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The challenge of changing teaching: investigating the interplay of external and internal influences during professional learning with secondary mathematics teachers

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Abstract Mathematics teaching at secondary levels has proven surprisingly resistant to change over the past century. This study draws on two theoretical models to investigate how the process of changing secondary teaching in algebra through school-based professional learning might occur, and its relationship to different external and internal influences on teachers and researchers. A cyclic change model is used to discuss three different change pathways that were found amongst six practising secondary teachers participating in an algebra teaching experiment, one phase of a larger design-based research project. Meta-didactical transposition is used to examine the dynamics between teachers and researchers and the institutional dimension of professional learning. Affordances and constraints related to the teachers' internal domains and social contexts in responding to professional learning opportunities are discussed. The bidirectional nature of brokering processes between teachers and researchers during professional learning is examined.

Keywords Teacher professional learning · Classroom experimentation · Meta-didactical transposition · Teacher beliefs · Algebra · Secondary mathematics

Introduction

Mathematics teacher education has been considered from many angles to study teachers' professional growth and how the process of learning to teach effectively might be facilitated with prospective and practising teachers. There are several conceptualisations of what teacher professional learning might look like and how it might be evaluated. The overarching aim is to improve student learning through improving teaching practice. Of all the school-related factors that influence student mathematics learning, teaching seems to be

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foundational (e.g. Hiebert and Grouws 2007; Swan 2007). In secondary classrooms over the past century, the traditional approaches for teaching mathematics have persisted for the most part with very little modification—the teacher defines the rules, demonstrates the procedures, and provides students with practice exercises. Changing mathematics teaching does not appear to be easy (Hiebert 2013). Studies have found that teachers' internal domain—their knowledge, beliefs, attitude, and identity—plays an important role in their teaching practice in the mathematics classroom (Campbell et al. 2014; Chapman 2014). External aspects of teachers' institutional environments also act to afford or constrain their ability to change their teaching (Arzarello et al. 2014; Clarke and Hollingsworth 2002).

Two aspects to consider in mathematics teacher education are what might motivate teachers to engage (or not) in learning and the process by which they might change (e.g. Gregoire 2003; Guskey 1986). Hiebert (2013) argued that trying to understand why mathematics teaching has resisted past efforts to change is the starting point for creating strategies for improving teaching. He offered three possibilities; first, that consensus has not been achieved in the profession on clearly defined learning goals for students. There are disagreements about what matters most, procedural fluency or conceptual understanding, and which topics to cover in an overcrowded curriculum. "With teachers teaching toward different goals, there is no way to share and accumulate what they are learning" (p. 47). Second, efforts have focussed on upgrading teacher *characteristics* (such as qualifications) rather than on upgrading the quality of teaching *methods*, perhaps because qualifications are easier to measure. Yet evidence has shown that changing qualifications has not significantly improved teaching. Third, not enough attention is paid to understanding mathematics teaching as a cultural activity handed down from one generation to the next. A teacher's own school experience is likely to be more potent than formal teacher training (Hiebert 2013). Prospective teachers do not seem to transfer their learning about teaching to their own classrooms when in service (Sfard 2005), perpetuating the historical cycle of persistent cultural practices. Other key considerations include the external characteristics of teachers' institutional contexts that might constrain them, whether or not certain types of professional development are effective for all teachers, individual teachers' responses, and the aim of long-term teacher growth rather than short-lived change.

