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Processes of decision-making by mathematics PD facilitators: the role of resources, orientations, goals, and identities

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Abstract
Facilitators of professional development (PD) for mathematics teachers currently gain increasing attention, as their practices are crucial for the success of spreading mathematics educational ideas and innovations into schools and strengthening the professional expertise of teachers. So far, mainly two components of facilitation have been empirically researched: facilitators' knowledge resources and facilitation moves. In this paper, we propose to develop a more comprehensive conceptual framework in order to explain facilitators' practices and underlying decisions. For this purpose, we lift Schoenfeld's ROG framework for teachers' decision-making (comprising resources, orientations, and goals) to the facilitator level, and add identity as a fourth component to obtain what we call the ROGI framework. We draw on data collected simultaneously in two PD projects for mathematics teachers in Germany and in Israel. Through three cases of facilitation, we illustrate how the adapted ROGI framework helps to understand processes of decision-making made by facilitators during mathematics professional development sessions.

Keywords Professional development of mathematics teachers · Facilitators' practices · Facilitators' decision-making · Adapting frameworks from the teacher level to the facilitator level

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Introduction

In recent years, there is a growing interest within the mathematics education community in the work of facilitators, i.e., those who lead PD programs for practicing mathematics teachers.[1] Facilitators are a specific and important subgroup of Mathematics Teacher Educators (MTEs), a term which, as Even and Krainer (2014) note, commonly refers to both those who educate prospective teachers (e.g., Appova & Taylor, 2019) and those who educate practicing teachers (e.g., Zaslavsky & Leikin, 2004). In this paper, we refer to the latter group. The interest in facilitators is linked to the need for increased understanding of the crucial role they play in processes of scaling up PD models (Cobb & Jackson, 2011; Rösken-Winter et al., 2015; Zehetmeier, 2015). A facilitator is a professional who manages the PD activities, sets norms for interactions, supports teachers' exchange of experiences and insights, monitors the discussion, and works with teachers toward the goals set for the PD. This is a highly demanding job, which requires preparation and support. Moreover, as with the job of a teacher, the experience gained through the work itself is expected to enhance professionalism, and in that regard the first year on the job is crucial in novice practitioners' learning process. Despite the growing literature on MTEs in general (e.g., Beswick et al., 2020), empirical studies that scrutinize facilitators' practices and the decisions underlying them are still scarce, and even more so when it comes to novice facilitators.

In this paper, we aim to contribute to this yet under-researched topic. We present cases of facilitation carried out by novice facilitators in two different contexts, and offer a conceptual framework that may help to explain facilitators' observed practices by investigating decision-making underlying them.

Existing research on facilitators

The job of mathematics PD facilitation is complex, as facilitators are expected to be committed to the goals and agenda of the PD program they represent, yet, at the same time, to be responsive to the idiosyncrasies of the specific context in which they act and to the needs of the participating teachers (Borko et al., 2014). In recent years, research on facilitation is accumulating in several directions:

- Facilitators' preparation (e.g., Borko et al., 2015, 2017, 2021; Karsenty, 2016, 2018a; Rösken-Winter et al., 2015);
- Facilitators' needed skills and knowledge (e.g., Borko et al., 2014; Elliott et al., 2009; Lesseig et al., 2017);
- Facilitators' evolving practices (e.g., Coles, 2019; Gibbons et al., 2021; Kuzle & Biehler, 2015; Prediger & Pöhler, 2019; van Es et al., 2014). These studies mostly focus on describing and categorizing enacted facilitation moves. Beyond this important focus, there is a scarcity of empirical studies that attempt to examine and interpret facilitators' decisions, whereas at the teacher level such research is already well established.

[1] Some other names used for this role are leaders (Borko, Koellner & Jacobs, 2014); mentor-teachers (Kuzle & Biehler, 2015); didacticians (Jaworski & Huang, 2014; PD providers (Even, 2005); teacher educators (Zaslavsky & Leikin, 2004) and other terms (see Even & Krainer, 2014).

One of the challenges faced by researchers in this emerging field is developing frameworks to conceptualize the profession of the facilitator and the process of becoming a facilitator. Karsenty (2020) reviews different ways by which such frameworks may be created, referring specifically to double-level frameworks, i.e., frameworks that can serve for both researching teachers and researching PD facilitators. Facilitators, like teachers, intend to support learning; on the classroom level, the learning content is mathematics, and on the PD level, it is mathematics teaching. Thus, many existing frameworks for researching facilitators were established by *lifting strategies* (Prediger et al., 2019). Under this term, Prediger et al. (2019) subsume all approaches that exploit analogies between teachers and facilitators in relation to research approaches, research questions or conceptual frameworks, established on the teacher level and adapted to the facilitator level (e.g., Borko et al., 2014; Jaworski & Huang, 2014; Karsenty, 2020). In the next section, we describe the double-level conceptual framework we developed by the lifting strategy for our research. As in other studies, the lifting must consider not only the similarities, but also the differences between teachers and facilitators.

The ROGI framework for understanding facilitators' practices and decisions

For the current study, we developed a framework for understanding the decisions underlying facilitators' practices (where practices are considered as the observable behavior and decision-making as the mental action leading to it). The basic assumption is that facilitators' decision-making takes place explicitly and implicitly, and shapes goal-oriented behaviors.

Based on this assumption, we chose to adapt Schoenfeld's (2010) well-known ROG framework, created to conceptualize teachers' decision-making. It is a suitable framework to be lifted to the facilitator level since it allows to explore analogies between teaching and facilitating as "knowledge-intensive goal-oriented behavior[s]" (Schoenfeld, 2010, p. 199). Schoenfeld's (2010) ROG framework for teachers' decision-making consists of a triad of constructs:

(1) *resources* (R): The intellectual, material, contextual, and social means on which the person draws;
(2) *orientations* (O): Overarching and domain-specific beliefs, values, preferences, etc.;
(3) *goals* (G): Explicit as well as implicit, often situational, purposes.

Schoenfeld analyzes the mechanisms of interactions between these three constructs in order to describe, explain and even predict teachers' decision-making processes.

When adapting the three constructs to facilitators, differences between teachers and facilitators were considered, in particular, the fact that facilitators' resources, orientations, and goals can refer to both the level of the mathematics classroom and the level of the mathematics PD. For example, mathematics teachers' knowledge resources include mathematical content knowledge (CK) and pedagogical content knowledge (PCK), e.g., knowledge about how students learn mathematics (Shulman, 1986). Facilitators' knowledge resources contain CK and PCK, as these are incorporated in the content knowledge for the PD level, but they also include PCK at the PD level, e.g., knowledge of how teachers learn the specific content of the PD (Borko et al., 2014; Carroll & Mumme, 2007; Prediger & Pöhler, 2019; Wood & Turner, 2015; Zaslavsky & Leikin, 2004). Thus, adapting the ROG

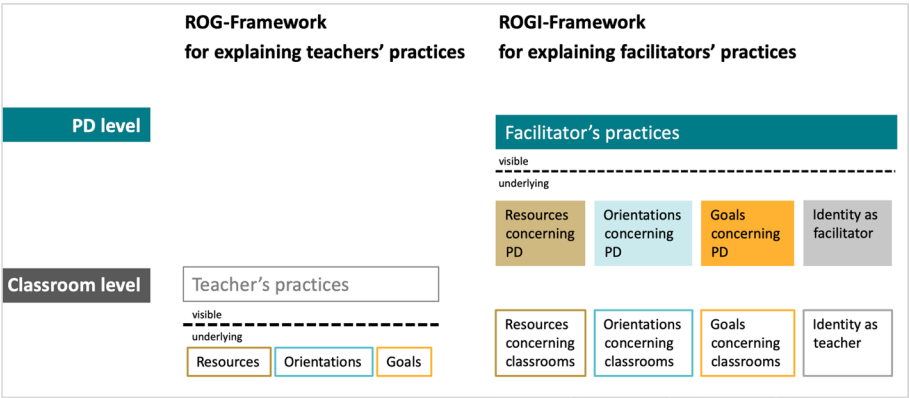

Fig. 1 Adaptation of the ROG framework into the ROGI framework

framework to facilitators requires the *nesting* of the classroom level within the PD level (Prediger et al., 2019), as Fig. 1 depicts:

