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Relating chains of instrumental orchestrations to teacher decision-making

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Abstract
The use of digital technology has the potential to support students' understanding in the mathematics classroom with the teacher playing a vital role. However, teaching with digital technology is not trivial, especially for teachers who are new to this. In this paper, we present an analysis of the enactment of a function lesson of a Sri Lankan mathematics teacher who used digital technology for the first time in her teaching. We combined the instrumental orchestration and ROG (resources, orientations and goals) frameworks into a conceptual framework to analyse her teaching. In particular, we used instrumental orchestration to identify how the teacher orchestrated the resources in her technology-rich classroom. This was combined with ROG theory to understand the reasons underpinning the decisions involved in moving from one orchestration to another. We demonstrate that this teacher showed diverse orchestrations and use the ROG framework to present these in the sequences in which they were used, formed into chains of orchestrations linked by goals. We propose that her didactical performance is a function of orchestration types over in-the-moment decision-making.

Keywords Orchestration chains · Instrumental orchestration · Decision-making · Resources · Orientations and Goals

Introduction

The potential of the use of digital technology (DT) resources to support students' mathematical learning has long been recognised (e.g. Kaput, 1992; NCTM 2008; Drijvers et al., 2013) with considerable research in this area over the past decade (e.g. Clark-Wilson et al.,

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2020). We have learned that the process of teaching with DT is not trivial, especially for teachers who are not familiar with new DT resources (Drijvers et al., 2013) and it may even require a complete change of mindset (Thomas et al., 1996). To illuminate these challenges, researchers such as Trouche et al. (2020) have conducted studies to identify and classify teacher activity with resources in a technology-rich classroom, leading to a typology of *instrumental orchestrations* (IOs) observed in teachers' practices. Notwithstanding this progress, how teaching unfolds in a DT-rich environment—referred to by some as a didactical performance—remains under-explored (Drijvers et al., 2019, 2020). To contribute to the development of this field of study, in this paper, we examine the didactical performance of a teacher in Sri Lanka, where the integration of DT into the mathematics curriculum and mathematics teaching is relatively new. We develop an argument for the importance of understanding not only orchestration types, but also the links between these as a lesson unfolds; and how these links, producing what we call temporal *chains of orchestration*, can be understood as a function of context and teacher decision-making in action.

We begin with a review of the IO literature, and its extension to include teacher decision-making to provide a conceptual framing for the paper and its focused research questions.

Literature review and theoretical orientation

Instrumental orchestration

The key theoretical construct that informs this paper, and one that has been widely used (Drijvers et al., 2020), is *instrumental orchestration* (IO) (Trouche, 2004). An instrumental orchestration is defined as the teacher's organisation of the available artefacts in a learning environment—in this case a technology-rich mathematics classroom—where a mathematical task is given in order to provide an external steering for students' instrumental genesis as they work on it (Trouche, *ibid*). Trouche (2004) introduced two elements within an IO. First is a *didactic configuration,* which includes the selection of various artefacts available in the environment and their layout. In the orchestral metaphor, this can refer to the selection of musical instruments and their physical arrangement on a stage prior to a performance; in a mathematics lesson, this could be the selection of available resources, such as DT tools, rulers, books, and their physical positioning in the classroom. Trouche's second element, an *exploitation mode,* is the way in which the configurations are organised into a plan: for the orchestra, this would be their musical *score* along with the conductor's ideas on how it should be played. For a mathematics lesson, this could include a lesson plan, a worksheet or extended task and the teacher's expectations and goals. However, as Drijvers et al., (2010, p. 215) note, "an instrumental orchestration is partially prepared beforehand and partially created 'on the spot' whilst teaching". Hence, they added a third element: a *didactical performance*, arguing that whilst an exploitation mode includes "[the] decisions on the way a task is introduced and worked through, on the possible roles of the artefacts to be played, and on the schemes and techniques to be developed and established by the students" (*ibid*, p. 215), this does not extend to actual *performance*, i.e. their use in practice. For them, a didactical performance encompasses all the in-situ decisions, including ad hoc ones, that the teacher makes whilst conducting the lesson according to the chosen didactical configuration and exploitation mode. This would include decisions such as "what question to pose now, how to do justice to (or to set aside) any particular student input, how to deal with an unexpected aspect of the mathematical task or the technological tool, or other emerging goals" (*ibid*, p. 215).

Amongst the three IO elements, a didactic configuration is a systematic arrangement organised before the actual lesson, i.e. in the planning stage, with the explicit intention, expressed in the exploitation mode, of implementing a given situation (Trouche, 2005). Thus, in the lesson, there is little or no opportunity to change these. In contrast, the didactical performance has greater flexibility since although some decisions are made during the exploitation mode, and others will be taken in the moment during classroom teaching. Hence, it cannot be fully planned, and its strong ad hoc aspect implies a temporal dimension, which, as we will show, is a crucial part of the investigation presented here.

These elements of an IO also indicate that a teacher is likely to perform qualitatively different types of orchestrations over the course of a lesson, depending on the DT tool used, the mathematical task(s) and the available resources. As noted in the introduction, research in the field has identified some of these, distinguishing further whether they are at the level of the whole class or individual learners (Drijvers et al., 2010; Drijvers et al., 2013; Tabach, 2011; Thomas et al., 2017). For instance, Drijvers et al. (2010) investigated the types of orchestrations teachers develop in a technology-rich classroom and related these to the teachers' views on mathematics education and the use of DT. They focused solely on whole-class teaching and identified six types of orchestrations, namely: *Technical-demo; Explain-the-screen; Link-screen-board; Discuss-the-screen; Spot-and-show* and *Sherpa-at-work*. Amongst these, the first three were categorised as teacher-centred, and the last three as student centred. They concluded that the types of orchestration teachers enacted aligned with their views on mathematics teaching and learning and of the role of DT therein. Thus, the evidence suggests teachers' beliefs, values, attitudes and confidence in using DT in mathematics teaching shape their IOs.

In another study, attending to individual orchestrations Drijvers (2011) introduced the orchestration *Work-and-walk-by*, which was demonstrated by teachers when the students worked individually or in pairs. Later, through a study with twelve Grade 8 mathematics teachers in the Netherlands, Drijvers et al., (2013) developed a more fine-grained taxonomy that included two additional whole class orchestrations: *Guide-and-explain* and *Board-instruction* and five categories for individual orchestrations: *Technical-demonstration; Guide-and-explain; Link-screen-paper; Discuss-the-screen* and *Technical-support*. Here, they were able to conclude from these that the teachers started with "orchestrations in which the digital technology plays a central role" (*ibid,* p. 997), but later in the teaching sequences, they had shifted to mathematics-centred orchestrations such as *Guide-and-explain*, relating these shifts to teachers' use of their pedagogical content knowledge.

Other researchers who have worked in this area have identified further orchestrations. For example, Tabach (2011) described an orchestration called *Not-use-tech,* which is when the teacher decides not to use technology even in a DT-rich classroom, a decision that is possible, notwithstanding the presence of a DT-rich teaching environment. Thomas, et al. (2017) identified another two types of orchestrations in their research involving innovative uses of DT in undergraduate mathematics courses namely: *Guide-to-investigate* and *Guide-to-maths-technique*. These orchestrations appear when the teacher or lecturer guides the students to activity on given mathematical tasks using available DT. These too are likely to be a function of teacher beliefs, values, attitudes and confidence, in this case with respect to the kinds of mathematical activity that they value.

All the studies discussed above, whilst building a typology of orchestrations, have at the same time explored how teachers' IOs relate to their teaching style. They point either implicitly or explicitly to teachers' decision-making, whether this is related to pedagogical approaches (teacher or student centred), or to mathematical activity (as techniques or inquiry), and to the place of the DT in learning. Whilst others, such as Besnier and Gueudet (2016) and Drijvers et al. (2019), have talked about sequences of orchestrations, they have identified throughout lessons, these have been treated as discrete entities without identifying how they might be linked together. Hence, Drijvers et al. (2019) made the telling comment that "Hardly any attention is paid to integrating them into instructional sequences. How can teachers sequence orchestrations into productive chains?" (p. 404). The focus on isolated orchestrations implies that there remains a need to investigate their integration in instructional sequencing (Drijvers et al., 2020) as well as paying more attention to the subject of the didactical performance element of the orchestrations. Hence, our focus on the continuity of these orchestrations during lessons, and their concatenation into chains, rather than simply noting that they comprise a discrete sequence. One aspect of this is the specific location of different orchestrations over the course of a lesson, whether and how they are linked and what we might learn from investigating *chains of orchestrations*. Our research questions below reflect this intent.

As we have argued, research to date points to, but does not elaborate, how teachers' beliefs, values, attitudes and confidence influence the didactic performance and hence the unfolding of a lesson over time. This is one key aspect of Schoenfeld's theorising of teachers' decision-making. We thus turn to this work, for further theoretical resources to draw on in our investigation of a particular teacher's chains of orchestrations.

Teachers' decision-making—the ROG framework

It is during the didactical performance element of an IO that a teacher's decision-making is crucial. Thus Schoenfeld's (2010) framework is highly relevant. According to Schoenfeld "teaching is a knowledge-intensive, highly interactive, dynamic activity" (2010, p. 136) and this is even more so when DT is integrated. He postulates that teachers' "in-the-moment decision-making is a function of their:

- resources (especially their knowledge but also the tools at their disposal),
- orientations (a generalisation of beliefs, including values and preferences), and
- goals (which are often chosen on the basis of orientations and available resources)" (*ibid*, p. 136).

