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Shared authority in the mathematics classroom: successes and challenges throughout one teacher's trajectory implementing ambitious practices

Jennifer Y. Kinser-Traut[1] · **Erin E. Turner**[2]

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Abstract
A critical role of mathematics teacher education is to equip teachers with understandings and ambitious practices that support effective mathematics teaching for students from diverse backgrounds, specifically connecting to children's mathematical thinking (CMT) and children's linguistic, cultural, and family funds of knowledge (CFoK). Drawing on data from a larger research project, TEACH Math, this longitudinal case study uses the lens of authority to examine one teacher's, Sena's, understandings and practices related to CMT and CFoK over a 4-year period, i.e., through mathematics methods, student teaching, and early career teaching. Findings identify changes made in understandings and practices related to CMT and CFoK, with a focus on the development of recognition and realization rules. Emphasis is placed on how authority was shared in the classroom and how this may have impacted one teacher's development of two very different learning-to-teach trajectories for connecting to CMT and CFoK. Implications for teacher educators focused on supporting novice teachers in developing ambitious mathematics teaching practices are discussed.

Keywords Authority · Equity · Elementary mathematics · Instruction · Teacher education · Teacher practice/classroom practice

Introduction

Prospective teachers (PSTs) tend to enter teacher preparation programs with limited experiences with students from diverse cultural, racial, and linguistic backgrounds (Bleicher 2011; Silverman 2010; Taylor and Sobel 2001), and beliefs and assumptions about diverse students that could undermine students' learning (Sleeter 2001). Specific to mathematics, PSTs have had limited exposure to interpreting children's mathematical reasoning (Jacobs et al. 2010) or connecting to the mathematical knowledge that children bring from experiences outside of school (Downey and Cobbs 2007; Foote et al. 2013). Thus, a critical role

✉ Jennifer Y. Kinser-Traut
jkt3@nyu.edu

1 New York University, New York City, NY, USA

2 University of Arizona, Tucson, AZ, USA

of teacher education is to equip teachers with instructional practices that support the learning of students from diverse backgrounds. Scholars have referred to such teaching practices as *ambitious and equitable* (Jackson and Cobb 2010), because they support "the learning of all students—across ethnic, racial, class and gender categories—and aim to deepen students' understanding of ideas" (McDonald et al. 2013, p. 385).

In mathematics, we argue that ambitious and equitable teaching practices include (a) connections to children's mathematical thinking (CMT) and (b) connections to children's linguistic, cultural, and family funds of knowledge (CFoK),[1] because these connections have been shown to support the learning, participation, and identities of diverse groups of students (Brenner 1998; Tate 1995; Turner and Celedón-Pattichis 2011). For instance, research has linked teachers' understanding of CMT to productive changes in teachers' knowledge and beliefs, classroom practices, and student learning (Carpenter et al. 1996; Fennema et al. 1993). These studies have defined CMT to include understandings about children's problem-solving strategies, common misconceptions, and frameworks for understanding problem structures and number choices. Prior research has also established the positive impact of teachers' connections to CFoK on students' mathematics learning (Civil 2002; Ladson-Billings 2009; Turner and Celedón-Pattichis 2011). While researchers have used CFoK in varied ways, sometimes to broadly describe the knowledge that children bring from experiences outside of school, CFoK is generally defined as the historically and culturally based knowledge, skills, and practices found in students' homes and communities (Civil 1994; González et al. 2001).

While research has explored PSTs' learning related to CMT (Philipp et al. 2007; Vacc and Bright 1999), and to a lesser extent, related to CFoK (Presmeg 1998), few studies have examined how PSTs learn to integrate CMT *and* CFoK in mathematics instruction. The TEACH Math project (Teachers Empowered to Advance Change in Mathematics)—in which connections to CMT and CFoK are introduced as integral parts of ambitious and equitable mathematics teaching—is a notable exception (Aguirre et al. 2013; McDuffie et al. 2014; Turner et al. 2012, 2016). Yet, few studies have followed novice teachers to examine how knowledge and practices related to CMT and CFoK develop over time. Such longitudinal research is important, because it enhances our understanding of whether and how teaching practices introduced in methods courses are taken up in early career teaching (Thompson et al. 2013). To address this critical gap in extant research, we present the case of one teacher, Sena, over a 4-year period. Sena is an illustrative case, as she both participated in Teach Math methods course *and* she was placed with a MT that supported understandings and practices consistent with those advocated in the methods course. By following Sena over time, this longitudinal case examines how ideas introduced in methods were taken up in her pedagogical practices. The following research question guided the study:

- What patterns, shifts, and/or differences in Sena's understandings and practices related to CMT and CFoK do we notice, if any, over time?

Theoretical perspectives

We begin with a general overview of frameworks for understanding how novice teachers take up practices learned in methods courses during early career teaching. Next, we argue for utilizing an *authority lens* to analyze teachers' understandings and practices

[1] Here, and in the remainder of the paper, the order of first CMT and then CFoK is not indicative of prioritizing one component of ambitious and equitable teaching over another.

related to CMT and CFoK. We conclude exploring how this lens informs our understandings of why teachers may take up these practices in contrasting ways.

Alignment of ideas from teacher preparation and early career teaching

From methods courses to student teaching

Research suggests that the transition from mathematics methods courses to student teaching is complex. For example, in a study focused on student teachers' efforts to select and adapt tasks from mathematics textbooks in ways that aligned with ideas from the methods course, Nicol and Crespo (2006) found inconsistencies in PSTs' practices—some PSTs deepened ideas from methods during student teaching, and others seemed to move away from such practices in the classroom context. Additional studies on this transition to student teaching have found similar conflicting results (e.g., Vacc and Bright 1999). Anderson and Stillman (2013) emphasized that variability in student teaching contexts (e.g., if mentor teacher "messages" about the students and their communities are consistent with principles from methods) may help to explain some of these conflicting results (p. 45). Additionally, they noted the positive impact of mentor teachers (MTs) who model methods-based teaching practices on PSTs' sense of preparedness to teach effectively in diverse, urban settings.

From methods courses to early career teaching

Other researchers have examined novice teachers' ability to *recontextualize*, or how teaching practices and ideas introduced in methods are enacted, or not, by early career teachers. Specifically, this paper draws upon Ensor's (2001) study which found that what was most likely to transfer were discrete activities from methods that PSTs reproduced in their own classrooms. Ensor examined a series of factors potentially related to the varying levels of take-up from methods (e.g., previous schooling experiences and school environment). While these factors were important, access to *recognition rules* and *realization rules* was decisive in explaining teacher take-up of practices. Here, similar to Ensor, we use *access to the rules* to refer to what extent "students were provided with the principles" of ambitious teaching practices *and* evidenced using them through talk or action (p. 314). Recognition rules refer to ways of talking about teaching practices that "enable student teachers to describe and evaluate 'best practices' discursively," whereas realization rules support "teachers to implement best practice in mathematics classrooms" (p. 315). In other words, an early career teacher may be able to *talk about* but not *implement* specific practices, thus highlighting the importance of realization rules. Ensor suggested that helping novice teachers develop habits of mind that allow them to *learn from* rather than simply *imitate* activities in classrooms would increase their access to realization rules. Specifically, methods courses that include opportunities to observe and/or apply specific practices in K-12 classrooms may support prospective teachers' development of realization rules and thus limit imitation.

Take-up of ambitious and equitable teaching practices

Adding to Ensor's work on realization rules, Thompson et al. (2013) explored novice teachers implementation of specific ambitious teaching practices introduced in methods courses, including *working on students' ideas* (p. 581). Thompson et al. argued that enacting ambitious teaching practices is often challenging for early career teachers because they have to negotiate differences between "two worlds" (Feiman-Nemser and Buchmann 1985), namely, the critical pedagogical discourse (of methods courses) and the contextual discourse (of elementary classrooms). They documented three ways novice teachers addressed this two-world problem: (1) integrating ambitious practices from methods courses into their own teaching; (2) compartmentalizing practice, or slowly implementing ideas and practices from methods as they develop practices congruent with both worlds; and (3) occasionally appropriating language (but not practices) from methods to "appease the instructors" (p. 598). The first group of novice teachers was able to go beyond replicating specific activities to implement ambitious practices through new activities they designed, or in Ensor's (2001) terms, they had access to recognition *and* realization rules, whereas the second and third groups (which comprised two-thirds of the participants) faced challenges recontextualizing what they learned during methods into K-12 classrooms. In summary, novice teachers are often able to *talk* about ideas and practice from methods courses (recognition), but more challenging is the *enactment* of ambitious teaching practices, such as connections to CMT and CFoK, in subsequent teaching (Ensor 2001; Thompson et al. 2013).

