Journal of Mathematics Teacher Education (2024) 27:235–255
https://doi.org/10.1007/s10857-022-09562-3

# Computational thinking practices as tools for creating high cognitive demand mathematics instruction

Kathryn M. Rich[2] · Aman Yadav[1] · Charles J. Fessler[1]

Accepted: 27 November 2022 / Published online: 12 December 2022


## Abstract
One characteristic of high-quality mathematics teaching is supporting students in engaging in tasks of high cognitive demand. In this paper, we explore relationships between two elementary teachers' efforts to integrate computational thinking (CT) practices—abstraction, debugging, and decomposition—into their mathematics instruction and their development of high-level tasks. Teachers engaged in professional development sessions about CT. Using their mathematics curriculum materials as a starting point, teachers then planned mathematics lessons to incorporate attention to at least one CT practice. Researchers transcribed their conversations and qualitatively coded the transcripts using an established framework for assessing the cognitive demand of tasks posed to students. Analyses of the planning conversations suggested that encouraging these teachers to examine their mathematics curriculum materials through the lens of CT practices supported them in adapting tasks from their curriculum materials in ways that raised the cognitive demand. Implications for the use of CT in elementary mathematics teacher education are discussed.

**Keywords** Mathematics education · Computational thinking · Cognitive demand · Curriculum materials

Supporting teachers in enacting ambitious instruction, characterized by providing opportunities for students to engage in rigorous content and disciplinary practices, is a perennial challenge in mathematics education (Lampert et al., 2013). Curriculum materials (CMs), including textbooks and teachers' guides, can be educative for teachers and support ambitious instruction (Davis & Kracjik, 2005). However, schools use a wide variety of CMs that differ in the amount of educative material available to teachers (Land et al., 2019). Moreover, the ways teachers interact with any CMs are critically important to determining the impact of the materials on instruction (Remillard, 2005). Teachers need support in adapting CMs to suit the particular needs of their students while still maintaining high-quality learning opportunities (Brown, 2009). Relatedly, they need strategies for maintaining students' engagement with high-level tasks during instruction. Teachers' attempts to support

✉ Kathryn M. Rich
richkat3@msu.edu

1 Michigan State University, East Lansing, MI, USA

2 American Institutes for Research, Chicago, IL, United States

students as they work on high-level tasks often inadvertently lower the demands of what students are asked to do (Stein et al., 1996).

Against this backdrop of the challenges of supporting ambitious mathematics teaching and learning, there are growing calls to bring computer science education to all students in K-12 (Vahrenhold et al., 2019). In elementary school contexts, a growing number of research projects have explored integrating computational thinking into mathematics instruction as a strategy to introduce students to computer science ideas (Gadanidis, 2017; Israel et al., 2015; Rich & Yadav, 2020). While there is variation in how *computational thinking* (CT) is characterized in different contexts and studies, CT is roughly defined as the set of thinking practices computer scientists use as they work (Wing, 2006). Several of these practices, such as decomposing problems into manageable parts and abstracting important information (Yadav et al., 2017), are highly aligned with the high-level thinking required to engage with high-quality mathematics tasks (Stein & Smith, 1998). While the trend of integrating CT into elementary mathematics began as an attempt to provide early, equitable exposure to computer science ideas, the similarities between CT ideas and descriptions of high-quality mathematics instruction raise questions about how CT could also serve as a tool for supporting mathematics teaching and learning.

In this paper, we present a post-hoc analysis of how two teachers adapted instructional tasks as they reviewed a lesson in their CMs through the lens of three CT practices: decomposition, debugging, and abstraction. Specifically, we examined how teachers' thinking about the CT practices related to changes in the cognitive demand of tasks they planned to pose to students. Through this analysis, we aim to open a conversation about how CT might be intentionally leveraged as a frame for supporting teachers' productive use of mathematics CMs to craft instruction.

## Literature review

This study draws on prior work in three areas: Teachers' uses of mathematics CMs, cognitive demand of tasks used in mathematics classrooms, and uses of CT practices in elementary classrooms.

### Teachers' uses of mathematics curriculum materials

Mathematics curriculum materials (CMs) can serve as supports for teachers in creating high-quality mathematics instruction (e.g., McGee et al., 2013; Stein & Kaufman, 2010). One way in which CMs can act as a support is by providing high-quality tasks or starting points for such tasks. For the potential of this support to be realized in practice, teachers must develop pedagogical design capacity, or "skill in perceiving the affordances of the materials and making decisions about how to use them to craft instructional episodes" (Brown, 2009, p. 29). That is, they must learn ways of interacting with CMs that allow them to thoughtfully choose among tasks and activities and adapt them in ways that support students' engagement with high-level thinking and productive mathematics.

Existing research shows teachers engage in multiple interpretative processes as they interact with mathematics CMs, including reading, evaluating, and adapting (Sherin & Drake, 2009). Moreover, teachers differ in the strategies they use to approach CMs. They may read different elements, evaluate using different criteria, or interact with the materials with different goals in mind (Remillard, 2012; Sherin & Drake, 2009). Not all strategies result in instruction that poses ambitious mathematical tasks to students. For example, Amador (2016) found several elementary teachers read CMs with attention to isolating elements that would prepare students for standardized assessments, skipping or ignoring many elements intended to support organization of instruction around student thinking.

By contrast, other studies have uncovered frames for teachers' interactions with CMs that are associated with instruction that engages students in high-level thinking. Stein and Kaufman (2010) compared two districts' implementations of *Everyday Mathematics* and *Investigations.* They found the teachers in the district with higher-quality implementation spent more time discussing the big mathematical ideas than the teachers in the other district. In another study, Choppin (2011) described how one teacher critiqued and adapted tasks in her CMs based on what she learned about student thinking in a previous enactment of the lesson. Similarly, Grant et al. (2009) found that when teachers read sample student dialogs in CMs with attention to anticipating how their own students might respond to tasks, they were able to guide class discussions about student strategies. Land et al. (2019) found a wide variety of CMs had elements that would allow teachers to open curriculum spaces, or spaces for children to connect their prior knowledge to mathematical ideas. The researchers developed a tool detailing strategies that could help teachers locate and leverage these curriculum spaces (Drake et al., 2015). One such strategy is to locate high-quality tasks and adapt them in ways that do not eliminate the opportunities for students to engage in high-level thinking.