His framework describes how the Resources, Orientations and Goals (ROG) of teachers are linked to the decisions that they make in the classroom. The basis of the theory is that teachers make decisions in order to achieve the goals they have set. This again links well with the concept of an IO, since the teacher's goals will form part of the plans made in the exploitation mode. Such goals, the things they want to achieve, may be immediate or longer term and conscious or sub-conscious. They may be perceived as linear that is where some must precede others, but often the order does not matter as long as they are all achieved. Larger goals, such as enabling students to investigate symmetry of Cartesian quadratic graphs, may be divided into sub-goals, such as providing understanding of axes, how to plot individual points and connect them, etc. Since the setting and achieving of goals are pivotal to teaching, it is important to understand how the goals arise and how they may be met. This is where resources and orientations play their part. For Schoenfeld and ROG, resources[1] comprise not only materials and tools but also teacher knowledge, as well as how time is functioning in their practice. The term orientation is "an inclusive term encompassing a group of related terms such as *dispositions, beliefs, values, tastes* and *preferences*." (Schoenfeld, 2010, p. 29). These perspectives on the world shape the way that teachers interpret and react to it, including the setting and prioritisation of pedagogical goals. The teacher then draws on available resources, both physical and cognitive, to achieve the goals they have set.

Schoenfeld (2010) argues that in routine situations, decision-making processes are automated and their actions may be a repetition of a previously implemented script. But in a new situation, the teacher's decisions could become more deliberate and so more visible, perhaps even puzzling to outside observers. For instance, consider an experienced teacher whose Mathematics Knowledge for Teaching (MKT) (Ball et al., 2008) is strong but is now teaching with a technological tool for the first time.[2] Although her MKT can be considered strong, her well-oiled routines of practice need to adjust and might well be *destabilised* in the new situation where the DT is integrated. Schoenfeld claims that the ROG theory has the capacity to describe and explain the decisions in new situations.

If we define an orchestration chain as a linked sequence of qualitatively different teacher orchestrations, where the links comprise intentional teacher actions, then, the idea of chains of orchestration presented here interfaces well with the ROG theory in at least two ways. First, the didactic performances of orchestrations involve the marshalling of resources in order to achieve goals. Second, the links in the chains represent the points at which there is a change of orchestration and hence, the teacher has made a deliberate decision at that point to direct available DT or other resources, especially their knowledge, in a particular manner towards accomplishing a goal or sub-goal. In the analysis of the vignettes below based on the lesson transcript data and the interviews with the teachers, we seek to infer what the decision at each point was and what its related goals, resources in use and orientations may have been.

In this paper, we analyse a lesson of a relatively new teacher, who is a novice with regard to using DT in her teaching, to understand her implementation—the didactic performance element—of her IOs. We combine aspects of the IO and ROG theories (Artigue et al., 2005; Prediger & Bikner-Ahsbahs, 2014; Prediger et al., 2008) to offer a conceptual framework (see Fig. 1) presenting chains of orchestrations as a function of types of orchestrations over decision-making.

As a contribution to the literature on IO, one of our aims was to identify different orchestration types a novice teacher practices in a novel context and to identify and describe links between the orchestrations and the role of decision-making in them. We set two research questions:

1. What orchestrations and chains of orchestrations can be identified in the practice of a teacher, who is a novice in using DT?

[1] Of course, both before and following the publication of ROG, the field of resources in mathematics education has developed substantively (e.g. Trouche et al., 2019). It is widely agreed that teachers' use of resources extend beyond the physical and material to include knowledge (for some e.g. Adler, 2000, 2012) and time (e.g. Adler, 2000; Gracin & Trupčević, 2022). For the purposes of this paper, and in our analysis, we focus first on ROG and its elements and return in the conclusion to reflect back the study reported, including ROG theory).

[2] We used MKT to refer broadly to mathematics teachers' content and pedagogical knowledge and not as a measure. MKT has implications for ROGs because teacher knowledge is considered as a resource.

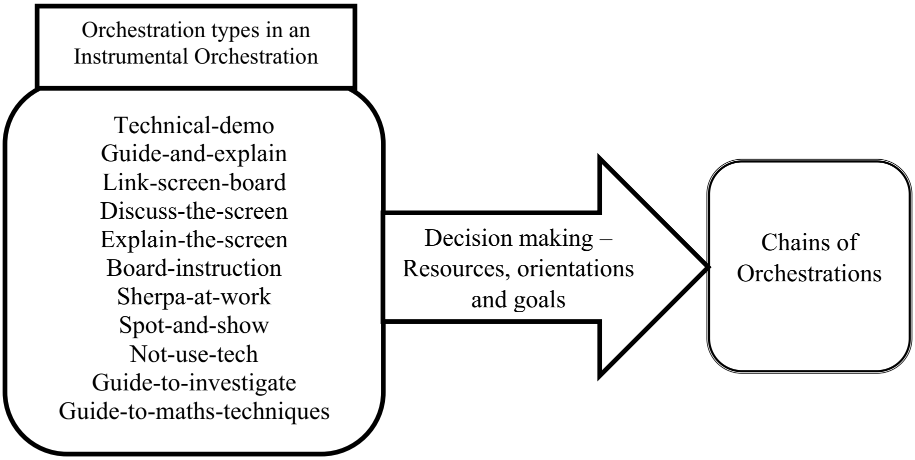


Fig. 1 Conceptual framework for chains of orchestrations as a function of types of orchestrations over decision-making

2. How do a teacher's resources, orientations and goals (ROG) inform her decisions in chaining instrumental orchestrations?

Method—data collection and analysis

This paper uses a case study of one teacher who taught a lesson on quadratic functions with GeoGebra to a class of 14 Grade 12 students.

The teacher, her context, the lesson and the data

The research presented here is drawn from a larger study (Ratnayake, 2018) conducted in Sri Lanka with 12 Advanced Level (AL) mathematics teachers, taking advantage of the first author's contacts there. All 12 who volunteered to participate had either very little or no experience in using technology in their teaching, forming a good fit with the goal of the wider study which was to assess the impact of a supportive professional development (PD) programme on DT task design and implementation (Ratnayake, et al., 2020). The teachers worked in four groups of three, first developing and then implementing DT algebra/functions tasks for Grade 12 (age 17–18 years) students as part of a DT-focused PD intervention. We focus on the implementation lesson planned by one of the groups and taught by a teacher from that group.

Grade 12 is the first year of AL and the students have selected further study in mathematics. From the four groups of teachers in the wider study, we focus on Group C, who designed the least rich task before the PD and modified it to become the richest task afterward (Ratnayake et al., 2020). The three teachers were in the same age group (30–40) having BSc degrees with a substantial component of mathematics, but fewer

Fig. 2 Classroom configuration

than five years of teaching experience. None of them had yet completed their professional teaching qualification (Postgraduate Diploma in Education) and thus pedagogically, can be considered relatively novice. Nelum (pseudonym) was the teacher from Group C who implemented the task they designed.

The lesson in focus was 40 min long and taught in Sinhalese. Additional data collection used instruments including individual and group interviews and a post-implementation discussion. The processes of task design and implementation were video-recorded, and the two interviews were audio-recorded. The recordings were transcribed and translated from Sinhalese to English. For this paper, we analysed the transcripts of the lesson, lesson planning discussion, the group interview and the post-implementation discussion, the worksheet, teaching goals and the lesson plan.

The process of data analysis

Our analysis includes the didactical configuration, the exploitation mode and the didactical performance with the teachers' decision-making, with didactical performance being the most extensive. We begin with a description of the first two, before turning to the detail on the lesson analysis.

The didactical configuration

In Nelum's school, most classrooms are not smart rooms. However, in this case, the didactical configuration of the IO for Group C included a smart room, equipped with a computer for the teacher, laptops for each of the 14 students, an inbuilt overhead projector, a screen and a whiteboard. Students sat in three rows (of 5, 5 and 4 from front to rear, respectively), and there was sufficient space for the teacher to walk between the rows (see Fig. 2).

The teacher could easily reach each student and observe their work with paper and pencil or laptop. However, she was not able to see students' screens from her computer. This configuration was both systematic and intentional on the part of the teacher.

Worksheet

1. Draw a rough sketch of the graph of the function $y = ax^2 + bx + c$ using GeoGebra
2. Observe the variation of the graph when the value of a changes.
3. How does the maximum and the minimum of the graph changes with the sign of a?
4. Get the value of $b^2 - 4ac$ for the values of a, b, c in the "Algebra view".
5. Change the values of a, b, c and observe the sign of the discriminant and observe whether the graph cuts the x axis or touches the x axis or neither cuts nor touches the x axis.

   What is the sign of the discriminant when the graph cuts the x axis at two distinct points?
6. What is the value of the discriminant when the graph touches the x axis?
7. What is the sign of the discriminant when the graph neither touches nor cuts the x axis?
8. (a) Using completing the square method rearrange the equation of the function $y = ax^2 + bx + c$ to get the above results algebraically.

(b) Draw the axis of symmetry using the input bar of Algebra view.

(c) What is the sign of $\left(x + \frac{b}{2a}\right)^2$ for all real values of x?

(d) What is the sign of $\left(\left(x + \frac{b}{2a}\right)^2 - \frac{(b^2-4ac)}{4a^2}\right)$ for all real values of x when the sign of $(b^2 - 4ac)$ is negative?

(e) Then, the sign of y changes according to the sign of a as:
When a is positive the sign of y is ______________.
When a is negative the sign of y is ______________.

(f) Write down how the graph changes with 'a' when Δ is negative. (Change the values of b and c to get negative values for Δ).