> ***Resources*** are enlarged to include relevant knowledge resources, material resources (e.g., facilitator guides provided by the PD team or the project's website), social resources (e.g., support provided by colleagues), etc.
> ***Orientations*** are enlarged to include not only teacher orientations toward student learning but also facilitator orientations toward teacher learning.
> ***Goals*** are enlarged to include content and process goals of the PD (Prediger & Pöhler, 2019), and atmosphere goals (e.g., maintaining a safe and respectful PD environment).
> ***Identity***: We propose to extend the ROG framework to include a fourth component—the facilitator's *identity*. The construct of identity has gained considerable attention of the mathematics education community in the last two decades (see, for example, the review by Darragh, 2016), and many different perspectives and definitions, as well as critical analyses thereof, have been published (Graven & Heyd-Metzuyanim, 2019; Sfard & Prusak, 2005). Herein, we use the term "identity" in the sense defined by Gee (2000, p. 99): "Being recognized as a certain 'kind of person', in a given context", also revoiced in the words of Darragh, (2016, p. 26), that "identity is a result of the process of identifying, whether this is self-identification or identification by others". We associate this definition with the perspective that identity involves the *positioning* of oneself within relevant environments and in relation to other participants in these environments. The term "positional identities" (Graven & Heyd-Metzuyanim, 2019, p. 368) is a useful one, and served several researchers who studied identities of students or teachers (e.g., Foyn et al., 2018; Losano et al., 2017). Losano and her colleagues have referred to positional identities, based on the ideas of Holland et al. (1998), as relating to "power, status, and the claim for material and social resources according to the social position of a person" (Losano et al., 2017, p. 292). They used the conceptualization of positional identities to explore the development of a novice mathematics teacher in her first year on the job. Other researchers have shown that the construct of identity is significant for understanding practices of mathematics teachers, in the particular context of participation in professional development communities (e.g., Gresalfi & Cobb, 2011; Hodges & Cady, 2012). Thus, adding identity as a fourth component of the suggested framework

is in line with empirical research on *teachers*, implying that it can be significant also for researching *facilitators*. Moreover, we note that this fourth component emerged quite straightforwardly in our research during the data analysis, when we sensed that the three ROG components were helpful and necessary, but not sufficient. Since new facilitators often have other roles (e.g., teachers, heads of mathematics departments, researchers, etc.) the "juggling" between these identities may affect their practices. It became evident that identity was prominent in the facilitators' narratives, yet was not entirely captured by looking at resources, orientations and goals. As a consequence, we began to explicitly search for facilitators' identity narratives, i.e., where they expressed the ways they recognize or position themselves or how they are recognized or positioned by others. In Sect. 3 we demonstrate the usefulness of identity in explaining decisions taken by facilitators within PD sessions.

We termed the modified framework for the analysis of PD facilitation as ROGI. Figure 1 presents a schematic summary of how ROG is lifted into, and nested within, the adapted ROGI framework.

In the following, we apply the ROGI framework to describe and explain cases of facilitation. Specifically, we present cases in which novice facilitators attempted to initiate, monitor, and sometimes re-direct teacher discussions within PD sessions. Managing discussions is of special interest, as leading productive discussions seems to be difficult especially for new facilitators (Borko et al., 2014; Schwarts, 2020). Thus, our research question is:

What drives the practices and decisions of novice facilitators while directing mathematics teachers' discussions within PD sessions?

To answer this question, we draw on data collected simultaneously in two PD projects, that are very different from one another in terms of cultural contexts, content, aims, and organizational framing. Within these differences, we attempt to distill common aspects that go beyond the specific features of each program. In a sense, this effort can be seen as addressing the call by Borko (2004), to conduct what she terms "phase 3 research"; in this kind of PD investigations, the goal is "to provide comparative information about the implementation, effects, and resource requirements of well-defined professional development programs […] [by] gathering and analyzing data from multiple professional development programs, as they are enacted by multiple facilitators at multiple sites" (Borko, 2004, p. 11). Such research is still scarce (see Sztajn et al., 2017), and although our scope here is modest—we restrict our perspective solely to the aspect of facilitating—we believe that this is an opportunity to gain new insights. As we shall show, despite the salient differences between the projects, there were apparent similarities concerning the facilitator profession which transcend context, goals, and frame of work.

Research context and methodology

Context: two PD programs for mathematics teachers

The investigation of novice facilitators' practices during their first year in this role was conducted comparatively in two PD programs, in Germany and in Israel. Both programs are research-based, and in both the implementation of in-service PD courses went through a process of upscaling, which necessitated the preparation of new facilitators.

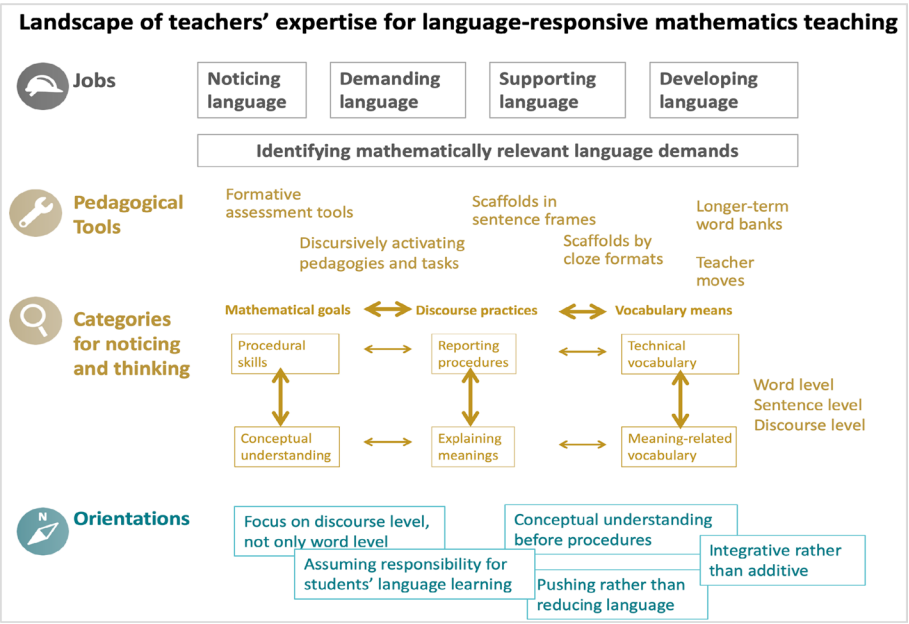

Fig. 2 Specified PD goals for language-responsive mathematics teaching (slide from facilitator preparation)

The German PD context: language-responsive mathematics teaching project

In the Language-Responsive Mathematics Teaching Project, the MuM research group in Dortmund (Prediger, 2018; Prediger & Zindel, 2017) designed and investigated a PD program for promoting mathematics teachers' expertise in fostering language in mathematics classrooms (Prediger, 2019). In five iterative design research cycles since 2014, four PD modules (of 3–4 h each) were developed to engage teachers in language-responsive teaching approaches. Figure 2 (from Prediger, 2019) shows the specified PD content goals for the teachers, related to five jobs: noticing, demanding, supporting and developing language, as well as identifying language demands. In addition, the teachers' pedagogical tools, categories for noticing and thinking, and orientations required for fulfilling these jobs, are presented.

Since 2017, teachers and university lecturers were qualified as facilitators for scaling up the program and were supported by designed PD curriculum materials, including worksheets and video clips for engaging teachers in PD activities (e.g., analyzing videos, analyzing students' products and task design), slides for providing some input, and a manual for facilitators informing about the underlying PD approaches and possibilities of adaptations. To date, more than 100 facilitators were prepared. The novice facilitators in the current study were prepared for facilitation within one year that included introductory courses and a professional learning community for preparing and reflecting facilitators' experiences, and also by tandem teaching in the PD, in sum 30 h of preparation.