Connections to CMT and CFoK

While these previous studies offer *general* understandings, what is missing is an understanding of how, and why, a teacher may successfully develop *selected* ambitious teaching practices (like connecting to CMT) and *not others* (like connecting to CFoK). To better understand these varying outcomes, we returned to the literature on teacher learning related to CMT and CFoK. A recent review of research in this area (Turner and Drake 2016) suggests that various factors may influence teachers' practices for connecting to CMT and CFoK. These factors include: teachers' identities and their orientations toward children and families from diverse cultural and linguistic backgrounds; extended experiences in contexts that support developing relationships with children and families; and access to specific activities, tools, or curriculum materials that support learning about CMT and CFoK (Burant and Kirby 2002; Downey and Cobbs 2007; Xenotos 2015). For example, Schultz et al. (2008) found that when the curriculum included explicit prompts to elicit students' ideas and experiences, novice teachers readily did so. Absent such prompts, connections to CMT and CFoK were less likely, suggesting that curriculum materials may play an important role in supporting take-up of these practices.

One theme that cut across the research was related to how novice teachers orient to students, families, and communities, and in particular, how teachers consider their own *authority*, and that of their students (Turner and Drake 2016). For example, Warfield et al. (2005) suggested that teachers' varying beliefs of their own and their students' authority could explain the differences in how teachers elicited CMT. More specifically, practices for connecting to CMT and CFoK require that teachers shift power relations and see students as having authority in the mathematics classroom (Campbel 1996; Wood and McNeal 2003). Thus, to better understand the *different and potentially inconsistent ways* that early

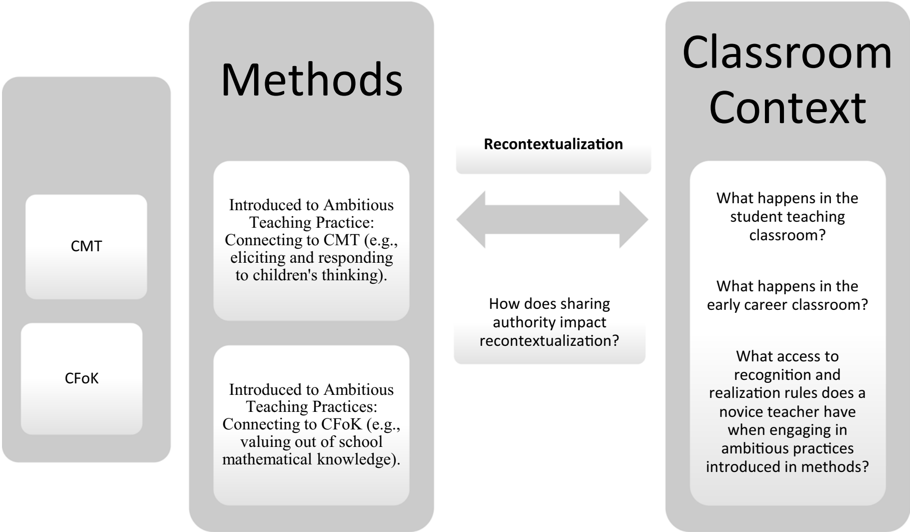

Fig. 1 Summary of our research focus

career teachers evidence recognition and realization rules in their take-up of practices for connecting to CMT and CFoK, we turn to the lens of *authority*. (See Fig. 1 for a summary.)

Authority in mathematics teaching

Building on Weber (1947), we define *authority* to mean the right, or power, an individual has, either given or assumed, to shape learning and events within the classroom. In mathematics classrooms, when teachers share authority and "see students as colleagues in learning," student understanding benefits (Amit and Fried 2005, p. 165). Drawing on Gerson and Bateman (2010), we use *sharing authority* to mean when students feel empowered to engage more fully in the learning. This happens when teachers attend to students' ideas and traditional teacher/student power dynamics are disrupted. For example, *sharing authority in the mathematics classroom* includes when teachers invite students to: determine correct answers or procedures; contribute mathematical ideas, discourse, and influence how these ideas are shared; and make connections between mathematics content and contexts outside of school (Gerson and Bateman 2010; Wagner and Herbel-Eisenmann 2014). Specific to our focus, how teachers share authority with students impacts teacher take-up of ambitious teaching practices (Amit and Fried 2005; Blanton et al. 2001; Hamm and Perry 2002).

Challenges to sharing authority

Sharing authority is challenging because "learning from student voices… requires major shifts on the part of teachers… about the issues of knowledge, language, power, and self" (Oldfather 1995, p. 87). In fact, because shifting authority toward students is such difficult work, "incremental shift in authority" is important to notice (Leonard et al. 2010; Hamm and Perry 2002). Outside of mathematics education, researchers have explored how teachers' orientations toward their students impact sharing authority. A key component is what

Cook-Sather (2002) referred to as "*authorizing students' perspectives,*" or viewing students as having knowledge and experiences to shape education (p. 3). This is particularly critical to mathematics teaching that connects to CMT and CFoK, as teachers need to see students as capable of generating mathematical ideas and bringing cultural and family experiences that support mathematics learning. For example, Planas and Civil (2009) found that tasks that invited students to share experiences from outside of school "open[ed] the channels of participation in the mathematics classroom" and were "a step toward changing both the students' and teacher's expectations" (p. 403) which began to shift the power in the mathematics classroom.

While novice teachers can develop orientations that honor children's reasoning in their talk (recognition rules), these orientations are often not evident in their practices (realization rules) (Sleep and Boerst 2012; Vacc and Bright 1999). In fact, in moment-to-moment classroom interactions, teachers may not recognize students' ideas as important mathematical contributions. Hand (2012) argued that understanding these in-the-moment distinctions requires attention to teachers' dispositions (i.e., orientations) toward students, including whether they view students' from a resource-based versus deficit-based perspective (Solorzano and Yosso 2001). Teachers often lack opportunities that would help them challenge dominant culture orientations that position some students as smart (white, middle-class, English-speaking students) and others as less capable (i.e., black, LatinX, low SES students). Thus, even well-meaning teachers may have orientations that privilege some students, or forms of knowledge and/or experience, over others.

In summary, sharing authority with students is a key to ambitious mathematics teaching that builds on students' knowledge and experiences. In this study, we use an authority lens to investigate patterns and shifts in one novice teacher's understandings and practices related to CMT and CFoK. To understand how the teacher recontextualized ideas and practices from teacher preparation into early career teaching we also attend to what the teacher was able to recognize and describe (recognition rules) and what she was able to move from discourse to practice (realization rules). (See Fig. 1.)

Methods

Participant and school contexts

Participant

Sena,[2] a first-generation Asian-American,[3] was a typical-age undergraduate student who attended a large university in the west. Sena was a former English learner and spoke a home language other than English with her family. She was part of a cohort of fourteen elementary education students seeking an elementary teaching certificate with an English as a Second Language (ESL) endorsement. The PSTs reflected diverse racial, ethnic, and linguistic backgrounds. The program included two semesters of methods and foundations courses, and associated field experiences, followed by a 15-week student teaching practicum.

[2] This and all other names are pseudonyms.

[3] This is a general descriptor to protect the identity of our participant.

We purposively selected Sena for this longitudinal case study examining how understandings and practices introduced in methods are taken up in subsequent teaching for two reasons. First, Sena reflected patterns of understandings and practices typical of PSTs who participated in the broader TEACH Math project. (See Aguirre et al. 2013; McDuffie et al. 2014; Turner et al. 2016). Second, across the methods and student teaching experiences, Sena worked with a MT who supported the practices advocated in the mathematics methods course. We were interested in how this *coherence between the worlds of methods and the field* (Zeichner 2010) might support Sena's recontextualizing between methods and the K-8 classroom.

Mathematics methods course

Methods courses were taught at a local elementary school and included frequent opportunities to interact with children and teachers at the school site. During each of the methods semesters, PSTs also completed a field experience practicum in a mentor teacher's classroom. Sena's mathematics methods course focused on developing PSTs' understandings and practices related to CMT and CFoK through instructional modules designed as part of the broader TEACH Math[4] research initiative (Turner et al. 2012). These modules were informed by research on Cognitively Guided Instruction (CGI) (e.g., Carpenter, Fennema et al. 1999) and Cultural Funds of Knowledge (Moll et al. 1992) and were aimed at helping PSTs to consider these different foci in their understandings and practices teaching mathematics.

Module activities included case studies of individual students (see Turner et al. 2016), critical analysis of classroom practice (see Turner et al. 2012), and investigations of mathematical practices in the community (see Aguirre et al. 2013). To support PSTs' recontextualizing of ambitious practices related to CMT, such as eliciting, and responding to children's thinking, PSTs conducted problem-solving interviews with individual students and planned problem-solving activities. To support recontextualizing of ambitious practices related to CFoK, PSTs interviewed children about experiences at home and in the community, including the ways that they used mathematics in daily practice. PSTs then planned mathematics tasks and outlined instructional suggestions that drew on what they learned about both CMT and CFoK. PSTs were able to enact some of the lessons and activities they planned with students in their field experience classrooms; however, this enactment was most common with problem-solving oriented activities and least frequent with lessons that connected in significant ways to home and community contexts.