In short, studies have supported the notion that teachers can choose high-quality tasks and modify the tasks in ways that maintain the intellectual challenge for students when they engage with CMs through lenses related to big mathematical ideas (Stein & Kaufman, 2010) or student thinking and knowledge (Choppin, 2011; Drake et al., 2015; Grant et al., 2009). Given the wide variety of CMs now available to teachers, it is important to identify "curriculum-proof" strategies (Taylor, 2016) that support teacher interactions with any CMs with attention to these ideas. In this paper, we explore CT practices as a potential framework for productive teacher interactions with CMs.

## Mathematics instruction and cognitive demand

Smith and Stein (1998) developed a framework for evaluating mathematics tasks according to their *cognitive demand,* or "what kind of thinking a task will demand of students" (p. 345). The four categories of the framework are shown in Table 1. Two categories—Doing

**Table 1** Levels of cognitive demand from Smith and Stein (1998)

| Task Category | Cognitive Demand | Typical Characteristics |
|---|---|---|
| Doing Mathematics | High | Do not immediately suggest a solution pathway<br>Require exploration of mathematical relationships |
| Procedures with Connections | High | Draw attention to concepts underlying procedures<br>Require exploration of mathematical relationships |
| Procedures without Connections | Low | Suggest algorithmic application of a procedure<br>Focus attention on correct answers, not concepts |
| Memorization | Low | Involve reproducing known facts<br>Does not allow for use of a procedure |

Mathematics and Procedures with Connections—are considered high cognitive demand, whereas the others—Procedures without Connections and Memorization—are considered low cognitive demand. Smith and Stein argued one strategy for creating high-quality mathematics instruction is to organize lessons around tasks of high cognitive demand to provide opportunities for students to grapple with important mathematics.

Relatedly, based on their observations of mathematics tasks being used in classrooms, Smith et al. (1996) argued that any mathematics task passes through at least three phases when used in classroom instruction: (1) the task as it appears in instructional resources, (2) the task as it is set up by the teacher in the classroom, and (3) the task as implemented by students in the classroom. A number of criteria can influence how the task is transformed as it moves from one phase to another. For example, a teacher's goals, content knowledge, and knowledge of students can affect how she plans to adapt the task described in CMs when she sets it up in her classroom. Classroom norms, student and teacher habits, and task conditions can affect how students take up the task the teacher sets up.

Stein et al. (1996) studied the cognitive demand of mathematics tasks as set up in classrooms and as implemented by students. They also looked for factors related to the decline of cognitive demand from set up to implementation as well as factors related to the maintenance of high cognitive demand. While around 75% of tasks were set up to be of high cognitive demand, the demand of more than half of these fell during implementation. Factors associated with a decline in cognitive demand included a tendency for teachers to step in and reduce the complexity of problems when students were struggling, a shift in focus from processes to answers, and providing too much or too little time for students to work. Factors associated with the maintenance of high cognitive demand included modeling of expert problem-solving practice and sustained presses for explanations.

### Computational thinking in elementary mathematics

The term *computational thinking* refers to the thinking practices used by computer scientists as they engage in their work (Wing, 2006; Yadav et al., 2017). While there is a great deal of variation about the details of what CT entails, there is consensus that CT "is the way of thinking used to develop solutions in a form that ultimately allows 'information processing' or 'computational agents' to execute those solutions" (Curzon et al., 2019, p. 515). Some of the most common practices associated with CT are decomposition (breaking a problem into smaller, more manageable parts), abstraction (simplifying complex problems by focusing on the most important elements), algorithms (developing sets of step-by-step instructions), and automation (turning the work over to a computational agent) (Yadav et al., 2017). Additionally, many definitions of CT include attention to debugging (systematic error correction) and pattern generalization (Grover & Pea, 2013). While some definitions restrict CT to work with computers (e.g., Denning, 2017), in this paper we adopt the more general perspective that focuses broadly on CT as problem solving in multiple contexts while drawing fundamental ideas and practices from computer science (Moore et al., 2020; Wing, 2006). To further explicate what CT opportunities might look like with versus without digital computing devices, we provide programming-specific examples and non-computer-based, or "unplugged" examples of three CT practices in Table 2.

A key result from a set of workshops convening experts in computational thinking was broad consensus that "the power of computational thinking is best realized in conjunction with some domain-specific content" (National Research Council, 2011, p. 9). A growing body of research has examined how CT might be integrated into elementary

**Table 2** Programming-specific and non-computer-based examples of CT practices

| CT Practice | Programming example | Non-computer-based example |
|---|---|---|
| Abstraction | Using a function call to simplify code by masking the complexity of the underlying function | Using a scale break on a graph to focus attention on a relevant portion of the data |
| Decomposition | Listing project components to be programmed and addressing one at a time | Breaking a complex calculation into multiple simpler calculations using place value |
| Debugging | Locating and fixing the source of a syntax error in the code | Locating and fixing an error in handwritten arithmetic or in a problem-solving strategy |

mathematics and the impact of this integration on teachers and students (e.g., Gadanidis et al., 2017; Israel et al., 2015). Elementary mathematics has been a popular area for CT integration for at least two reasons. First, several lines of research demonstrate a natural and intuitive fit between mathematics and computer science. Papert's (1980) work with the Logo programming environment explored how children could use computer science ideas to learn mathematics. An analysis of the K-5 Common Core State Standards for Mathematics showed the standards contained multiple contexts for exploring CT ideas related to sequence, repetition, and conditionals (Rich et al., 2020). Elementary teachers perceive these connections, as well. Interviews with elementary teachers about the prospect of CT integration revealed they made many more connections to their mathematics teaching than to their science teaching (Rich, Yadav, & Schwarz, 2019). Teachers in another study who chose their own points of integration with the elementary curriculum commonly chose mathematics (Duncan et al., 2017).