When Δ is negative:

- Graph lies above/below the x-axis when a is positive.
- Graph lies above/below the x-axis when a is negative.

9. Fill the blanks using the observations of the graph and the results obtained from rearranged function.
   a. If 'a' is positive and $b^2 - 4ac$ is negative then the function is __________ for all real values of x.
   b. If 'a' is negative and $b^2 - 4ac$ is negative then the function is __________ for all real values of x.

Fig. 3 Task implemented by the group

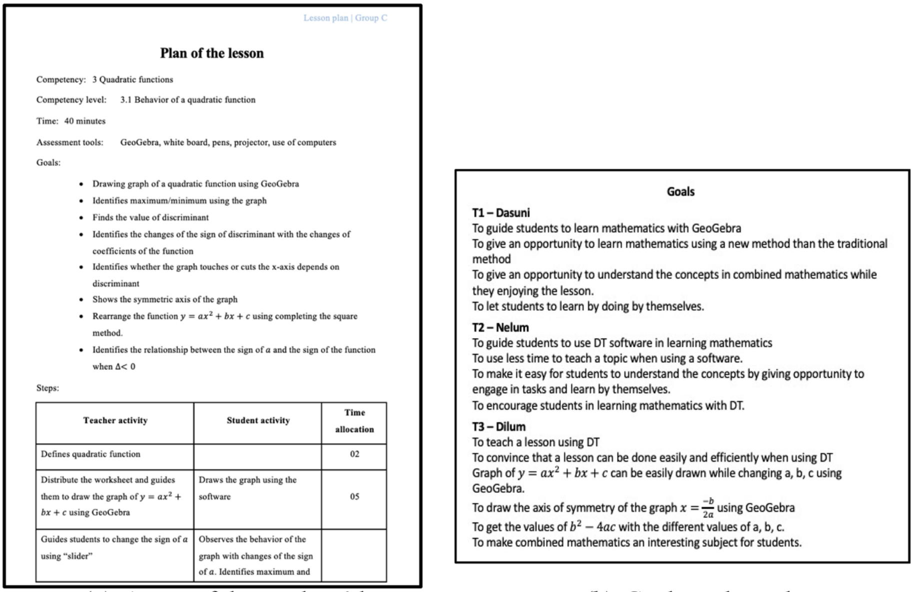
The exploitation mode

The exploitation mode comprised a number of documents, along with intentions for their use and proposed decisions for the implementation. This was governed by the group's choice of a mathematical focus for the DT task they designed, which was to help students identify properties of the function $f(x) = ax^2 + bx + c$ when $\Delta < 0$. Figure 3 shows the task that was prepared during the exploitation mode for Nelum to implement.

In addition to the worksheet to scaffold the task, they produced a lesson plan including the intended learning goals (see Fig. 4a) and a written list of the difficulties that students might face when they worked on the task along with possible remedies the teacher could take to overcome them. Apart from these documents, they also wrote individually what they saw as the teaching goals of this lesson (see Fig. 4b). The exploitation mode also comprised their joint decisions that students would work on the task individually and that the lesson would include both paper and pencil and DT work.

The didactical performance and the teacher's decision-making

When the teacher enacted the lesson, we noticed that she made decisions based on the exploitation mode as well as in-the-moment decisions. These decisions were the



Lesson plan | Group C

Plan of the lesson

Competency: 3 Quadratic functions

Competency level: 3.1 Behavior of a quadratic function

Time: 40 minutes

Assessment tools: GeoGebra, white board, pens, projector, use of computers

Goals:

- Drawing graph of a quadratic function using GeoGebra
- Identifies maximum/minimum using the graph
- Finds the value of discriminant
- Identifies the changes of the sign of discriminant with the changes of coefficients of the function
- Identifies whether the graph touches or cuts the x-axis depends on discriminant
- Shows the symmetric axis of the graph
- Rearrange the function $y = ax^2 + bx + c$ using completing the square method.
- Identifies the relationship between the sign of a and the sign of the function when $\Delta < 0$

Steps:

Teacher activity	Student activity	Time allocation
Defines quadratic function		02
Distribute the worksheet and guides them to draw the graph of $y = ax^2 + bx + c$ using GeoGebra	Draws the graph using the software	05
Guides students to change the sign of a using "slider"	Observes the behavior of the graph with changes of the sign of a. Identifies maximum and	

(a): A part of the teachers' lesson plan including learning goals.

Goals

T1 – Dasuni
To guide students to learn mathematics with GeoGebra
To give an opportunity to learn mathematics using a new method than the traditional method
To give an opportunity to understand the concepts in combined mathematics while they enjoying the lesson.
To let students to learn by doing by themselves.

T2 – Nelum
To guide students to use DT software in learning mathematics
To use less time to teach a topic when using a software.
To make it easy for students to understand the concepts by giving opportunity to engage in tasks and learn by themselves.
To encourage students in learning mathematics with DT.

T3 – Dilum
To teach a lesson using DT
To convince that a lesson can be done easily and efficiently when using DT
Graph of $y = ax^2 + bx + c$ can be easily drawn while changing a, b, c using GeoGebra.
To draw the axis of symmetry of the graph $x = \frac{-b}{2a}$ using GeoGebra
To get the values of $b^2 - 4ac$ with the different values of a, b, c.
To make combined mathematics an interesting subject for students.

(b): Goals each teacher wrote individually.

Fig. 4 **a**: A part of the teachers' lesson plan including learning goals. **b** Goals each teacher wrote individually

reasons for her moves from one orchestration type to another. We coded the classroom teaching using IO types to identify those she practised during the lesson. Although types of IOs were easy to identify, we had to infer and then code, the resources, orientations and goals related to her decisions. For that we analysed documents developed in the exploitation mode, namely the worksheet, the lesson plan, goals of teaching written by each teacher; together with transcripts of the lesson planning, the discussion on students' difficulties and the two interviews (pre- and post-implementation).

Our analysis proceeded by dividing the lesson into episodes based on the worksheet and then, analysing each episode. From the content of the worksheet, the first episode involved recalling prior knowledge and introducing the lesson. Therefore, we did not consider this for the analysis.

Identifying episodes

The episodes are related to the learning goals in the lesson plan (Fig. 4a) and also reflect the written teaching goals (Fig. 4b). For instance, one of the learning goals in the lesson plan is for students to identify *'the changes of the sign of discriminant with the changes of coefficients of the function'*. To achieve this goal, the teachers set sub-goals. In this case, the worksheet sub-goals, some in question form, were: *Change the values of a, b, c and observe the sign of the discriminant and observe whether the graph cuts the x-axis or touches the x-axis or neither cuts nor touches the x-axis. What*

Episode		Related learning goal/s of the lesson plan	Activities in the worksheet to achieve these goals
1	Introduction, connecting to the previous lesson		
2	Behaviour of the quadratic function in the form of $f(x) = ax^2 + bx + c$ with the changes of sign of a	• Drawing graph of a quadratic function using GeoGebra • Identifies maximum/minimum using the graph	1. Draw a rough sketch of the graph of the function $y = ax^2 + bx + c$ using GeoGebra 2. Observe the variation of the graph when the value of a changes. 3. How does the maximum and the minimum of the graph changes with the sign of a?
3.1	Calculating the value of $b^2 - 4ac$ for a range of values	• Finds the value of discriminant	4. Get the value of $b^2 - 4ac$ for the values of a, b, c in the "Algebra view".
3.2	Linking the discriminant with the behaviour of the graph	• Identifies the changes of the sign of discriminant with the changes of coefficients of the function • Identifies whether the graph touches or cuts the x-axis depends on discriminant	5. Change the values of a, b, c and observe the sign of the discriminant and observe whether the graph cuts the x axis or touches the x axis or neither cuts nor touches the x axis. What is the sign of the discriminant when the graph cuts the x axis at two distinct points? 6. What is the value of the discriminant when the graph touches the x axis? 7. What is the sign of the discriminant when the graph neither touches nor cuts the x axis?
4	Completing the square – rearranging the function [paper-pencil work, algebraic representation]	• Rearrange the function $y = ax^2 + bx + c$ using completing the square method.	8. (a) Using completing the square method rearrange the equation of the function $y = ax^2 + bx + c$ to get the above results algebraically.
5	Axis of symmetry [algebraic and graphical]	• Shows the symmetric axis of the graph	(b)Draw the axis of symmetry using the input bar of Algebra view.
6	Behaviour of the graph with the sign of a when $\Delta < 0$	• Identifies the relationship between the sign of a and the sign of the function when $\Delta < 0$	(c) What is the sign of $\left(x + \frac{b}{2a}\right)^2$ for all real values of x? (d) What is the sign of $\left(\left(x + \frac{b}{2a}\right)^2 - \frac{(b^2-4ac)}{4a^2}\right)$ for all real values of x when the sign of $(b^2 - 4ac)$ is negative? (e) Then, the sign of y changes according to the sign of a as: When a is positive the sign of y is ______________. When a is negative the sign of y is ______________. (f) Write down how the graph changes with 'a' when Δ is negative. (Change the values of b and c to get negative values for Δ). When Δ is negative: • Graph lies above/below the x-axis when a is positive. • Graph lies above/below the x-axis when a is negative. 9. Fill the blanks using the observations of the graph and the results obtained from rearranged function. a. If 'a' is positive and $b^2 - 4ac$ is negative then the function is __________ for all real values of x. b. If 'a' is negative and $b^2 - 4ac$ is negative then the function is __________ for all real values of x.