A detailed analysis of Sena's participation in these modules is outside the scope of this paper. However, prior studies on the implementation of these modules (including in Sena's methods course) found that many PSTs evidenced connections to CMT or CFoK in planned mathematics lessons, yet these connections were often emergent (i.e., problem contexts that reflected familiar locations in the community) and/or uneven in specificity (e.g., specific connections to knowledge of CMT paired with general assumptions about contexts that would be of interest to students). Fewer PSTs evidenced meaningful connections both to mathematical practices in homes and communities *and* to specific knowledge of children's reasoning and strategies (Aguirre et al. 2013; McDuffie et al. 2014; Turner et al. 2016), suggesting that a more robust form of this practice may require additional scaffolds and/or develop over time.

[4] For more information, see the project Web site: Teachmath.info.

Methods and student teaching practica

Sena's methods and student teaching practica occurred at Mountain Vista, a K-8 school serving predominantly Mexican/Mexican-American, low-income communities. (77% of students were LatinX, 97% qualified for free/reduced lunch; www.greatschools.org.) Mountain Vista focused on mathematics and used the standards-based *Investigations* curriculum (TERC 2008). For both practica, Sena worked with Ms. Soto, an experienced MT, widely regarded as an effective teacher of mathematics. Ms. Soto, who was Mexican-American, shared a cultural background with her students and developed relationships with students' families and the community. Ms. Soto was also skilled at engaging students in rich mathematical discourse and connecting mathematics to students' "experiences and to the real world" (ST, Interview-MT). Ms. Soto's alignment with practices advocated in methods offered Sena additional access to recognition and realization rules for connecting to CMT and CFoK.

Early career teaching

Upon graduation, Sena relocated to a nearby metropolitan area and taught at Edison, a K-6 charter school focused on core academic subjects. The school's official curriculum was the teacher-directed Core Knowledge Sequence (Hirsch 1995) which Sena chose to supplement with Investigations (TERC 2008). Edison's student population was 95% Hispanic, and 94% of students qualified for free or reduced lunch (www.greatschools.org). Throughout her first 2 years of teaching, Sena communicated and visited Ms. Soto on a regular basis, further supporting Sena's efforts to recontextualize as she developed her own practice.

Case study methods

We employed case study methods (Stake 2013) to investigate patterns in Sena's understandings and practices for connecting to CMT and CFoK in her early career mathematics teaching. We first sought to understand Sena in each distinct time frame, or context, of learning to teach (e.g., methods, student teaching, early career teaching). Next, we examined the longitudinal development of Sena's understandings and practices; this was the *quintain*, or object of study (Stake 2013). In other words, Sena's case represents a longitudinal study of an early career teacher's access to recognition and realization rules for connecting to CMT and CFoK, with a specific focus on how sharing authority may support these practices.

Data collection

Data sources included: (a) methods course assignments related to CMT and CFoK; (b) transcripts of interviews during the methods semester, student teaching, and first 2 years of teaching; and (c) scripted field notes (FN) and detailed analytic summaries of mathematics lessons observed during student teaching and first 2 years of teaching. (See Table 1.)

Table 1 Data sources

Context	Data sources (number and type)
Methods course	Methods course assignments (11 documents) Beginning and end of semester interviews
Student teaching	Observation of two mathematics lessons Per lesson: Scripted FNs, analytic summaries Lesson plan Pre- and post-observation interviews End of semester interview with Sena End of semester interview with Ms. Soto
Early career: Year 1 and Year 2	Observation of 16 mathematics lessons (spread across each year) Per lesson: Scripted FNs, analytic summaries Pre- and post-observation interviews Beginning, middle, and end of the Year 1 and 2 interviews

Again, our purpose in this analysis was to capture teaching moments over time—snap shots of Sena's developing practice—and *not* to focus solely on a single stage in her development, such as methods or student teaching

Data analysis

Our three-phase data analysis followed the principles of analytic induction (Bogdan and Biklen 2003). Differences in interpretation were discussed and clarified by research team members, and patterns and claims were triangulated across multiple data sources (Marshall and Rossman 2010).

Phase 1: Initially, all data were chunked by time period (e.g., student teaching) and open-coded (Strauss and Corbin 1990) for understandings and practices related to CMT and CFoK. Understandings and practices related to CMT included such things as: (a) eliciting or responding to student thinking, including student confusion, (b) anticipating and reflecting on students' strategies, (c) considering problem structures and number choices, and (d) focusing on correct answers versus student reasoning (indicating a lack of focus on CMT). Understandings or practices related to CFoK included instances such as: (a) connections between mathematics and home/community contexts, (b) reflections on problem contexts, and (c) orientations toward mathematical knowledge and practices in students' homes. See the Appendix for additional examples. The above lists are not exhaustive, but reflect examples evident in the data.

Phase 2: Next, we completed a second layer of open-coding and labeled each data unit (from phase 1) as evidencing recognition rules and/or realization rules. For example, if the data unit included talk about the importance of students sharing their mathematical thinking, we labeled it as evidencing recognition rules, and if the data unit focused on Sena's action in the classroom to support mathematical discussion, we labeled it as evidencing realization rules. We also identified how Sena talked about *and* enacted sharing, or not sharing, authority in each data unit. Talk and practices related to sharing authority, or lack thereof, related to CMT included such things as: (a) eliciting students' ideas about how to solve a problem (sharing authority), (b) encouraging students to answer each other's mathematical questions (sharing authority); and (c) directing students to use a specific method to solve problems (not sharing authority). Talk and practices related to sharing authority, or lack thereof, related to CFoK included such things as: (a) asking students how they or their families use a particular math concept or skill at home (sharing authority); (b) allowing students to define parameters of a problem or activity based on their own experiences or assumptions (sharing authority); and (c) telling students how they will use a particular math idea in the “real world” (not sharing authority). See the Appendix for additional examples. This list is not exhaustive, but again reflects examples evident in the data.

Phase 3: We then completed *analytic memos* examining Sena’s trajectory both within each time period (methods, student teaching, years one and two teaching) and across time. First, both authors wrote memos for the same small set of coded raw data (i.e., methods assignments) with a focus on patterns or contrasts in Sena’s connections to CMT and CFoK, including how each data set evidenced recognition and/or realization rules, or sharing authority. We continued in this manner until our selection of data excerpts, analysis and interpretation were consistent across authors (after writing analytic memos for 7 data sets). We then divided the remaining data sources and individually produced memos for each small set of raw data. We met frequently to share data excerpts and interpretations and discuss questions that arose. In total, we produced 32 memos for small sets of raw data.

Next, we produced analytic *memos across memos* that summarized Sena’s perspective and understandings (i.e., recognition rules), enacted practices (i.e., realization rules), and sharing of authority related to CMT and CFoK at four specific points in time (i.e., methods, student teaching, Year 1, Year 2). Memos also noted ambiguities or tensions within a strand (i.e., CMT) or between strands (i.e., CMT compared to CFoK). This resulted in eight memos (i.e., two for each of the four time periods). Finally, we wrote longitudinal memos that traced patterns and progressions related to Sena’s connections to CMT or CFoK across time. This included attention to Sena’s: recognition rules, or talk around CMT/CFoK; realization rules, or implementation of connections; how authority was shared; and any tensions. This resulted in two longitudinal memos, one focused on connections to CMT, the other on CFoK. These memos formed the basis for our findings.

Findings

We present Sena’s experience as a novice teacher chronologically. For each period, we discuss her perspective and understandings (i.e., recognition rules) and enacted practices (i.e., realization rules) related to CMT and then CFoK, with attention to when and how she shared authority.

Methods semester

Recognition and realization rules related to CMT in methods

Sena entered the mathematics methods course with non-traditional perspectives on mathematics, noting there are “so many different ways to approach mathematics, it’s not just the rules.” Furthermore, she was “excited to see what my students come up with” as “children think a lot more differently than adults do about situations" (Interview-beginning[5]). While Sena began methods with productive orientations related to CMT, she lacked practices to enact this orientation and in her initial interactions with students often emphasized correct answers over student thinking.

Sharing authority by eliciting and valuing diversity of CMT: methods Throughout methods, Sena sought to share authority with students as she learned to elicit and interpret students' mathematical thinking. For instance, reflecting on problem-solving interviews she conducted, Sena realized that strategies other than traditional algorithms, including strategies generated by students, could be valid. She explained: "I realize now after our readings, that this way [the case student's method] is also correct—[an] invented algorithm" [Reflection-case study]. This example demonstrates how the course readings and the Student Case Study (a methods course assignment) supported Sena in further developing her recognition rules for sharing authority with regard to CMT.