Second, theoretical and empirical analyses have pointed to affordances that CT brings to problem solving in K-12 STEM contexts (e.g., Weintrop et al., 2016), in K-12 mathematics contexts (Kallia et al., 2021), and particularly to elementary mathematics classrooms (Nordby et al., 2022). For example, one line of research has examined connections between mathematics problem-solving processes and the CT idea of levels of abstraction (Rich & Yadav, 2020; Rich, Yadav, & Zhu, 2019). Another study used Scratch activities with fifth and sixth grade mathematics students to explore "the hypothesis that an unfamiliar problem domain can be better approached by students who have been taught to deconstruct mathematical concepts and logical sequences into the simple steps to be understood by a computer" (Brown et al., 2009, p. 3). Students who completed the Scratch activities, which were intentionally designed to support mathematical problem-solving skills, increased their problem-solving skills at a greater rate than a control group. This result suggests the decomposition ideas embedded in CT may support mathematical problem solving.

While he was not focused specifically on elementary school, Pérez (2018) argued that CT could be a means for mathematics educators to support students in developing dispositions necessary for working through high cognitive demand tasks, such as a tolerance for ambiguity and persistence on difficult problems. As an extension of this point, we argue that Curzon et al.'s (2019) description of CT, as quoted above, resonates well with the distinctions made between high and low cognitive demand tasks in the Math Task Framework (Table 1). CT is not focused on executions of rote, already-defined procedures, as described in the Procedures without Connections category, but rather focused on developing such procedures (e.g., through decomposition and debugging) or meaningfully choosing and matching procedures to new problems (e.g., through abstraction and pattern matching), as described in the Doing Mathematics and Procedures with Connections categories (Smith & Stein, 1998). Researchers have also argued that CT can support teachers to explicitly teach metacognitive strategies to students, which can improve students' mathematical problem solving (Teong, 2003; Yadav et al., 2022).

In response to the synergy between CT and high-cognitive demand thinking, and given that existing research suggests there has been successful integration of CT ideas into elementary mathematics instruction, unpacking ways CT might be leveraged to support teachers and students in engaging in rich mathematical tasks is an important research area for further exploration.

## Study purpose and research questions

In this study, we examined whether and how focus on three CT practices—abstraction, decomposition, and debugging—supported teachers in adapting tasks from their CMs to have high cognitive demand. The broader project of which this study is a part (described further below) focused on understanding how elementary teachers incorporated CT into their mathematics and science teaching. Preliminary examination of the study data suggested that one of the unanticipated effects of focusing teachers' thinking on CT as they planned mathematics lessons was that teachers used CT ideas as tools to raise the cognitive demand of tasks they found in their CMs. Based on an analytic memo (Maxwell, 2015) documenting this unexpected result, we conducted a detailed, post-hoc analysis of how CT related to cognitive demand in two focal teachers' mathematics lesson planning. We addressed the following research question:

> How, if at all, did examining mathematics CMs through the lens of CT practices support two elementary teachers in adapting tasks to maintain or increase their cognitive demand?

## Conceptual framework

This study utilized the *math task framework* (Smith et al., 1996), which consists of the levels of cognitive demand in Table 1 and the two phases of task transformation, from *instructional materials* to *set up by the teacher*, and from *set up by the teacher* to *implemented by students*. First, we categorized the tasks presented in the participants' CMs and the versions of the tasks they planned to present to students according to the levels of cognitive demand to gain insight into how a focus on CT practices may have shaped teachers' choice and adaptations of tasks. Then, we utilized the phases from the math task framework to guide further analysis, focusing on the transition from the first to the second phase. To study the transition from tasks as given in CMs to tasks as set up by the teacher, we analyzed the planning conversations of the participants as they incorporated CT into their lessons. We used this analysis to describe how consideration of CT practices shaped teachers' goals for the lessons and the ways they considered the kinds of thinking students would do.

## Methods

### Study context

The CT4EDU project was an NSF-funded research partnership between a university in the Midwestern United States and a nearby, urban and suburban intermediate school district. The purpose of the project was to support elementary school teachers to incorporate CT into their mathematics and science teaching. The CT4EDU project focused on four big ideas in CT, *abstracting* important information from situations, *decomposition* of complex problems into simpler parts, finding and leveraging *patterns,* and *debugging,* or finding and fixing errors. These CT practices were chosen based on pre-interviews with the teacher participants (Rich, Yadav, & Schwarz, 2019), who were able to see connections between these four practices and Common Core State Standards for Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) as well as their own mathematics teaching practices.

The first year of the project, from which this study's data is drawn, focused on supporting teachers in using CT in unplugged contexts—that is, in creating CT-infused experiences for their students that did not involve use of computers or programming. The project team viewed this unplugged approach a potential "onramp" for teachers and students to using CT practices with digital computers. For example, supporting students to decompose mathematics problems into parts (see rightmost column of Table 2) could prepare them to later break computational problems into parts (see middle column of Table 2) and eventually to develop modular computational solutions—a computational problem-solving practice identified by Weintrop et al. (2016) in their CT in Mathematics and Science Practices taxonomy for secondary students.

The project team introduced CT practices to the participating teachers in a three-day professional development workshop. During this workshop, teachers and PD facilitators first shared their current understanding of CT and its component practices and then discussed the value CT practices could add to math and science instruction, both as flexible problem-solving skills and early exposure to computer science ideas. Next, teachers explored three kinds of CT activities: general, math-related, and science-related. The general CT activities were embedded in everyday contexts that did not specifically relate to any curricular topics. For example, teachers wrote directions for a partner to draw a predetermined design to support their thinking about *decomposition.* They revised their directions after their partners attempted to follow them as an exercise in *debugging.* The math-related and science-related activities were designed to highlight how the CT practices could be embedded in typical math or science lessons. For example, teachers explored how students might go about *decomposing* the complex task of placing fractions and mixed numbers on a number line. They also discussed how they might use *pattern recognition* to help them draw conclusions from data they collected during a science experiment.

