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One university's story on teacher preparation in elementary mathematics: examining opportunities to learn

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Abstract

This multi-case study examines how three elementary mathematics methods instructors, in the same teacher education program, provide their prospective teachers with learning opportunities. Qualitative data were collected through interviews, classroom observations, and artifacts. The findings suggest that the instructors' beliefs associated with teaching philosophies influence both the content that prospective teachers have the opportunity to learn (*what*) and the nature of the prospective teachers' opportunities to learn (*how*). Through analytic induction, three assertions were developed to understand and explicate: similarities in opportunities to learn, differences in opportunities to learn, and perceptions about the purpose of the methods courses across the three cases. Specifically, the first assertion examines how all three methods instructors focused on developing conceptual understanding and combating mathematical misconceptions for which prospective teachers most often experience opportunities to learn through representations and approximations. The second and third assertions place more emphasis on differences across the cases based upon observed instructor actions and their beliefs. This study is significant because it helps us gain a deeper understanding about prospective teachers' opportunities to learn within one teacher education program, and therefore, may point toward what can be done in the future to better prepare teachers in elementary mathematics education and the development of ambitious instruction. Additionally, this study unpacks how prospective teachers in the same teacher education program may have varying experiences and thus varied access to opportunities to learn.

Keywords Teacher education · Elementary mathematics · Opportunities to learn · Prospective teachers · Qualitative multi-case study

Introduction

This multi-case study is part of a larger, longitudinal study of pre-service teacher preparation in elementary mathematics and English Language Arts (ELA) in five preparation programs across three states in the USA. This study specifically examined how elementary

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mathematics methods instructors provided prospective teachers (PTs) with learning opportunities within their courses. Recent literature (e.g., Lampert et al. 2013) unpacks ambitious pedagogies in mathematics teacher education; yet, as documented by Clift and Brady (2005), there is still much to learn about the various instructional strategies employed by mathematics methods instructors as well as the learning opportunities offered to PTs (Cavanna et al. 2017). Therefore, the study reported here is significant because it examined PTs' opportunities to learn (OTL) ambitious pedagogies within one teacher education program, and therefore, may point toward what can be done in the future to better prepare teachers in elementary mathematics education.

The focus of this study is on instructional strategies, used synonymously with teaching practices. In particular, the focus is on instructional strategies that method instructors used in their methods courses that resulted in learning opportunities for PTs. This research unpacks both the content that PTs had the opportunity to learn and the nature of the PTs' opportunities to learn across the different cases (methods instructors). The findings are summarized by three assertions which examined similarities in OTL for PTs across the cases, differences in OTL across the cases, and the methods instructors' beliefs about the purpose of the methods courses.

Literature review

In mathematics education research, the terms ambitious instruction and standards-based teaching practices are often used together or in place of one another. Within Literature Review, I examine these constructs and discuss how observation measures of standards-based teaching practices, such as Mathematics-Scan, can be used to identify ambitious mathematics instruction. These measures can be used qualitatively to examine the ambitious practices of elementary mathematics methods instructors. Provided that the instructors' teaching practices translate into observable actions that provide learners with opportunities to engage in mathematical practices or behaviors, one can further discuss PTs having OTL ambitious instruction. The latter part of Literature Review examines research from mathematics teacher education that focuses on how mathematics teacher educators' (or methods instructors') "use of instructional strategies, curriculum, and content is mediated by their mathematical knowledge and beliefs about teaching and learning" (Beswick and Goos 2018, p. 419). Further, aligning with literature on beliefs, this study suggests that PTs' OTL are highly influenced by their instructors' beliefs about effective mathematics teaching practices.

Ambitious instruction

The construct of ambitious instruction is well supported in the literature on teaching (Franke et al. 2007; Grossman et al. 2014; Lampert et al. 2011; Thompson et al. 2013). The larger, longitudinal study, under which this study is nested, defines ambitious instruction as teaching that promotes students' deep, conceptual understanding of academic content and procedural fluency and helps them meet rigorous learning goals. Ambitious instruction involves high expectations which are evident in tasks chosen by teachers, how students are supported to engage in those tasks, and how teachers respond to students' mathematical thinking (Kazemi et al. 2009). It enables students to develop their own solutions to tasks, to provide justifications for their solutions and respond to critiques, and to compare different solution methods. Further, it is important to acknowledge that the term ambitious pedagogies is often used interchangeably with standards-based teaching practices (e.g., Lampert et al. 2010).

Standards-based refers to teaching practices that provide learners with opportunities to engage in mathematical practices or behaviors (e.g., math discourse, representation) as outlined in the National Council of Teachers of Mathematics (NCTM) process standards (NCTM 2000; Walkowiak et al. 2018). These process standards focus on problem solving, reasoning and proof, communication, connections, and representation. Such teaching practices are reflected in more recent standards (e.g., mathematical modeling and argumentation) in the USA released by the National Governors Association Center for Best Practices and Council of Chief State School Officers (2010; Walkowiak et al. 2018). All of these standards focus on teaching practices that support conceptual understanding. Numerous observation measures have been developed to measure for standards-based mathematics teaching practices including Mathematics-Scan.

Mathematics-Scan

Mathematics-Scan (M-Scan) is an observational instrument that measures the extent to which instruction includes standards-based mathematics teaching practices that are characterized by students' opportunities to engage in mathematical behaviors as outlined by standards documents (e.g., NCTM 2000; Walkowiak et al. 2019). Walkowiak et al. (2018) note that although the vision for M-Scan was grounded in *Principles and Standards for School Mathematics* (NCTM 2000), the measure highly overlaps with *Principles to Actions* (NCTM 2014) that was published after the development of the instrument and outlines "eight effective teaching practices." Lampert et al. (2010) claim that the vision in these NCTM documents outlines ambitious instruction (Walkowiak, et al. 2018). Furthermore, Walkowiak et al. (2018) state that "the four domains of M-Scan are tightly connected to these… features of ambitious instruction" (p. 462). The four domains of M-Scan include: task selection, use of representations, discourse, and coherence; the domains further break down into nine dimensions (as seen in Table 1 in the findings for Assertion 3). For definitions of each domain and dimension, see Berry et al. (2017). When using the coding rubric to collect quantitative data, the dimensions are each coded on a scale of 1 to 7 with descriptors of low (1–2), medium (3–5), and high (6–7). For further detail regarding the measure's validity and score reliability, see Walkowiak et al. (2014). This study draws upon M-Scan as an observation measure qualitatively given its emphasis on students' opportunities to engage in ambitious pedagogies in mathematics education (later discussed in the methods).

Opportunity to learn

The definition of OTL has varied substantially since the 1960s. Carroll (1963) "defined OTL as the amount of time allocated to the learner for learning a specific task" (Tate 2001, p. 1019). Husén (1967) referenced to OTL as how accurately the curriculum taught matched that "assessed by achievement tests" or the quality of the instruction (Tate 2001, p. 1019). Tate (2001) focused upon how time, quality of instruction, and technology influence students' understanding of science. Furthermore, Tate et al. (2012) discussed how OLT traditionally (e.g., Tate 2001; Tate and Rousseau 2007) focused upon content exposure and coverage, content emphasis, and quality of instructional delivery; they felt that other factors such as time and quality factors linked to science, technology, engineering, and mathematics (STEM) education needed to be explored. Additionally, Schmidt et al. (2011b, c) claimed that content coverage variation across districts and states had the largest impact on a student's opportunity to learn and that this affects academic achievement. Although these studies looked across different variables that influenced OTL, they were broadly concerned with factors that impact students' understanding within a discipline (or across disciplines) and that translated into their academic achievement.

