Cover · Copy · Compare · Math to Mastery · Mathematics · Interventions - Generalizations

Introduction

Many students in the educational system do not obtain math skills at the appropriate grade level, placing them at risk for negative outcomes including failure to meet state-derived benchmarks on high stakes testing, retention, and dropout (Rhymer et al. [2000\)](#page-14-0). Reports estimate that approximately 5–8% of school-aged students have skill deficits in math (Geary [2004](#page-14-0); Kosc [1974](#page-14-0)). Math skill deficits were also found to be the second highest factor in the identification of a learning disability (Kavale and Reese [1992](#page-14-0)). These findings suggest that effective interventions are needed to prevent and/or remediate math skill deficiencies.

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Several procedures such as repeated practice, corrective feedback, and modeling have been found to increase accuracy and automatic responding to basic mathematics facts (McCallum et al. [2004](#page-14-0); Skinner et al. [1989;](#page-15-0) Skinner and Smith [1992\)](#page-15-0). Additionally, these procedures have been shown to elicit high rates of active academic responding, which can increase both response speed and maintenance, contingent on accurate responses (Skinner et al. [1989\)](#page-15-0). One specific intervention designed to address mathematics skill deficits is Cover-Copy-Compare (CCC). CCC is an effective mathematics intervention designed to improve both accurate and fluent responding across a variety of mathematics calculation skills (Skinner et al. [1989,](#page-15-0) [1993;](#page-15-0) Grafman and Cates [2010](#page-14-0)). CCC is a self-managed intervention that provides a series of learning trials within a short period of time. Overall, CCC has been cited as a pragmatic intervention that has been effective in addressing mathematics skill deficits as well as problems in spelling and writing skills (McLaughlin and Skinner [1996](#page-14-0); Saecker et al. [2009](#page-14-0)).

Several factors may contribute to CCC's effectiveness. First, the brief time required for each CCC learning trial allows students to complete many learning trials in a minimal period of time. However, providing many learning trials will only increase accuracy levels if students are practicing accurate responses. The immediate self-evaluation component of CCC likely prevents students from practicing inaccurate responses. Immediate self-correction ensures that the last response within each CCC learning trial is correct. Therefore, self-correction may prevent future errors due to recency effects. Responding immediately after looking at the academic stimuli also minimizes response errors during CCC learning trials (Skinner et al. [1997](#page-15-0)).

Math to Mastery (MTM) is another intervention that has shown promise in applied settings. MTM is a structured intervention package that includes previewing problems, repeated practice, immediate corrective feedback, summative and formative feedback, and self-monitoring of progress (Doggett et al. [2006\)](#page-14-0). MTM effects on student fluency have been examined in clinic settings (Hoda [2006\)](#page-14-0) as well as school settings (Miller [2007;](#page-14-0) Mong [2008\)](#page-14-0). In these studies, MTM effectively enhanced students' fluency in basic calculations. A rapid increase in level from baseline to an intervention phase was observed in most of the participants in these studies. MTM was also effective in promoting the generalization of skills from instructional to grade-level material.

The combination of several components appears to contribute to MTM's efficacy. In the MTM intervention package, modeling appears to be an effective component because it provides an example of the correct manner in which to solve math problems, the appropriate rate at which the math problems should be solved, and the expected digits correct per min (DCPM) to achieve mastery and discontinue the session. Repeated practice and immediate corrective feedback have also been shown to be effective at increasing academic performance (e.g., Anderson [1982;](#page-13-0) Darch et al. [1984](#page-14-0); Greenwood et al. [1984](#page-14-0); Harber et al. [2003,](#page-14-0) [2004;](#page-14-0) Skinner and Shapiro [1989\)](#page-15-0). In relation, the combination of repeated practice and immediate corrective feedback components is believed to be responsible for improvement in math fluency. Furthermore, goal setting, progress monitoring, self-charting, and reinforcement for achievement of a mastery criterion have been found to be

effective interventions for improving a number of behaviors including academic skills deficits (e.g., Ayllon and Roberts [1974](#page-13-0); Coding [2003](#page-14-0); Brown et al. [1986;](#page-13-0) Jackson and Mathews [1995;](#page-14-0) Rhymer et al. [2000](#page-14-0); Skinner et al. [2000\)](#page-15-0). By setting precise, clearly defined goals, students are made aware of the expectations placed upon them. Progress toward these goals can then be monitored in a manner in which the student knows exactly where he/she stands in relation to achieving the set criterion.