Lenses for the teacher level	What the lenses unpack
Mathematical and meta-mathematical ideas	The space of relevant ideas and concepts that underlie the topic of a mathematics lesson; meta-mathematical ideas (e.g., one counter example is sufficient to refute a conjecture) that are employed within the lesson.
Explicit and implicit teacher goals	Possible goals that may be attributed to the teacher, on the basis of actions or decisions observed in the lesson, as well as pros and cons of preferring certain goals over others.
Classroom tasks and activities	Features of the tasks and activities and how they are enacted in the lesson, including whether and when this process develops differently than expected ("a posteriori task analysis").
Teacher-student interactions	How the teacher poses further questions to those included in of the task; listens to (or ignores) comments or difficulties raised by students; manages discussions; delegates responsibilities in the process of knowledge generation.
Teacher dilemmas and decision-making	Teacher decisions prior to and during the lesson; situations of dilemma (i.e., when there is no evident optimal course of action) that the teacher seems to be facing during the lesson, and possible pathways that can be offered to resolve these dilemmas, while considering consequent tradeoffs.
Teacher beliefs about mathematics teaching, how students learn and the teacher's role	Orientations, beliefs and values that may be attributed to the teacher; implicit messages that may be conveyed to students through the teacher's communications and actions.

Fig. 3 The six-lens framework used in the VIDEO-LM project (Karsenty, 2020)

The Israeli PD context: VIDEO-LM

VIDEO-LM (Viewing, Investigating and Discussing Environments of Learning Mathematics) is a PD project running in Israel since 2012. The project's main aim is to promote reflective skills of secondary mathematics teachers about their practice, through peer discussions around videotaped lessons of unknown teachers. The discussions are guided by a six-lens framework developed for this purpose. The lenses are mathematical and meta-mathematical ideas; goals; tasks; interactions; dilemmas and decision-making; and beliefs (Arcavi & Karsenty, 2018; Karsenty & Arcavi, 2017; Karsenty, 2018b). These lenses were designed to direct participants' attention to different features of teacher actions and decisions, as a vehicle for discussing relevant issues at the core of the mathematics teaching practice (Fig. 3).

As of 2015, the project team had designed and implemented a system for preparing and supporting lead-teachers to become VIDEO-LM facilitators (Karsenty, 2016, 2018a) in

order to meet the increasing demand for PD courses. Two cohorts of the facilitator preparation course were conducted, (each consisting of 30 h, spread over one year within 7 monthly sessions) with 52 participants in total. The novice facilitators received Web-based materials and a personalized support provided by a VIDEO-LM team member throughout their first year of facilitation.

Methods of data collection

The data presented in this paper relates to three PD sessions, two held in Israel and one in Germany (see the contexts presented above). These were selected from the larger corpus of collected data, which encompassed multiple means, as follows:

- *Video-recordings of the facilitators' PD courses.* The novice facilitators were videotaped during PD sessions they conducted, at least 3–4 h per facilitator (or tandem). The video-recordings were aimed at capturing facilitators' practices.
- *Video-recordings from video-stimulated post-PD reflections*. The facilitators reflected on the videos from their PD sessions, in a meeting with a researcher, based on a semi-structured protocol (adapted from Speer, 2005). These meetings were aimed at eliciting the facilitators' perspectives and interpretations regarding their observed practices. At any point the facilitator could stop the video and comment on whatever he/she wished to discuss.
- *Reflective journals*. Facilitators wrote journals, describing the plan and the intended goals before each PD session, and their reflection after the session. These journals were aimed at gaining insights into facilitators' decision-making processes throughout the PDs. In some cases, written journals were substituted by additional reflection sessions with a researcher.
- *Pre-, post-, and follow-up questionnaires*. At the beginning and end of the first year of facilitation, and 8 months after the end of the first year, each facilitator filled a written questionnaire. Questionnaires prompted facilitators' input about various aspects of their role (e.g., motivation, goals, reservations and difficulties, facilitation dilemmas, etc.).

These multiple sources of data provided different kinds of information regarding the facilitators' resources, orientations, goals, and identities, and allowed for triangulation.

For this paper, we chose three cases, focusing on how facilitators manage teacher discussions during PD sessions. The criterion for case selection was their potential to illustrate interesting phenomena in discussion management by novice facilitators. Many of the documented cases, both in Israel and in Germany, were suitable for this purpose. We selected from among them three that highlight different facilitators' practices and decisions. Other cases will be reported elsewhere.

Methods of data analysis

The qualitative thematic analysis was conducted in parallel for both projects by the respective research teams and was discussed in joint meetings between teams. It consists of three steps across the four data sources stated above:

Step 1. In the videotaped PD sessions, practices of managing discussions were identified, and relevant sections were transcribed.

Step 2. The transcribed episodes, as well as the three other sources of data (video-stimulated reflections, questionnaires, and journals) were analyzed using the ROGI framework (see Figure 1), with the aim of depicting the facilitator's decision-making processes underlying his/her observed practices, and bringing them to the fore as explanatory means. Goals and resources were often explicitly referred to by facilitators, yet could also be inferred by examining facilitators' conduct within the PD. Orientations were inferred mainly by articulations in which facilitators talked about teaching, facilitation, pedagogies, etc. (e.g., "teachers should be thoroughly acquainted with the material they teach, and beyond it as well"; "my job as a facilitator is to hear the teachers"). Identities were inferred, in line with our definition of identity stated earlier, from instances where facilitators either expressed how they recognize or position themselves, or how they are recognized or positioned by others, (e.g., "I understand myself not only as a moderator […] I try to become part of the group"; "everyone is much more experienced than me").

Step 3. The attributed resources, orientations, goals and identities were triangulated and validated. Only those aspects that were identified by the researchers within at least two data sources were included in the final interpretation. Each case analysis was conducted and confirmed by at least two researchers, to enhance the reliability of interpretations.

Herein we use the following abbreviations: T1, T2, etc. denote different teachers participating in a PD session; PQ denotes pre-questionnaires; R denotes video-stimulated reflection (e.g., R2–60 denotes transcript line no. 60 in the second reflection session).

Empirical analysis and findings

Josh's case: between eliciting and 'telling'

Josh is a secondary mathematics teacher in his 4th year of teaching, a graduate of a prestigious teacher education program. He serves also as a pedagogical director and is highly regarded in his school. Josh conducted his first VIDEO-LM PD in his own school, with 6 participating mathematics teachers, most of them with teaching experience of over 30 years. In the second session of the PD, Josh screened a 12th grade lesson in Analytic Geometry on the canonical equation of a circle, filmed in a low-track class. The filmed teacher starts by inviting two students to the board. One student fixes one end of a ribbon onto the whiteboard, and the second stretches the other end and marks points while she moves it around. The class guesses that the lesson's topic is *circles*. Then, students attempt to define a circle.

The following exchange occurred in the PD after the teachers watched this aforementioned classroom video episode:

300 Josh: […] Can we conclude what are his [the teacher's] goals in the lesson from these 3 minutes? And if so, what are they?

301 T1: The definition of circle, first of all, from these 3 minutes, it's...

302 T2: He wanted to illustrate for them what a circle is. Make it vivid.

303 Josh: Okay. To illustrate for them...

304 T2: How I create a circle, how I construct a circle…

305 Josh: [*To T1*] So again... you said?

306 T1: The definition of a circle. Which is actually...

307 Josh: The definition of a circle.

308 T2: How do I construct a circle in... in reality.

309 T1: […] we talk of a set of points… that they would understand what is the set… what set of points exactly…

310 Josh: But who defined the circle here?

311 T1: Ah…

312 Josh: There is no actual definition yet, right?

313 T1: No.

314 Josh: […] Who spotted the lesson's topic? Did the teacher tell them what the topic of the lesson was?

315 T1: The children shouted it out... they said "it's a circle".

316 T3: No, they said it…

[…]

320 T1: They recognized the figure...

321 Josh: They said "it's like a compass"... "ah, then our topic is... the circle".

321 T1: Right.

323 Josh: That is to say, it's possible that one of his goals is also maybe that the students… the issue will be raised by the students…

Whereas Josh starts with an open question (Turn 300), and at the beginning re-voices what teachers say (Turns 303 and 307), his questions become more and more closed-ended (in Turns 310, 312, 314), and the discussion concludes with Josh himself providing an answer (Turn 323) to the question he brought up (in Turn 300), regarding the filmed teacher's goals. Interestingly, Josh directs the participants' attention to the filmed teacher's pedagogy, i.e., to have students actively construct an idea and connect it to their previous knowledge, while Josh himself acts in an opposite manner, as he is the one to provide the answer that apparently he wants to hear. We named this practice *eliciting the answers expected in the session plan*.