While there is variability in definitions for OTL, this study used Schmidt and colleagues' framework for OTL, defining it as "the content to which future teachers are exposed as a part of their teacher preparation programs" (Schmidt et al. 2011b, c, p. 140). This framework differentiates the content that PTs have the OTL within their mathematics teacher preparation coursework based upon four categories: mathematics, mathematics pedagogy, general pedagogy, and practical experience (Schmidt, Blömeke, et al. 2011a; Youngs and Qian 2013). Therefore, this framework for OTL aligns with the purpose of the study, which is to examine PTs' OTL ambitious teaching practices in elementary mathematics methods courses, and is concerned with factors that impact PTs' ability to demonstrate expertise within the teaching profession. OLT is observable based upon the actions and teaching practices of the instructors and the behavior of the PTs. This study examines both the content (what) that PTs have the OTL and the nature of the PTs' learning opportunities (how).

Examining learning opportunities

What prospective teachers have opportunities to learn. In addition to being competent with indicators of ambitious instruction and standards-based teaching practices, PTs should have opportunities to develop and apply mathematics knowledge for teaching (MKT). The development of MKT for PTs is foundational to teaching for conceptual understanding.

Mathematics Knowledge for Teaching. There has been an ongoing discussion about a gap in knowledge for how to best prepare pre-service teachers (Boyd et al. 2009), specifically in mathematics education. Deborah Ball fostered this discussion by elaborating on how preparation should focus on mathematics content knowledge (MCK) and pedagogical content knowledge (PCK). Ball (1990) expanded upon the work of Shulman (1986) with content knowledge and his conceptualization of PCK by adapting the theoretical framework to mathematics specifically. MCK has been defined as "a comprehensive understanding of breadth, depth, connectedness and thoroughness" of mathematics (Ma 1999; Hine 2015b, p. 2). PCK is defined as "knowing a variety of ways to present content and assisting students in deepening their [mathematical] understanding" (Hine 2015a, p. 483).

More recently, there is increasing support for developing mathematics knowledge for teaching (MKT) with prospective teachers, especially in elementary education (Delaney et al. 2008). MKT incorporates PCK and subject matter knowledge (SMK). Within MKT, SMK is inclusive of common content knowledge, specialized content knowledge, and horizon content knowledge, and PCK includes knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum (Ball et al. 2008). Developing MKT in PTs is foundational to their readiness to teach elementary students for deep conceptual understanding and to interpret and respond to their mathematical misconceptions.

Teaching for Conceptual Understanding. The National Research Council (NRC 2001) acknowledges the evolving, historical meaning of "successful mathematics learning" (p. 115). In particular, the NRC (2001) states that during the first half of the twentieth century, success was defined by the usage of computational procedures of arithmetic, particularly in pre-kindergarten through eighth grade. Though there have been numerous definitions of success since that time, when references are made to "traditional" mathematics, they generally acknowledge a dominant, historical emphasis on procedures, memorization, and formulae (Barkatsas and Malone 2005; NCTM 2014). NCTM (2014) claims that beliefs from the "traditional lesson paradigm" are unproductive when they hinder effective teaching practices and limit students' OTL (p. 9). In contrast, NCTM (2014) advocates for productive beliefs that align with ambitious instruction as seen within the "eight mathematics teaching practices." Likewise, the NRC (2001) claims that for anyone to successfully learn mathematics, they must develop *mathematical proficiency* including conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (stands are interwoven and interdependent).

There is a focus in mathematics education (e.g., Garofalo 1992; NCTM 2017; NRC 2001) on teaching for conceptual understanding, which has been defined as "an integrated and functional grasp of mathematical ideas" (NRC 2001, p.18). Teaching for conceptual understanding means that teachers have to be prepared to confront mathematical misconceptions by discouraging the use of isolated facts, methods, and rules for the sake of efficiency (e.g., Cardone and MTBoS 2015; Karp et al. 2014). For example, "rules" such as "always take the bigger number minus the smaller number" that appear in instruction can create mathematical misconceptions and become problematic when students progress to more advanced mathematics where the rule expires (as with the introduction of integers). Sometimes, teachers may not even recognize how they are supporting misconceptions because these practices are so entrenched in traditional mathematics teaching. Teaching for conceptual understanding drives ambitious instruction and standards-based teaching practices.

Prospective teachers' opportunities to learn. Two *pedagogies of practice* emerging from the work of Grossman et al. (2009) include *representations* and *approximations.* Cavanna et al. (2017) define representations as teachers (i.e., pre-service and/or novice in-service teachers) having the opportunity to watch and/or read about others engaging in teaching practices. Therefore, representations occur through modeling, generally by a mentor or instructor, watching teaching videos, examining vignettes, and/or other forms of observation. It is important to note that such representations of teaching are different from mathematical representations such as symbols, graphs, pictures, words, charts, diagrams, and physical manipulatives used to demonstrate mathematics concepts (Berry et al. 2017). *Approximations* entail having safe places to practice what PTs will actually be expected to do as teachers (Grossman et al. 2009). Approximations can appear in various forms such as practicing classroom management, enacting teaching episodes, creating and/or grading assessments, and making lesson plans.

Instructors' influences on PTs' learning opportunities

In this study, I have chosen to describe the participants as mathematics methods instructors, but it is important to point out that such terminology is interchangeable with mathematics teacher educators (MTEs). Beswick and Goos (2018) draw attention to the various ways in which MTE knowledge has remained an under-researched area despite such importance in the field. A dearth of research has focused upon MTE knowledge despite its potential to influence instructional strategies, curriculum, and content (e.g., Li and Superfine 2016).

MTE knowledge has been defined as a kind of meta-knowledge that includes knowledge that mathematics teachers would need (including but not limited to the content and instructional strategies for PTs previously discussed); however, MTEs would need to have a rather different understanding of such knowledge (Beswick and Chapman 2012; Beswick and Goos 2018). Additionally, Beswick and Goos (2018) argue that research on MTE knowledge should include MTEs' beliefs about the nature of the content that they are teaching to PTs (e.g., Aydın et al. 2009; Callingham et al. 2012; Lovin et al. 2004) and how such beliefs may influence their instructional strategies.

Questions surrounding the influence of teachers' beliefs upon their teaching practices have been an ongoing area of research in mathematics education at the elementary and secondary levels. NCTM (2014) stated that "Teachers' beliefs influence the decisions that they make about the manner in which they teach mathematics" (p. 10). Philipp (2007) defined beliefs as "psychologically held understandings, premises, or propositions about the world that are thought to be true" and that are generally organized into belief systems based upon similar constructs or ideas such as effective mathematics teaching (p. 259). Cooney et al. (1998) stated that when examining belief structures, "the constructs of quasi-logical and psychological strength are quite different" (p. 309). Thus, method instructors may believe that it is important to provide PTs with opportunities to use mathematical manipulatives, but if instructors' beliefs about mathematics teaching practices are not psychologically strong, then they may not follow through with creating learning opportunities within the classroom. This is important in this study because instructors' psychologically strong beliefs can directly impact OTL for PTs. Likewise, just because a belief has strength in one context does not mean that it will have as much strength in another context (Cooney et al. 1998). In their literature review, Bishop et al. (2003) examined various ways in which researchers identified similarities and differences between beliefs and values. Building upon this literature review, I am using the term *belief* (as opposed to values) based upon the context-dependent nature of judgments made about effective teaching practices. Likewise, I take the position that beliefs are more cognitively ingrained and harder to change than attitudes and perceptions (Philipp 2007).

For beliefs about mathematics teaching practices, I examined instructor beliefs about the content that should be taught and the ways in which such content should be delivered. Thus, when examining how method instructors' beliefs about mathematics teaching practices impact instruction and PTs' OTL, this study considered both what the instructors were saying and what was observed in their actions. In the findings (Assertions 1 and 2), this study specifically reports on instructor beliefs that were considered psychologically strong (given enactment in their instructional strategies), supported with evidence, and demonstrated consistently by participants' actions. The instructors' beliefs about mathematics teaching practices are manifested in their actions as they instruct PTs, and such actions resulted in OTL for PTs. This is significant because it helps explain how instructors' beliefs impacted their instructional choices related to content (coverage and exposure) and how that in turn influenced PTs' learning opportunities related to content.