Both CCC and MTM interventions include modeling, practice, immediate corrective feedback, and reinforcement components. Despite these similarities, the implementation of each of these intervention components is quite different between the two interventions. For example, the modeling and feedback components of the CCC intervention are delivered via the worksheet; whereas in MTM, the correct answers are provided by the interventionist.

Additionally, although both interventions provide repeated practice, the CCC intervention requires additional practice only for incorrect responses whereas MTM requires additional practice for incorrect and/or slow responses. Similarly, both interventions also provide reinforcement, but CCC does not provide social reinforcement that is provided by the interventionist in MTM. Despite similar core components, the variations in method of delivery of each strategy (e.g., modeling, practice, immediate corrective feedback, and reinforcement) could yield different results in math performance and require comparison. Given these differences, it is hypothesized that MTM may be more effective for the development of skill acquisition primarily due to the fact that MTM requires additional practice for incorrect and/or slow responses.

Although both CCC and MTM have some evidence supporting their efficacy, further research is warranted to examine the relative effects of one intervention package versus the other with regard to individual student performance. Furthermore, although there is a wide base of research to support the use of specific instructional procedures for promoting the acquisition and fluency of math skills (Skinner [1998\)](#page-15-0), research regarding instructional practices to systematically program for the generalization of math skills is less developed (Poncy et al. [2010](#page-14-0)). In the current study, we extend the research on CCC and MTM by comparing the effects of the two interventions on basic math fluency and generalization. Specifically, we address the following question: What are the relative effects of MTM and CCC on DCPM and errors of three second-grade students struggling in mathematics?

Method

Participants and Setting

Participants were three second-grade students from a suburban school district in the Southeastern United States with approximately 1,000 students (grades Pre-K-4); 47% of whom qualified for free or reduce-price lunch and 32% of whom were from ethnic minority backgrounds. Brandon was a 7-year-old Caucasian boy, Kathy was an 8-year-old Latin American girl, and Mike was an 8-year-old African American boy. The students who participated in the study provided student assent, and parental consent was obtained consistent with procedures outlined by the university Institutional Review Board.

In addition, the referred students were not receiving special education services or other mathematics interventions according to school records. The students for this study were identified based on teacher recommendation and on their performance in math as measured by three curriculum-based measurement (CBM) probes previously administered during local norming procedures. Three different secondgrade-level multiple skill probes were used. The local norming probes were generated using the Curriculum-Based Assessment Math Computation Probe Generator available on the website www.interventioncentral.org (Intervention Central n.d.) and were timed for 2 min. State benchmarks were used to determine which skills were representative of each grade level. Based on district level identification procedures, students identified were those who scored in the lowest quartile in each classroom based on the local norms. The second author served as the interventionist and implemented all intervention and progress-monitoring procedures. All sessions were conducted in an empty resource room.

Materials

Calculation problems were also generated using Curriculum-Based Assessment Math Computation Probe. The program allows the user to design worksheets requiring the use of specific skills. State benchmarks were used to determine which skills were representative of each grade level. The program was then used to create a worksheet specific to a particular grade level and state benchmark. The computer program randomized (a) the order of problems within a worksheet and (b) the order of the factors within each problem. All worksheets included a number to identify the worksheet within the grade level and blank lines for name, date, and examiner.

The MTM instructional worksheets contained 24 problems of the targeted single skill with 6 rows of 4 problems. On each CCC worksheet, 4 calculation problems with correct answers were listed on the left side of the worksheet, whereas the same calculation problems without answers were listed on the right side of the worksheet. For each session, 6 CCC worksheets (24 total problems of the targeted single skill) consisting of different problems were used. Generalization grade-level worksheets were comprised of 24 mixed skill problems.