This article discusses the findings from one phase of a design-based research project, in which six practising mathematics teachers participated in an algebra teaching experiment with their Year 7 classes over several weeks. The study sought to examine the process of onsite collaborative professional learning with secondary teachers, and internal and external influences within the teachers' environments, to understand more about affordances and constraints in seeking to change teaching. The study focussed on changing algebra teaching because of widespread calls to develop a wider and deeper view that promotes algebraic thinking rather than only narrow symbolic manipulation (e.g. Kaput 2008; Rakes et al. 2010). It investigated teachers learning and experimenting with a teaching approach for early algebra noticeably different from the previously described traditional approach. It involved the incorporation of a series of hands-on problem-based pattern generalisation tasks connecting multiple representations of the same functional relationship—descriptive generalisations, tables of values, symbolic equations, and graphs. The study's intent was to consider how any changes to the teachers' approaches in their algebra teaching might relate to changes to their subject-specific beliefs, their current teaching methods, their involvement in classroom experimentation, and the dynamics of their interactions with the researcher (author) and the other participating teachers. The study used a theoretical model for the process of a mathematics teacher's professional

growth (Clarke and Hollingsworth 2002) to examine the relationships among four possible change domains (external, personal, practice, and consequence) during participation in the collaborative project. It also used the more recent model meta-didactical transposition (MDT), which explicitly considers the influence of institutional constraints on teachers and the interactions between researchers and teachers in professional learning (Arzarello et al. 2014). An accounts-of-practice methodology (Simon and Tzur 1999) was used in the discussion to differentiate between the teachers' expressions and the researcher's interpreted observations of their conversations and actions during lessons and meetings. The research questions for this study were: (1) What types and sequences of changes were perceived during the teachers' participation in collaborative professional learning? and (2) How might aspects of each teacher's internal (personal) domain and external change environment have facilitated or constrained their access to or utilisation of opportunities to change their algebra teaching practice? The following section presents details on the context for the study and the theoretical framework on teacher professional development that informed its design.

Context and background

There are differing perspectives on how to theorise the professional learning of teachers and then provide professional development opportunities that might be effective for improving teaching. The following four subsections review the literature for: framing professional learning; the theoretical framework that shaped the study's analysis; studying the influence of teachers' personal domains; and researching teachers' professional learning on algebra.

Approaches to framing professional learning

The literature on the professional development of teachers interprets the notion of "change" in various ways. Clarke and Hollingsworth (1994) identified six views: as training; as adaptation; as personal development; as local reform; as systematic restructuring; and as growth or learning. Johnson (1996) conceptualised teacher professional development as "opportunities for learning" that are "embedded into the on-going work of the school" (p. 12). This perspective distinguishes between "one-shot" professional development programmes based on a deficit training-mastery model, and change as a complex and continuing process of improving. Change as growth or learning considers that "teachers change inevitably through professional activity; teachers are themselves learners who work in a learning community" (Clarke and Hollingsworth 2002, p. 948). Professional learning is seen as including teachers' ongoing development of knowledge and effective teaching practices, which ultimately lead to improved student outcomes. Teaching practices encompass teachers' repertoire of professional behaviours: their pedagogical actions and approaches in the classroom when interacting with their students, as well as activities outside of the classroom, such their lesson planning and assessment actions.

As with the many theories of student learning—behavioural, constructivist, sociocultural—how the learning of teachers is conceptualised will influence how professional development programmes are designed, implemented, and evaluated for effectiveness. *Cognitive* theories of learning focus on an individual teacher's acquisition of knowledge or skills (for example, Hill et al. 2008; Sfard 1998; Shulman 1986), and many studies have

sought to measure changes to teachers' knowledge. *Situated* perspectives on learning focus on teachers' "participation in socially situated practices" (Lave 1996, p. 150). Knowledge is not so much a commodity as an action, and learning involves actions—*doing*, rather than having (Sfard 1998). Some studies have framed their research from this situated perspective, yet have also paid attention to increasing an individual teacher's knowledge (for example, Putnam and Borko 2000; Kazemi and Franke 2004). Other studies have identified with a cognitive paradigm, yet have employed participatory activities (for example, Zwiep and Benken 2013).

Viewing teachers as learners and schools as learning communities enables a shift from thinking of professional development as "programmes that change *teachers*, to teachers as active learners shaping their professional growth through reflective participation in professional development programmes and in practice" (Clarke and Hollingsworth 2002, p. 948). The ultimate aim is helping teachers change their teaching practices rather than seeking to change the teachers per se, which Hiebert (2013) argued would be more likely to succeed.