Purpose and research questions

The purpose of this study was to examine PTs' opportunities to learn ambitious pedagogies in elementary mathematics methods courses at Robin University (pseudonym).

Specifically, this study focused on PTs' learning opportunities that stemmed from the instructional strategies used by their methods instructors. Further, this study considered ways that instructor beliefs about effective mathematics teaching practices influenced PTs' OTL. My research questions were as follows:

- What instructional strategies did the three elementary mathematics methods instructors implement and which strategies led to learning opportunities for prospective teachers?
- How did instructor beliefs about effective mathematics teaching practices influence learning opportunities for prospective teachers?
- What did these elementary mathematics method instructors believe the purpose of elementary mathematics method courses to be?

Methodology

Access and Role Chosen

During the Spring of 2017, I joined the larger, longitudinal research project that focused on the development of *ambitious instruction* in elementary mathematics and ELA. That study followed 175 elementary PTs from five teacher preparation programs in three states (in Northeast, Midwest, and Southeastern USA) into their first and second years of teaching. As part of the larger study, I focused on one of the cooperating universities and contacted methods instructors for interviews to discuss their course content and instructional strategies. During these interviews, I became curious as to how their actual or observed instruction compared with their self-reported accounts. This curiosity stemmed from learning about their stated beliefs related to effective teaching practices.

Site and sample

Robin University is nestled in a southeastern state located in a rural city (population of 50,000), and it is known for its large, well-respected, 5-year elementary teacher education program. The sample for the summer interviews included five methods instructors (three mathematics and two ELA) and two individuals in administrative positions. For the purpose of this study, I focus on the sample of three elementary mathematics instructors due to my interest in mathematics education. It should be noted that all of the instructors were White; they had all taught at the elementary level prior to becoming methods instructors; and their ages ranged from late-30s to late-50s.

The names of these instructors were William, Brittany, and Megan (pseudonyms). William had completed his M.Ed. and had been nationally recognized as an exemplary elementary teacher prior to coming to Robin as an instructor; Brittany was an associate professor who had completed her Ph.D.; and Megan was an assistant professor who had completed her Ph.D. It should also be noted that the elementary mathematics methods instructors taught two different courses throughout the semester: *Mathematics Education for Children I* and *Mathematics Education for Children II*. These are consecutive courses taken during PTs' fourth and fifth years of their teacher education program. To incorporate multiple perspectives, I also interviewed two administrators, Sadie (coordinator of the elementary education program) and Chelsea (elementary department head).

Data gathering procedures

Interviews. Prior to conducting classroom observations, I interviewed the three elementary mathematics methods instructors. Email was used to contact each instructor directly to arrange these interviews which occurred during the summer of 2017 via FaceTime. An interview protocol was used to guide this process (see "Appendix"). The interviews were all recorded and transcribed, which allowed me to conduct member checking, and ranged from 60 to 90 min with each participant. These initial interviews served as preliminary data and helped to focus the observations, given that I was originally interested in examining how the instructors' personal accounts of their instruction compared to my observations of their instruction.

Following classroom observations, additional interviews were scheduled. The second round of interviews was necessary in order to have the instructors further unpack some of the instructional strategies and learning opportunities that were noted within the observations. These interviews were all approximately 45 min, and were conducted remotely, using FaceTime. Additionally, two of the administrator interviews were conducted in person by one of the principal investigators (PI) for the larger study; he recorded both of these interviews and I transcribed the data for analysis. Further, I conducted a second 30-min in-person interview with Sadie to further unpack her role at the institution. These interviews speak to the context of issues of alignment within the teacher education program.

Classroom observations. Throughout the Fall 2017 semester, I conducted eight classroom observations of the three elementary mathematics method instructors, accounting for 20 cumulative hours of observations. The intent was to observe all three instructors for approximately the same amount of time, but due to scheduling conflicts, there were three observations of William's classes, three observations of Brittany's classes, and two observations of Megan's classes. The courses were at various times, on different days of the week, yet all classes were two-and-a-half hours long. During each observation, I took detailed double-column fieldnotes, describing the instructors' actions and their students' reactions to their instruction. I transformed most of these fieldnotes into write-ups on the same day that the observation occurred or soon after. An observation protocol drawing upon M-Scan's domains and dimensions of ambitious instruction was used to keep the study's purpose at the forefront and to conceptualize the implementation of such teaching practices. Assertion 3 (see findings) includes explicit references to examples of M-Scan domains and dimensions across the three mathematics methods courses.

Other reportable events and documentation. Lastly, I used "other reportable events" as data. These included informal conversations with the participants, e-mail correspondence, and/or notes from brief scheduled meetings with participants. Further, while I was in the field observing classroom instruction, the methods instructors provided me with copies of handouts used in instruction and I received their consent to take photographs of some of the activities and student artifacts.

Researcher positionality and researcher as instrument

As a former mathematics teacher, I am invested in work that focuses on teaching practices in prekindergarten through 12th grade which make mathematics more accessible, equitable, and empowering for learners. Therefore, I find value in teacher education programs that provide PTs with opportunities to learn ambitious mathematics teaching practices. It should be disclosed that my doctoral advisor is one of the developers of M-Scan; because of my training with this observation measure, I was originally asked to join the research team for the larger, longitudinal project.

Given that this is an interpretivist study, it is pivotal that I acknowledge my role as the researcher as instrument. As the instrument, I both administered the protocols and made interpretations of the data throughout every step of the process. Thus, as Emerson et al. (1995) argue, my own "assumptions, interests, and theoretical commitments enter into every phase of writing…and influence decisions that range from selecting which events to write about to those that entail emphasizing one member's perspective on an event over those of others" (p. 167). Though social science work is by its very nature subjective, I have tried to the best of my ability to engage in a systematic reflective and iterative process, in which I confronted my own assumptions and triangulated data (Erickson 1986). In this study, I did not come into the work with a pre-existing theory that informed my research design and data analysis. Instead, I took a grounded theory approach in which I tried to let the data generate theory (Corbin and Strauss 1990).

Credibility

Based on my unique "insider" and "outsider" perspective, I believe that my own positionality contributed to credibility in this study. I was very much an insider having been a mathematics teacher, having a degree in mathematics, and being a recent PhD graduate. On the other hand, I was an "outsider" because of my lack of familiarity with the teacher preparation program at Robin University. Thus, I had the ability to interpret these data in a distinct way. On another note, it should be acknowledged that although I did not attend Robin University, I grew up in the area where it is located and later worked as a high school mathematics teacher in the same county where many Robin PTs were placed for practica and student teaching. This insight enabled me to be invested in and knowledgeable about the PTs' experiences outside of the classroom. I kept a methodological journal documenting the entire process and decisions made along the way, which helped me maintain an audit trail to ensure trustworthiness. Further, I attempted to adhere to Erickson's (1986) suggestions for maintaining and establishing credibility.

Data analysis procedures

This study drew upon Erickson's (1986) model of analytic induction. As indicated previously, following each observation, fieldnotes were transformed into write-ups, and all of the audio from the interviews in the first round were transcribed. From write-ups, transcripts, and other reportable data and documentation, analytic memos were written intermittently to document emerging themes and inferences. I listened to the audio from the second round of interviews repeatedly to document inferences and supporting evidence which appeared in my final analytic memo. Data sources were triangulated and re-read and re-coded to document emerging patterns and assertions. I compared confirming evidence and disconfirming evidence for each emerging assertion and continued to adjust the assertion until I had accounted for all evidence. I thought through assertions extensively including connections between assertions. Additionally, I engaged in peer debriefing to ensure trustworthiness. Throughout this inductive process, data were reduced to three assertions for the focus of this paper. In the findings, the assertions are supported with evidence in the form of quotes, observational data, vignettes, and documentation of other reportable events.