Dependent Measures and Scoring Procedures

The number of digits computed correctly per min (DCPM) served as the primary dependent variable for this study. A digit was scored as correct when the appropriate number was written in the proper column (Shinn [1989\)](#page-15-0). Based on guidelines by Shapiro [\(2004](#page-15-0)), DCPM were calculated by dividing the number of digits correct by the total number of seconds and multiplying by 60. Errors per min (EPM) served as a secondary dependent variable. Responses were scored as errors if incorrect digits were written below the line or if digits were written in the wrong place or omitted.

Experimental Design and Procedures

An alternating treatments design (ATD) was used within each intervention phase with four phases consisting of the following: (a) curriculum-based assessment (CBA), (b) baseline, (c) CCC, and (d) MTM. The ATD has a number of distinct advantages as it does not require withdrawal of treatment and allows for treatment comparisons more quickly than in a withdrawal design (Hayes et al. [1999\)](#page-14-0).

Only one intervention was implemented per day, and the order of presentation was counterbalanced across sessions. The order of the first intervention sequence was randomly chosen for each participant. Each intervention was implemented during an equal number of days with the intervention phase lasting 28 days. Prior to the session, students were told specifically what intervention would be administered. Each intervention's probes were printed on different color paper (blue for CCC, red for MTM) to help the students further differentiate between the interventions (Codding et al. [2007;](#page-13-0) Montarello and Martens [2005\)](#page-14-0).

CBA

The students' instructional levels on mixed skill grade-level probes were established using standardized CBA procedures outlined by Shapiro ([2004\)](#page-15-0). According to Burns et al. [\(2006](#page-13-0)), a student's independent instructional level was the point in the curriculum where he or she could complete math problems with 14–31 digits correct if enrolled in Grades 1–3. Mastery performance was achieved if a student obtained 31 or more digits correct. All students' instructional level was identified as 2nd grade; Brandon obtained 15 DCPM with 6 EPM, Kathy obtained 16 DCPM with 3 EPM, and Mike obtained 18 DCPM with 1 EPM. The specific instructional skill identified for remediation with Brandon and Kathy was addition with one, two-digit number plus one, one-digit number without regrouping (i.e., $22 + 5$). Error analysis revealed both Brandon and Kathy committed the majority of their errors on this type of problem. This finding was also confirmed by their classroom teachers. Mike's instructional level intervention included subtraction using two, two-digit numbers without regrouping (i.e., 49–31). Despite his relative lack of errors, Mike's teacher reported that he frequently struggled with subtraction problems involving two, two-digit numbers without regrouping.

Baseline

During each session, students attempted to complete one CBM probe consisting of 24 problems in the targeted single skill according to Shinn's ([1989\)](#page-15-0) instructions. Each student also attempted to complete a single mixed skill grade-level generalization probe. Students received no intervention during the baseline phase. Both baseline probes were administered for 2 min.

Cover, Copy, Compare Intervention

All students used CCC to practice mathematics facts at their respective instructional levels. CCC consisted of 5 steps (Codding et al. [2007](#page-13-0)): (a) look at the problem with

the answer, (b) cover the problem with an index card, (c) write the solution to the problem without the answer, (d) uncover the problem and solution, (e) compare your answer. If the students' answer did not match the pre-recorded answer, students were told to repeat the steps. Prior to implementation, the examiner explained all five steps and demonstrated the steps before the first session. A 2-min CBM probe in the target skill was administered using standard instructions (Shinn [1989\)](#page-15-0) after the 5-min practice session.