Fig. 5 Relationship of the seven episodes to the intended goals and student activities

	Mathematics-centred	DT-centred	Connecting-maths-and-DT
Whole-class	Mathematics related whole class instructions	DT related whole class instructions	Whole class instructions connecting mathematics and DT work
Individual	Mathematics related Individual instructions	DT related individual instructions	Individual instructions connecting mathematics and DT work

Fig. 6 Categories of the orchestrations

is the sign of the discriminant when the graph cuts the x-axis at two distinct points? What is the value of the discriminant when the graph touches the x-axis? What is the sign of the discriminant when the graph neither touches nor cuts the x-axis? These episodes and their relationship to the intended learning goals and planned worksheet activities are shown in Fig. 5.

Coding episodes

In order to code the lesson episodes, orchestration type, resources the teacher used and the mode of instruction were attended to. For the latter, we made the following distinctions: the teacher gave instructions either to the whole class or to individual students; instructions were mathematics related, DT related or connecting mathematics and DT work. This gave rise to the six categories of orchestration types this teacher practised, as shown in Fig. 6.

Orchestration	Example	Indicator
Guide-to-investigate	Ok now, what is the next one [in the worksheet]? Observe the behaviour of the graph, whether the graph cuts x −axis, touches the x −axis or neither touches nor cuts and the value of the discriminant when the values of a, b, c are changing. Did you observe? What is happening? When you change the values of a, b, c like this. Change the values and see.	This is a whole class instruction related to mathematics. Teacher guides the students to follow the worksheet and investigate the relationship.
Explain-the-screen	[teacher moves the sliders in her computer] Now in this example, here are the values for a, b, c. the graph is entirely above the x-axis. So what is the sign, the sign of the discriminant? Negative. The sign of a is positive. So we have a concaved up graph.	This is a whole class instruction connecting maths and DT work and also connecting different representations - the graph, sliders and the numerical value of Δ. 3 different representations – graphical, algebraic and numerical
Board-instruction	b equals 2, c equals 1 [$b = 2$. $c = 1$]. Use these values and see. [teacher writes on the board $a = 1, b = 2, c = 1$]. In this instance, the graph touches the x-axis and we can see that the value of the discriminant is 0 as well.	Whole class instruction on connecting maths and DT.
Technical-support	T to S6: like this, right-click here and you get this. We can either remove or have the axes. Now we have [the axes], right? Here we have the axes now. This is your algebra column. You have closed that. That's why it was disappeared. Now enter the values here. a equals 1, b equals 2 and c equals 1 [$a = 1$, $b = 2$, $c = 1$].	An individual instruction on technical issues. Teacher provides the instruction while looking at student's screen and student does the changes accordingly.
Technical-demo	Now, change a now change c. And see the changes of the sign [of the discriminant] [Teacher works in S6's computer, changing the values of a, b, c and then guiding S6 to change the sliders to observe the sign of the discriminant]	This is a technical demonstration to an individual student. Teacher works on student's laptop and demonstrates how to do this.

Fig. 7 Indicators of the codes for orchestrations

To code the IOs, we used the different types of orchestrations that have been identified (e.g. Drijvers et al., 2013; Thomas et al., 2017), as in Fig. 7.

After coding the whole lesson in terms of orchestration types, we analysed the teacher's moves from one orchestration to another. These movements were more frequent in some episodes than others. So, to identify and examine these movements, we drew what we have defined above as *Orchestration Chains:* a sequence of IOs together with teacher decision-making links. As discussed above, others have noted the existence of sequences of orchestrations (Besnier & Gueudet, 2016; Drijvers et al., 2019). In contrast, an orchestration chain is a linked sequence of qualitatively different teacher orchestrations. The links are crucial and comprise intentional teacher actions, such as decisions and the reasons for them. To understand the nature of the complex temporal flow of decisions and orchestrations, a deeper analysis was needed. In each episode, we identified the moves from one orchestration to another and used the ROG framework to infer what decisions she made for such movements and why.

To code for ROG, we analysed various documents, including transcripts of discussions on planning the lesson, using codes for resources, orientations and goals. We considered material resources such as the worksheet, lesson plan, GeoGebra, whiteboard, the screen and laptops but placed our emphasis on teacher knowledge,[3] such as mathematical content knowledge; pedagogical knowledge; knowledge from the planning discussion; knowledge of students' prior work; knowledge of working with GeoGebra and knowledge of time as a constraint, (coding these R1, R2 etc.). The teacher's beliefs and values of teaching and learning mathematics with DT and confidence in teaching with DT were the orientations Nelum and her group members showed (O1, O2 etc.). The goals were arranged under three categories namely mathematical, DT related and pedagogical (G1, G2, G3) as in Figs. 10 and 14. As we explain below, we zoom in on two episodes to develop our argument for the significance of orchestration chains and present the detailed ROG coding for these (and not the entire lesson).

Results and discussion: Nelum's didactical performance

We begin with an overview of the lesson, and categorisation of all orchestration types followed by detailed discussions of chains of orchestrations of two episodes.

Overview of the lesson

The lesson was on the effect of sign of a on the sign of $f(x) = ax^2 + bx + c$ when the discriminant is negative. Nelum followed the structured worksheet that embedded the DT task they developed. First, we provide an overview of the content of the lesson. At the beginning of the lesson, a structured worksheet for the task was distributed amongst the students. The DT they used in this lesson was GeoGebra 4. Nelum started by introducing the general form of a quadratic function followed by referring to the worksheet and explaining how to enter the function into GeoGebra, first by writing the commands on the board—*Board-instruction* DT related—and then demonstrating how to enter the function—*Technical-demo*. The worksheet was structured in a way that the students would first observe the relationship between the sign of a and the concavity of the graph. According to the Sri Lankan curriculum, this is taught in Grades 10 and 11. However, this was the first time that the students had learned about the behaviour of a quadratic function in its general form $f(x) = ax^2 + bx + c$. Therefore, the teacher seems to have used GeoGebra to confirm their existing knowledge and then to generalise it. The first sub-goal was thus to have students observe the relationship between the sign/value of the discriminant and the position of the graph relative to the x-axis. Following this the students were guided to rearrange the function by hand using the method of completing the square. The new arrangement of the function was then used to find and draw the axis of symmetry of the graph and to identify the relationship between the sign of a and the sign of $f(x)$ when $\Delta < 0$. They then used DT to confirm this relationship.

[3] Whilst there are other important resources used in the lesson, such as a whiteboard, computers, GeoGebra, pens, in the ROG, we focus on knowledge-based resources, which was the original intention in Schoenfeld's (2010) theory, where, for example, we see the heading "On Resources (With a Focus on Knowledge)" (p. 25).

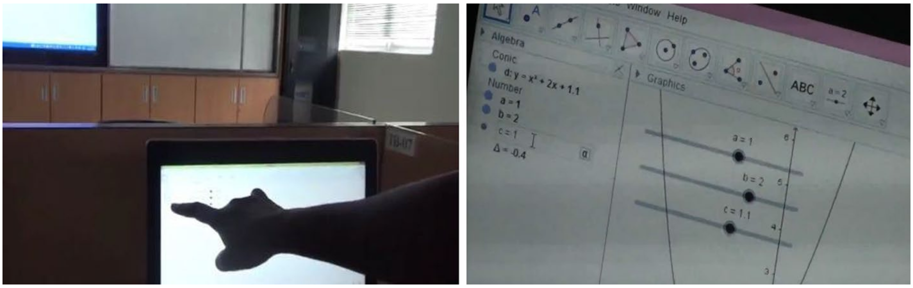


Fig. 8 Nelum explains the screen to S6

Categorisation of orchestrations

With respect to orchestration types in the literature, this teacher enacted the following eight in her lesson: *Board-instruction*, *Guide-and-explain*, *Guide-to-investigate*, *Technical-demo*, *Technical-support*, *Explain-the-screen*, *Discuss-the-screen* and *Link-screen-board*. All except *Technical-support* were seen as whole-class orchestrations. The individual orchestrations were *Board-instruction, Guide-and-explain, Technical-demo, Technical-support* and *Explain-the-screen*. According to Drijvers et al. (2013), *Explain-the-screen* is categorised only as a whole-class orchestration. However, we noticed that Nelum explained to an individual student what was happening on her (S6's) laptop screen to connect mathematics and DT work (see Fig. 8). For example, in the following utterance, she explained what is seen on S6's screen and points out the relationship between the value of the discriminant of the function, arising from the values of the sliders *a, b* and *c*, and where this is visible on the screen. So, we propose that *Explain-the-screen* orchestration type can also be practised as an individual orchestration.

T to S6 Now can you see the value of delta here? [points at the screen] This is the value according to the current values of your variables.

Further, as seen in Fig. 6, we organised these types of orchestrations into three categories, mathematics-centred, DT-centred or connecting-maths-and-DT, based on the nature of the instruction. For example, Nelum used a *Guide-and-explain* orchestration as whole-class mathematics-centred, whole-class connecting-maths-and-DT, individual mathematics-centred and individual connecting-maths-and-DT. We then drew the chains, with their links, to illuminate the sequence of moving between the orchestrations for each episode. The categorisation of orchestrations helped us to understand these sequences in terms of where the teacher moved, from which type of orchestration to which, and why, and what intentional activity, such as crucial decision-making, occurred between them. For instance, in the second and the seventh episode, Nelum's orchestrations were limited to whole-class orchestrations. In the second episode, these included mathematics-centred, DT-centred and connecting-maths-and-DT orchestrations. The last episode, which is a short one, focused on the final question of the worksheet, a wrap-up and a connection to the next lesson. Here, her orchestrations were limited to whole-class connecting-maths-and-DT types. This is not a surprise, because in the last episode, the students were expected to connect the rearranged version of the function and the changes of the graph to generalise the sign of $f(x)$ when $a > 0$ and $\Delta < 0$ and the sign of $f(x)$ when $a < 0$ and $\Delta < 0$.