Findings

From the data analysis process, three assertions were developed to understand and explain the following: similarities in opportunities to learn, differences in opportunities to learn, and perceptions about the purpose of the methods courses across the three cases.

> Assertion 1: Similarities in Opportunities to Learn: Prospective teachers experienced some similarities in opportunities to learn across the three classrooms consistent with the similar beliefs and teaching philosophies of their instructors.

What prospective teachers had the opportunity to learn. In my time in the field, I began to notice that all of the mathematics elementary instructors were engaging in instructional strategies that were student-centered, open-ended, inquiry-based, and highly interactive. Although there were evident differences in instruction based on instructor beliefs (which will be unpacked in Assertions 2), all three instructors focused on teaching students for conceptual understanding such that during every observation, I saw instructors pushing back against teaching procedures and rules for the sake of efficiency. William claimed, "You have to slow down to go faster." All three instructors discussed how these "rules" or memory tools had to be accompanied with activities that build conceptual understanding; otherwise, they would lead to mathematical misconceptions, especially if the rules expired as the students advanced through mathematics. The following vignette shows a typical interaction when unpacking mathematical misconceptions. Though the vignette addressed events in Megan's classroom, such instructional strategies were seen in the classrooms of William and Brittany as well.

> Megan has just finished showing her students a short video clip about fractions. The video has upbeat music, jokes, and intriguing visuals, but the PTs are supposed to be debating its worth. The video sings a tune to the rule for changing a mixed fraction to an improper fraction. So, for instance, when looking at $2\,\frac{3}{5}$, one must multiply two by five and then add three to get 13 for the numerator, and know to keep the same denominator, such that the answer becomes $\frac{13}{5}$. Megan shows another video of a student struggling to remember the rule, for the same task, the student ends up getting $\frac{11}{5}$ when she multiplies two by three, and then adds five, getting the incorrect numerator. A discussion follows the videos, but it becomes clear that the girl in the second video was taught a rule, without further instruction to build conceptual understanding. The PTs unpack what the teacher could have done differently.

When addressing the question, *what* are PTs having the OTL, teaching for conceptual understanding and combating mathematical misconceptions was by far the most emphasized content by all three participants.

How prospective teachers had the opportunity to learn. The two most prevalent types of instructional strategies used were *representations* and *approximations*.

Representations. Representations were defined as modeling for PTs either effective or desirable ways to teach and/or modeling undesirable instructional strategies (typically followed by critique). Representations came from video footage, audio, readings, and/or peer presentations. Thus, representations took various forms across the three classrooms. In the vignette that was previously presented, Megan showed a video clip to get her PTs to critique instructional strategies that were based on memorization or rules that would expire. However, just as Megan and her colleagues were seen incorporating representations to discourage certain teaching practices, I more often observed them modeling creative and interactive lessons that could be incorporated into the PTs' practica, field placements, and/or future careers. Below is a vignette from Megan's classroom which provides an example of how PTs were provided with OTL through representations (though there are certainly many other dimensions of ambitious instruction present).

> Megan has just dispersed manipulatives and supplies out to her students based upon table groups, which have three to four individuals. "Everyone gets either a piggy or a dinosaur and place your player anywhere on the game board." The game board is a labeled one hundred, square grid. The students have charts for documenting their scores. They are further instructed to put their three quarters, five pennies, and two dimes anywhere on the board, and to give each participant four cards, followed by placing the stack of cards in the center. The cards have integers on them and the students can use as many or as few as they want (within each hand; so, one to four) to try to land on a spot with money and collect a coin. The students are told, "Always redraw to have four cards in your hand at all times. Your goal is to get the most money!" The students play this game for about five minutes. Megan has Sesame Street music, from a YouTube source, playing in the background while they work. Megan uses a classroom management strategy to get the students to stop playing. For a short time, Megan breaks role playing, as do the students, and they discuss the benefits of having elementary children play such a game, including: disguising an integer lesson, practicing adding up the value of money, and using critical thinking skills. Then, Megan switches back into the role of an elementary teacher, talking about another game and how to play it. She tells the students that in this game, the piggies are the food source or the prey and the dinosaurs are the predators. Megan instructs, "The dinosaur picks his/her place on the board first. The dinosaur always has five cards and the piggy has six, and the piggy always goes first. This is a partner's game. If the piggy is still alive when I call time (5 min) then the piggy wins, otherwise, well otherwise, the piggy got eaten when the dinosaur landed on his/her square, so the dinosaur is the winner." The last couple minutes were filled with laughter, Sesame Street, and 20-year-olds who were able to pretend to be children if only for a while.

There are several aspects in this vignette which deserve to be unpacked. First, the vignette shows how Megan, the methods instructor, transitioned into an elementary teacher. When she modeled for the PTs, she modeled everything from how to pass out the manipulatives, the delivery of instruction, language and speech with elementary students, classroom management, and the creation of a welcoming, child-friendly environment (with music). Second, the PTs reacted to such representations by embracing the role of being elementary students. Though I have chosen data from Megan's classroom to demonstrate how PTs learned through representations, it is important to note that all three instructors were seen using such strategies. In fact, Megan explained that the games unpacked in the vignette came from William who had used them in his own teaching practices as an elementary teacher.

William often spoke about how vital these learning opportunities were for PTs; he believed that if they had to engage in such instructional strategies as if they were elementary students, then they would be more prepared to meet the needs of their own students and acquire a more informed understanding when teaching. While the PTs may not have tested all of the strategies and activities that they contemplated enacting with elementary students, they at least had opportunities in their courses to begin to think through how such activities might be perceived by children.

Approximations. Approximations were defined as having students or PTs do tasks that they would be expected to do as teachers in the field. For the *Mathematics Education for Children II* courses taught by William and Brittany, approximations most often appeared in the form of mini-lessons. For the mini-lessons or teaching episodes, the PTs had to plan lessons (based upon a particular topic) and present them as if they were in the classroom with elementary students. William stated, "… [mini-lessons are] about giving you practical experience, not challenging your classmates…play the role that you think that you need to play. This is practice." In the *Mathematics Education for Children I* courses taught by Megan and William, approximations often appeared in the form of planning for lessons, creating mathematical games, writing word problems and assessments, seeking out and critiquing specific curriculum resources, and various other classroom management strategies.

There were some differences in the ways that the methods instructors viewed the relationship between representations and approximations. For instance, William believed that the first course in the sequence should focus on representations with fewer approximations and that the second course (part II) should scaffold, with more emphasis on approximations, thereby relinquishing more responsibility. On the other hand, Megan felt that representations and approximations had to be balanced across both course levels but with varying intensity.

As previously noted, many of my observations captured the act of peer-teaching episodes. The following excerpt was taken from an observation in William's course on a day when the content and curriculum focused on measurement.

> Natalie (PT) begins by passing out popsicle sticks and a worksheet to her "first-grade" students. She asks the students to join her on the carpet, while leaving their supplies at their desks. Next, she asks for a student volunteer to be the "object of measure." Jen volunteers and lays flat on the carpet while everyone else explores how many popsicles sticks in length Jen might be from head to toe. Natalie intentionally leaves gaps in the sticks to initiate a conversation, regarding whether this is a proper way to measure. Once they agree upon a length using popsicle sticks, Natalie informs them that they can go back to their desks and use popsicle sticks to measure the classroom objects indicated on the worksheet with a partner. While the students work collaboratively, Natalie walks around to assess progress. When she feels confident with their work, she gives them the rest of the worksheets; one has them measure using cubes and paper clips, and the other, includes pictures of classroom objects for which they are asked to estimate which object is larger and then compare actual measurements. (Observation, September 11, 2017).