Later in the summer, the teachers involved in the project were asked to plan one math lesson and one science lesson, starting from CMs, that incorporated at least one of the CT practices ideas mentioned above. As part of a second three-day professional development workshop, teachers spent a day planning a math lesson. In the morning, teachers looked for opportunities for students to engage in CT practices within the existing lesson and noted where they thought each practice was already present or could be added. In the afternoon, they created more detailed lesson plans that outlined what they (the teachers) would do in the lesson, what students would do, and how they might informally assess student learning. On the last day of the workshop, teachers shared their lesson plans with the whole group and reflected on the lesson-planning process.

Each teacher enacted their planned lesson in the first three months of the school year. All the teachers introduced the four CT practices to students prior to teaching the lessons they planned at the professional development workshop. Both of the participants in this study had student-facing CT posters hanging in their classrooms. These posters were created by another project participant and served as a communal reference to the four practices. All teachers were encouraged, but not required, to incorporate explicit references to CT into their instruction. Note that although the participants introduced all four CT practices to their students, *patterns* did not come up in the data we analyzed for this study, so the analysis focuses on abstraction, debugging, and decomposition.

## Research design

Given the post-hoc nature of the study and limited existing knowledge of whether and how teachers think about mathematics CMs from a CT perspective, we used an exploratory case study approach (Yin, 2017). Our case study proposition was centered around examining CT as a tool for examining mathematics CMs and CT for supporting cognitive demand. We chose two teachers as the focus and unit of analysis in our case study.

### Participants

Two of the 11 teachers from the larger project were chosen for inclusion in this study. These two teachers, Alice and Cindy (pseudonyms), were from the same district and were using *Math Expressions* (Fuson, 2012), the set of CMs mandated by their district. Specific notes about why each of these teachers made for interesting cases to examine are included below.

Alice was a fourth-grade teacher with 15 years of experience. We chose Alice as a case because initial viewing of Alice's conversations during the professional development seminar suggested deep engagement with her CMs as well as with the task of incorporating CT into the mathematics lesson. She served as a strong example of how CT might be used as a lens for supporting thinking about student strategies. Alice worked on the lesson-planning task with a mathematics specialist from her district and a member of the project staff. She taught at a school where 76% of students qualified for free and reduced lunch and 53% were non-White.

Cindy was a fifth-grade teacher. She had five years of experience, but the year prior was her first year in her current district and her first year in fifth grade. Because there were two other fifth-grade teachers from her district participating in the project, Cindy worked in a small group with those two teachers as well as two project-affiliated facilitators (including the first author). We chose to focus on this group because they held an extended discussion of the CMs in general as well as the format and content of the specific lesson they focused on. Their conversation was a strong example of how CT could function as a tool for supporting careful pedagogic reasoning on the part of teachers. We chose to focus on Cindy, as opposed to the other group members, because Cindy dominated the conversation—she spoke 593 sentences during the discussion, more than double the 283 and 129 spoken by the other two teachers. Thus, we had more insight into Cindy's thinking than to the thinking of the other teachers. Cindy taught at a school where 63% of students qualified for free and reduced lunch and 42% were non-White.

### Data

We drew on two primary data sources for this study. First, we examined the lessons from *Math Expressions* that served as the starting point for teachers. Second, we transcribed and analyzed video recordings of the planning conversations of the two relevant small groups during the math planning day of the professional development workshop. We also transcribed and analyzed the recording of each of the focal teachers sharing their lesson plans with the full group on the last day of the workshop.

### Analysis

Analysis occurred in two steps: assigning levels of cognitive demand and examining the transition from CM tasks to tasks set up by teachers.

First, we assigned a level of cognitive demand to each teacher's core task at two points: as presented in the CMs and as planned by the teacher. We examined the *Math Expressions* lessons that served as starting points for Alice and Cindy, as well as the particular student tasks that were the focus of their conversations. Two researchers independently classified these tasks according to the levels of cognitive demand, using Smith and Stein's (1998) description of each level as a guide (See Table 1). The researchers agreed on the classification of one teacher's task and resolved a disagreement about the other through discussion. Next, we created descriptions of the tasks these teachers planned to pose to students, piecing together details from the planning conversations and classroom videos. Two researchers also classified these tasks according to cognitive demand, with agreement on each.

Second, we analyzed the transcripts of the planning conversations to understand the processes by which the tasks as presented in the CMs were adapted for use in the classroom. The first author read the transcripts in their entirety, highlighting all the decisions these teachers made during planning. Next, she identified the decisions that related to changes to the task. For each of these decisions, she examined the justifications and explanations the teachers articulated during the decision and coded them according to whether and how teachers' considerations of CT practices influenced the decision, and if so, which practices. An explicit mention of one of the CT practices or direct reference to one of the handouts containing descriptions of the CT practices was considered potential evidence of influence of these practices on the teachers' reasoning. Decisions were coded as influenced by CT when the teacher (1) made the decision shortly after expressing a desire to incorporate a CT practice and then related the decision to the practice's description, (2) described how a proposed change to a task would provide opportunities for students engage in a CT practice, or (3) described how her thought process connected to a CT practice as she reflected on the lesson-planning process.

To increase the dependability of the coding results and manage any interpretation biases, we used a process of dual coding. When the first author's coding was complete, the full transcripts and a list of each teacher's decisions was presented to the third author along with a code book giving one example each of a decision related and unrelated to task transformation and specifying the criteria for coding a decision as influenced by CT. The third author coded the decisions according to whether they related to task transformation, whether they were influenced by CT practices, and if so, which practices. Because the focus of this analysis is on examining decisions that related to task transformation *and* were influenced by CT, we looked for agreement on tasks that were coded as both of these. That is, when one researcher coded a decision as related to task transformation *and* influenced by CT, but the other did not, this was considered a disagreement. When both researchers coded a decision as *either* not related to task transformation or not related to CT, or when they both coded a decision as related to task transformation *and* influenced by CT, this was considered agreement. This analysis resulted in 84% agreement and a Kappa value of 0.60, indicating moderate to substantial agreement. Discrepancies were resolved through discussion. When coding was completed, we wrote narrative summaries of how CT played a role in teachers' decisions about task transformation. Narrative summaries of this analysis appear in the results.