Each episode of the lesson started with a whole-class mathematics-centred orchestration type, except in episode 3.1 —calculating the value of $b^2 - 4ac$ for a range of values—which started with a whole-class DT-centred orchestration and was the last episode where only whole-class connecting-maths-and-DT orchestrations took place. From our analysis, we can observe that the teacher provided more support for DT work, both whole class and individual, in episodes 2 and 3.1 than elsewhere. The teacher knew that this was the first time her students had worked with GeoGebra, so she likely decided to provide more support with technical issues in the early part of the lesson. In the later episodes, she did not use *Technical-demo* for the whole class other than on one occasion during episode 5, which focused on the axis of symmetry. However, she provided individual support in technical issues (*Technical-demo* and *Technical-support*) in each episode. During the post-implementation discussion, she claimed that "they struggled with the technology. I think it is because this is the first day…They are not familiar with the commands in GeoGebra. So, mainly the girls, they struggled a lot with entering the equation." This implies that since the students needed continuous support the teacher had to provide technical support individually throughout the lesson. In particular, this reflects how Nelum's lack of confidence in her students' ability to use DT influenced her decisions to provide individual technical support. Further, she tended to focus more on connecting-maths-and-DT in later episodes.

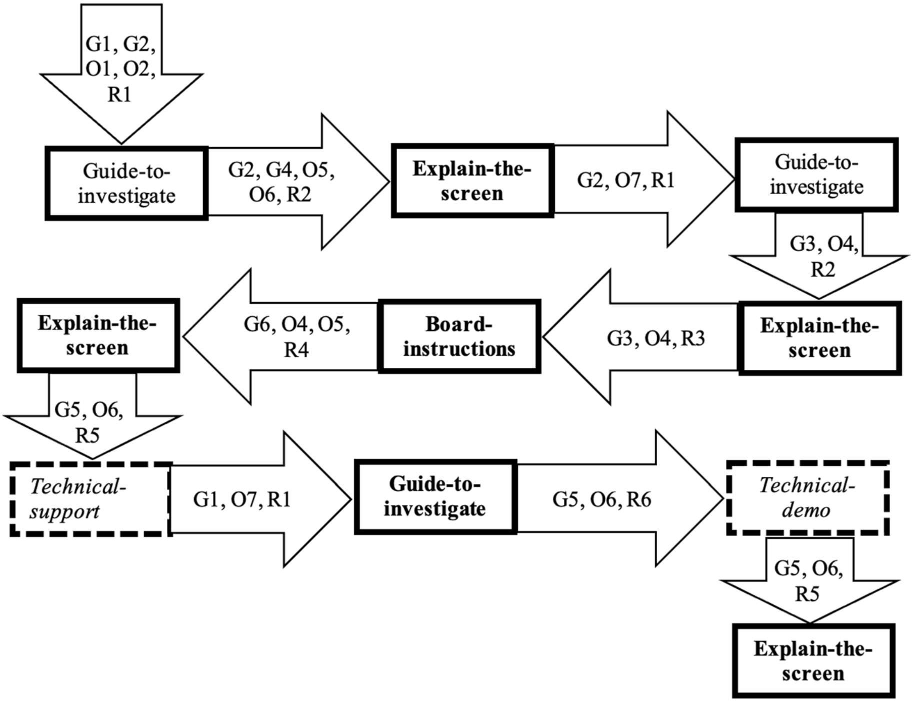
Throughout the whole lesson, it was clear that the worksheet, which linked each episode, and Nelum's observations of students' work (both by-hand and DT) exercised a considerable influence on the teacher's decision-making. We discuss this in detail with two examples in the next section.

The chains and teacher's decision-making

In this section, we discuss how the teaching goals, along with decisions from the exploitation mode, teacher beliefs, values and confidence and available resources including her knowledge and time factor as a constraint influenced the teacher's in-the-moment decision-making. Episode 3.2 is a key episode in this lesson. In it, the students needed to link the value of the discriminant with the behaviour of the graph and then later use these results to accomplish the main goal of the lesson (the focus of episodes 6 and 7). Whilst episode 5 is not linked to the final goal of the lesson, it provides an interesting story of connecting students' prior knowledge of the axis of symmetry with its equation in the context of a quadratic function in its general form. Thus, we deliberately chose these episodes to illuminate Nelum's decision-making with the different types of orchestrations worthy of attention.

Orchestration chains

The orchestration chains introduced above comprise two elements—nodes and links (connections between nodes). The nodes represent the types of orchestrations whereas the links represent the intentional moves the teacher makes, based on her ROG, including decisions. The next node (or the action—orchestration type) is thus a function of her decision-making. We illustrate the results of our analysis of the nodes and links below. First, we consider episode 3.2.



	Mathematics-centred	**DT-centred**	**Connecting-maths-and-DT**
Whole class	Bold outline normal letters	Bold outline italic letters	Bold outline bold letters
Individual	Dotted outline normal letters	Dotted outline italic letters	Dotted outline bold letters

Fig. 9 Episode 3.2—An example of an Orchestration Chain and a key for its nodes

Episode 3.2

In Fig. 9, we can see Nelum's orchestration chain during episode 3.2 with the ROG-based links and an interpretation of the nodes. The content of the links can be seen in Fig. 10.

Episode 3.2 is related to *linking the discriminant with the behaviour of the graph* and it covers questions 5, 6 and 7 of the worksheet and addresses two main goals in the lesson plan:

- Identifies the changes of the sign of the discriminant with the changes of coefficients of the function.
- Identifies whether the graph touches or cuts the x-axis depends on the discriminant.

In episode 3.1 the students entered the equation $\Delta = b^2 - 4ac$ in the input bar. So, now they could see the value of the discriminant in the algebra column and how it changed when they moved the sliders for a, b, c. They could also observe the corresponding changes

Goals	
G1	Mathematical – to help students observe and identify the relationship between the sign/value of the discriminant and the position of the quadratic function graph relative to the x-axis
G2	Pedagogical – to guide students to identify when the graph of a quadratic function cuts, touches or neither cuts nor touches the x-axis
G3	Pedagogical – to provide suitable examples to help the students see that when $\Delta = 0$ the graph touches the x-axis
G4	Pedagogical/DT – to guide students to use GeoGebra to see the relationship between the sign/value of the discriminant and the position of the graph of a quadratic function relative to the x-axis
G5	DT – to provide support to overcome technical issues with GeoGebra – e.g., changing the values of a, b, c on algebra view
G6	DT – to help students obtain the value of $\Delta = b^2 - 4ac$ from different values of a, b, c, using sliders
Orientations	
O1	A belief that DT has value in teaching and learning mathematics
O2	It is very important to achieve the lesson goals
O3	It is more important to achieve mathematical goals than to provide time to struggle with DT-related work
O4	A lack of confidence in students' abilities to move sliders and get different graphs on their own, such as a graph touching the x-axis
O5	Values linking different mathematical representations, such as graphs and algebra
O6	Values using GeoGebra to identify the relationship between the discriminant and the position of the graph relative to the x-axis.
O7	Believes it is important for students to work at their own pace whenever possible
Resources	
R1	Worksheet
R2	Mathematical Knowledge – how to connect different representations such as algebra and graphs
R3	DT Knowledge – students' anticipated difficulties in getting a graph to touch the x-axis by changing the sliders
R4	Mathematical Knowledge – the relationship between $\Delta = b^2 - 4ac$ and the position of the graph relative to the x-axis for different values of a, b, c for quadratic functions
R5	Pedagogical Knowledge – how to respond to students' DT work on entering the values $a = 1, b = 2, c = 1$ in the algebra column of GeoGebra
R6	Pedagogical Knowledge – how to respond to students' DT work on moving sliders to $a = 1, b = 2, c = 1$ (to get a graph that touches the x-axis)
R7	Mathematical Knowledge – the relationship between the changes of value c and the position of the graph of quadratic functions relative to the x-axis

Fig. 10 Specific resources, orientations and goals seen in episode 3.2

in the graph. Nelum started the episode by saying "ok, now what is the next one [in the worksheet]? Observe the behaviour of the graph, whether the graph cuts the x-axis, touches the x-axis or neither touches nor cuts and the value of the discriminant when the values of a, b, c are changing. …". This decision to consider Q5 of the worksheet was motivated by the orientation O1—her belief that DT has value in teaching and learning mathematics with the goal G1—to help students observe and identify the relationship between the sign/

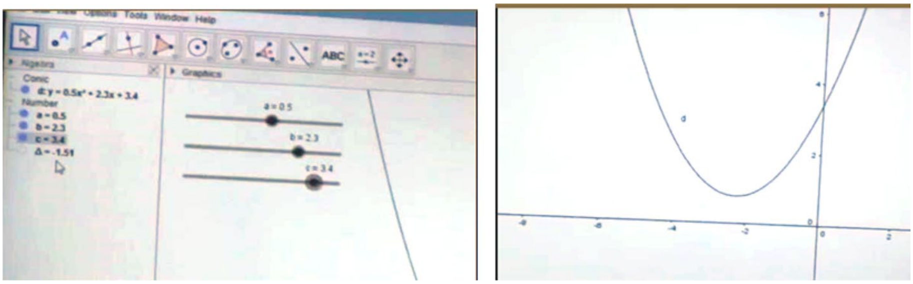


Fig. 11 Screenshots of one student's screen parallel to the teacher's instruction in line 68

value of the discriminant and the position of the quadratic function graph relative to the x-axis and employed the worksheet (R1) as a resource (see Fig. 10). Here, as decided in the exploitation mode, she guided the students to follow the worksheet—*Guide-to-investigate whole-class maths-centred*—and decided to explain using some examples on her screen so the students could observe the connections between the graph, algebra and numerical value of the discriminant (see line 68 of the transcript below and Fig. 11 for students' work parallel to teacher instructions). Her lack of confidence in students' abilities to move sliders and get different graphs on their own seems (O4) to have influenced her decision.