Although this lesson was condensed due to the limited time that was available for each presentation, it was evident that Natalie had thought through and planned extensively for the lesson. In the mini-lesson that followed, another PT, Taylor, became unnerved as she realized that she had planned for the wrong mathematical content, liquid volume, instead of length measurement. She had planned an activity to model the volume of a cup, pint, quart, and gallon. However, she was so anxious that she did not teach, but instead talked through what she would do, such as discussing real-world applications/connections and water pouring demonstrations. Then, Taylor suggested having her students use construction paper to build a "Gallon Man," for which she shared various images on the projector. Gallon Man or King Gallon is a memorization tool, which does not help students build conceptual understanding but instead simply helps them memorize differences among a gallon, quart, pint, and cup and their proportional relationship to one another. William was strategic about addressing this "tool" after the presentation; in a caring way (in an effort not to negatively impact his rapport with Taylor), he pointed out to the class that Gallon Man was a memory tool, just like others that he had discouraged using with students. He emphasized that if such tools were used, they had to be accompanied by instructional strategies that built conceptual understanding. Later, William told me that he had never had a PT do so poorly.

I chose to include evidence from both Natalie's and Taylor's teaching-episodes because such evidence speaks to potential differences in learning opportunities. William was purposeful in allowing PTs to plan and make choices on their own accord; he gave them little guidance unless they asked for it. Thus, in his course, he aimed to structure practice which modeled the profession, and performing poorly highlighted areas in need of improvement. So, even though Natalie did well, demonstrating instructional strategies that she had previously learned, Taylor had a very different experience which highlighted areas of improvement. This created opportunities for Taylor to reflect upon her own performance as well as opportunities for William to think about what he could do to further help Taylor. Although Brittany also assigned teaching episodes in a very similar fashion instructionally, she provided PTs with far more support and guidance throughout the planning process (her focus on planning is unpacked in Assertion 3). Thus, with approximations, it is important to consider how much help PTs obtained along the way and how this impacted their OTL.

> Assertion 2: Differences in Opportunities to Learn: Prospective teachers experienced some differences in opportunities to learn across the three classrooms consistent with differing beliefs and teaching philosophies of their instructors.

When the methods instructors were asked about their teaching philosophies, they each placed emphasis on different instructional strategies. Likewise, the observations revealed that even though the instructors collaborated extensively, they each had their own approach to and beliefs about their roles which impacted *what* and *how* PTs have the OTL.

Case 1: William. William had taught third grade for 11 years followed by one year teaching kindergarten students prior to beginning his career at Robin University. Thus, his beliefs about teaching were highly influenced by his time working with children. He particularly elaborated on the desire to develop mathematical mindsets among his elementary students and PTs; he wanted them to see themselves as "Doers of Mathematics." Compared to the other instructors, William placed greater attention on the NCTM process standards. In round two of the interviewing process, he mentioned how this focus on the NCTM process standards gave PTs a place to start as they reflected upon their own implementation of instruction as well as equipped them with professional mathematical language. In the following observation, a PT presented a lesson on angles such that her students were first tasked with exploring angle properties through visual illustrations. Through exploration and peer collaboration, the students collectively identified angles. The latter part of the lesson included an activity in which the students used protractors to measure angles taped to their desks. As the lesson came to an end, the following occurred:

> The presenter stands in the center of the room and reflects upon her performance, sharing her thoughts with William and her peers. Then, each table group, which has

been preassigned an NCTM process standard, discusses feedback for the presenter. Individuals are responsible for writing down feedback on post-it notes, but there is also whole-class discussion during which each group shares their feedback directly with the presenter, as well as with the rest of the class. (Observation, September 25, 2017).

This observation excerpt is important because it modeled *what* the PTs were having the OTL as it related to the NCTM process standards as well as *how* they were having OTL through feedback and reflection. For two of my observations (five hours in the field) of William, I saw this reflection and feedback process occur four times with various PT presenters; additionally, he claimed that this occurred with every PT. In this way, William intentionally created learning opportunities by having the PTs reflect upon their own teaching both informally and formally (the written submitted portion of this assignment), and by contemplating peer and instructor feedback following the lesson. The noteworthy part of this process is that often times, peer feedback required PTs to defend their instructional decisions and to think critically about the choices that they made and/or the revisions that they would make in the future.

Case 2: Brittany. Brittany was the most experienced professor and had spent a great deal of time working in the field of mathematics education. Her personal journey had taken a winding path. Compared to the other two methods instructors, she placed more value on MCK. When asked about her teaching philosophies and beliefs, she stated, "And I guess, this sounds really generic but I just want them to understand the math that they're teaching. And something else that's really important to me that I emphasize, I want them, if they're afraid of fractions or 3-D pieces, I want them to educate themselves about it." William made the following comment (about Brittany) when asked to address the degree of alignment of teaching philosophies within the program: "I know there's a teacher or a professor who I have worked with, who in the years that I've been there, [I] have kind of helped shift her focus from teaching [more] content to more methods." It is important to acknowledge that William was not claiming that content is not important, but rather that in the methods courses taught, there should be an emphasis on MKT as a whole. This was especially the case since the PTs had to take three content mathematics courses prior to enrollment in *Mathematics Education for Children I*. Brittany's beliefs translated into *what* PTs had the OTL given that it was more common to see her going over and deriving traditional algorithms with PTs than was the case for the other methods instructors. In the following observation excerpt, Brittany tried to help her students understand the mathematical relationship of computing the area of various geometric shapes.

Brittany draws a rectangle on the board and asks the PTs how they calculate the area of a rectangle. The PTs respond, "Area = length × width." Brittany proceeds to give them a four by three rectangle to construct on their geoboards. After examining the area of the rectangle, Brittany prompts the PTs to figure out the area of the triangle created by the diagonal. This leads into a discussion about how the area of a triangle equals $\frac{1}{2}$base × height or $\frac{1}{2}$length × width when referencing back to the rectangle. Brittany gives the PTs a couple of "challenging" triangles on the geoboards to determine the area. Next, she models a parallelogram on the geoboard under the document projector (see Fig. 1); she proceeds by drawing an interior altitude, creating a triangle and models how the triangle can just be moved (relates to transformations in discourse) to create another rectangle. This leads to the implication that they can find the area for a parallelogram using the same formula as that for a rectangle. Brittany

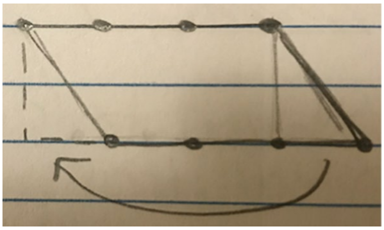


Fig. 1 Relating area of parallelogram to area of rectangle

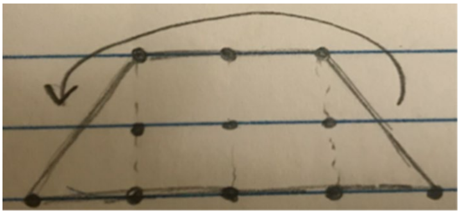


Fig. 2 Relating area of isosceles trapezoid to area of rectangle.

> continues on with the lesson by modeling a similar strategy for isosceles trapezoids (Fig. 2) but then states, "trapezoids don't always look like this." So, she has the PTs work with a partner using their geoboards to examine an example of a trapezoid that is not isosceles, and they unpack the area formula once again as it relates to a rectangle (Fig. 3). She then has the PTs go back to the area formula for an isosceles trapezoid and compare how it relates to a parallelogram (Fig. 4). Brittany states, "If we are going to use formulas, we need to know where they come from." (Observation excerpt, October 3, 2017).

Although Brittany's beliefs about methods courses could be termed "traditional", given her tendency to place more emphasis on content and formulas, she spoke openly about not wanting her students to enact traditional forms of teaching in which the mathematics was presented in a procedural manner without instructional strategies to support conceptual understanding.