Math to Mastery Intervention

All students used MTM to practice mathematics facts at their respective instructional levels. The following steps of intervention were based on the *Math* to Mastery intervention manual developed by Doggett et al. [\(2006](#page-14-0)): (a) the examiner completed each math problem on the probe while the student followed along, (b) the student then practiced completing each math problem on the math probe in a series of 1-min trials until a mastery criterion of 32 digits correct was obtained, (c) while the student was completing each problem, the interventionist followed along, marking digits in error and giving immediate corrective feedback, (d) immediately after each 1 min math trial, the examiner calculated and informed the student of his or her DCPM for the trial while also offering ample social praise for effort and performance on each trial and recording DCPM and EPM, (e) the student completed the self-monitoring chart at the end of each 1-min math trial to visually display his or her ongoing performance. On average, the MTM sessions lasted 10 min for each participant. A 2-min CBM probe in the target skill was administered using standard instructions (Shinn [1989\)](#page-15-0) following the intervention.

Generalization

Generalization of students' math fluency was assessed using multiple skill gradelevel probes similar to those used in baseline. Problems included: addition with two, one-digit numbers with and without regrouping (i.e., $3 + 4$, $9 + 7$), addition with one, two-digit number and one, one-digit number without regrouping (i.e., $11 + 3$), addition with two-digit numbers without regrouping (i.e., $20 + 10$), subtraction with two, one-digit numbers (i.e., $7 - 4$), subtraction with one, two-digit number and one, one-digit number without regrouping (i.e., $15 - 4$), and subtraction with two, two-digit numbers without regrouping (i.e., $27 - 13$). The mixed skill gradelevel probes were administered after a student completed each intervention sequence (CCC and MTM).

Follow-Up Data

To examine the maintenance of skills, follow-up data were collected 6 days and 18 days after the last intervention day. A novel single skill instructional probe was used to assess maintenance of the targeted skill. Brandon and Kathy completed follow-up probes assessing addition with one, two-digit number plus one, one-digit number without regrouping (i.e., $22 + 5$), while Mike completed follow-up probes assessing subtraction using two, two-digit numbers without regrouping (i.e., 49–31).

Treatment Integrity

Gresham, Gansle, and Noell ([1993\)](#page-14-0) defined treatment integrity as the degree to which treatments are implemented as they were designed. Poor treatment integrity can compromise the validity of the findings of an experiment. If change occurs during the intervention that was not implemented as designed, one cannot conclude the change was a result of the intervention. Therefore, treatment integrity was evaluated during 33% of the sessions evenly distributed across all phases of the study based on completion of a checklist completed during the session. Three separate checklists (Baseline, CCC, MTM) were used to assess the treatment integrity. Each checklist was comprised of the necessary steps to complete each phase of the study. Treatment integrity was calculated by the number of items on the checklist completed correctly divided by the total number of items on the checklist and multiplied by 100. Treatment integrity for this study was 100%.

A second observer was also used to assess treatment integrity for a minimum of 35% of the sessions evenly distributed across all phases of the study. The second observer marked all items on the treatment integrity checklists that were correctly completed. IOA for the treatment integrity was calculated by dividing the agreed upon number of steps completed for each session divided by the number of available steps to complete for each session and multiplying this ratio by 100. Average IOA was 96%.

Interscorer Agreement

The primary author designed a list of scoring instructions for the mathematics probes used in the investigation. Two scorers and the primary author scored a sample of 15 probes independently. Interscorer agreement was calculated by dividing the number of agreements plus disagreements per digit by the number of agreements plus disagreements per digit and multiplying by 100. The rules were clarified and revised until there was at least 90% agreement on a set of 45 sample probes. Scorers were then cleared for scoring of the probes used in this study. The primary author was routinely available to discuss their questions regarding scoring specifics. Approximately 33% of the total probes were independently scored by the two scorers across all phases of the study. Actual interscorer agreement for this study was 98%.

Social Validity

To measure the social validity of the interventions, students were administered a seven-item questionnaire adapted from the Children's Intervention Rating Profile (CIRP) (Turco and Elliott [1986\)](#page-15-0). The CIRP was also modified to assess students' acceptability of math interventions in previous studies (e.g., Arra and Bahr [2005\)](#page-13-0). The questionnaire consisted of seven items rated on a Likert scale of 1 (strongly disagrees with the statement) to 6 (strongly agree). Total acceptability scores on the modified CIRP for the CCC intervention were 38 (Brandon), 35 (Kathy), and 36 (Mike) of 42, where 42 indicates the most acceptable score possible. All students strongly agreed that they liked the CCC intervention, felt that the CCC intervention would help them in school, and did not think there were better ways to help improve their math fluency. Total acceptability scores on the modified CIRP for the MTM intervention were 34 (Brandon), 34 (Kathy), and 33 (Mike) of 42. All three students agreed that they liked the MTM intervention and felt that the MTM intervention would help them in school.