Explaining Josh's observed practice

To explain Josh's practice in this PD section through ROGI analysis, we aimed to identify Josh's resources, orientations, goals, and identities. We included data from the video-stimulated reflection session with Josh, as well as his pre-questionnaire, to reveal these elements.

Josh's identities. In R1, Josh stopped the video after the aforementioned exchange, and said:

R1–60 Josh: […] I don't like these moments, when I have to raise something, because it becomes more "a teacher" and less "a facilitator".

R1–62 Josh: […] When I am there [in the PD], especially in my position compared to them […] everyone is much more experienced than me... I'm not aiming at introducing something new to them [...]. And if we compare it to teaching, I don't want to transfer any knowledge, I don't want to tell them, "Look, this thing is happening", I expect that they will bring things up following my guiding words...

These quotations reveal that Josh is not pleased with his practice in this episode, and that he identifies it as something he would do as a teacher but should not do as a facilitator. He makes a clear distinction between these identities and the practices they yield: as a mathematics teacher, he sees his job as "transferring knowledge", whereas as a facilitator his goal is to allow teachers to "bring things up". This distinction was also expressed in Josh's pre-questionnaire:

Item 6 in PQ: In your opinion, is there a difference between a good teacher and a good facilitator?

PQ Josh: These are two roles that require very different abilities: A good teacher in my view is someone who comes with the knowledge and a goal of explaining it and conveying it to those he faces, meaning that most of the focus is on him. A good facilitator is required to be a guiding factor […] it's not his role to bring the knowledge he possesses or to advocate the merit of his own practices.

Thus, it appears that in Josh's identity, the tension between teacher and facilitator is still unresolved. On the one hand, he explicitly wishes to avoid bringing his teacher identity into the PD, perhaps since he positions himself as less experienced than his peers ("everyone is much more experienced than me… I'm not aiming to introduce something new to them", R1–62). On the other hand, the PD transcript indicates that Josh does not act as "a guiding factor", as he thinks a facilitator should. As we show next, this tension is manifested also in contradicting goals.

Josh's goals. Josh's declared goal is that teachers will "bring things up", rather than be told "Look, this thing is happening" (R1–62). He specifically does not wish to "transfer any knowledge" to teachers, yet this is eventually the practice he enacted in the episode. This contradiction points to the possible existence of a competing goal, that is perhaps more implicit, yet dominant. The words "I have to raise something" (R1–60) may shed light on such a goal. The articulation '*have to*' indicates that Josh saw no alternatives for action, which implies he was compelled to achieve something that for him was most important. To understand this "something", three citations from the post-PD reflection session are helpful (emphasis added):

R1–3 Josh: I come quite ready, all the things the teachers say, I have them already in my notes, and they [the teachers] only meet my expectations, what I pre-prepared.

R1–54 Josh: So, here I don't know if I had other things on the page that were *supposed* to come up.

R1–215 Josh: If I had time, I would look at the page, see what else she *should* say and direct her there.

These citations suggest that Josh pursuits the goal of having teachers arrive at certain outcomes in the discussion, that are pre-specified in his session plan. This goal is clearly in tension with the goal of managing a discussion in which teachers freely bring their own input. We further see this tension when examining the element of resources.

Josh's resources. Josh explained that when preparing for the PD sessions, he used the Observer Guide (OG), a document analyzing the lesson that appears in the VIDEO-LM website:

R1–79 Josh: What mainly helps me to prepare the session is […] the OG, […] things that I wouldn't have noticed on my own, ah…

R1–99 Josh: I don't come with prior knowledge, again, the only knowledge I have is from the OG, […] but… it's not exactly knowledge, there is no absolute truth in there, so it's hard to call it knowledge […] And many of the questions I ask, maybe even almost all the questions, are, if I see that they didn't talk about some of the things that are written there, so I direct them through questions to talk about these things.

Josh attributes great importance to the ideas in the OG, and refers to them as some sort of "knowledge". This reveals a possible challenge in the transition from teaching to facilitating: switching the learning content from mathematics to mathematics *teaching*, changes 'the rules of the game'. These two learning contents have a different epistemic status: within school mathematics, it is the 'authority' and rigor of mathematics itself that ultimately establishes the correctness of a statement, whereas within mathematics teaching

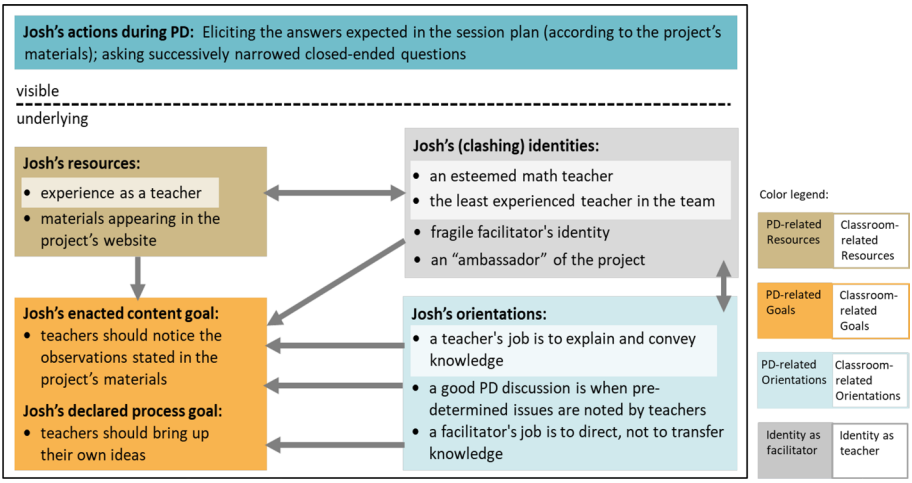

Fig. 4 Summary of the ROGI analysis of Josh's case

discussed in a PD, no explicit rules regulate what is considered the "right" answer for questions such as "how should we teach canonical circles?". Encountering this epistemological discrepancy is perhaps inevitable when shifting from a mathematics teacher to a facilitator, and may have led to the incoherence in Josh's goals and actions. In the lack of an "absolute truth", the OG becomes Josh's central source of knowledge as if it substitutes the rules of mathematics. Although Josh is aware of the epistemological difference ("there is no absolute truth in there, so it's hard to call it knowledge", R1–99), he still considers the OG ideas necessary for the discussion, as reflected in the words "supposed" and "should" in R1–54, 215 above.

Josh's orientations. A strong orientation that seems to be held by Josh is depicted in the quotations already brought above: Josh appears to believe that *a good discussion in a PD is one in which all the ideas he (or the session plan) aims for should come up* (either by the teachers or by himself). This is further evidenced in the following citation:

R1–273 Josh: If I want to define what is a good [PD session] […] it's whether during the discourse the issues from the OG appear. This is for me a good-enough indication, even a very good indication.

Consequently, Josh appears to believe that his job as a facilitator is to direct teachers to notice and voice out ideas appearing in the OG, which are perceived by him as the desired PD content. This orientation brings us back to identity. We can interpret Josh's belief in light of another identity he holds, that of an "ambassador" of VIDEO-LM, who represents the project's agenda.

To sum up Josh's case: Josh's identity as a facilitator is still in the making; he is not pleased with himself when watching the PD episode, and is disappointed to acknowledge that he was not acting as a facilitator but rather as a mathematics teacher, a stance he was trying to avoid in the first place. Josh acknowledges the differences between a teacher and a facilitator, differences that originate from two sources: the different epistemic status of the learning content (mathematics vs. mathematics teaching) and the differences in the target audiences. We suggest that Josh's little experience as a facilitator, combined with his identity as a person representing the project and faithful to it, compelled him to act more upon

his identity as a mathematics teacher, and thus to prefer the content goal of the ideas noted in the OG, over the declared process goal of encouraging teachers' own contributions.