> …I want them to know that they don't have to do it all their first-year teaching, but don't get caught in a rut because it's really, really easy to go back to the traditional way of teaching, because it's easy. I know, I did it. And I'm embarrassed. I'm glad that we didn't have social media back in the late 80s because I'd hate to think of what, you know, my students [would have said] and what a horrible math teacher I was.

Brittany intentionally tried to counteract her more traditional beliefs, which was evident in *how* she provided PTs with OTL. For instance, she always incorporated a plethora of hands-on-activities that PTs could potentially take directly into their own field placements. In many ways, these activities could be classified as representations (Assertion 1); however, they were distinct from those that appeared in William's and Megan's courses due to the range of content and pedagogy covered. For instance, Brittany was the only instructor who I observed deliberately trying to use technology (other

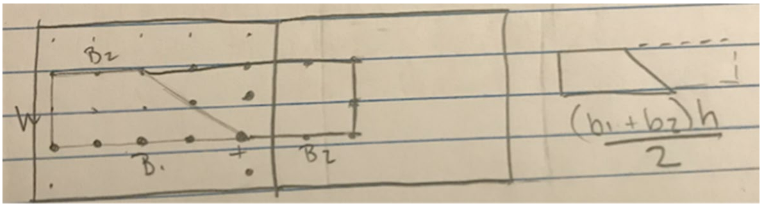
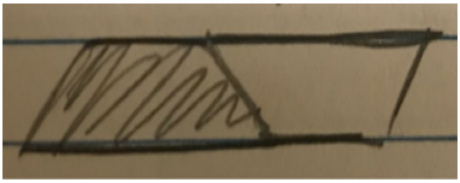


Fig. 3 Relating area of trapezoid to area of rectangle

Fig. 4 Making connections between area of isosceles trapezoid and area of paralleogram



than supplementary video clips) to support instructional strategies; she did a mini-lesson using applications on I-Pads to explore transformations of two-dimensional figures (after the PTs had explored such content using hands-on methods). Additionally, some of her lessons were interdisciplinary, showing PTs how they could integrate ELA and mathematics. An example of this was when she had her students make their own tangrams and then create images using their tangrams from *Grandfather Tang's Story* (Tompert and Parker 1990), as Brittany read the story aloud.

Case 3: Megan. Megan stood apart from her colleagues because of her focus on research. When asked about her beliefs and teaching philosophies, she almost always drew upon literature in the field; this literature was incorporated into her courses and became part of *what* the PTs had OTL. In the following excerpt, Megan discussed beliefs that she hoped to instill in PTs.

> I do a lot of belief research, … I liked Ernest 89′s definition of beliefs in terms of mathematics. I want them to understand that it's a problem-solving approach, … it's a man-made skill where they can actually come together and construct for themselves, and there's multiple ways to be able to solve any problem. So, going off of that, and then their teaching, it's again from Ernest's framework. I like Liebman [and the notion of being a] facilitator where they basically, instead of lecturing up there at top, they actually work with their students. They actually understand how to get them to understand the topic through their own learning. I was teaching learning as an active construction of knowledge, and it's not a passive construction.

There were certainly similarities among the three of the instructors and how they discussed wanting their PTs to develop self-efficacy with regard to their ability to teach mathematics. However, Megan continually make these statements while referencing research studies. Additionally, Megan was more accepting of multiple strategies and mathematical representations which not only appeared in *what* PTs had the OTL but also *how* PTs were provided with OTL. It is important to note that r*epresentations* in this case does not refer to the pedagogy of practice discussed in Assertion 1, but rather to mathematical representations used to demonstrate mathematics concepts (Berry et al. 2017). In Megan's classroom, PTs were instructed to work in small groups to develop their own word problems (based upon specified criteria) and then represent the problem-solving involved with various representations. Later, the small groups presented their problems and representations to the class. Photographs were taken to document and capture PTs' work samples. These presentations were very informal and conversational in nature, but they provided an important space in which PTs could reflect upon what goes into facilitating ambitious mathematics instruction. The group work and presentations also created a link between research (their readings for class) and practice. This type of opportunity was a common occurrence in her course.

> Assertion 3: The three methods instructors expressed a common purpose of seeking to influence their prospective teachers' mathematical mindsets. However, the methods instructors each expressed different (from one another) dimensions of ambitious instruction when interviewed about the purpose of the elementary mathematics methods courses.

Though there were certainly similarities across the three cases, as witnessed in their desire to influence PTs' mathematical mindsets (e.g., Boaler 2016), there were noted differences in how the methods instructors talked about and acted upon the purpose of the methods courses. These differences are unpacked by looking at each case or instructor separately in this assertion. It is important to note that although the instructors emphasized certain dimensions of ambitious instruction more than others when discussing their course purpose, this does not mean that each did not address all dimensions of ambitious instruction at various points throughout the semester. Even though this assertion focuses heavily on interview data, it also compares participant interview data about course purpose to what was observed in the classrooms (which was unpacked in Assertions 1 and 2).

Case 1: William. When William was asked to discuss the primary purpose and objectives of his course, he discussed how he "focus[es] on teaching through the NCTM process standards, problem-solving, [and] reasoning." Based upon William's beliefs, examined in Assertion 2, this was not surprising. However, William continued to discuss how he hoped that such instructional strategies would help his PTs develop a more positive perspective on what mathematics education can look like in the classroom. In the following excerpt, he commented on how many of the PTs in his courses had low self-efficacy and fixed mathematical mindsets due to their own experiences as students. For William, the challenge was to have his PTs develop growth mindsets and envision what mathematics could be.

> I'm trying to get our candidates … They come in a lot of them so like beaten down and [they] don't have a good mindset about what they can do mathematically, so I try to let them see that there are different ways to teach and this idea of teaching students by just forcing procedures on them isn't the way to go and let's make this an interactive environment. Let's let the students problem-solve and talk about what they're doing and focus on their ideas and their solutions, make math accessible to them that way…It's important that we kind of try to break this chain and give these kids, make them comfortable and confident with their ability to do this and you're just kind of steering the ship as the teacher.

William's account emphasizes how he hoped that his courses and instruction would contribute to breaking the cycle of fixed mathematical mindsets. He acknowledged that his PTs were products of their own experiences, but without an intervention, many of them would continue to teach mathematics with an emphasis on procedural knowledge and skills while simultaneously reinforcing the belief that only "some people are good at mathematics."

> I want you to walk out of my class with the idea of I wish somebody would have taught me math the way that you're saying we should teach math, so that when they get in, I tell them on the first day, "Look, I could put you all on a school bus right now and take you to the closest elementary school and you could all pick up the manual and stand in front of the class and have the kids follow a set of steps, a set of procedures, but that's not what we're here to do."

This quote not only illustrates William's purpose for the class in regard to having his PTs strive to teach mathematics in ways that were more inquiry-based, hands-on, and exploratory, focusing on conceptual understanding, but also begins to highlight what William considered to be ambitious instruction. In the second round of interviews, William unpacked how his definition of ambitious instruction meant having integrity or doing the right thing, simply because you know that it was the right thing to do. To William, ambitious instruction was about going that extra mile, even when you knew that there was no incentive to because that was how you "produce kids who are functioning at a deeper level and who have a deeper understanding." To William, this began with giving students *cognitively demanding* tasks which required them to grapple with *problem solving* while having to engage in *explanation and justification* of their reasoning. Note that the italicized terms correspond to dimensions of ambitious mathematics instruction. Thus, I argue that William placed the most emphasis on task selection and discourse when discussing the course purpose during his interview; however, this emphasis did not necessarily emerge from observational data.