Sena's increasing understanding of and attention to children's diverse ways of reasoning were also evident in course-related teaching activities. For example, when she planned and taught a whole group problem-solving lesson at the end of the methods semester, Sena invited students to solve multi-digit multiplication problems (i.e., 78×7) using several methods. She noted, "I even asked the students to verify their work using more than one strategy" (Reflection-lesson). On another problem, 18×26, she asked students to estimate the product and celebrated that "students got different answers depending on how they solved it. This created a great discussion amongst the students because they were each trying to argue their own position" (Reflection-lesson). Sena repeatedly communicated that "all students" were mathematically capable (e.g., Interview-beginning), and that she expected each student to reason through problems. She emphasized her desire to share authority in the mathematics classroom, stating, "I don't want to be, the teacher that holds the authority of the math, I want students to figure it out. … through their own reasoning what the right answer is" (Interview-final).

Challenges in sharing authority when eliciting CMT: methods While Sena was developing understandings and practices for sharing authority related to CMT, she still faced challenges. For example, during mathematics lessons, Sena's responses to children's contributions were inconsistent. When students' initial ideas were unclear or incorrect, she was sometimes uncertain about how to probe students' thinking further. She noted, "When [students] got a higher estimate than the exact answer, I kind of struggled with how to present the information needed without giving them the answer" (Reflection-lesson). Sena also acknowledged the challenge of responding to children's diverse ideas: "I'm going to have 28 different views/experiences coming in and I have to understand and… be able to guide each one individually and as a group depending on what they're bringing" (Interview-final).

Perhaps as a response to this uncertainty about unclear or divergent ideas, during whole group discussions, Sena essentially handed authority over to students. Although Sena's MT played an active role in discussions (Observations, FN), Sena's perception was that discussions were student-led, meaning students described "why they did what they did and then

[5] All data sources are from the designated period of time (i.e., Methods), unless otherwise noted.

if [other] students disagree they can ask questions," with little to no participation on the part of the teacher (Interview-final). Sena reflected this perception in discussions she led.

Overall, during the mathematics methods semester Sena willingly shared authority (or handed over authority), not wanting to be positioned as *the* authority in the classroom. This appeared generative in increasing access to recognition rules. In other words, the more Sena shared authority, the more she learned about CMT, which in turn reinforced her developing understandings (recognition rules) that children are competent math problem solvers who can generate a range of viable solution strategies.

Recognition and realization rules related to CFoK in methods

During methods, Sena was less certain about the role CFoK should play in her mathematics teaching. While the methods course advocated connecting mathematics instruction to students' experiences outside of school, and while Sena was open to this idea, she had never experienced such connections in her own schooling. She explained: "None of my teachers tried to connect mathematics to my culture; … if these connections were made it would have been easier for me to understand because I could have had some sort of bridge" (Autobiography).

Teachers as authority on connections to CFoK: methods When Sena envisioned making connections to students' out-of-school experiences, she did so in ways that maintained the teacher (rather than students and/or their families) in a position of authority. Sena elaborated this position when she reflected on the lesson she taught at the end of methods, "I tried to connect [the lesson] to their lives by explaining when they would use estimation…but it was difficult for them to grasp" (Reflection-lesson). While Sena seemed to develop recognition rules for practices advocated in methods (i.e., she talked about connecting mathematics lessons to children's experiences), she positioned herself as the authority and the one responsible for making such connections, which led to realization rules that limited her opportunities to learn about students and their experiences.

Sena also maintained a position of authority as she considered how to connect her mathematics teaching to family activities. She described specific things that families should do to support school-based learning at home (i.e., homework), and viewed her role as "telling" or "explaining" to parents the mathematics done at school. Sena seemed unaware of the many ways (other than homework) that families may already support students' mathematics learning, via daily family activities that involve mathematical practices (i.e., cooking or small businesses). Yet, methods course activities aimed at fostering strength-based orientations toward children's families challenged some of Sena's assumptions. For example, in the case study activity, Sena "realized that my assumptions about [case study student] were not entirely true. She seems to have an [educationally] encouraging mom and brother at home" (Reflection-case study). This recognition of family strengths was a potentially pivotal moment for her that might have challenged her position of authority and encouraged her to learn more about students' families and communities.

In summary, Sena entered methods valuing student's mathematical ideas and then during methods deepened her understanding of the importance of children's diverse mathematical strategies, as advocated in methods. Her efforts to share authority during methods appeared generative—the more she shared authority the more she learned about CMT. Yet, Sena's practices for eliciting and responding to CMT were inconsistent, as she was still developing realization rules (Ensor 2001). In contrast, Sena struggled to understand connections CFoK and was inconsistent in her orientations toward families. At times Sena recognized families' support of student learning and was open to connecting mathematics instruction to students' experiences outside of school (recognition rules). Yet, by positioning herself as an authority on such connections, she limited her opportunities to notice or elicit ways that students and families engaged in mathematics outside of school. This in turn challenged her ability to develop recognition and realization rules for connecting to CFoK in her mathematics instruction. Therefore, during methods Sena began developing different trajectories when utilizing ambitious teaching practices.

Student teaching findings

During her final semester in the teacher education program, Sena spent 15 weeks completing a full-time student teaching practicum in Ms. Soto's classroom. Sena recognized that she was "lucky" to be placed with a MT whose practices were consistent with those advocated in methods, "*It makes a huge difference*…so for me to transfer that to my classroom was a lot quicker than if I had just been taught it, and I never saw it" (Y2, Interview-final). Ms. Soto agreed that the consistent connections to CMT made it easier for Sena, explaining "the transformation [Sena] made was amazing" (Interview-MT). However, Ms. Soto struggled to explain how she supported Sena's understandings and practices related to CFoK. Similar to the methods semester, Sena evidenced further contrasting trajectories of connections to CMT and CFoK during student teaching.

Realization rules for eliciting CMT in student teaching (ST)

Across the student teaching semester, Sena increasingly embraced structured student-driven discussions over teacher-directed explanations, suggesting a willingness to share authority with students, and recognition that students can support one another in productive struggles with challenging mathematics. For example, during one observed lesson, she adapted an activity from the *Investigations* curriculum (TERC 2008) to decrease teacher explanations and increase opportunities for students to share thinking. She launched the lesson with a whole class discussion to elicit student reasoning, because "I can ask questions that get them to start thinking…and then when they work in their groups, they can rely on each other" (Observation-final, pre-interview). Again, this insight proved to be generative, as the more Sena shared authority by eliciting students' justifications and reasoning (not just the steps they followed to solve the problem) the more evidence she gathered of their problem-solving capacities, which seemed to help her recontextualize ideas and practices from methods.

Limited access to realization rules for facilitating mathematics discussion: ST In other instances, Sena turned over all the authority to students during discussions, a practice carried over from the methods semester. As a result, the discussion often veered away from students' thinking. During one observed lesson, students spent more time discussing topic sentences and proper spelling than strategies or mathematical ideas (Observation#1, FNs). This inconsistency may have been a result of Sena not understanding the nuanced skills (realization rules) her MT used to select, sequence, and connect student strategies to support the development of mathematical ideas (Smith and Stein 2011).

While Sena recognized that Ms. Soto "put in a lot of effort [at the beginning of the year]" to set up successful discussions, her impression was that later in the year, Ms. Soto just relied on students. Sena explained, "[Ms. Soto had] the students show their work, [she didn't] really say anything" (Observation#2, pre-interview). Sena appeared to emulate this practice: "I just sit there. … [I] let them do the work" (Observation#2, pre-interview). In other words, while Sena valued whole group discussion advocated in her methods course, her strategies were informed by her *own* interpretation of Ms. Soto's practice, which led Sena to turn all authority over to the students.

During student teaching, Sena continued to be unsure about how to respond to unanticipated, unclear, or incorrect student thinking. At times, she resorted to maintaining (vs. sharing) authority by focusing less on students' ideas and more on correct answers. During one observation, a student claimed that '4/8 = ½.' Another student questioned this answer, asking "Isn't 2 half of 4? Shouldn't it be 2/4?" Sena responded by asserting "2/4 is half. So ½ is the same. Both of your ways are correct" (Observation#2, FN). Sena maintained authority by resolving the different answers, missing an opportunity to learn about students' mathematical reasoning. Yet, after the lesson she brainstormed possible reasons for the student's question (Observation#2, post-interview).