Following this decision, as planned, Nelum, in line with G2 and G4, encouraged the students to see the connections between where the graph sits (entirely above the x-axis), the sign of the discriminant ($\Delta < 0$) and the connection between the sign of a ($a > 0$) and the concavity of the graph (concave up) (O6). The orchestration type she practised here was *Explain-the-screen whole-class connecting-maths-and-DT*. Further, she used the DT to demonstrate the connection between algebraic (changing the sliders), graphical (position of the graph) and numerical (value/sign of the discriminant in the algebra column) representations (O5), as seen in Fig. 11. She used her pedagogical knowledge to guide the students to connect graphical and algebraic representations to see the relationship between the relative position of the graph and the signs of a and the discriminant (R2).

68 Now in this example, here are the values for a, b, c. The graph is entirely above the x-axis. So what is the sign, the sign of the discriminant? Negative. The sign of a is positive. So, we have a concave up graph

After this *Explain-the-screen* orchestration, she allowed students to keep working on their own (*Guide-to-investigate* orchestration). We infer that the reasons for this decision were her pedagogical goals to guide students to identify when the graph of a quadratic function cuts, touches or neither cuts nor touches the x-axis (G2) and to provide time to work their own (O7). Whilst the students were working on their own, she moved towards the class and stopped at the front row and asked "Did you observe a case when the graph touches [the x-axis]? Is there anybody who observed that?" Although the students' responses are inaudible, we can assume that they said "no" based on the teacher's response, seen in line 72, "No? Ok, wait I will give you some values".

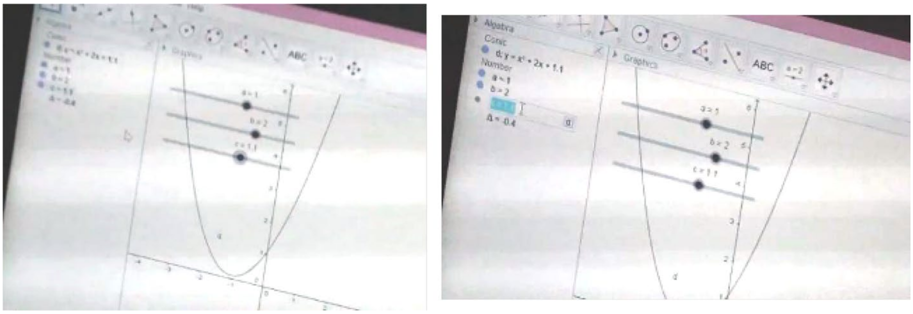


Fig. 12 An example of students' work on changing a, b, c to 1, 2 and 1 to get a graph touching the x-axis

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72 Now let's change the value of a and consider a concave down graph. Now, what happened? The sign of the discriminant is positive. When I am changing the value of c, can you observe the graph move up and down? Like this. … Is there anybody who observed that? No? Ok, wait I will give you some values

73 It is possible to get a graph touching the x-axis by moving the graph up and down. But sometimes it's not very easy. So let's substitute some values and try. Here, in this column on your left-hand side what is called *Algebra*, right click here, let's take a equals 1, b equals 2 and c equals 1 [$a = 1, b = 2, c = 1$]. Now can you see? … I gave this example because it is not very easy to get a graph [touching the x-axis] by changing the sliders

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The teacher did not check with the rest of the class. Since she did not receive the positive affirmation she sought, she provided remedial direction and acknowledged this decision in the post-implementation interview. Here she moved back to *explain-the-screen* orchestration and showed how to change the values of *a, b, c* to specific values—$a = 1, b = 2, c = 1$. Nelum made a quick decision to cut-short students' investigation and provided remedial to her anticipated difficulty of the students in getting a graph touches the x-axis. Hence, we can infer that Nelum made the in-the-moment decision to change the approach based on students' responses to her question and the knowledge of preparation to react to students' difficulties (R3). Whilst at first, she decided to provide students with time to work their own, she suddenly changed her mind and her goal (G3), driven by her lack of confidence in the students' instrumentation (O4), was now to provide suitable examples for the case $\Delta = 0$. She may have expected problems here since this was a difficult point that the group of teachers had anticipated, as recorded in an exploitation mode document. In this they wrote, "Difficulties may arise in considering the case $\Delta = 0$ (eg., $a = 1, b = 2, c = 1$)ε, recognising that students could encounter a problem here when they worked on the task. These teachers decided to try out the task themselves and their own experience of working on the task, placing them in the students' position, forced them to anticipate the same difficulty for the students.

Dilum: And when the discriminant is zero might be a problem.
Nelum: Ah yes.
Dasuni: Getting a graph when the discriminant is zero.
Dilum: Yes. To find. So we can give an exemplar value set as a guide. To support them.
Dasuni: Yes they can zoom out and see.
Nelum: Then.. What were the values we used?
Dasuni: $b = 2$. And $a = 1, c = 1$

Nelum: We can use this as an example.

Nelum then moved back to the class and observed that S2 was trying to enter the values that she had demonstrated (line 73). At that point, she likely realised that it would be better to write the values on the board for all the students. Following that in-the-moment decision (G3, O3, R4), she moved to a *Board-instruction* orchestration (line 75). Figure 12 shows how a student changed the values of a, b, c to $1, 2, 1$ in the algebra view to get a graph touching the x-axis. Here, the teacher made the decision based on her observations of students' work on their laptops, and hence, it was necessarily ad hoc rather than from the exploitation mode. After writing the values on the board, she moved back to her computer and used an *Explain-the-screen* orchestration to show how to get different graphs by changing only the slider for c and described the case when $c = 1$, where the graph touches the x-axis. She guided students to observe the three representations, graphical, algebraic and numerical in this instance too (R4, O5). This decision could be related to her previous decision of writing the values on the board. Apart from this, it looks like she also wanted to show them how to get the graphs touching the x-axis by changing the sliders (G6). Her lack of confidence in students' ability to do DT work themselves (O4) would have influenced this decision.

74 T to S2: 1.[teacher is observing S2's screen]

75 T to the WC [whilst walking to the board]: b equals 2, c equals 1 [$b = 2.c = 1$]. Use these values and see. [teacher writes on the board $a = 1, b = 2, c = 1$]. In this instance, the graph touches the x-axis and we can see that the value of the discriminant is 0 as well

Nelum then moved around the class and observed each student's screen (R5) and provided support where necessary rather than allowing them to struggle and find solutions to DT issues themselves (G5, O3). Interestingly, all the individual orchestrations Nelum practised in this episode were DT-centred. She engaged in *Technical-support* and *Technical-demo* orchestrations to overcome technical issues that may have caused a delay in accomplishing her mathematical goals. Another interesting decision she took was guiding the students to follow the steps in the worksheet at their own speed. For example, whilst she was helping an individual student (S6) with technical issues, she paused for a moment and said "those who have observed that can go to the next step. What we have in the next? What is the sign of the discriminant when the graph cuts the x-axis at 2 points?" and kept supporting the same student (S6) that she was helping. In addition to the worksheet (R1) and the overall mathematical goal (G1), her belief in the importance of providing opportunities for students to work at their own pace whenever possible (O7) might have influenced this decision. This type of decision-making was visible in all the episodes of this lesson. Finally, she moved to a *explain-the-screen* whole-class maths-and-DT-connect orchestration and guided students to observe the pattern of the movements of the graph with the changes of slider c (R7). She seemed to value linking different mathematical representations, such as graphs and algebra (O5) to achieve pedagogical and DT goal: Pedagogical/DT—to guide students to use GeoGebra to see the relationship between the sign/value of the discriminant and the position of the graph of a quadratic function relative to the x-axis (G4).

Overall, although this was more of a teacher-centred lesson, it appears that Nelum also appreciated the rate of student's individual progress. It was likely that having a structured worksheet as a resource helped Nelum to make these decisions (Nelum at the group interview before the task implementation lesson: "And when doing the lesson, we thought to

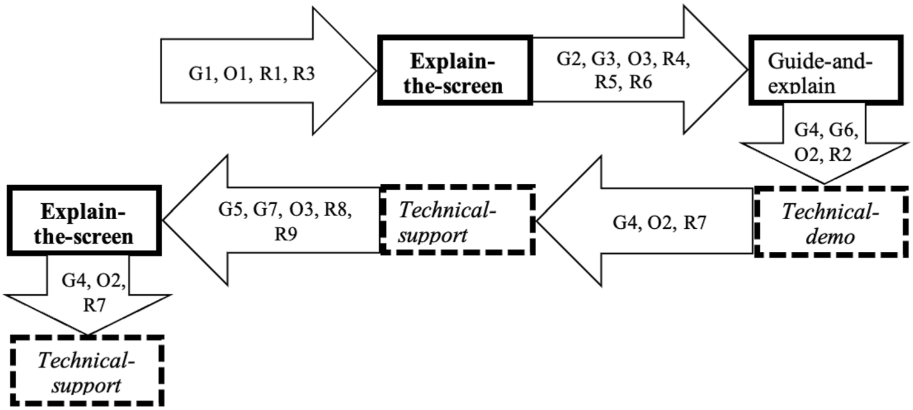

Fig. 13 Chain of orchestrations of episode 5

give a worksheet. So that it would be easier to guide them step by step. We thought it is better than giving all the instructions by the teacher"). The chain in Fig. 9 shows that Nelum's observations of students' work lie behind many decisions in this episode, and this is visible in other episodes too. It appears that teacher observation of student work and the students' responses to teacher questions are important influences in the formation of decisions regarding sub-goals, in alignment with elements of the exploitation mode (such as prior planning) and individual teacher orientations, such as confidence in teaching with DT.