Case 2: Brittany. As discussed in relation to Assertion 2, Brittany described having been in the field long enough to see a shift in how elementary mathematics methods courses had been taught. In her first interview, she mentioned how when she first arrived at Robin, she focused more on MCK than PCK in her methods courses. She commented that, "they were more traditional." However, because of revised program requirements in content courses and the addition of a second elementary mathematics methods course, she now felt as if she had more time to focus on PCK. This dilemma between time spent on MCK and PCK was further elaborated upon when Brittany was asked what she wanted her PTs to learn and be able to enact from her methods courses as seen in the following excerpt.

> Mainly, I want them to, and I hate to say, it's not that I focus on algorithms, but unfortunately, because of standardized testing, I have been forced to really address algorithms. But I want them to [first] understand what's going on when they solve traditional algorithms or algorithmic, you know, when they use an algorithmic procedure. I want them to understand what's going on, and to also know that there are different procedures or algorithms that can be used to solve the same problem. And it's really difficult because they already know the traditional way. And I mean, to me, that's one of the biggest challenges that I have, getting them to set that aside and focus on the different methods.

In this passage, Brittany's emphasis on MCK is evident, but it is also clear that she wants the PTs to develop conceptual understanding that can be fostered in their own classrooms, with their own elementary students. This ties back to Assertion 1 and the unified focus on creating OTL that unpack what it means to teach for conceptual understanding. In response to the question about the purpose of her method courses explicitly, Brittany once again talked about understanding the mathematics, but she then continued to elaborate on how her secondary purpose involved helping PTs learn how to seek out resources especially when they lacked confidence in their understanding of the content.

Brittany's focus on seeking out resources, such as practitioner-based journal articles, related to her emphasis on planning. She believed that one develops ambitious instruction (informed by her own understandings of terminology) through purposeful planning.

> … you can accomplish ambitious instruction by being well planned. [The participant was questioned about what is important to plan for] … you want to start with a measurable objective. And I have to help my students to understand what that even means… You have to plan for that and it has to be important to you, to plan it because it's too easy otherwise. To me, it is really easy to be an elementary teacher if you don't do a good job, you've got the text books that do it all for you, but if you want to be a good teacher, you've got to spend the time planning.

Brittany's comments and actions revealed her beliefs related to the purpose of her methods courses, specifically, the development of MCK, the awareness of available teacher resources, and how to plan for ambitious instruction. Brittany acknowledged that ambitious instruction takes time for novice teachers to develop.

Reflecting on the dimensions of ambitious mathematics instruction, Brittany' goals for the courses focused on coherence and included engaging in thoughtful planning within the *structure of a lesson* as well as adhering to *mathematical accuracy*. Brittany's expressed emphasis on planning was described in relation to Assertion 1, based upon her focus on facilitating the planning process for the peer-teaching episodes, and her emphasis on MCK was unpacked in Assertion 2, especially in the excerpt which described her derivation of area formulae.

Case 3: Megan. Megan explained that part of the purpose of her elementary mathematics methods course was to address PTs' beliefs about mathematics. One of her course assignments included a mathematics autobiography in which PTs confronted their own beliefs as they related to their experiences with mathematics education. She stated, "(A) lot of the pre-service teachers, have negative views of mathematics in general and negative views of the way they've been taught mathematics." Thus, a goal of her course was to help PTs develop a more informed understanding of different methods and strategies for helping all learners see themselves as "doers" of mathematics.

Additionally, some of the instructional strategies that Megan stressed as being important for the purposes of the course aligned directly with the dimensions of M-Scan for ambitious mathematics instruction. Megan's account of the purpose of her course focused on what she did in her own instruction and the assignments given to PTs to provide OTL.

> I talk about first off what are effective mathematical questions? What do I mean by that? I go by Boaler and Brodie's framework (2004), which is talking about what are the different mathematical questions that we have to ask. Yes. Then we talk about multiple representations. This is one that I felt like a lot of my students didn't get to see, so they have to first off ask multiple questions. They have to actually bring in multiple representations whenever they're teaching the concept. They need to think about the level of cognitive demand, and make sure it's a high level even if it's Smith and Steins (1998) level of cognitive demand framework. Either it's doing mathematics or procedures with connections. Either one was fine, I just wanted them to be able to see what that looked like, and then have to make those mathematical links. Mathematical links, I am kind of referring to the Boaler and Brodie connection of what is linked to mathematical concepts.

This excerpt shows that Megan's expressed course purpose aligned with the instructional strategies witnessed during classroom observations and unpacked in Assertions 1 and 2. For instance, Megan's instruction focused heavily on (a) the development of various questioning techniques as they related to fostering classroom *discourse* and the *presence of student explanation and justification*; (b) the presence of *multiple representations* inclusive of *mathematical tools*; and (c) the *selection of cognitively demanding tasks*. Figure 5 features PTs' group work addressing such dimensions. Also, as seen in Assertion 2, Megan often grounded her OTLs for PTs in research, which appeared in her interview excerpt through her description of Boaler and Brodie's (2004) framework and Smith and Stein's (1998) framework. Although Megan did not talk about Cognitively Guided Instruction (CGI) in her interviews explicitly, observational data revealed that she placed considerable importance on CGI which was consistent with her emphasis on question types, cognitive demand, and multiple representations.

Table 1 further illustrates the dimensions of ambitious instruction that each instructor described when explicitly asked about course purpose. The table also indicates that through various means, all of the instructors addressed all indicators of ambitious instruction.

Table 1 Elementary math method instructors' course purpose within the dimensions of ambitious instruction

Mathematics-scan dimensions	William's emphasis	Brittany's emphasis	Megan's emphasis
Task selection			
(1) Cognitive demand	p/m/o	m/o	p/m/o
(2) Problem solving	p/m/o	m/o	m/o
(3) Connections and applications	m/o	m/o	m/o
Representations			
(4) Use of representations	m/o	m/o	p/m/o
(5) Use of mathematical tools	m/o	m/o	p/m/o
Discourse			
(6) Mathematical discourse community	p/m/o	m/o	p/m/o
(7) Explanation and justification	p/m/o	m/o	p/m/o
Coherence			
(8) Structure of lesson	m/o	p/m/o	m/o
(9) Mathematical accuracy	m/o	p/m/o	m/o

Specified as course purpose by instructor during interview(s): p

Mentioned in interviews: m

Observed in classroom instruction: o

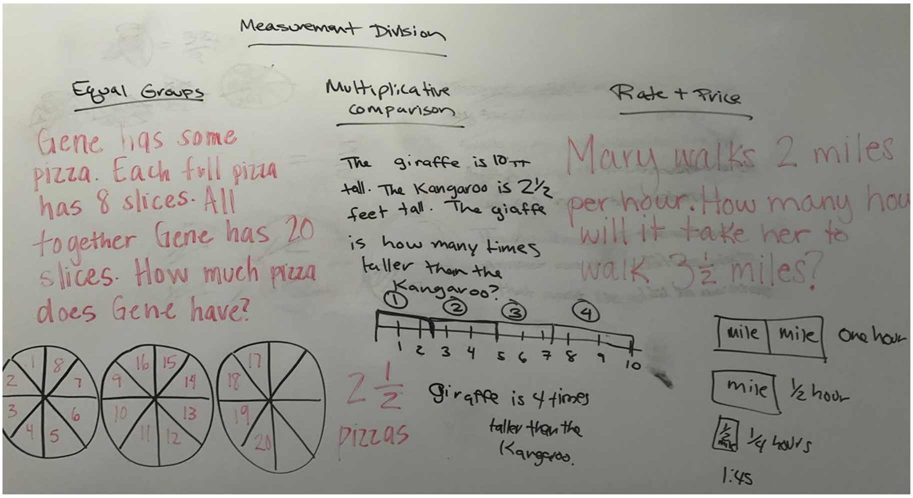


Fig. 5 Prospective teacher work samples in Megan's course

Limitations

Based upon 20 hours of observation, I believe that I observed "typical" instruction for each instructor. The instructors also confirmed that I observed typical instruction. However, it is possible that the instructors engaged in activities that they thought that I wanted to see. On another note, the context of this study, and how it is nested within a larger, longitudinal study, may have impacted the nature of the data collected. During the timeframe that I was in the process of performing observations, one of the PIs on the larger study went to Robin University and gave a brief presentation on some of the larger study's initial findings as it related to the institution's teaching candidate survey data. The PI discussed survey data that addressed what PTs had said about their OTL in the teacher education program at Robin University for both mathematics and ELA. This presentation caused skepticism with my participants as they questioned the overall purpose of the larger, longitudinal study and how data were going to be used given that comparisons were made across disciplines and institutions in the presentation. Thus, this presentation alone may have impacted instruction delivery by making the instructors more cognizant of the types of learning opportunities that they were providing PTs.