Data Analysis

Because the MTM intervention is relatively new, it may require more scrutiny than established interventions (Beeson and Robey [2006\)](#page-13-0). Thus, additional data analytic techniques were employed to examine the efficacy of the intervention in addressing the math fluency of the students. Effect size was estimated using the percentage of non-overlapping data points (PND) (Olive and Smith [2005](#page-14-0)).

PND is calculated by dividing the number of non-overlapping data points with baseline by the total number of intervention data points. Because this study was designed to improve student academic skills, the highest baseline data point was used to establish the overlap of baseline data points with intervention data points. Benchmarks for PND scores have also been established by Scruggs and Mastropieri [\(2002](#page-15-0)). Specifically, PND scores below 50% suggest an ineffective intervention effect, PND scores between 50 and 70% suggest that a questionable intervention effect, PND scores between 70 and 90% suggest an effective intervention effect, and PND scores above 90% suggest a very effective intervention effect.

Results

Each student's DCPM on the targeted single skill is presented in Fig. [1](#page-8-0). During baseline, Brandon obtained a mean of 15.8 (range 15–17) DCPM. He displayed a gradual increase in level, trend, and variability for both CCC and MTM across the intervention phase. After the first four intervention sessions, divergence between the interventions was observed with Brandon obtaining a mean of 24.7 (range 17–33) DCPM under the MTM intervention compared to a mean of 21.3 (range 16–27) DCPM under the CCC intervention. Brandon achieved mastery performance (32 DCPM) on the final three sessions of MTM whereas he failed to achieve mastery performance under the CCC intervention. Brandon achieved mastery performance during both MTM follow-up sessions when compared to his performance remaining relatively level below the mastery criterion during the CCC follow-up sessions. The PND was 79% for CCC, suggesting that it was effective at increasing Brandon's fluency. The PND was 86% for MTM, suggesting that it was slightly more effective at increasing Brandon's fluency than CCC.

Kathy obtained a mean of 19.5 DCPM (range 18–20) during the baseline phase. She displayed a gradual increase in level and trend for both CCC and MTM across

0 1 2 3 4 -5 -6 -7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 **SESSIONS**

Fig. 1 Digits correct per minute (DCPM) across baseline, interventions (Math to Mastery (MTM) and Cover, Copy, Compare (CCC)), and follow-up phases for Brandon, Kathy, and Mike

the intervention phase. Divergence occurred during sessions 11–13 and sessions 15–18 as MTM yielded higher fluency rates during the earlier sessions and CCC yielding higher fluency rates during the latter divergent sessions. Overall, Kathy obtained a mean of 28.9 (range 22–38) DCPM during the CCC intervention and 29.6 (range 22–37) DCPM during the MTM intervention. She achieved and maintained mastery performance during the final five CCC sessions and the final seven MTM sessions. Both interventions were successful at maintaining Kathy's mastery performance during the follow-up sessions. The PND was 100% for CCC, suggesting that it was effective at increasing Kathy's fluency. The PND was 100% for MTM, suggesting that that the two interventions were equally effective at increasing Kathy's fluency (Fig. [2\)](#page-9-0).