Josh's resources also limit his practices: as a teacher, he is used to "come with the knowledge", yet as a facilitator, his preparations heavily lean on the newly-acquired OG content. This seems to have played an additional role in his recruitment of practices that align with his teacher rather than facilitator identity, and further explains his adherence to the session plan: as a teacher he is expected to cover certain topics each lesson, and this expectation appears to have percolated into his PD management, where he felt obliged to cover the OG ideas.

The ROGI analysis of Josh's case is summarized in Fig. 4 (one-head arrows denote influence of underlying aspects on others, two-head arrows denote interrelationships between aspects, as described in the analysis). Josh's case reveals the conflicting forces with which a novice facilitator deals, and how these forces may hinder the realization of his original goals, as happened in most of Josh's second session. Considering Josh's multiple identities is prominent in understanding his conflicting orientations and goals (i.e., teachers would bring their own ideas vs. teachers would arrive at pre-determined ideas), and in pointing to what drove his enacted decision (i.e., the dominancy of his teacher identity).

Nina's case: using the lenses to encourage teacher reflection

Nina is an experienced teacher who serves as head of the mathematics department in her school, where she also facilitated a VIDEO-LM PD. This was her first time as a VIDEO-LM facilitator, and the episode below is taken from the second session of the PD, with five participating teachers. The group watched the first 20 min of an advanced 10th grade lesson, *Introduction to Calculus*. In the video, the students attempt to sketch the graph of $f(x)=4(x-1)^3-3x+4$, without any prior knowledge of calculus, and the teacher leads them to conclude that they lack tools for this task. Before the video was screened, Nina gave each teacher two different 'lenses', written on cards, to think about while watching the lesson. At some point, Nina re-screened a 30-s clip, where several students discuss aloud whether finding the intersection points of $f(x)$ will be sufficient for sketching the graph, while the teacher is seen drawing an axes-system on the whiteboard and avoiding taking part in this discussion.

After the teachers watched again this clip, the following exchange occurred:

262/264 Nina: What happened here now? […] What didn't she do?

265 T1: She didn't intervene in their conversation.

266 Nina: She didn't intervene in their conversation, okay? What was their conversation about?

267 T1: What helps me and what doesn't help me.

[…]

270 T2: It's not a straight line, it's not a parabola, it's not a circle...

271 Nina: And what does she choose to do here?

272 T1: To be silent.

[…]

279 Nina: […] Can we think what was the purpose of this avoidance?

280 T2: To empower them. [...] She trusts them, [...] that is, he [the student] said it already so why should she say it again... Once it's closed and the statement is correct, then... it's powerful that she doesn't... ah… intervene, and also, do you need the teacher's approval? No.

281 Nina: So can I connect this to the second lens you have? What was your second lens?

282 T2: Beliefs about mathematics teaching. Ah [...] her beliefs?

283 Nina: Hers, hers.

284 T2: Yes, so... she actually lets them, ah... I don't know if it's to teach each other, but she actually gives them some kind of, ah... she empowers them in such a way that later she... they can also themselves... take their part in the lesson, ah… without her.

285 Nina: Who holds the knowledge? Who is the source of authority and knowledge so far? From what we have seen...

286 T2: Well, overall, in mathematics teaching, a teacher comes and provides knowledge to his students.

287 Nina: Is this the [filmed] teacher's belief?

288 T2: No, I'm saying… it's…

289 Nina: Is that your belief?

291 T2: No... it's a common thing. Now, what is she doing? […] She lets them discuss and close the discussion, and then she goes on like nothing happened, when she's actually aware of them enriching each other without her.

[…]

294 Nina: Okay [looks at all participants], what else, in your opinion is the [filmed] teacher thinking… about mathematics teaching? Things that... her beliefs?

295 T3: When it comes from the students […] it's better than… like, it must be absorbed well.

296/298 Nina: Instead of telling them... a, b, c...

299 T3: Although now, for me, she shattered something... Like, to give an example, in 8th grade I taught a new topic [...] I wrote on the board and one student said, "I want to [do it], I want to". So I said to him "listen, when I teach something for the first time I want the whole class to hear it from me" […] The first time I teach how to do it, I want it to come... that the students will hear it from me […] and she is really not like that. Like, she has no problem, she will go with the flow, okay...

As can be seen, Nina first asks teachers to describe the filmed teacher's decision not to intervene in the students' conversation. Then, she presses further to uncover the goal for this decision. From Turn 281 onward, Nina connects the ideas of Teacher 2 to the lens of 'beliefs', and directs the teachers, through a sequence of questions, toward articulating the observed teacher's, and the participants', beliefs. In Turn 299, a profound reflection emerges by Teacher 3. Her words ("now, for me, she shattered something") suggest that she realized her practice was not the only possible way to act. We named Nina's practice in this episode *leading teachers toward reflection*.

Explaining Nina's observed practice

Here too, we use the reflection session with Nina to shed light on her possible resources, orientations, goals, and identity.

Nina's orientations and goals. After watching this episode, Nina stopped the video and commented:

R1–154 Nina: 'For me, she shattered something' Teacher 3 says [...] Clearly it's linked to her beliefs about teaching mathematics, but she herself doesn't think it's linked...

R1–158 Nina: Because we often don't think about why we do what we do... like if you ask me what my beliefs are... I'll tell you a, b, c. And then if you ask me how I teach the

straight-line equation, I'm not sure there would be a correspondence between what I told you and what I do. And now she is suddenly urged to think about it, and it shakes her, even shutters something for her.

R1–163 Nina: As a facilitator I feel that this question […] 'who is the source of knowledge in class?' is a question that opened a meaningful space here for insights of each one about themselves.

In this vignette, Nina expresses what we might refer to as "meta-orientation", i.e., a belief about beliefs. She raises the idea that beliefs guiding people's actions may remain latent within their cognizance: First, she refers to a possible lack of awareness of Teacher 3 regarding her own beliefs ("it's linked to her beliefs about teaching mathematics, but she herself doesn't think it's linked", R1–154); then, she generalizes this idea ("we often don't think about why we do what we do", R1–158), and finally, she exemplifies it referring to herself ("I'm not sure there would be a correspondence between what I told you and what I do", R1–158). Turn R1–163 reveals that this meta-orientation is linked to how Nina perceives her role as a facilitator, seeing it as one of her jobs to encourage teachers to explicate their beliefs, thus opening "a meaningful space […] for insights of each one about themselves" (R1–163). Moreover, she acknowledges that the uneasy feelings that Teacher 3 describes can be seen as productive in this process ("now she is suddenly urged to think about it, and it shakes her", R1–158). We note that the goal that Nina related to, i.e., having teachers talk about their beliefs, resonates with the VIDEO-LM declared goal – to help teachers reflect on their own practices, through the observation of other teachers and the analysis of goals, dilemmas, beliefs, etc. ascribed to the observed teacher (Karsenty & Arcavi, 2017). In the PD episode, Nina consistently pursues this goal. Turn 281 is key in this endeavor: it is at this point that Nina connects between possible purposes for a certain teacher action, and the lens of beliefs. At first, she insists on focusing on the filmed teacher's beliefs (Turn 283), but then she provokes the teachers, in what seems to be a deliberate move, to examine if they are talking about the filmed teacher or about themselves (Turns 287, 289).

However, Nina has other goals as well, explicitly stated in the pre-questionnaire:

PQ Nina: I would really like to act within my team at school, to turn it into a team that learns more and advances toward quality teaching, and I think that such a PD, and especially with me, the head of department, as the facilitator, can be effective in pursuing this goal. […] I want to create a shared language, through the VIDEO-LM language, for future conversations within peer-observation, peer-feedback and the feedback I give my teachers.