## Results

### Alice

Alice was working from a fourth-grade lesson on estimation and mental math. Table 3 shows the initial tasks posed in the CMs and the task Alice planned during the professional development and set up in the classroom. Both tasks on the left can be classified as Procedures without Connections. Students are asked to produce two answers to the first task—an estimate and an exact total—but not to explain their reasoning. Students are asked to provide a solution method and a yes-or-no answer for the second task. Attention to method has greater potential for raising the cognitive demand than attention only to the answer, but students are only asked what Tomas can do, not why. The suggested discussion topics in the teacher text have the potential to push students to think conceptually, but the tasks themselves do not rise to the level of Procedures with Connections. Moreover, Alice described her previous enactments of this lesson as focused on students rounding to estimate, then finding the exact answer using another (likely algorithmic) method. As she noticed language in the CMs suggesting students mentally adjust their estimates to find an exact total, Alice said, "I just teach them to find the exact and the estimate. So, I'm guessing what they mean by that is you're adjusting the estimate by finding the exact." Thus, there is some evidence indicating that Alice's previous ways of setting up the task in her classroom, as well as the task as presented in the CMs, might have been classified as Procedures without Connections.

The task on the right in Table 3, by contrast, can be classified as Procedures with Connections. Students are asked to make sense of why the problem's narrator and his friend might be disagreeing about their estimates. While the problem suggests using rounding and addition procedures to make sense of the situation, students must think about the impact of estimating via rounding to the nearest hundred on the real-world context. Thus, the planned version of the task has a higher level of cognitive demand than the tasks as posed in the CMs.

**Table 3** Alice's starting tasks and task as set up in the classroom

| Starting Tasks from *Math Expressions* | Task as Set Up by Alice |
|---|---|
| The best selling fruits at Joy's Fruit Shack are peaches and bananas. During one month Joy sold 397 peaches and 412 bananas. (a) About how many peaches and bananas did she sell in all? (b) Exactly how many peaches and bananas did she sell? | My friend gave me $930 to purchase items for a trip. The exact costs are $651 for his plane ticket, $112 for clothes, and $156 for meal gift cards. I rounded the amounts and added them to get an estimate of $1000 for the total cost. I told my friend he did not give me enough money, but he said I was wrong. I rounded the costs to the nearest hundred and added them like this: 700 + 100 + 200 = $1000. Can you help me figure out what I did wrong? Did he give me enough? How did I round incorrectly? |
| Tomas has $100. He wants to buy a $38 camera. He also wants to buy a $49 CD player and 2 CDs that are on sale 2 for $8. How can Tomas figure out whether he has enough money for all four items? Does he have enough? | |

## Role of CT in transition from task in CMs to task set up by Alice

Three of Alice's decisions while transforming the task were influenced by consideration of CT practices. These decisions are in Table 4.

First, Alice decided which task from the CMs she would use as the main task in her lesson. She primarily attended to the two tasks shown at the left of Table 3 and decided to start with the latter task. This decision was driven by a desire to give students an opportunity to engage in *decomposition* of a problem into parts or steps. She felt the two-part format of the first task did the decomposition for students: "I feel like now, looking at this, this wouldn't be good because they're kind of giving it to them. You know, they're telling them how to break it down." She felt the latter task left that work to the students: "So this would be a little bit more complex because it's not telling them how to break it down." This shows how Alice used attention to decomposition as a general strategy for choosing a focal task.

Second, Alice decided to change the statement of the problem to make sure it prompted a discussion about different possible estimates and how those estimates differ from the exact total. Alice claimed that when she taught the lesson in the past, discussions of over- and under-estimation came up naturally, but she wanted to build the discussion more intentionally into the lesson. Speaking of the task on the bottom left of Table 3, Alice said: "If I added the question… how much extra will he have left over… 'cause I feel like I just automatically go there. Is this an overestimate or is it an underestimate. So maybe just build that in." One factor driving this decision was a desire to provide an explicit opportunity for *debugging*. Of this decision, Alice later said, "And that would get, like you said, into the debugging." This part of the discussion eventually led to the statement of the problem shown at the right of Table 3.

Third, Alice decided to change the numbers in the task to be in the hundreds. According to Alice, *Math Expressions* directed students to always round to the highest place value—two-digit numbers to the nearest 10, three-digit numbers to the nearest 100, and so on. She expected students to use this rounding technique as they made estimates and felt that changing the numbers to be in the hundreds would lead to estimates farther from the exact total: "These numbers aren't gonna have them overestimate. So maybe change them so that the numbers are higher? So then it's like 138? And then if they're only doing 100…" Alice trailed off without completing this sentence, but as the discussion continued, it became clear she was reasoning that if students rounded to solve the problem as stated in Table 3, they would round to the nearest 10 and end up with an estimate only $5 away from the exact total. But if she changed one of the costs to $138, they would round to the nearest hundred and end up with an estimate at least $38 from the exact total. Estimates that are further away from the total, reasoned Alice, could lead to a discussion of *debugging* because poorer estimates would increase the likelihood of making a problematic decision based on those estimates, requiring students to consider what went wrong. These latter two decisions show how Alice saw a natural fit between debugging and estimation.

### Cindy

Cindy was working from a fifth-grade lesson on fractions greater than 1. Table 5 shows the student tasks as posed in *Math Expressions* and the tasks Cindy planned during professional development and set up in the classroom. The tasks on the left can be classified as Procedures without Connections. Students can complete the page by following the procedures given in the examples, without thinking conceptually about the mathematics. The task on the right, by contrast, can be classified as Procedures with Connections. Students must provide two additional representations of a particular fraction greater than 1, given either a picture, sum of unit fractions, or fraction. They are prompted to think about the whole when they have to decide how many squares (or other shapes) to draw in their pictures and how many unit fractions to ring in the second column. Thus, the planned version of the task has a higher level of cognitive demand than the tasks as posed in the CMs.

#### Role of CT in transition from task in CMs to task as set up by Cindy

Cindy made five decisions influenced by her attention to the CT practices that led to the transformation of the task (see Table 6).