In fact, during student teaching Sena began to reflect more on moments when students' understandings did not coincide with her predictions. At times, these reflections strengthened her recognition rules related to the importance of probing and understanding CMT, which in turn may have supported efforts to connect to children's thinking in instruction (realization rules). In other instances, Sena's reflections were less fruitful. For example, Sena believed the previously mentioned group discussion focused on spelling and topic sentences "Went very well. …students shared what they thought and it was the students giving feedback … more than the teacher talking" (Observation#1, post-interview). Similar to methods, there was a disconnect between Sena's *desire* to share authority in mathematics discussions, and the *understandings* and *skills* needed to effectively do so. In other words, Sena may not have had full access to recognition rules and/or sufficient realization rules to support the methods aligned ambitious practice of mathematical discussions during student teaching (Ensor 2001). However, she sought to share authority with her students with regard to mathematics. Similar to methods this effort appeared to be generative, as it helped her begin to develop *some* nuanced practices (including reflection) to successfully elicit and respond to CMT in ways that further supported sharing authority.

Implementing ideas for eliciting CFoK in student teaching (ST)

While Sena continued to wonder about making connections to students' experiences, she struggled to make mathematical connections to CFoK and to share authority with her students (and their families) in this realm.

Outsider and orientation challenges: ST Perhaps because of her experiences in methods, Sena continued to want to learn about her students' culture. She recognized that her MT connected students' real-world experiences with mathematical content (recognition rules) and described the support Ms. Soto offered Sena when lesson planning, "she always gives me ideas of how to relate it back to the students" (Interview-final). Yet Sena struggled to understand how *she* might make these connections (realization rules) because of her limited knowledge about students, their families, and communities. As an Asian-American, Sena was apprehensive about her outsider status:

> There's a lot that I'm still learning. My teacher, she's obviously part of that culture so she can relate very well to the students…So, I mean, you know, there's- they have parents that- sell tamales. And I'm like "What are tamales?" Like just little things that I have no clue about the culture. (Interview-final).

Sena was concerned that she would not "know the best way to connect the mathematics [to the students' CFoK]" (Interview-final). Her outsider status appeared to limit Sena's access to recognition and realization rules, and contrary to principles taught during methods, Sena understood that it was the teacher's *responsibility* to make connections between students' experiences and mathematics which dissuaded Sena from eliciting student ideas (i.e., realization rules).

Sena's orientation toward students' communities and cultures also discouraged opportunities to learn more about students' out-of-school experiences. For instance, Sena believed that while support for education was a core value of her own culture, this was not necessarily true for her students, "I think in my culture, you know, education is the most important thing. … some of these kids may not get that support from home" (Interview-final). Unfortunately, these deficit-based views were at times reinforced by Sena's mentor, who Sena recounted as noting: "[The] majority of these parents they don't have that high of standards for their child when it comes to education… " (Interview-final). While Sena recognized that connecting to students' CFoK was important, she did not often talk about families from an asset-based perspective and did not evidence foundational recognition rules for this practice advocated in methods.

Fun and motivating connections: ST As a result of her limited understandings about students' families and communities, Sena opted for more surface-level connections that might be engaging or "fun" for students.

> We were getting taught this [connections to CFoK in methods] and I was like, "Oh that's great! I want to try and include that." But I think this year, [I] realized it's not always… feasible. … So, last week … we had the students do a scavenger hunt of eggs and then they had to get the jellybeans and create a pie chart … it wasn't necessarily school and community but… enjoyable mathematical activities…(Interview-final).

Sena's decision to emphasize "feasible" and "fun activities" over connections to students' CFoK was reinforced by her observations. For example, Sena observed her mentor teacher relate fraction concepts to sharing a candy bar (Observation#2, post-interview) and noticed that students were more engaged when problems had their names, stating "they love talking about themselves …. when they know, it's about them, it's like, "Oh!" (Interview-final). As a result, Sena began to change names and contexts in word problems, a practice she continued in her early career teaching. In summary, during methods and continuing into student teaching Sena's dominant approach to connect to CFoK was through fun and/or familiar contexts, perhaps because she found making meaningful connections to students' experiences to be challenging.

Caveat and possible growth: ST During one observation, Sena did share authority with students when she elicited their experiences using fractions outside of school at the beginning of a lesson. Sena was surprised by how many ideas students shared (i.e., using fractions in cooking, building, and measuring). This recognition that students use and make connections to mathematics outside of school was the beginning of a potential shift for Sena. Reflecting on this instance, she noted:

> When you figure out what it is that the students know and what their culture is, when you tie it in … it's incredible. … being able to see it [my MT connecting to students' CFoK]…makes a huge difference. (Interview-final)

However, in the lesson debrief it appeared Sena's view of students' families was still a barrier to eliciting and connecting to CFoK. She explained, "maybe understanding how big 1/4 really [is], or 1/2, they can bring that home and when they are cooking with their parents they'll know 1/4 is the smaller one than 1/2" (Observation#2, post-interview). Sena's focus remained on how *school* learning could support students' activity outside of school, versus how students' experiences outside of school could be resources for their work in her classroom.

In other words, Sena maintained authority over connecting mathematics to students' experiences and had limited access to recognition rules that might help her to learn from students and families. However, Sena recognized that student engagement increased via the *small advances* she made (i.e., learning about tamales) and the connections that she witnessed from her MT. This reflected a tension evident throughout student teaching—Sena wanted to connect to students' CFoK, as advocated in methods, but lacked both recognition and realization rules for accomplishing these connections. As a result, she did not regularly share authority with students, thus limiting what she could learn from them about their families, communities, and cultures.

In summary, across student teaching Sena evidenced a stark contrast between her access to recognition rules to CMT, which positioned children as authorities capable of generating strategies and solutions, and her limited access to recognition rules in connecting to CFoK, which positioned the teacher as the authority responsible for making these connections. During student teaching Sena was solidifying the disparate trajectories for the two ambitious teaching practices.

Early career teaching: crystallizing identity as a teacher

Following graduation, Sena taught fourth grade (first-year teaching) and third grade (second-year teaching) at Edison Primary. The majority of her students were LatinX, emerging bilinguals. As we saw in Student Teaching, during early career teaching Sena continued to share authority with and make connections to CMT, but struggled to share authority with and make connections to CFoK.

Recognition and realization rules related to CMT in ECT

Sena regularly elicited students' thinking as part of the lesson, explaining "[I go] straight to the kids" as, "no one's going to be able to tell me better [about their mathematical thinking] than they would… It's just interesting to see how different students will answer," (Y1, Interview-middle). For example, during a lesson on patterns: "[I] went up to a group, and I knew the pattern…[but] when the student explained it, I was, like, 'Oh, my gosh. I didn't even see that. You're absolutely right'" (Y1, Observation-Fall, post-interview). Sena then incorporated what she learned from her students into her instruction stating, "for some lessons, I can change my guiding questions just because I know how the kids might respond"

(Y1, Observation-Spring, post-interview). Here Sena's reflections were generative in developing realizations rules that further shared authority in her mathematics classroom.

Yearly progression of realization rules connecting to CMT—authority sharing: ECT Interestingly, during both her first and second year of early career teaching, we noted a progression of authority sharing across the year: Sena began each year with teacher-centered instruction and then progressed toward student-centered problem-solving lessons (i.e., realization rules advocated in methods). For example, during the beginning of Sena's second-year teaching, she enacted teacher-directed lessons "showing" her third-grade students how to add numbers on an open number line (Y2, Observation-Fall#1) and how to multiply (Y2, Observation-Fall#2). The following example of highly scaffolded questions to guide students to expected answers was typical early in the year (Y2, Observation-Fall#1, FN):

Sena: What is 30 plus 7?
Students: 30 plus 7 is 37.
Sena: 37 is what? [Points to vocab list.]
Students: SUM!
Sena: What do we use to find the sum?
Student 1: Addends.
Student 2: Place value strategy.
… [Sena leads another guided example]
Sena: Are you ready to try one on your own?
Students: Yes!

While Sena appeared not to access realization rules consistent with mathematics methods as she maintained authority in the mathematics classroom during these early-in-the-year lessons, i.e., driving strategies used and summarizing key ideas, she still attended to CMT. For example, she carefully considered number choices to ensure accessibility for students (Y2, Observation-Fall#2, post-interview) and included small group discussions so all students had a "chance to explain their thinking" (Y2, Observation-Fall#1, post-interview).

As the year progressed, Sena shared more authority through adapting curriculum to ensure that students had frequent opportunities to discuss their thinking. In contrast to student teaching, Sena no longer "sat back" and offered the students full authority during group discussions. Rather, she elicited students' ideas, noticed students' misconceptions, and worked to engage them in mathematical reasoning. Her access to realization rules for group discussion and ability to share authority were evident during a discussion on writing equations to represent word problems. Sena strategically picked students to present their work "who were missing specific pieces or had errors in their answers to promote rich mathematical discussion" (Y2, Observation-Fall#3, post-interview). As Sena shared more authority with students, she moved away from a focus on correct answers and refrained from confirming answers for students (Y2, Observation-Fall#3, FNs). This again suggests that sharing authority is a generative practice in increasing access to both recognition and realization rules.