During baseline, Mike obtained a mean of 20.8 (range 19–22) DCPM. He displayed a gradual increase in level and trend for both CCC and MTM across the

Fig. 2 Errors per minute (EPM) across baseline, interventions (Math to Mastery (MTM) and Cover, Copy, Compare (CCC)), and follow-up phases for Brandon, Kathy, and Mike

intervention phase. Divergence between the interventions was initially slight but increased over time as Mike obtained a mean of 26.9 (range 21–35) DCPM under the CCC intervention compared to a mean of 31.1 (range 24–38) DCPM under the MTM intervention. Mike achieved mastery performance on the final three CCC sessions whereas he achieved mastery performance during the final eight sessions under the MTM intervention. Mike maintained mastery performance during all follow-up sessions across both interventions. The PND was 67% for CCC, suggesting that it was effective at increasing Mike's fluency. The PND was 100% for MTM, suggesting that it was more effective at increasing Mike's fluency than CCC.

Both interventions were effective in decreasing errors as Brandon displayed an immediate decrease in both level and trend across both interventions when compared to baseline. He committed a mean of 4.3 EPM (range 4–5) during baseline, a mean of .79 EPM (range 0–2) under CCC, and a mean of .64 EPM (range 0–3) under the MTM intervention. During the follow-up phase, Brandon maintained low error rates only making one error in the first CCC follow-up.

Both interventions were also effective in decreasing errors for Kathy as she displayed an immediate decrease in both level and trend across both interventions when compared to baseline. She committed a mean of 2.3 EPM (range 2–3) during baseline, a mean of .57 EPM (range 0–2) under CCC and a mean of .29 EPM (range 0–1) under the MTM intervention. During the follow-up phase, Kathy made a single error in the second follow-up session of both CCC and MTM.

Due to Mike's general lack of errors across baseline and intervention, it is difficult if not impossible to determine whether either intervention had an effect on his error rate. During baseline, Mike committed a mean of .3 EPM (range $0-1$). He committed a mean of .1 EPM (range $0-1$) during the CCC intervention and a mean of .07 (range 0–1) EPM during the MTM intervention. He did not make any errors during any of the follow-up sessions.

Generalization data are presented in Fig. [3.](#page-11-0) All participants displayed increases in mathematics fluency and decreases in errors from baseline to intervention on the grade-level mixed skill probes. Brandon's DCPM increased from a baseline minimum score of 11 DCPM with to an intervention maximum score of 23 DCPM. He decreased his EPM from a baseline maximum of 6 EPM to an intervention minimum of 0 EPM. Despite this increase, Brandon was unable to achieve mastery performance on the generalization probes.

Kathy's DCPM increased from a baseline minimum score of 12 DCPM with to an intervention maximum score of 32 DCPM. She decreased her EPM from a baseline maximum of 5 EPM to an intervention minimum of 0 EPM. Kathy achieved mastery performance on the final two generalization probes.

Mike's DCPM increased from a baseline minimum score of 14 DCPM with to an intervention maximum score of 32 DCPM. He decreased his EPM from a baseline maximum of 1 EPM to an intervention minimum of 0 EPM. Mike achieved mastery performance on the twelfth generalization probe but was unable to maintain his performance through the thirteenth and fourteenth probes.

Discussion

The purpose of this study was to examine the effects of the Cover, Copy, Compare (CCC) and Math to Mastery (MTM) interventions on the fluency of three secondgrade students. The current study extends the research on MTM, a relatively new intervention, by comparing it to an intervention with a large research base (CCC), demonstrating that MTM was as effective if not more so than CCC for increasing the students' fluency and accuracy. Both interventions were successful for all three students as they each increased their performance as measured by DCPM. Both interventions were also successful in decreasing errors for the two students who

Fig. 3 DCPM and EPM on mixed skill grade-level generalization probes across baseline and intervention phases for Brandon, Kathy, and Mike

consistently committed errors. The interventions may have also had an effect on Mike's errors, but this was difficult to determine due to his general lack of errors across baseline and intervention phases.

Although both interventions were effective, MTM appeared to be slightly more effective for two of the three students. Both Brandon and Mike increased their mean DCPM more under the MTM intervention than the CCC intervention when compared to baseline. MTM also produced larger effect sizes for MTM for Brandon and Mike. Both interventions yielded similar results for Kathy, although it should be noted that MTM again produced marginally higher mean DCPM and effect sizes.