Episode 5

The second episode we highlight here is related to the teachers' stated pre-implementation goal involving using the DT for students "To draw the axis of symmetry of the graph $x = \frac{-b}{2a}$ using GeoGebra." In addition the group listed, as follows, the axis of symmetry amongst the points they expected students to find difficult.

1. To identify $x = \frac{-b}{2a}$ as the axis of symmetry in the rearranged formula of the function (using completing the square method)
2. To show the axis of symmetry on the graph.

The goal found its way onto the student worksheet in the form "8(b) Draw the axis of symmetry using the input bar of Algebra view."

The episode begins with the teacher deciding to use a whole class *explain-the-screen* mathematics-centred orchestration (see Fig. 13) with the sub-goal of reminding the students what they had done to find the axis of symmetry of a specific function in a previous lesson (G1-see figure 14) (line 102 of the lesson transcript). Here, the teacher points to her written work on the whiteboard and the DT work on the screen. She connects the algebra and the graph and reminds students' of their prior work.

Goals	
G1	Pedagogical – to help students recall the facts about the axis of symmetry of quadratic functions from a previous lesson
G2	Mathematical – to have students identify the algebraic equation of the axis of symmetry, $x = \frac{-b}{2a}$, of a general quadratic function
G3	Mathematical – to help students see the connection between the algebraic equation of the axis of symmetry in the rearranged form of the function and the line on the graph
G4	DT – to help students draw the axis of symmetry, $x = \frac{-b}{2a}$, of the graph of a quadratic function using GeoGebra, avoiding input problems
G5	DT – to help students change the values of a, b, c in the algebra view of GeoGebra
G6	DT – to emphasise the need for precision in the entry of GeoGebra functions
G7	Mathematical – to use the graph of a specific, previously seen, quadratic function, $f(x) = 4 - x^2$, and relate it to the general form of a quadratic function $f(x) = ax^2 + bx + c$
Orientations	
O1	It is very important to achieve the lesson goals
O2	Lacks confidence in student ability to input the equation $x = -\frac{b}{2a}$
O3	Values linking different mathematical representations, such as graphs and algebra
Resources	
R1	Worksheet
R2	DT Knowledge – potential student difficulties in entering the equation $x = -\frac{b}{2a}$
R3	Mathematical Knowledge – that the graph of $x = -\frac{b}{2a}$, where a, b, c are the coefficients of a general quadratic function, gives the axis of symmetry of its graph
R4	Pedagogical Knowledge – how to connect board work on rearranging the function using the method of completing the square with DT work
R5	Pedagogical Knowledge – how to connect paper and pencil work with DT work
R6	Mathematical Knowledge – how to connect different representations such as algebra and graphs
R7	Pedagogical Knowledge – observations from students' work on their screens when inputting the equation $x = -\frac{b}{2a}$
R8	Mathematical Knowledge – how to relate the general form of quadratic function with specific quadratic functions, such as $f(x) = 4 - x^2$ to $f(x) = ax^2 + bx + c$
R9	DT Knowledge – use of GeoGebra commands to get the function $f(x) = 4 - x^2$ by changing the sliders

Fig. 14 Resources, orientations and goals influenced for decisions in episode 5

102a How did we get the equation of the axis of symmetry? Do you have the function that we considered earlier? The one we rearranged using completing the square method? In the previous lesson, we completed the square of x squared minus $2x$ plus 3 [$y = x^2 - 2x + 3$]. [connecting the knowledge of finding the axis of symmetry for specific functions to the graphs of the function in the form of $f(x) = ax^2 + bx + c$] Didn't we? How did we find the axis of symmetry of that?

She then moves on to the next sub-goal from the exploitation mode, which was to identify algebraically the equation of an axis of symmetry for a general quadratic function and to relate it to the graph (G2, G3). We note that the axis of symmetry is a geometric concept and so it requires careful linking to the algebraic representation, which is an algebraic formula. Nelum articulates this through the words 'draw the axis' but 'write the equation'.

102b Inside the whole thing squared. I think I erased that. Let's consider this general form. How can we find the axis of symmetry here? From x equals minus b divided by 2a [$x = -\frac{b}{2a}$]. In the same way, if we draw the axis of symmetry of the graph that you can see on the screen, then, we can write the equation of the axis of symmetry

The primary sub-goal (G4) of this part of the lesson, again decided in the exploitation mode, was to draw the axis of symmetry for a general quadratic graph using GeoGebra. So, based on that, the teacher makes a decision to move back to the DT in order to link the representations, relating the algebra to the graph (R6, O3). This quickly moves to a *Guide-and-explain* DT-centre dorchestration whereby students are directed to "Go to GeoGebra" themselves and told how to input the formula $x = -\frac{b}{2a}$. This shows she lacked confidence in the students' ability to enter the function themselves (O2). We also see that Nelum's personal instrumentation was not sufficiently honed to get this input right first time, and her final version of $x = -(\mathrm{b}/(2a))$ is still a little inelegant, since the outer brackets are unnecessary here.

102c, 103 In the same way, if we draw the axis of symmetry of the graph that you can see on the screen then we can write the equation of the axis of symmetry. Go to GeoGebra again. Then, in the input bar, we can input x equals, we also got this from completing the square, our equation of the axis of symmetry is, minus b divided by $2a$. We know that from completing the square form that the axis of symmetry is x equals minus b over$2a$. [$x = -\frac{b}{2a}$]. So input here, type x equals minus b over $2a$ [$x = -\frac{b}{2a}$] in the input bar, x equals minus [$x = -$], we need to use a bracket here b divided by $2a$ [$\frac{b}{2a}$] [she typed $x = -(\frac{b}{2a})$ and it didn't work. Then, she tried to change it by adding another bracket around $2a$ and it worked, i.e. –(b/($2a$))]

During the post-implementation discussion Nelum answered a question from the researcher regarding this decision—"why did you put a bracket for $\frac{b}{2a}$ and write a negative sign out of the bracket?"— she replied that "At that moment I also got stuck. The equation I typed didn't work. This worked. So I used it". This evidences her lack of confidence in her knowledge of GeoGebra commands, which influenced her pragmatic in-the-moment decision on how she wrote the equation for students to enter. It seems that Nelum did not want her students to struggle with the same issue that she faced. Thus, rather than providing opportunities for students to attempt their own function entry she decided to show how to enter it and her beliefs about students' confidence (O2) likely also influenced this decision.

This then ran into a *Technical-demo* DT-centred whole class orchestration with the goal of cementing in place the need for care in the entry of the function (G4, G6), this time also adding an explicit multiplication sign, *.

109a If you don't give the command correctly, you won't get the axis correctly. Therefore, when you type, do it like this:x equals minus add a bracket b, we denote division like this, open another bracket, 2 multiplied by a. [teacher writes on the board: $x = -(b/(2 * a))$] like this. Otherwise, the equation you get may not be that of the axis of symmetry

Now, in line with the planned implementation, Nelum made the decision to try to get the students to use the DT sliders to see how the axis of symmetry changes as *a, b* and *c* change (R2).

109b Then, you also can observe the changes of the axis of symmetry with the changes of a, b, c. Use the sliders for that. [the teacher is helping the students individually]

Whilst the students are working on their own Nelum kept helping them individually on technical issues in entering the equation —*Technical-support* DT-centred individual orchestration. She made the decision who needs what technical support by observing their screens and asking demanding questions such as "did you get the axis? Where?" (R7). Her goal was to help students draw the axis of symmetry, $x = \frac{-b}{2a}$, of the graph of a quadratic function using GeoGebra, avoiding input problems (G4) and she used observations from students' work on their screens when inputting the equation $x = -\frac{b}{2a}$ (R7) to make the decision because she was not confident enough in students' abilities to input the equation $x = -\frac{b}{2a}$ (O2). Soon after she saw that one student was not getting the correct axis, "No, wrong. This is not the axis of symmetry." So, she made the impromptu decision to move to another whole class *Explain-the Screen* Connecting-maths-and DT orchestration using her computer and a previously used example function, $y = 4 - x^2$. However, this orchestration is very much mathematics-centred and rather than trying to have students use the sliders to obtain pseudo-continuous changes in the graph she resorted to a single, discrete example. This is clearly a move away from the power of the DT and was a pivotal attempt to link the DT to the vital mathematical goals, which she believes to be of primary importance.