Discussion and implications

The first two research questions align directly with Assertions 1 and 2. In Assertion 1, I unpacked similarities identified across the three elementary mathematics methods instructors' instructional strategies, examining both *what* PTs had an OTL as well as *how* they were provided with OTL. Specifically, the assertion focused on teaching for conceptual understanding and combating mathematical misconceptions (*what* they had the OTL) (Karp et al. 2014) for which PTs most often experience OTL through representations and approximations (*how* they had the OTL) (Grossman et al. 2009). In Assertion 2, I examined each case to unpack differences identified in instructional strategies and OTL for PTs. In Case 1, William focused more than the other instructors on OTL that addressed the NCTM process standards (what; NCTM 2000) through reflection and feedback (how).

In Case 2, Brittany created OTL that placed more emphasis on MCK (what), whereas both William and Megan thought more holistically about MKT (Ball et al. 2008). Additionally, Brittany created OLT through hands-on activities, technological applications, and interdisciplinary instructional strategies (how). In Case 3, Megan created OTL related to research in the field (what), often having PTs engage in group work linking research and practice while focusing on the presence of multiple mathematical representations (how; Berry et al. 2017). In both Assertions 1 and 2, beliefs (Philipp 2007) about effective mathematics teaching practices were evident in the actions of the instructors, resulting in various learning opportunities for PTs. As the researcher, I want to be clear that I am not claiming that any of the instructors were engaging in preferential OTL, but rather that different OTL were present for PTs.

Assertion 3 focused on the third research question, addressing how the different methods instructors perceived the purpose of the elementary mathematics method courses. Within this assertion, I analyzed each case separately and attempted to demonstrate how each method instructor's perception of the purpose of the methods courses aligned with the dimensions of ambitious mathematics instruction as indicated by M-Scan (Walkowiak et al. 2019); in doing so, I indicated differences across the three cases. While Assertions 1 and 2 drew heavily upon observation data and what the instructors did in their classrooms, Assertion 3 unpacked how the methods instructors expressed their course purpose (via interview data) and how that compared with what they were seen doing (Assertions 1 and 2). It is important to acknowledge that not all of the instructors' descriptions of course purpose were supported in their actions; thus, they would not be considered psychologically strong beliefs as defined in this study (Philipp 2007).

The findings from this study add to research literature (e.g., Cavanna et al. 2017; Clift and Brady 2005) that examined the range of teaching practices in elementary mathematics methods courses and OTL for PTs. For example, this study provides insight into the content covered in these methods courses and the pedagogies of practice (Grossman et al. 2009) used. Further, this study, like others (e.g., Koedel et al. 2015), continues a much larger conversation about an overarching critique in our field regarding how OTL may vary across methods courses and how PTs who attend the same teacher education program may have very different experiences and OTL. In particular, this study highlights how differences in OTL seem to be linked to individual instructors' beliefs related to effective mathematics instruction. This is pivotal in helping identify ways to support PTs' development of ambitious instruction and deserves further attention in mathematics education research especially when contemplating the ways in which generalizations are sometimes made at the program level when unpacking teacher preparation.

Furthermore, the findings from this study raise questions about the role of MTEs' knowledge. More work is needed to understand how MTEs become knowledgeable and form beliefs about the types of learning opportunities that they provide for PTs, and how such knowledge influences MTEs' instructional strategies (Beswick and Goos 2018). Specifically, this study brings up questions about beliefs about how elementary mathematics is best learned, why conceptual understanding is important, and goals and purposes in designing such courses. Further implications for this study involve examining the ways in which networking within a community of practice between the MTEs influence instruction (e.g., Krainer 2001; Wenger 1998). Findings from this study support the call (Beswick and Goos 2018) for the need of large-scale studies and collaborations both nationally and internationally in mathematics teacher education to better understand MTEs' knowledge and how it influences OTL for PTs.

Appendix

First interview protocol

1. Tell me a little about your educational background and experiences in teaching and teacher education (What is your degree in? How long have you been teaching the methods course? Do you have elementary school teaching experience? Please describe (length, location, grade level, subject if departmentalized))
2. Describe the major objectives of the methods course as you see them (Note: if there is more than one required methods course in mathematics or ELA at their university, ask them to indicate which course they teach and when in the program it is offered; Probe for whether they primarily focus on helping candidates develop knowledge, instructional strategies, skills, practices, approaches)
3. How would you characterize the overall approach to teaching that you seek to develop among the prospective teachers through the course? (Probe for their beliefs/philosophy regarding purposes of teacher education; Probe for their perception of any differences between how the program expects them to teach the course and their approach to teaching the course)
4. What major instructional strategies do you want prospective teachers to learn and know how to enact? Why do you focus on these strategies?
5. How do you engage prospective teachers in learning these strategies? (Note: If they have taught the course multiple times, probe for their most recent experience teaching the course. Note: Ask them to send us/review their course syllabus and major assignments prior to the interview)

   a. What kinds of activities do you use to help them learn about these strategies?
   b. What kinds of activities do you use to help prospective teachers build their skill in enacting these strategies?
   c. How do you assess the prospective teachers' knowledge and skills enacting these strategies?

6. What are the major assignments and how do they relate to the major objectives? (Ask participant to take you through the assignments to explain the following:)

   a. Major goals and objectives
   b. How prospective teachers are prepared to complete the assignments
   c. How assignments are assessed
   d. How assignments are related to student teaching or clinical practice.
   e. How assignments relate to capstone project or other program-wide assessments?

7. How does this course fit into the goals and guiding principles of the larger teacher education program? (Probe for connection to other courses, field experiences)
8. How would you characterize the prospective teachers' knowledge of mathematics (ELA)? For example, what is the range of mathematical content knowledge? What types of backgrounds in mathematics do students typically have? Do you address content knowledge in your course? Please describe.
9. What kinds of interactions do you have with cooperating teachers? With university supervisors? (Probe for frequency, types, and content of interactions?

10. Do you typically visit your methods students during practicum or student teaching placements? (Probe: If they do visit their methods students in schools, ask what they focus on during these visits)

Second interview protocol

Approximations

1. How do you support students in preparation for teaching episodes? (assumption based on observations)
2. How do the teaching episodes influence your instruction?
   a. Probe for impromptu teaching

Representations

3. Please talk about the various ways in which you model teaching for your students.

External Context-Surrounding Teacher Preparation

4. How do you select classroom activities? What influences your selection of these activities?
   a. Probe for depth versus breadth
   b. Probe for focus on standards versus time for more hands-on-tasks on topic

Addressing Mathematical Misconceptions

5. What is the relationship between misconceptions and mathematics content knowledge (MCK)?

Classroom Environment

6. How would you describe your rapport with your students? How does this manifest in classroom management?

What should be done moving forward?

7. In your opinion, what should be done to help prospective teachers moving forward?
   a. Probe for classroom management and curriculum implementation
   b. How do we have students buy-into implementation of non-traditional forms of teaching?

Ambitious Instruction

8. How would you define ambitious instruction in your own words?

    a. How do you implement this in your class?

Class Purpose

9. What is the purpose of the class?

    a. What do you want students to walk away with at the end of the semester?

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