Potential challenge in sharing authority at ECT school Interestingly, Sena's focus on CMT was not widely supported at her school. Rather, the instructional coach recommended the "I do, we do, you do" model.[6] While this stance may have challenged her efforts to recontextualize ideas from methods, Sena was given the freedom to teach differently, because as she stated, "they've seen my test scores, so they don't usually bother me too much about it, it's good" (Y2, Interview-final). As part of her strong recognition rules for CMT Sena challenged the status quo of mathematics instruction at her school and frequently adapted the curriculum so that lessons allowed for shared authority (Y1, Observation#1, pre-interview). She explained:

> enVision[7] is fine, but it's not very inquiry based. When I try and get students to be more hands on and figure it out themselves, it's hard because I don't have all the resources that would be ideal for doing that. I think that's my biggest setback right now…. (Y2, Interview-middle).

Despite these challenges, Sena evidenced access to realization rules as she planned and enacted lessons that had multiple entry points and encouraged diverse ways of thinking. For example, she "looked at some of the word problems from Investigations" to design a set of word problems that would be open-ended and relatable (Y2, Observation-Spring#3, pre-interview). As she presented these tasks she reminded students that diverse ways of reasoning were valued, "We are going to do some math problems [and] you could use one of these strategies, any of these strategies. Is there a right or wrong strategy? [No!] It does not matter as long as you show me the evidence" (Y2, Observation-Spring#3, FN). By the end of each school year, Sena regularly shared authority, by inviting diverse ways of thinking and eliciting reasoning from students.

Possible rationale. One explanation for Sena's trajectory of increasingly sharing authority across the school year is that she was recontextualizing recognition and realization rules to a school that advocated teacher-directed instruction. Sena believed that she needed to scaffold students' transition from "traditional" (advocated by the school) to problem-solving-based instruction (Sena's classroom) where authority and power were shared. She explained:

> At the beginning of the year these kids, they're brilliant, but they're used to 'I do, we do, you do.' Me presenting something and having them run with it was so hard. It took them a whole day and a half [to solve a problem] just because they kept pushing back and I said, "I'm not telling you." (Y1, Observation-Fall, pre-interview)

Sena noted that while students were initially more comfortable with traditional teaching practices (teacher as authority), they eventually embraced mathematical discussion and "were happy because they understand what they were doing" (Y2, Interview-final). In summary, Sena developed a skilled (and evolving) practice of connecting to and eliciting CMT. Furthermore, through her efforts to share authority with her students—even positioning herself as learning from and with students about mathematical reasoning—she may have increased her access to realization rules (i.e., to notice, elicit, and connect to CMT), and therein, her engagement in ambitious teaching practices.

[6] In this model, the teacher demonstrates a problem, the class does a problem together, and then the students work on their own.

[7] enVision is a K-8 mathematics curriculum published by Pearson (enVision Math Common Core 2012).

Access to recognition and realization rules connecting to CFoK in ECT

Unlike her trajectory related to CMT, Sena continued to face challenges describing and enacting connections to CFoK, due in part to her limited understandings about her students and their families.

Increased access to recognition rules: ECT. As a novice teacher, when asked to reflect on connections to children's out-of-school experiences, Sena frequently noted, "this question is always hard" (Y2, Observation-Spring#2, post-interview)—indicating her limited access to recognition rules. Yet she sometimes generated ideas for ways she could connect to children's experiences, even if those ideas were rarely enacted in her lessons. For example, following a lesson on area and perimeter, Sena recalled that one of her students had experience farming, including how to fence plots of land: "I remember him telling me this story [about his family and farming]. … it was just something that he brought with him." She suggested making connections to this knowledge explaining, "How do we figure out how much fence we need? That defines the perimeter. That's one thing I'm hoping to tie in … with both perimeter and area" (Y2, Observation-Spring#2, post-interview). While we do not have evidence that Sena enacted these connections, she began to consider ways to connect to students' experiences—indicating an increase in her access to recognition rules.

Swapping contexts, limited access to realization rules: ECT. During early career teaching, Sena extended her practice of "swapping contexts" in textbook word problems for contexts that were familiar to her students. Sena referred to this teaching move as "something simple" and "quick," explaining, "Yeah. I just throw it [something I know about student interest] into the word problem or just something that they can relate to" (Y1, Interview-beginning). Sena argued that this teaching move supported understanding because students were able to visualize the situation in the problem:

> If I were to say a store that they've never been to, it wouldn't click for them; but when I say a store that they do know they can imagine themselves going in, find[ing] things. They have that background knowledge … It makes the problem come more to life. (Y2, Observation-Fall#3, post-interview)

While Sena was potentially supporting student understanding by connecting to children's experiences in a familiar community location, she was not eliciting or connecting to ways that children and families might engage in mathematics outside of school. This is an important nuance to the recognition rules surrounding CFoK.

During a second-year lesson, Sena created division word problems about packaging tamales into bags to sell. The problem stated, "3rd grade is selling tamales after school. They have 36 tamales. Miss Sena put the tamales in bags. She put 6 tamales in each bag. How many bags of tamales does she have?" (Y2, Observation-Fall#3, handout). Sena explained, "the problem [includes] something that they have a connection with" (Y2, Observation-Fall#3, pre-interview). Other problems included erasers, marbles, cookies, pencils, and crayons, items that students might purchase at a neighborhood store. Students responded positively and seemed interested in what and who the problems were about (Y2, Observation-Fall#3, FN). As students worked on the tasks, one student wondered whether the problems were actually real, asking, "Did you really do these things?" When Sena quickly noted that no, the problems were all "just pretend," the student looked visibly disappointed (Y2, Observation-Fall#3, FN). Yet Sena maintained that while the problems did not reflect "real" situations, adapting problem contexts to include familiar situations was still effective, as it supported student sense making. Returning to the tamale example, she explained, “when it’s something they’ve seen and they know, ‘Oh, I can put tamales in bags,’ then it makes it easier to think through the problem.” (Y2, Observation-Fall#3, post-interview). In other instances, Sena adapted problem contexts to reflect her own experiences. This was a strategic move, aimed at sharing information about her life outside of school, and in particular details about activities and interests that might connect to students. For instance, during a fractions lesson in her first-year teaching, she explained, “I’m using myself in a problem. … The kids know that I love carne asada, so I decided to put that in a problem because all the kids get excited” (Y1, Observation#2, FN).

Sena’s ECT practices may have been impacted by her school environment. While the school maintained positive views of students and their capabilities, parents were not included in classroom-based learning activities (e.g., parents were rarely allowed past the front office). Instead, the teacher was responsible for learning, and parents were asked to support school-based learning via practice activities at home (i.e., homework) (Y1-Y2, Observations, FNs). In summary, Sena viewed making connections between the mathematics she taught in school and students’ interests or activities outside of school as part of *her* role as the teacher (rather than something the students might contribute). In fact, she often described this practice as “making the connections *for* them, so that they can understand,” implying that students would be unaware of the relevance of mathematics in out-of-school contexts without her explicit intervention. This view indicated the challenge Sena faced in recontextualizing the ideas and practices advocated for in methods (i.e., learning about students’ mathematical activities outside of school) in her classroom context.

Potentially pivotal moments: ECT. In the few instances when she did access realization rules and invite students to generate connections (i.e., Y2, Observation-Spring#2, FN), Sena seemed genuinely surprised at what students were able to generate. After the lesson, she explained, “I wasn’t expecting as many [examples of patterns in the world] as I got, … I’ll be honest. … just being able to hear all these different ones definitely made a difference” (Y1, Observation#2, post-interview). In fact, Sena marked this moment as pivotal, noting that in future lessons, she would “not underestimate how much they can actually bring in from their homes, because even something as random as patterns—they still could make connections to” (Y1, Observation#2, post-interview). However, across her ECT Sena did not evidence significant shifts in accessing realization rules advocated for in methods, instead, she maintained authority and continued to make surface-level connections to children’s interests. In other words, Sena did not evidence consistent recognition and realization rules for connecting to CFoK.

In summary, during ECT Sena evidenced a pattern of increasingly sharing authority during mathematics lessons across the school year, in support of her efforts to recontextualize in a direct instruction school environment. She thoughtfully adapted and brought in curriculum to expand spaces for CMT and her reflections on lessons further supported her development of realization rules. Again, sharing authority seemed productive, increasing Sena’s access to recognition and realization rules. For CFoK, Sena’s limited knowledge about students and their families continued to pose a challenge. She continued to view connections to students’ experiences as *her* responsibility (she maintained the authority) and yet had limited realization rules for learning about students’ experiences outside of school. By the end of her second-year teaching, Sena had developed two very different trajectories for engaging in the methods advocated ambitious practices of connecting and eliciting CMT and CFoK.