Thus, Nina aligns her goals as head of the mathematics department with her goals as a VIDEO-LM facilitator. This alignment can explain her coherent persistence, throughout the episode, to use the lens of beliefs as a springboard for teacher reflection.

Nina's resources and identity. Nina has a profound knowledge about the six lenses. This is evident from the first part of this PD session (prior to the cited episode), when she devotes a considerable amount of time to explaining and exemplifying each of the lenses. Other knowledge resources available to Nina are the VIDEO-LM documents such as the OG, which – similarly to Josh – Nina mentioned in the pre-questionnaire as a source for preparing PD sessions. Yet, she also wrote in the questionnaire that "in any subject, I can find relevant materials in textbooks and various activities on the web". This indicates that Nina's span of accessible resources for facilitation is not limited to those she had acquired during her training in VIDEO-LM. Moreover, her stating of such resources suggests that she sees their use as legitimate, i.e., her facilitation role is viewed as necessitating the integration of external resources with those provided by the project. This can be interpreted as pointing to both confidence and flexibility, and is coherent with the multiple identities that

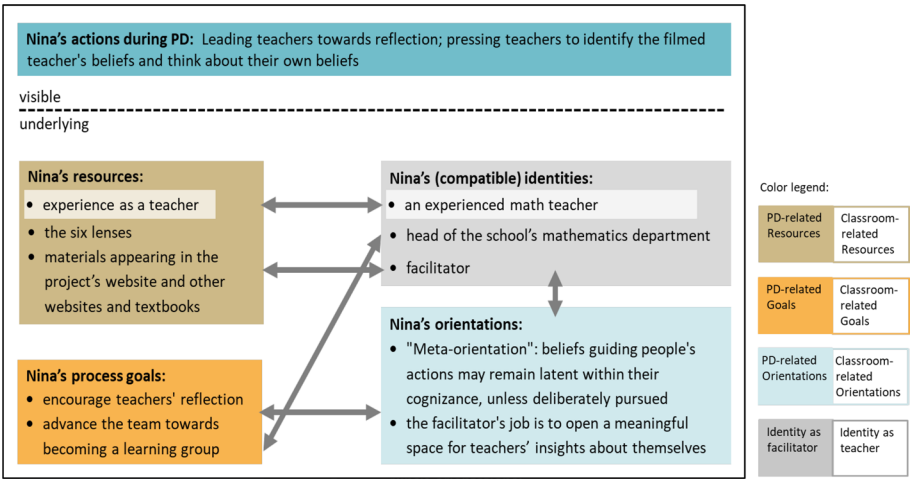

Fig. 5 Summary of the ROGI analysis of Nina's case

Nina holds, as an experienced teacher, head of department, and facilitator. She seems to be moving flexibly between these identities. For example, in the interview vignette above, she switched freely between her identity as a teacher ("if you ask me how I teach the straight-line equation", R1–158) and her identity as a facilitator ("As a facilitator I feel that this question […] opened a meaningful space", R1–163). The fact that in the pre-questionnaire she interweaves the VIDEO-LM goals with her own goals as head of department, further supports the assertion that her multiple identities effectively serve her role as facilitator, rather than clash with one another, as we saw in Josh's case.

Figure 5 summarizes the ROGI analysis of Nina's case.

Christin's and Alice's case: directing teachers' focus toward the PD content goals

The case from the German research context stems from the first PD session that the facilitators Christin and Alice have conducted together. The PD participants were 17 secondary mathematics teachers from two schools. Whereas Christin has much experience as a researcher on the language-responsive classrooms and as a university lecturer, she has limited experience in classrooms and PDs. In contrast, Alice is a secondary mathematics teacher in her 11th year of teaching and with much experience as a facilitator for other mathematics PD contents, but is new to the topic of language-responsiveness.

The major PD content goal of the session was to support teachers in identifying mathematically relevant language demands (see Fig. 2). For this purpose, categories are to be introduced which can guide and focus teachers' categorial perception and identification, and especially direct them not to focus only on the word level (i.e., teaching vocabulary) but also on the discourse level (i.e., engaging students in discourse practices).

Christin's adaptive shifts in steering the discussion

The PD episode selected for this case starts 1.5 h after the PD session began, and the discussion is facilitated mainly by Christin. In a video-based activity, a long-term learning

progression was discussed for different discourse practices in early algebra, which starts in finding and describing pattern in arithmetic tasks in Grade 3 (e.g., connections between $5+7=12$; $6+6=12$; $7+5=12$; $8+4=12$; etc.) and advances gradually toward describing generalized algebraic patterns in Grade 7 (e.g., $(5+x)+(7-x)=5+7$), with increasing complexity of the involved discourse practices of describing patterns. The episode starts when Christin asks the teachers to find similar long-term trajectories of discourse practices (like describing patterns) in other mathematical topics from their everyday mathematics teaching:

1 Christin: You can think about it briefly again. … Perhaps you have a similar example in mind, for a topic you are currently working on, a topic which is currently in the focus of your classes. Which are the relevant discourse practices and which language means are needed in this case, though? You can start thinking about it. How could you transfer the example to your class?

[…] [Four minutes chatting phase]

2 Christin: Ok, so what have you discussed? [16 sec murmuring] Which group starts to summarize for everybody what you have discussed briefly? Of course, it is not yet completely mature after these few minutes.

3 T1: Well, we said that for every mathematical topic there are always words which repeatedly occur. I remember, we have already made another school-based PD. That's why we haven't written many things down, now. There was again a glossary with your words which you can use again and again. Well, "reducing", "increasing". And at school we have a vocabulary notebook that every student should write. And there you could add words in the end. They can open it, when they have to articulate procedures.

4 Christin: Mhm. This would be on the level of word banks, again, though. [6 seconds break] Which discourse practices are especially important for you? In your math classrooms? What do you often demand from your students? [5 seconds break]

5 T2: For me, summarizing something by using own words is very important. Because I think, if you are able to do this, you probably understand the process. If you can report it with your own words, not with the given ones, but with slightly different words. I often do it like this.

6 Christin: And for the others?

7 T3: Explain why you do it like this […].

8 Christin: Mhm. Which role does "writing" thereby play in your mathematics classrooms?

9 T3: They have to write down again and again, every once in a while, how and why they do it. Which ideas and solution approaches did they have.

Turn 3 shows that some teachers have not met Christin's challenging demand of adopting long-term perspectives on discourse practices, but still work on the word level ("for every […] topic there are always words which repeatedly occur. […] we have a vocabulary notebook that every student should write", Turn 3). Christin seems to recognize this state of teacher learning. She responds to T1 by acknowledging it and linking her contribution to the categories and tools introduced before ("This would be on the level of word banks, again, though", Turn 4). With her succeeding question ("Which discourse practices are especially important for you?", Turn 4), she tries to redirect teachers' focus of attention to the discourse level.

Following these moves, the teachers shift their attention to discourse practices ("summarizing something", Turn 5; "explaining why", Turn 7). Instead of referring to these contributions explicitly and exploring them with respect to a long-term progression, Christin asks the teachers about the role of writing activities in their mathematics teaching (Turn 8).

When a teacher provides a highly important distinction between two discourse practices ("write … how and why they do it", Turn 9) she does not exploit this contribution in the following (non-printed) turns. She seems to have taken the decision to reduce the demand, from reflecting on long-term progressions to simply identifying discourse practices. We named Christin's practice in this episode *adaptive redirection of teachers' attention*.

Explaining Christin's observed practice

This descriptive account of Christin's facilitation practice can be explained through a ROGI analysis, using further data from the reflection sessions and the questionnaires:

In the post-PD reflection session, Christin and Alice extensively discussed their impression that the teachers did not contribute much and that teachers also considered much as trivial. In this context, Christin also summarized the PD episode:

R1–181 Christin: But also when they had to transfer it [early algebra example of long-term progressions of discourse practices] to their own teaching […] they contributed nothing. […] The feeling that everything is trivial [for them] […] was somehow simultaneous with the feeling that they are cognitively overloaded.

Her impression that the teachers did not see the depth of the topic and that her request to adopt long-term perspectives on discourse practices overstrained the teachers, resonates with her resources and orientations, as analyzed below.