One possible reason that MTM was more effective than CCC for two participants is the repeated practice component of MTM, not included in CCC. During MTM,

students repeatedly completed the same worksheet until they reached mastery level before collecting intervention data, which may indicate the effects of repeated practice during MTM. These results may also support the fact that repeated practice is crucial for building automaticity in students with calculation deficits (Hasselbring et al. [1988\)](#page-14-0).

Another factor for consideration is the efficiency involved in each intervention. Intervention efficiency was calculated by determining the difference between the mean follow-up DCPM for each intervention and mean baseline DCPM. This increase in rate was then divided by the total time spent in each intervention (10 min in MTM or 5 min in CCC multiplied by 14 sessions). During MTM Brandon increased .11 DCPM of instruction (6.6 digits increase per h of instruction) when compared to his CCC increase of .14 digits increase per min (8.4 digits per h). Kathy experienced a .09 DCPM increase under MTM (5.4 digits per h) when compared to her .17 DCPM increase (10.2 digits per h) under CCC. During MTM Mike increased .08 DCPM (4.8 digits per h) when compared to his CCC increase of .15 DCPM (9.0 digits per h). Using this method of estimating efficiency, all three students achieved greater increases in DCPM under the CCC intervention. This finding should be noted as teacher time required for CCC is approximately half of what is required for MTM.

The social validity of the interventions may also have been affected by the time involved. MTM took approximately twice as much time on average to implement when compared to CCC. This may be one reason why all three participants rated CCC as the more likeable intervention as the intervention process was generally over sooner than MTM. These findings may indicate that costs were needed to obtain benefits in this study and that even an intervention that demands additional costs may be recommended for students with math deficits to gain sufficient levels of benefits in typical school settings, where resources are limited.

Although this study contributes to our understanding of CCC and MTM on mathematics fluency, there are several limitations that should be noted. First, threats to internal validity (e.g., maturation and ongoing class instruction) and threats to external validity (e.g., three students in general education) should be stated. Replication of this study with populations with diverse characteristics (e.g., age, grade, gender, special education classification) may address these issues.

Second, all intervention sessions were conducted by the second author, who has had several years of didactic training in academic interventions with an emphasis in curriculum-based measurement. Although this may have increased the study's internal validity, it somewhat limits the generalizability of the findings to use of the intervention package by personnel without such training and applied experiences. However, it should be noted that both CCC and MTM have been shown to have been easily mastered by teachers who had no previous training in curriculum-based measurement procedures.

Third, the results are limited in their generalizability to mathematic skills other than the basic addition and subtraction facts used in the current study. Future research should examine and compare the effects of both interventions on other areas of math such as multiplication and division.

Fourth, because generalization probes were administered after each pair of intervention sessions (CCC and MTM), increases may have been partly a function of combined practice under the two training procedures and the types of operations included on the mixed skill probes. As such, any increases should be interpreted with caution due to potential multiple treatment interference.

Fifth, no-training control probes were not collected. Future researchers should utilize combined adapted alternating treatments design with a multiple-probe design to simultaneously investigate and compare two treatments while implementing a control condition (Skinner and Shapiro [1989\)](#page-15-0). This design allows for the comparison of two distinct interventions on equivalent sets of instructional items while accounting for history and spillover effects through the use of a continuous control condition (Sindelar et al. [1985\)](#page-15-0).

Sixth, because the researchers were not blind to the purpose of the study, individual bias may have been a limitation. However, it should be stated that both authors have received training in both interventions and have used both CCC and MTM on numerous occasions in applied settings.

Both CCC and MTM are multicomponent instructional procedures. Future researchers should conduct component analysis studies to determine which component or combination of components caused the increases in accuracy and fluency. Additionally, researchers should consider how manipulating components can enhance learning rates.

In summary, both the CCC and the MTM interventions were successful in increasing mathematic fluency for all three students when compared to baseline. However, the results indicate that MTM was more effective than CCC for two students while the other student also performed marginally better under the MTM intervention. Given these results, it appears that MTM shows potential for use as an effective strategy for use with elementary school students who do not possess the academic skills to be deemed fluent with basic addition and subtraction computation.

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