Discussion

In this article, we presented the longitudinal case of one early career teacher, Sena from her mathematics methods experience through her second year of classroom teaching. (See Fig. 2 for a summary.) Consistent with findings from other research, Sena entered her teacher preparation program with limited understandings about students' mathematical thinking (e.g., Jacobs et al. 2010), as well as limited experience and, at times, deficit-oriented beliefs and assumptions about students from diverse backgrounds (e.g., Sleeter 2001). Yet Sena also brought a positive orientation toward children, and their capacity as mathematical learners, and a willingness to learn more. To support prospective teachers in these areas, Sena's teacher preparation program emphasized recognition and realization rules for ambitious and equitable teaching practices, which included connections to CMT and CFoK (Turner et al. 2012).

During methods and student teaching, Sena's MT consistently supported Sena's efforts to connect to CMT, facilitating a productive synergy (or context for recontextualizing) between methods and the field (Anderson and Stillman 2013). Sena's MT also modeled and supported connections to CFoK, though with less consistency. While teaching at Edison Primary, Sena consistently lacked support for *both* realms. Despite similar support for both realms throughout her 4-year trajectory (i.e., the dual *focus on both* CMT and CFoK in her methods course, and the *lack of attention* to both constructs in her early career school context) Sena evidenced markedly different trajectories for connecting to CMT and CFoK.

More specifically, across the 4 years, Sena refined practices, and increased her access to realization rules, for eliciting and connecting to CMT by *sharing authority* with her students. Despite occasional setbacks, Sena persisted in sharing authority through continuous attention to and reflection on CMT. Yet, when connecting to

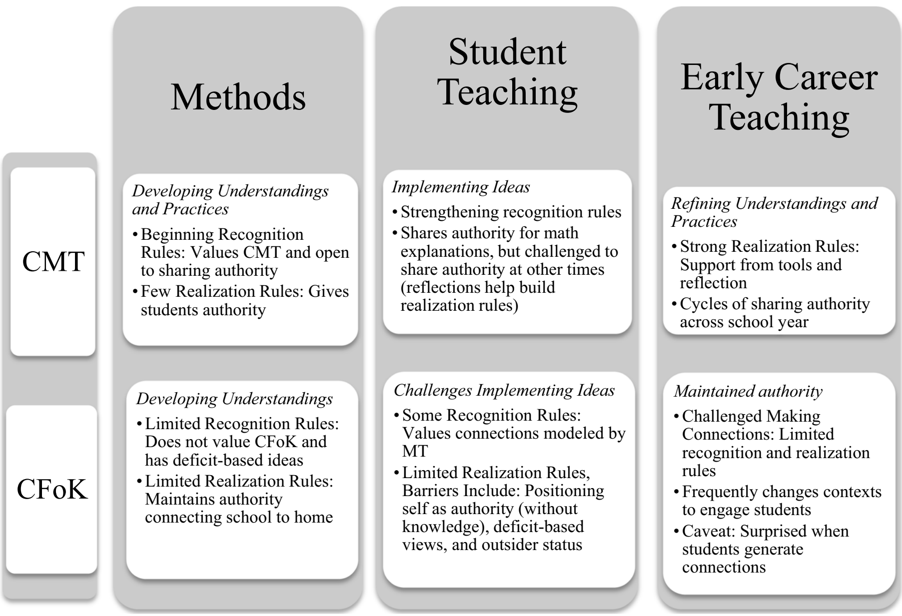

Fig. 2 Summary of Sena's trajectory

CFoK, Sena tended to *maintain authority* during lessons, and rarely elicited students' ideas or experiences—indicating she lacked access to recognition and realization rules for learning about and connecting to CFoK. Instead, she emphasized making "fun" connections to interest students. These contrasting trajectories offer an opportunity to better understand how different ambitious teaching practices are taken up (or not) given similar foundational and early career experiences.

Learning to share authority: development of two different trajectories

Sena's development of practices for eliciting and connecting to CMT highlights the challenge novice teachers face when recontextualizing what they have learned in methods to the classroom (Ensor 2001). For example, Sena—building on her positive orientations—further developed recognition rules during methods for sharing authority by eliciting and responding to students' mathematical ideas. However, when she tried to replicate similar discussions in student teaching she simply turned authority over to students, which undermined her effort to elicit and respond productively to CMT, indicating she lacked access to realization rules. It took time and continued reflection through her initial years of teaching to develop realization rules for these practices.

In contrast, while Sena had, and further developed, practices for connecting to students in her lessons (e.g., superficial connections to students' interests), these practices did not resonate with the principles advocated in her methods course, indicating she lacked access to recognition and realization rules related to CFoK. Sena's understandings were shaped, at times, by deficit-based views of students' families, and as such, her mathematics lessons did not include regular opportunities to elicit and connect to children's out-of-school experiences. In addition, she seemed to focus solely on math as defined in the school curriculum, and in turn she may not have seen the knowledge and experiences that children brought from outside of school as relevant. With this, she maintained authority over determining what to include (or not include) in the school math curriculum. Her limited focus on school math seemed to make it difficult for her to see the knowledge and experiences that children brought from outside of school as relevant. Given her limited knowledge of students, her focus on school mathematics, and her stance that she, as the teacher, should maintain authority when connecting mathematics lessons to experiences outside of school, Sena developed practices that emphasized cursory family connections (i.e., word problems about familiar food) and/or "fun" activities. While we observed two potentially pivotal moment (i.e., students offered examples of fraction and pattern use at home), this practice was not sustained. Figure 2 shows how this trajectory developed.

Similar to Ensor (2001) we found that Sena, given these contrasting trajectories, had selective recruitment of practices and ideas from methods. We suggest that the reason she did not equally enact all practices or ideas aligns with how Sena shared, or did not share, authority in her classroom. For example, Sena regularly shared authority related to mathematical strategies and ideas, and her lessons evidenced frequent connections to CMT. In fact, Sena's reflections on lessons suggested that the more she positioned herself to share authority, the more she was able to notice, elicit, and connect to CMT. This suggests that the sharing of authority was a catalyst in developing understandings and practices in connecting to CMT. In contrast, Sena's understandings and practices for connecting to CFoK positioned her as the authority, despite (or perhaps because of) her own identification as an outsider. This positioning appeared to prevent Sena from further developing practices to elicit CFoK. While she was making small shifts in her teaching practice, shifts in authority were less evident. In short, her recognition and realization rules for connecting to students' CFoK reflected a lack of sharing authority that may have hindered her capacity to learn about and connect to CFoK in more meaningful ways. This is not to say Sena was not willing to share authority, but rather she may have faced difficulty developing this practice given the implicit—rather than explicit—focus on sharing authority in methods. Also, she may have needed more time to take up the practices.

Sharing authority, a generative practice. With regard to Sena's CMT trajectory, her efforts to share authority, reflect on her teaching, and to learn from her students were generative (Rodgers 2006). Similar to Franke and Kazemi (2001) we define generative to be when teachers integrate their new knowledge (or practices) with existing knowledge (or ideas), thereby building on the access they have to recognition and realization rules for ambitious practices from methods. Therefore, an important contribution of this study is, as PSTs learn to share authority it may increase access to recognition and realization rules for connections to CMT and CFoK, and therein support generative growth of ambitious practices. In other words, sharing authority may be a driver in developing understandings and practices connecting to CMT and CFoK.

Tools to support sharing authority. Similar to Thompson et al. (2013) we argue that integration of ambitious practices may be supported by access to tools that support enacting and revising practices. Figure 2 represents how tools supported Sena's development of practices connecting to CMT. Specifically, the curriculum, *Investigations*, which Sena used during both methods and student teaching, supported her efforts to elicit and connect to CMT, by offering tasks and lessons that created space for students' thinking (realization rules), and teacher notes that supported her developing recognition rules (Bartell et al. 2017; Schultz et al. 2008; Thompson et al. 2013). Furthermore, through her use of Investigations across multiple settings (i.e., methods course, student teaching school, and early career classroom), even when contexts did not offer broader support for the curriculum (i.e., ECT school), Sena was developing an identity as a teacher who connects to CMT (Thompson et al. 2013). In turn, this identity may have further strengthened her commitment to connect to CMT. This finding builds on Drake and Sherin's (2006) importance of interactions between teachers' use of curricula and teachers' developing identities.