First, Cindy decided to teach the *Math Expressions* lesson in two parts. During the first part, she would focus on helping students create and interpret representations of fractions greater than 1. Only during the second part would she focus on interchanging fractions and mixed numbers: "We're talking about a visual representation and a numerical representation [*The numeric representation she is referring to is a sum of unit fractions*]. But there's also the whole being able to say it both ways as a mixed number and an improper fraction. So I don't know if that would be activity 2?" Adding more focus on the other representations was a significant shift from the CM lesson, which framed the main goal as converting

**Table 4** Alice's decisions while planning how to set up the task

| Description of Decision | CT Practice Influencing Her Thinking |
|---|---|
| Select particular tasks from within the CM lesson | Decomposition |
| Change the statement of the question | Debugging |
| Change the numbers in the problem to be in the hundreds | Debugging |

between fractions and mixed numbers. As she reflected on her lesson plan, Cindy credited this decision to thinking about *decomposition*:

> I think it's even the CT is helpful to me as the teacher, in a sense that I'm now looking through a finer lens at the lesson itself and thinking, gosh, the workbook does go in this order, this fast. But really breaking it down and trying to think like the students are, and really think about what challenges they have. And how I can decompose the lesson itself into smaller pieces that I know that they can handle or that will build more sequentially for them to help them get to that end goal.

Similar to Alice, Cindy's consideration of decomposition led her to think about the complexity of the task students were to complete. Unlike Alice, however, she chose to focus on one aspect of the task. In this case, rather than taking away opportunities for students to make sense of a task, the teacher decomposing a task on students' behalf led to more opportunities for students to think conceptually, as shown by the changes Cindy made to the task. Relatedly, in this passage, Cindy explicitly says that CT supported her in looking at her CMs differently, supporting the idea that CT served as a productive lens for her.

Second, Cindy decided to launch the lesson by showing students one representation at a time (picture, or sum of unit fractions) and having a class discussion about how they could change one representation into the other:

> I would present those two. A visual and the numerical, but using different problems. On one slide you'd have the visual, and you'd ask them how much is here. Then on the next slide would be just the numerical and say, how much is this? And then tie it together and say, ok, let's go back to the first example where we saw these, this visual. Could we show this one using unit fractions? And then go back to the numerical one and say, could we model this?

Cindy felt that this would keep the focus on the first big idea she identified in this lesson: understanding what happens and what it looks like when a fraction is greater than 1. This

**Table 5** Cindy's starting tasks and task as set up in the classroom.

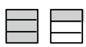
<table>
<tr><th>Starting Tasks from Math Expressions</th><th>Task as Set Up by Cindy</th></tr>
<tr><td>Change each mixed number to a fraction.<br>Example:<br>$2\frac{1}{2} = 2 + \frac{1}{2} = 1 + 1 + \frac{1}{2} = \frac{2}{2} + \frac{2}{2} + \frac{1}{2} = \frac{5}{2}$<br>$3\frac{2}{5}$ = ____   $2\frac{3}{8}$ = ____<br>(four additional problems are given)<br>Change each fraction to a mixed number.<br>Example:<br>$\frac{13}{4} = \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = 1 + 1 + 1 + \frac{1}{4} = 3\frac{1}{4}$<br>$\frac{10}{7}$ = ____   $\frac{12}{5}$ = ____<br>(four additional problems are given)</td><td>Fill in the missing parts of the chart. In the unit fraction column, draw a ring around the whole.<br><table><tr><th>Picture</th><th>Sum of Unit Fractions</th><th>Fraction</th></tr><tr><td></td><td>$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$</td><td></td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td>$\frac{12}{5}$</td></tr></table>(additional rows were given)</td></tr>
</table>

**Table 6** Cindy's decisions while planning how to set up the task

| Description of Decision | CT Practice(s) Influencing Her Thinking |
|---|---|
| Divide the lesson into two parts, with the first focusing on representing fractions greater than 1 | Decomposition |
| Launch the lesson by showing a visual representation or a sum of unit fractions and asking students how to change one to the other | Abstraction |
| Change the format of the student page to include visual representations and sums of unit fractions | Abstraction, Debugging |
| Change the numbers in examples to be less than 4 | Abstraction, Debugging |
| Encourage students to circle unit fractions that make up a whole | Abstraction, Decomposition |

decision effectively transformed the lesson from focusing on converting between fractions and mixed numbers to converting between more conceptual representations. As she shared her lesson plan with the group, Cindy justified this choice as follows: "[W]e really wanted to emphasize that understanding. Not just them counting them up or using that algorithm to just quickly switch back and forth."

During the conversation that led up to this decision, Cindy connected the greater focus on making sense of representations to the CT idea of *abstraction*: "The abstraction though, I liked the discussion where we talked about building from the concrete, the building, to the drawing, to the visualizing, and then eventually moving them to being able to compute on their own." As the discussion moved into the specifics of the lesson, the idea of having students connect the visual and unit fraction representations came up. Cindy then connected the class discussion of the representations back to abstraction as she reflected on her lesson plan:

> Yeah, that abstraction is heavy. Even having them consciously aware of what that abstraction feels like and looks like here. To have that discussion when you go from the visual to the sum of unit fractions or the mixed number and really highlighting that idea.

During the discussion, she had begun to think about symbolic representations of fractions as an abstraction and developed ways to have students use other representations to highlight the important information about fractions (e.g., the relationship of the denominator to the size of the pieces in a whole). This decision highlights how Cindy saw a natural fit between abstraction and the mathematical content of fractions, similar to how Alice saw a natural fit between debugging and the specific mathematical content of estimation.

Cindy's third and fourth decisions were closely related to the second. Third, she decided to incorporate the pictures and sums of unit fractions into the student page so that students' independent work would more closely mirror the class discussion. She ended up creating a four-column chart as shown at the right of Table 5. Fourth, Cindy chose to limit the examples the class discussed together, and the problems on the student page, to numbers less than 4. She did this so that drawing models and writing sums of unit fractions remained a viable strategy: "I don't like when they put like, 20 in there. Because if you're trying to shade and model, it's like, what? 20 circles? Come on now. Even 9 is too much. Can we stick with like, 1, 2, or 3?" (The student page in *Math Expressions* included numbers such as $20\frac{3}{4}$ and $\frac{56}{6}$ as problems.)