Christin's orientations. Christin's decisions seem to be based on her general orientation that facilitator adaptivity is crucial for supporting teachers' learning and for avoiding cognitive overload (R1–181). Christin implicitly shows this orientation again and again during the reflection session. For example:

R1–22 Christin: We weren't prepared for such extreme adaptations. You should have jumped into another PD module to do this [the language-responsive mathematics teaching] for a concrete content. The general sensitization for the relevance of language-responsiveness was just there.

Christin's orientation becomes explicit at the end of the reflection session when she articulates her own learning need with regard to subsequent PD sessions:

R1–312 Christin: I would like to become more flexible in reacting to teachers' needs, whatever they may be.

Her own expressed feeling of still insufficient adaptivity may also be linked to the observation that she has refrained from commenting on the contributions made by teachers in Turns 7 and 9, where further distinctions were offered.

Christin's resources. Christin's first prompt in the PD episode ("Perhaps you have a similar example in mind, for a topic you are currently working on", Turn 1) draws upon her (topic-independent) repertoire of moves, specifically here the move of connecting newly-introduced ideas to teachers' experiences, which she describes in her reflection:

R1–181 Christine: What I tried again, sometimes, in order to activate the teachers' ideas [...] somehow the idea of developing language, whether they should do it somewhere else, too. […] Or when they noticed that this is sometimes relevant, that I have to build on a certain language […] whether they already know that somehow. […]

However, her decision to transfer the long-term trajectories of discourse practices was far-reaching and rested upon her earlier impression that many teachers had already reached the discourse level. She articulated this impression in the reflection session:

R1–22 Christin: At the beginning [of the PD session] during the Concept Mapping [the introductory activity for noticing teachers' state of learning] they already mentioned a lot, not only on the word level as expected, but mainly on the discourse level.

This reflection also provides evidence for Christin's topic-specific knowledge resources about teachers' typical learning pathways: Focusing on the word level is teachers' typical starting point, thus reaching the discourse level is a first learning goal. Noticing that many teachers seem to have already reached this goal (R1–22) appeared to have encouraged Christin (in light of her adaptive orientation) to steer toward a next step in teachers' learning trajectory, adopting long-term perspectives on students' discourse practices. However, her adaptive orientation also calls for further flexibility, this time in the opposite direction: Her knowledge about this learning trajectory is also the knowledge resource on which she draws after recognizing that not all teachers have reached the discourse level (in Turn 3). Hence, she decides to reduce the complexity of the discussion by focusing only on discourse practices (from Turn 4 on) without their long-term progression (originally addressed in the video the teachers watched before Christin posed the question, Turn 1).

Christin's goals. From Christin's utterances during the PD session and the reflection session, we can infer various possible goals that she probably pursued. She follows the content goal of the PD material, that teachers should internalize relevant categories of the language-responsive mathematics teaching, such as word, sentence and discourse levels (this goal would resonate with the redirecting move in Turn 4). However, she extends this introductory content goal at this early stage toward the more ambitious goal that teachers should generally be able to link theoretical aspects with their practical experience (a process goal possibly underlying the move of connecting ideas; Turn 1); and hence could also sequence different discourse practices (content goal, Turns 1 and 4).

These situative goals could mainly be characterized as content goals and are in line with the PD learning goals that Christin stated in the pre-questionnaire:

PQ Christin: Teachers should become aware of language requirements in mathematics classrooms. […] Teachers should become familiar with relevant categories of the language-responsive mathematics teaching […]

But pursuing content goals is also an issue of timing: According to her PD session plan, the categories for distinguishing between explaining meanings and reporting procedures were planned to be treated in detail in a subsequent PD activity. So, not taking up too early seeds of this distinction in teachers' utterances (Turns 5 to 9) was instrumental to avoid interfering with the subsequent activity.

Christin's identity. Christin's strong emphasis on formatively assessing teachers' ideas about the content goal can be explained by her background as a researcher and a university teacher educator. Within this double identity, Christin conducts research on teachers' learning pathways. Her own identity as a participant-oriented facilitator is therefore strengthened by this background.

Alice's prioritization of atmospheric goals

The presented PD episode ends with an additional contribution by Christin's co-facilitator, Alice:

10 T4: With regard to that, we can probably do more. Also with writing continuous texts, that really takes a lot of time. When you then do this you can't let the texts keep half complete. Instead, strictly speaking, you already have to look that you correct something.

That is very time-consuming. Especially because you can't realize a mutual control because they don't find most of the difficulties.

11 Alice: I think that is really our problem as teachers, as math teachers. Where do we set our priorities? And if you take always everywhere everything into account, it is barely possible to integrate this in regular sessions of 45 minutes.

In many occasions of their co-facilitation practice, Christin and Alice highly value each other's expertise and complement each other. In this case, however, Alice did not relate to Christin's question on important discourse practices. Probably she has not immediately recognized Christin's situative changed content goal, which she spontaneously adapted with regard to teachers' learning pathway. Instead, Alice picked up a teacher's concern about time constraints in mathematics classrooms: "With regard to that we can probably do more. Also with writing continuous texts, that really takes a lot of time" (Turn 10) and the related need for setting priorities.

Explaining Alice's observed practice

Alice's resources Alice's contribution could be based on her knowledge about the relevance of motivational aspects for teachers' acceptance of PD courses (*resources within the pedagogical knowledge*). As Alice expresses in the post-PD reflection session, this knowledge arises, among other things, from her own PD experiences as a participating teacher:

R1–248 Alice: I always need something for myself if I attend such PD sessions […]. I always need something that I experienced before or could experience in the future and not only, without judging it, specialized knowledge. But rather take the specialized knowledge into my classroom.

*Alice's goals and orientations*It could be assumed that through her contribution, Alice pursues the atmospheric goal that teachers should feel respected in PD sessions. Presumably this is connected to her orientation, that creating a safe and respectful PD environment is a relevant basis for teachers' learning and for integrating new ideas into classrooms. However, Alice's decision at this point, to prioritize an atmospheric goal over the content goals set earlier by Christin, demonstrates that co-facilitators can sometimes complement each other yet at other times overtake each other's agenda.

Alice's identity In contrast to Christin, Alice positions herself in this moment as an experienced teacher who knows the everyday problems of teaching, hence in an identity of a teacher, rather than only a facilitator or a researcher. The teacher identity seems to be a strong and meaningful one for Alice, as revealed in the foIllowing citation from the reflection session:

R1–225 Alice: This is often like this. That if somebody contributes something, I remember a situation in my teaching, reflect on myself and then use it as a starting point and would like to offer it to them. I understand myself not only as a moderator, who provides input. Instead I try to become part of the group, for keeping it lively after all.

Figures 6 and 7 summarize the ROGI analyses for Christin and for Alice, respectively.

Discussion and conclusions

In this paper, we pursued the research question of what drives the practices and decisions of novice facilitators while directing mathematics teachers' discussions within PD sessions. To answer this question, we gathered empirical data in two different PD contexts, and

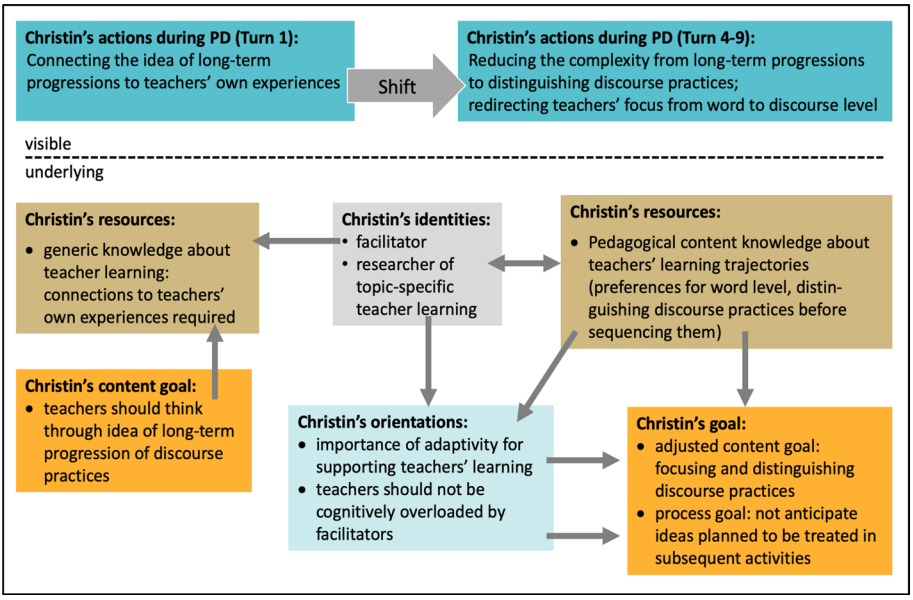

Fig. 6 Summary of the ROGI analysis of Christin's case (two actions between which a shift takes place)