While tools, such as curriculum materials, can support teachers recontextualizing (Ensor 2001; Thompson et al. 2013), supports for connections to CFoK in mathematics are often lacking. Even experienced teachers can struggle when attempting to connect to students' CFoK because they are still developing understandings and practices (Turner et al. 2012). Perhaps if during methods and student teaching, Sena had developed an affinity to a curricula (or other tool) that offered Sena support in connecting to CFoK (e.g., prompted Sena to elicit students' experiences related to particular mathematical ideas), she might have done so more readily. In the absence of such tools and sufficient specific experiences to understand what it means to share authority with regard to CFoK, Sena struggled to develop recognition and realization rules with regard to CFoK. Rather, she relied on swapping contexts in word problems to *engage* students. Furthermore, her early career teaching school context did not encourage teachers to connect to students' out-of-school experiences in mathematics teaching.

Implications

Given the challenges novice teachers face in recontextualizing, our findings suggest that mathematics teacher educators could support novice teachers in methods courses by attending more explicitly to both CFoK and to authority. Here, we foreground implications focused on: (a) opportunities to understand and experience how connecting to CFoK supports learning school mathematics, (b) explicit conversations about sharing authority with students, and (c) recontextualization support for ECTs.

Despite mathematics methods course activities purposefully designed to support connections to CMT and CFoK, the access to realization and recognition rules for connections to CFoK were not sufficient for Sena to regularly engage in this ambitious teaching practice. In other words, novice teachers (including Sena) may leave math methods courses with limited understandings about CFoK or may see connections to CFoK as an add on or optional component of teaching mathematics. For example, Sena referred to connecting to CFoK, as "something simple" and "quick" (Y1, Interview-beginning). This case study highlights the need for mathematics educators to provide additional support for recontextualizing practices related to CFoK. Ways to do this may include observing connections to CFoK in the elementary classroom, encouraging MTs to talk explicitly about CFoK with PSTs, and increasing PSTs' focus on understanding ways that mathematics is used in homes and communities.

Added to this, while Sena's methods course offered a variety of understandings and practices that had an implicit component of sharing authority, explicit conversations about sharing authority were infrequent. We argue that explicitly offering recognition and realization rules around sharing authority could support recontextualization as it may offer a generative approach to connecting to and eliciting CMT and CFoK. Furthermore, our findings suggest that focusing on, practicing, and *explicitly* labeling recognition and realization rules around sharing authority in the classroom may limit the selective recruitment of methods-based practices. For example, working with PSTs and ECTs to adapt CFoK-focused tools offered in methods to ensure shared authority may support a resource-based orientation toward CFoK (Thompson et al. 2013). This explicit focus may also increase the use of tools, further supporting teachers in sharing authority by learning *from* students, families and communities, and facilitate the connection between math content and CFoK.

Support for recontextualizing tools from methods into ECT contexts, may encourage ECTs eliciting students' experiences outside of school. This additional recontextualization support in using and adapting tools from methods may be particularly important for PSTs who see themselves as outsiders to students' communities and/or struggle to implement the tools from methods into their teaching context. Furthermore, ensuring that PSTs have an opportunity to use these tools *in* a classroom could support access to realization rules and increased implementation of these practices in their ECT classrooms. In turn, sharing authority may offer a generative approach to increasing novice teachers' access to recognition and realization rules (i.e., connecting and eliciting CMT). In other words, focusing specifically on sharing authority in the methods class could provide novice teachers with a framework, or lens, to increase their access to ambitious teaching practices.

Finally, offering support to ECTs for recontextualizing shared authority may be essential. For example, creating communities of practice where ECTs focus on sharing authority through common readings and lesson studies may also support the access of these realization and recognition rules (Feiman-Nemser 2001). This may be particularly valuable if offered during the induction period (first two years of teaching) when novice teachers are seeking to recontextualize in a new school not necessarily aligned with their methods course ideals. In summary, we suggest that teacher educators increase novice teachers' opportunities to understand and experience how CFoK supports learning school mathematics, offer sharing authority as a generative practice for novice teachers and offer recontextualization support during the first two years of teaching.

Conclusion

In this case study, we found that Sena's willingness to share authority was generative and may have been a catalyst further supporting her development of recognition and realization rules for connecting to CMT. However, Sena's contrasting trajectory for connecting to CFoK highlights the need for further research on how novice teachers develop understandings that encourage sharing authority with their students. Additionally, we suggest that teacher educators develop accessible and adaptable tools to support novice teachers in learning to share authority. Finally, this was only one case, and further research on connections between sharing authority and ambitious teaching practices is needed.

In conclusion, we highlight the practice of *sharing* authority with one's students as a generative practice to engage in ambitious mathematics teaching practices. To develop more equitable learning opportunities mathematics educators should strive to provide prospective teachers with experiences to view their students as having knowledge and experiences to shape education (Cook-Sather 2002, p. 3)—specifically seeing students as capable of generating mathematical ideas and bringing cultural and family experiences that support mathematics learning.

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Appendix: Example of coded data from phase 1 and phase 2

Coding Category	Example from Data
Understandings and Practices Related to CMT	
a. eliciting or responding to student thinking	"We are going to do some math problems [and] you could use one of these strategies, any of these strategies. Is there a right or wrong strategy? [No!] It does not matter as long as you show me the evidence" (Y2, Observation-Spring#3, FN)
b. anticipating and reflecting on students' strategies	"I realize now after our readings, that this way [the case student's method] is also correct—[an] invented algorithm" (Methods, Reflection-case study)
c. considering problem structures and number choices	Sena revised number choices in mathematics problems, to ensure accessibility for all students: ". . .I realized after Student M shared I only had them go down or add 10. I was like, okay, maybe I shouldn't do 20 yet, because they might not know. I did 15 or 18 and then the next time I said, let's try it, so I did 20, 22 or something, and they all got it. . . Eventually I got all the way up to 60" (Y2, Observation-Fall#1, post-interview)
d. focusing on correct answers versus student reasoning (indicating a lack of focus on CMT)	". . .Another child comes up and says that 4/8 = ½. A child questions this answer and says 'isn't 2 half of 4? Shouldn't it be 2/4?' Sena responds by saying '2/4 is half. So ½ is the same. Both of your ways are correct.'" (Student Teaching, Observation#2, FN)
Understandings and Practices Related to CFoK	
a. connections between mathematics and home/community contexts	"Sena asks the students when we use addition in the real world, when you are outside with your parents. St: I seen it when I was walking to the park." (Y2, Observation-Fall#1, FN)
b. reflections on problem contexts	"We were getting taught this [connections to CFoK in methods] and I was like, "Oh that's great! I want to try an include that." But I think this year, realized it's not always.. feasible. So I, I think what I've learned is you can still incorporate those ideas in a smaller setting. So last week we did, it wasn't school and community, but we had um.. the students do a scavenger hunt of eggs and then they had to get the jellybeans and create a pie chart. . .So it wasn't necessarily school and community but,.... I guess providing them with enjoyable mathematical activities..." (ST, Interview-Final)
c. orientations towards mathematical knowledge and practices in students' homes	". . So maybe understanding how big, you know, 1/4 really means or 1/2, *they can bring that home and when they are you know, cooking with their parents or something.*. That they do. It'll- they know 1/4 is the smaller one than 1/2." (ST, Observation #2, post-interview)

Talk and Practices Sharing Authority Related to CMT	
a. eliciting students' ideas about how to solve a problem (sharing authority)	Sena invited students to solve multi-digit multiplication problems (i.e., 78 × 7) using several methods. She noted, "I even asked the students to verify their work using more than one strategy" (Methods, Reflection-lesson).
b. encouraging students to answer each other's mathematical questions (sharing authority)	"I can ask questions that get them to start thinking...and then when they work in their groups, they can rely on each other" (ST, Observation-final, pre-interview)
c. directing students to use specific method to solve problems (not sharing authority)	Sena enacted a teacher-directed lesson "showing" her third-grade students how to add numbers on an open number line (Y2, Observation-Fall#1): **Sena:** What is 30 plus 7? **Students:** 30 plus 7 is 37. **Sena:** 37 is what? [Points to vocab list.] **Students:** SUM! **Sena:** What do we use to find the sum? **Student 1:** Addends. **Student 2:** Place value strategy. …. [Sena leads another guided example] **Sena:** Are you ready to try one on your own? **Students:** Yes!
Talk and Practices sharing authority related to CFoK	
a. asking students how they or their families use a particular math concept or skill at home (sharing authority)	"Sena asked the students to identify the patterns they see in their everyday lives in an effort to help them see a connection to the mathematics patterns they were solving." (Y1, Observation-Fall, FN)
b. allowing students to define parameters of a problem or activity based on their own experiences or assumptions (sharing authority)	Connecting to students' real-world experience of farming: "How do we figure out how much fence we need? That defines the perimeter. That's one thing *I'm hoping to tie in* ... with both perimeter and area" (Y2, Observation-Spring#2, post-interview)
c. telling students how they will use a particular math idea in the "real world" (not sharing authority)	"I tried to connect [the lesson] to their lives by explaining when they would use estimation…but it was difficult for them to grasp" (Methods, Reflection-lesson)

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