She related these decisions to abstraction in a similar way to how she related her lesson launch idea to abstraction. She felt that giving students opportunities to translate between visual representations would help them better be able to abstract the important information from symbolic representations of fractions. She also noted that the revised student page offered opportunities for *debugging*. Of the new format of the student page, she said, "I think the debugging we have somewhat too, in terms of giving them the chart. Because they were, they can compare with a partner or work with a partner." She reasoned that because more of the students' work and thinking would be visible, they would be better able to notice and correct any mistakes they might make. Similarly, of the number choices she said, "And then the debugging as well. I think giving them more opportunities to reflect on what they're doing, and giving them some tools to use to kind of find their mistakes." The "tools" she is referring to are the visual representations, which are only possible with smaller numbers.

Lastly, Cindy decided to ask her students to draw a circle around the whole(s) as they wrote out sums of unit fractions. For example, when writing $\frac{5}{4}$ as $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$, she encouraged students to circle the first four fourths to show that those add up to 1 whole. This decision came about as she thought about both *decomposition* and *abstraction.* Early in the conversation, Cindy noted how conversions between mixed numbers and fractions required students to decompose whole numbers: "So I'm thinking that the decomposition is big in this concept. Just because they do have to take the 2 and 2/3 and they have to break it down into $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ and they have to know, how many unit fractions is that?" Later, however, she reflected that the sum of unit fractions may be overwhelming to students. She discussed the idea of circling the unit fractions as a way to simplify the unit fraction representation:

> [W]e've got all these fourths lined up. That's kind of overwhelming to look at. And at first glance we might think, well gosh, at first glance I've got no idea how much all of those are. Is there a way that we can simplify that? By, oh yeah, I know that four of those fourths, if I put those together those are going to equal 1. So let's circle this.

The planning tools available to teachers during these planning conversations identified abstraction as a process of simplification. This passage shows another way that Cindy took up this description of abstraction and saw it as a natural fit with fractions. When she imagined completing the task from a student perspective, she thought the representation $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ was "overwhelming" because it was difficult to immediately make sense of how many fourths were there and how the total related to the whole. She believed asking students to put a ring around the first four fourths would support them in seeing the total in a simplified, more digestible way—as one whole plus an extra fourth.

## Discussion

As the two participants in this study planned lessons from their mathematics CMs with CT practices in mind, they made adaptations to raise the cognitive demand of tasks the CMs suggested posing to students. In particular, attention to *decomposition* and *debugging* contributed to Alice setting up a Procedures with Connections task rather than a Procedures without Connections task. Thinking about *abstraction* contributed to Cindy *decomposing* the lesson and transforming a Procedures without Connections task into a Procedures with Connections task. She created a new student page in part because she felt it would provide more opportunities for *debugging*. In the sections that follow, we discuss the ways in which the reasoning fostered by examining CMs through the lens of CT both echoes other studies of productive teacher CM use and suggests an affordance that is specific to CT.

### Similarities to other studies of CM adaption

For both teachers, examining their CMs through the lens of CT practices assisted them in planning high cognitive demand tasks for their classrooms using strategies identified in prior research. In Alice's case, her dual focus on the CT practices of decomposition and debugging supported her in thinking deeply about how students would approach various tasks—a strategy for adapting tasks demonstrated in past research (Choppin, 2011; Grant et al., 2009; Stein et al., 1996). Specifically, thinking about whether students would have opportunities to decompose problems led her to consider the impact of the CMs breaking problems into subparts for students—which is one way of lowering the cognitive demand of a task by changing a challenge into a nonproblem (Stein et al., 1996). Thinking about reasons why students might need to debug led Alice to consider how she expected her students to approach rounding problems—in this case, by always rounding to the highest place value—and the impact that approach may have in a real-world context. She subsequently designed a task that she hoped would prompt students to think about the implications of their strategies, not just to blindly apply those strategies.

In Cindy's case, her consideration of the CT practices of decomposition, abstraction, and debugging supported her in thinking deeply about the big mathematical ideas in her lesson—another strategy for adapting tasks demonstrated in past research (Stein & Kaufman, 2010). As she considered numerical representations of fractions and mixed numbers as an abstraction, she began to consider the multiple mathematical ideas that are encapsulated in those representations. For example, the notation $\frac{7}{5}$ is intended to communicate multiple ideas, even when one considers only the part-whole interpretation of fractions: wholes are divided into 5 equal parts, we are considering 7 of those parts, and the fact that there are more than 5 parts means that the number $\frac{7}{5}$ is greater than 1. The number $\frac{7}{5}$ is an abstraction that captures all this information, but Cindy realized she did not think students would be able to "see" all this information in a strictly symbolic representation without more experience with other representations. Through her consideration of abstraction, Cindy identified multiple big mathematical ideas in the lesson, which she then used to decompose the lesson into parts. Once she had settled on one big mathematical idea for the first part of the lesson—the "greater-than-1-ness" of mixed numbers and fractions with greater numerators than denominators—she designed student tasks and a lesson launch that focused on this idea.

Thus, CT practices could potentially be used as a framework for supporting teacher reasoning about CMs that leads to more opportunities for students to engage in high cognitive demand tasks. Using CT practices for this purpose may be particularly useful if schools or teachers are looking for strategies that could apply across subjects instead of introducing a framework specific to mathematics.

### A specific affordance of CT as a lens for examining curriculum materials

While much of Alice and Cindy's pedagogical reasoning can be mapped onto strategies for CM adaptation described in other research, this study also suggests how using CT practices as a lens for examining CMs might support a novel kind of reasoning. Specifically, we theorize that the CT practices of abstraction, decomposition, and debugging might serve to bridge the two previously described strategies—focusing on big mathematical ideas and considering student thinking—in teachers' thinking as they plan with CMs. Specifically, teachers took up CT practices in ways that provided opportunities for them to see how their students could engage in with the mathematical ideas. By using CT as a lens for analyzing CMs, teachers reflected on how their students may engage in thinking processes, or metacognitive strategies (Yadav et al., 2022), that both increase the cognitive demand of the tasks and make the tasks accessible to their students.