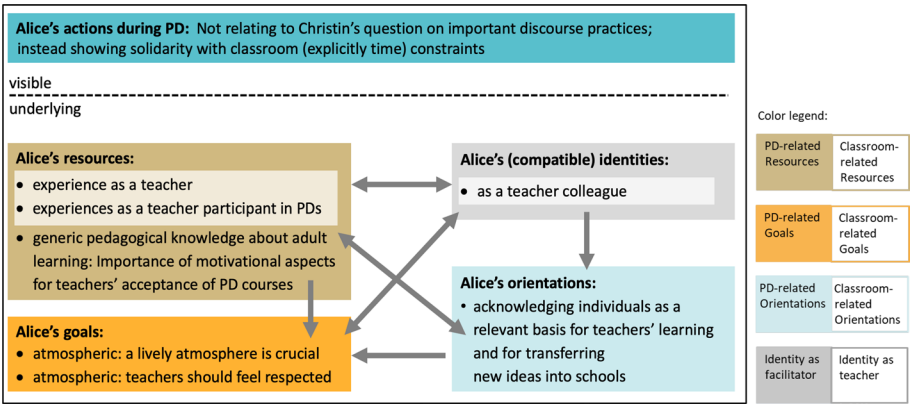

Fig. 7 Summary of the ROGI analysis of Alice's case

developed a conceptual framework in order to understand and explain identified practices of facilitation that emerged from this data. The suggested framework, ROGI (Resources, Orientations, Goals, Identities), is a result of adapting Schoenfeld's (2010) well-established ROG framework by lifting it from the teacher level to the facilitator level, to create a double-level framework (Karsenty, 2020; Prediger et al., 2019). While the original ROG components were already shown by Schoenfeld (2010) to be influential in practitioners' decision-making, the added construct of identity enriches the analysis by highlighting the decisions of facilitators as driven also by social, institutional and personal positioning.

The lion's share of the paper was devoted to a ROGI analyses performed in three cases, demonstrating that practices employed by novice facilitators while managing teachers'

discussions can be explained through a scrutinized look at how facilitators' decisions are affected by the resources available to them, the orientations they hold about teaching mathematics and about facilitation, the combination of goals they set for the PD, their various identities, and the interrelations between all these:

- In the first case (Josh), the visible practice was a facilitator's attempt to elicit answers expected in the session plan, at the expense of seizing opportunities to build on teachers' independent input. The ROGI analysis revealed the tension between conflicting goals and identities ascribed to the facilitator, which, combined with limited facilitator resources, explain his in-the-moment-decision to strictly adhere to the pre-prepared session plan.
- In the second case (Nina), the visible practice was a facilitator's use of a sequence of questions to press for teacher reflection, resulting in a meaningful insight by one of the participating teachers. In this case, the ROGI analysis suggested that the coherence existing between the facilitator's multiple identities and goals, the flexible nature of her resource use and her well-developed orientations have allowed her to support a process of generating reflection.
- In the third case (Christin and Alice), the visible practice was a facilitator's adaptive redirection of teachers' attention, while reducing the complexity of the content goal presented, followed by the co-facilitator's choice to prioritize an atmospheric goal. The ROGI analysis here uncovered that the two co-facilitators differ considerably in their identities, and that their different ensuing resources and orientations can explain why they decided to pursue different goals in their shared PD session.

Taking a comparative look at these cases may allow us to unpack differences and similarities between the four facilitators, and to distill issues of facilitation that the ROGI analyses enable us to illuminate.

Firstly, we see a spectrum of orientations regarding roles that facilitators should assume. Josh emphasizes the directive role of the facilitator, whereas Christin centralizes the facilitator's ability to be adaptive. Nina sees it as one of her roles to create a meaningful space for teachers' insights to arise, and Alice views herself as responsible for a safe and respectful PD environment. These different roles may be realized, if they resonate with the facilitators' multiple identities (as with Christin, Alice and Nina) or remain unfulfilled, due to identity conflict (as in Josh's case). Thus, we can explain different facilitators' handling of similar situations. For example, both Josh and Christin have experienced a certain disappointment when they did not receive the answers they expected. However, Josh's reaction was to narrow down his questions and ultimately provide the desired answer himself, whereas Christin shifted her goal and re-focused the discussion around another, less demanding, aspect. We suggest that what drove these different facilitators' practices can be understood in light of the ROGI analyses. Josh's orientation of the facilitator as a guiding factor was not realized, because his identity as a teacher overtook his identity as a facilitator and because his resources as a facilitator were limited. In contrast, Christin's orientation on adaptivity matched her resources, stemming from her identity as a researcher of teacher learning, hence she could exploit these resources and handle the situation according to her adaptivity orientation.

Secondly, in terms of goals, it is interesting that the four facilitators focused each on one kind of goals, either PD content goals (Christin and Josh) or process and atmospheric goals (Alice and Nina), sometimes with enacted goals that deviate from the originally articulated goals (as in Josh's case). Particularly for novice facilitators, it seems to

be challenging to find an appropriate balance between fidelity (pursuing the PD learning goals and the agenda of the PD session plan) and flexibility (assuring adaptivity to teachers' learning pathways). Thus, an important implication is that support given to new facilitators needs to explicitly address this challenge.

A similar challenge exists with respect to balancing identities; for novice facilitators, reaching a stable identity (possibly including different aspects) and being able to move flexibly between different identities, is far from being trivial (see Schwarts, 2020). Within the current analyses, this is particularly apparent for Josh with his clashing identities, and Alice, whose dominant teacher identity led her to open a path of discussion straying from what her co-facilitator had intended. Again, we draw consequences for the support given to facilitators. As Schwarts (2020) notes, facilitators' mentors should aim to assist their beginning peers to realize that "conflicts between identities should not be resolved with one identity taking over the other, but rather, holding both identities can enrich practice and keep the facilitator practice-oriented while working with her colleagues" (p. 547).

Thirdly, if we look at the resources recruited by the four facilitators, the current analyses reveal that facilitators refer to different resources (i.e., different kinds of knowledge, experiences, materials). The use of these resources is often closely linked to facilitators' dominant identities, without always being in line with their ambitions. For example, Josh's case shows that knowledge at the classroom level is not enough; new facilitators might benefit from perceiving such knowledge as an important basis which is *part* of the knowledge at the PD level. In other words, they should become aware of the nested structure of their knowledge, meaning that their knowledge about classrooms is relevant yet needs to be extended by knowledge about teachers' learning pathways and other aspects of pedagogical content knowledge at the PD level. This also has important consequences for facilitator preparation programs.

To conclude, this paper offers a conceptualization of facilitators' practices and decisions. Our main claim, which we attempted to substantiate through the presented data analyses, is that the ROGI framework is a useful conceptual tool for researching the profession of PD facilitators, specifically facilitators' evolving practices. One of the compelling features of ROGI, in our perspective, is that it allows also for studying facilitators' development and change over time, as our research team has recently done (Schwarts et al., 2021). Further research is needed for exploring how using the ROGI framework can inform approaches for preparing and supporting new facilitators, e.g., by making some of the resources, orientations, goals or identity conflicts more explicit for new facilitators. We thus hope that the framework will be used not only as a means for analysis but also as a design instrument that guides facilitator educators' decisions.

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