114 If you want to draw a particular graph, you can change these a, b, c values here [pointing to the algebra view] and can get the graph from this standard form. Say we want to draw y equals 4 minus x squared [writes on the board: $y = 4 - x^2$]. We drew this in the previous lesson. What should we do to draw this? Here, we need to input negative 1 for a, what is the value of b? zero. The value of c is positive 4. $a = -1, b = 0, c = +4$. You can observe this as well. This is one of the graphs we drew. a is negative 1, the value of b, in this instance, is 0, the value of c is positive 4. Here, we see the same graph we drew yesterday. So like this, we can get the other graphs also using the general form of the function

Afterward she decided to give some time for students to work on this and provided *Technical-support* for those students who needed it (G4, O2, R7). As in episode 3.2 she did not wait until all the students achieved this goal but rather guided the students who had accomplished the goal to move to the next step on the worksheet. The final comment here "So like this, we can get the other graphs also using the general form of the function." appears to imply that what has been done was sufficient for the teacher to think that the goal of drawing the axis of symmetry had been accomplished and she was ready to move on to her next goal. It would seem that this was driven by the belief that the essential mathematical point had been made, which over-rides decisions about DT use, as we saw in episode 3.2. Overall, it is can be seen that the teacher provided opportunities for the students to work on their own (following decisions made during the exploitation mode's lesson plan discussion) but moved to different orchestration types based on her observations of student work and their responses to her questions. She made these decisions to support students to overcome DT issues so they could achieve the mathematical goals rather than struggling with the technology.

Answering the research questions

In this paper, we have presented our analysis of a function lesson enacted by a teacher using DT for the first time in her teaching. This analysis is useful in describing the actions and decisions of this teacher who we considered strong in aspects of MKT (Ball et al., 2008) but novice in teaching mathematics with DT and relatively novice in pedagogical knowledge (Shulman, 1986). Our goal was to understand what we could learn from her orchestrations, in particular to answer two research questions: 1. What orchestrations and chains of orchestrations can be identified in the practice of a teacher, who is a novice in using DT? and 2. How do a teacher's resources, orientations and goals (ROG) inform her decisions in chaining instrumental orchestrations?

The first question relates to describing the teacher's orchestration of classroom activities and what underpins their implementation. In terms of the categorisation found in the literature (e.g. Drijvers et al., 2013; Thomas et al., 2017), this teacher employed eight different types of orchestrations in her lesson—*Guide-and-explain, Guide-to-investigate, Board-instruction, Technical-demo, Technical-support, Explain-the-screen, Discuss-the-screen* and *Link-the-screen* and enacted both whole-class and individual types of orchestrations. According to Drijvers et al. (2013), *Explain-the-screen* and *Board-instruction* are whole-class orchestrations. But as seen in our analysis, this teacher's practice provides evidence that these two types can also be individual orchestrations. Furthermore, we propose (as in Fig. 6) that the types of orchestrations can be further categorised as *maths-centred*, *DT-centred* and *connecting-maths-and-DT*. For instance, we found *Guide-and-explain* and *Guide-to-investigate* orchestrations were practised for maths-centred as well as connecting-maths-and-DT for whole-class and individual instructions. Furthermore, *Board-instructions* were used for both maths-centred whole-class and individual instructions and DT-centred whole-class instructions. Since the teacher was a novice in using DT in teaching, it is not surprising to see evidence that her personal and professional instrumental geneses (Drijvers et al., 2010; Haspekian, 2014) could have been stronger and as a result there were several false moves in her instrumentations for her students, especially with regard to the format of function inputs. For the second part of this research question about whether chains of orchestrations could be identified, we can provide a definite yes and these have been exemplified above.

Our key finding is that the teacher orchestrations were not isolated, but rather formed part of an orchestrational sequence (Drijvers et al., 2019), or more precisely, a chain, where the teacher often moved from one orchestration to another. To illustrate these moves, we introduced the notion of *chains of orchestrations*. As we have previously noted, these differ from simply sequences and hence from the descriptions of Besnier and Gueudet (2016) and Drijvers et al. (2019), in that a key element is that a chain consists of a single composite item made up of linked parts that comprise intentional teacher activity. We have identified a number of these chains in the practice of this teacher and have described two of them in detail above.

The second research question involves an explanation of how the orchestrations were linked by the teacher's decision-making, and we return to that idea below, to answer how

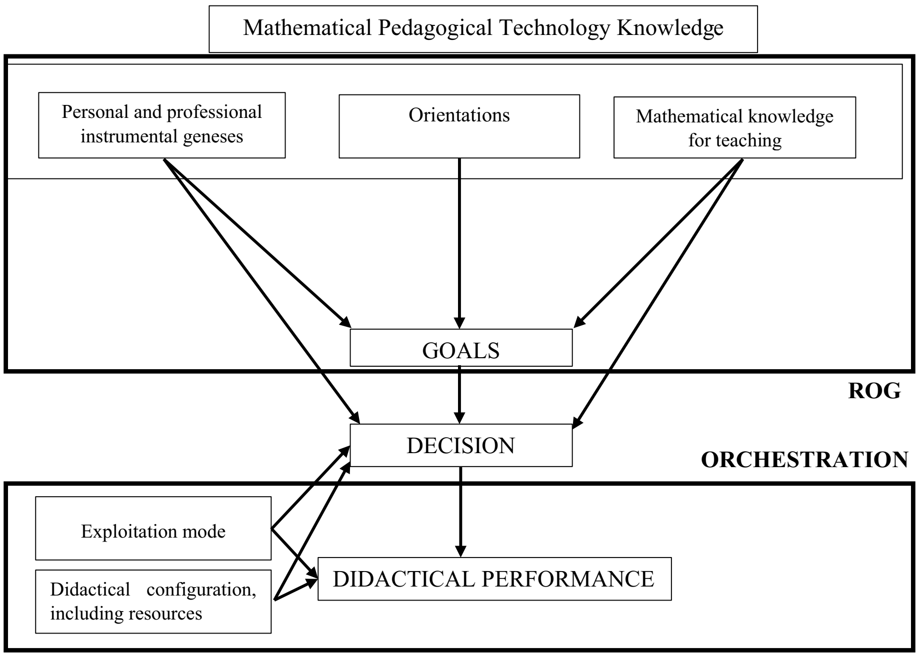

Fig. 15 Our hypothesis for a tentative model of the factors influencing decision-making and classroom orchestrations.

the teacher's resources, orientations and goals (ROG) informed her decisions in chaining instrumental orchestrations.

This novel concept of temporal orchestration chains proposed here not only proved to be a valuable analytical tool but also allowed us insight into the decision-making process of the teacher. We have seen that each new orchestration was directly preceded by a decision and that we were largely able to infer this and the reasons for it based on analysis of data from the exploitation mode of the IO, which included goals decided on by the group, and the teacher's reflections. An analysis of in-the-moment decisions (during the didactical performance) showed that they were influenced by decisions made during lesson planning, including anticipated students' difficulties, as well as teacher observations of student work (both DT and by-hand) and their responses to the questions the teacher asked. In the two episodes we presented in this paper, we were able to identify the decisions made, the goals and orientations driving them and the resources drawn on to accomplish the goals. It became clear that the teacher allowed students to work on their own (a decision made during the exploitation mode) but provided a lot of technical support to help them see the connections between different representations and to connect DT and by-hand work. It is notable that her main goals were mathematical, and she provided support to overcome technical issues in order to achieve these. Whilst valuing the use of DT to connect different representations in GeoGebra and by-hand work, the teacher's lack of confidence in students' instrumental geneses seem to influence many of her decisions. We argue that identification of the sequences of IOs alone, without the links, would not enable one to make these types of claims that the chains allow.

Overall, one thing that has emerged from our study is the opportunity to hypothesise a tentative model of the role of orchestrations and decision-making for a novice user of DT that may have application to other DT users. In summary, as seen in Fig. 15, we agree with Schoenfeld (2010) that decision-making is driven by goals and that these goals are primarily set in accord with the teacher's orientations. In turn, the decisions prompt the didactic performance element of the orchestrations that the teacher intentionally initiates to achieve the goals. However, the picture appears not to be quite so clear cut as this. Since the teacher had both mathematical and DT goals that were running in concert, sometimes these conflicted. At these points when a decision was to be made, two other aspects of her Mathematics Pedagogical Technology Knowledge (MPTK) (Thomas & Palmer, 2014), other than her orientations came into play. These were her personal and pedagogical instrumental geneses (Haspekian, 2014) and her MKT. In the orchestration chain, we see that her strong MKT as a resource sometimes caused her to instigate a *maths-centred* orchestration or a *connecting-maths-and-DT* orchestration in order to move the lesson towards achieving the primary goals related to mathematical understanding of the concepts in the lesson. It has been suggested (Thomas & Yoon, 2014), and we agree that a useful PD strategy to address the issue of classroom conflicts would be to assist teachers to become more aware of their ROG and its influence on in-the-moment classroom decisions. nb As previously mentioned, in the figure above, we focus primarily on knowledge-based resources (especially mathematical, pedagogical and technological), which was the original intention in Schoenfeld's (2010) theory)

Conclusion

In conclusion, in line with Drijvers and et al., (2013), we have described the kind of practices and orchestrations novice teachers (with respect to DT) may develop when seeking to use digital resources in their mathematics classroom. We have shown that these orchestrations are relatively wide-ranging, and, importantly, we have been able to highlight the chains of orchestrations employed along with the in-the-moment decisions that accompany changes in direction in the teacher's orchestrations. Analysing these chains has required combining and coordinating the IO and ROG frameworks. Our attempt at employing these frameworks together (Artigue et al., 2005) has proved extremely useful and in doing so, we have taken a few tentative steps towards combining and coordinating them (Prediger et al., 2008), fitting together elements from the different theories. We coordinated orchestrations and the ROG to produce a conceptual framework which presents chains of orchestrations as a function of IOs over teacher's decision-making in action. This has enabled us to describe how the decision-making in this teacher's didactical performance was influenced by available resources, by pedagogical goals, student learning goals and her beliefs about using DT in learning mathematics, as well as her own confidence in DT use and her beliefs about her students' confidence in using DT in learning mathematics.

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