For example, Alice's initial consideration of the CT practice of debugging led her to identify the big mathematical ideas she wanted to focus on—specifically, considering whether estimation strategies led to over- or under-estimates and what the impact of the differences between an estimate and an exact answer might be. She felt intentionally focusing on this big idea, instead of expecting it to crop up in the discussion, was a way to highlight debugging opportunities. As she imagined how such a lesson might play out, Alice considered the impact of students' rounding strategies in a way she had not in previous years of teaching the lesson and created a version of the task that would prompt students to reflect on those strategies metacognitively. For Alice, thinking about the CT practice of debugging first supported her in focusing on a big mathematical idea, then led her to consider student thinking—linking the two CM adaptation strategies discussed in prior research.

Relatedly, as she began to think of symbolic fractions as abstractions, Cindy realized there were multiple conceptual ideas students were expected to grapple with in her lesson and decided to spend more time on a particular one ("greater-than-1-ness" of mixed numbers and their equivalent fractions). As she focused on "greater-than-1-ness," she considered how a student might become overwhelmed when examining a long string of summed unit fractions and incorporated the strategy of asking students to draw a ring around unit fractions summing to a whole. For Cindy, abstraction served both as an avenue to identify the multiple big ideas included in her lesson and a prompt for considering how students might interpret a novel representation and how she could support them to track and document their thinking about that representation. Abstraction linked her consideration of big mathematical ideas and student thinking.

In sum, the novel contribution of CT to teachers' reasoning might not be in supporting them to consider big mathematical ideas (Stein & Kaufman, 2010) or student thinking (Choppin, 2011), but in providing a connection between these two strategies for CM adaptation. These two teacher planning strategies are connected to the *topics* students explore and the *processes*—including metacognitive processes—students might use to solve problems. Interestingly, the strategies have a loose correlation to the two types of integrated CT and math activities articulated by Nordby et al. (2022) in their recent review of computational thinking in the primary mathematics classroom: activities that focus on CT skills (or topics) and activities that focus on CT processes such as communication and exploration. When teachers used CT practices as lenses for examining their mathematics CMs, they created tasks that supporting topical explorations *and* anticipated student thinking processes, suggesting the tasks would support several types of student learning. Future research could further investigate whether CT practices serve this dual role for other teachers. If so, CT practices could serve as a powerful lens for teacher examination of CMs that elicits multiple kinds of productive thinking from teachers that lead to rich learning experiences for students.

The available data do not allow us to empirically examine *why* the lens of CT practices offered novel support in connecting student thinking to big mathematical ideas. Through our team discussions, we have speculated that the added value may come specifically by framing the CT practices as coming from computer science rather than discussing them as mathematical practices. While the CT practices highlighted here bear a strong resemblance to disciplinary practices used in mathematics, the import of these ideas from computer science may have aided the teachers in engaging with them in different ways that supported new kinds of pedagogical thinking. Decomposition, for example, is an idea discussed in the Common Core State Standards for Mathematics (CCSSI, 2010), but only in reference to decomposing mathematical objects such as numbers or geometric shapes. Computer scientists, by contrast, tend to discuss decomposition of *problems* (Yadav et al., 2017). This broader nature of the object being decomposed supported Alice in thinking about decomposing the steps of a problem (rather than a number into place-value parts) and supported Cindy in moving beyond thinking about decomposing a fraction into unit fractions into thinking about decomposing the multiple mathematical ideas in her lesson. We believe that the nature of CT practices as being unique to computer science, a subject area not expected of elementary teachers, allowed them to see familiar practices in new ways and make their thinking visible during the lesson co-design sessions without the fear of being judged about their knowledge of mathematical content or pedagogical practices. As computer science education continues to emerge as a unique research area, mathematics teacher educators may benefit from cross-disciplinary conversations that offer new perspectives on existing ideas.

## Conclusions and future directions

This study illustrated at least one potential use of CT practices to support elementary mathematics education that is worthy of further investigation. Specifically, CT served as a productive lens through which elementary teachers could examine their CMs and adapt tasks to be of higher cognitive demand. Teachers created these higher-level tasks by using the CT to focus on big mathematical ideas, anticipate student thinking, and consider the connections between the mathematics and student thinking. Examining CMs through the lens of CT may therefore serve as a curriculum-proof strategy (Taylor, 2016) for supporting teachers in planning high-quality mathematics instruction.

At least three lines of future research in this area are warranted. First, researchers should develop and study professional development experiences for teachers that include attention to both CT practices and the importance of high cognitive demand tasks. Explicitly designed learning experiences for teachers that address the connections between these concepts may better support teachers in using CT practices as a tool for creating and enacting high cognitive demand mathematics tasks. Second, teachers should supplement the analyses of planning conversations in this study with interviews or other data sources that will more explicitly contribute to understanding how teachers are using CT practice to shape their instructional planning. Lastly, further research should explore teachers' classroom implementation of their lessons incorporating CT to investigate whether CT practices help sustain high cognitive demand of tasks during instruction.

## Limitations

This small exploratory study had several limitations. We were able to conduct the detailed analysis of the work of only two teachers who may not be representative of the project or their school or district. The analytical frame of the mathematics task framework (Smith & Stein, 1998) provides a useful way to consider student and teacher thinking, but its cognitive demand categories are coarsely grained and do not support a fine-grained analysis of the impact of CT.

**Acknowledgements** The authors would like to thank Dr. Corey Drake for her comments on an earlier version of this manuscript.

**Funding** This study is supported by the National Science Foundation under grant number 1738677. Any opinions, findings, or recommendations are those of the authors and do not necessarily reflect the views of the National Science Foundation.

**Availability of data and material** Not applicable.

**Code availability** Not applicable.

## Declarations

**Conflicts of interest** The authors declare no conflicts of interest.

**Ethics approval** The Institutional review board at the university at which this study was conducted approved the study.

**Consent to participate** Participants gave informed consent to participate in the study.

**Consent for publication** Not applicable.

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