Some earlier theoretical perspectives on professional development viewed changing teachers' beliefs and attitudes as a necessary precursor to changing their teaching practices. They conceptualised professional learning as a linear process in which external inputs, such as a practising programme, lead to changes in teachers' beliefs and attitudes, and then to changes in teachers' practices. An alternative linear model placed change to teachers' beliefs *after* changing teaching practices, with teachers noticing improved student outcomes first (Guskey 1986; Shaw and Jakubowski 1991). Teacher beliefs encompass a diverse range of types and contexts in the literature, and there are many theoretical perspectives on beliefs and their relationship with teachers' actual practice. In this study, the teachers' beliefs were seen as including their views on what matters most in mathematics education—their learning goals for students (Hiebert 2013), or on how students learn best specifically in a subject domain such as algebra (Nathan and Koedinger 2000), or to the efficacy of particular classroom practices for effective student learning (Clarke and Hollingsworth 2002). Their beliefs were also viewed as likely to relate to their mindsets about students' intelligence and ability to learn mathematics (Dweck 2010) and to their attributions of student behaviour or outcomes (Wilson et al. 2014).

According to some mathematics education researchers, teachers need a compelling reason to change, perhaps some sort of perturbation, since they are not quick to alter their existing teaching practices. Cobb et al. (1990) argued that gaining an initial sense of commitment from teachers is needed so that they develop the motivation to even consider changing their teaching practice. They then need to see evidence of improved student learning *before* they change their beliefs or attitude: "they believe it works because they have seen it work" (Guskey 2002, p. 383). Teachers in numerous studies were found to attend to traditional achievement measures of students (test or examination scores) and also a wide range of student behaviours, motivation levels, and attitude, when drawing conclusions about the effectiveness of a new classroom practice (Cobb et al. 1990; Guskey 2002).

Although some models for professional learning have represented the process of change as linear, other theoretical perspectives have conceptualised professional learning as a more complex, and perhaps cyclic process. Huberman (1992, 1995) proposed a variety of cyclic models for different approaches to professional development, and which also involve ongoing classroom experimentation. He suggested that models for teacher professional development need to resonate with how teachers typically "tinker" in their

classrooms individually and that classroom experimentation, along with involvement with colleagues, is more likely to engage passive or disinclined teachers.

Theoretical framework for the study

The study reported on in this article sought to investigate the process of changing teaching using two theoretical models developed in recent years by mathematics education researchers as part of the shift of attention from dealing with student learning difficulties in mathematics to improving teaching practices (Sfard 2005). The first is a theoretical processual model developed empirically from multiple studies of Australian mathematics teachers (Clarke and Hollingsworth 2002) and used in this study as an analysis tool for exploring different change pathways of the teachers during their participation. The second is a descriptive and interpretative model known as meta-didactical transposition (MDT) developed by Italian researchers over several decades (Arzarello et al. 2014) and used in this study for examining teachers' institutional constraints and affordances that might affect changes to their teaching approaches. These models are each described in turn.

Clarke and Hollingsworth's (2002) interconnected model for professional growth incorporates four change domains analogous to Guskey's original model but arranged nonlinearly to allow for additional and multiple pathways that seem to model more realistically teachers' individual changes. Figure 1 presents a diagram of the model.

Change can be located in any of the four domains, and a change sequence is defined as an occasion when change in one domain is demonstrably connected to change in another domain. The *external change domain* includes a wide range of sources of information or stimulus, for example, participation in a practising programme, professional reading, conversations with a colleague, and involvement with researchers. The *personal (internal) change domain* incorporates the development of new knowledge for teaching, and change in beliefs and attitude. Professional experimentation in the *change domain of practice* refers to different types of practice, not just teaching in the classroom, for example, assessing students' written responses to a new type of task.

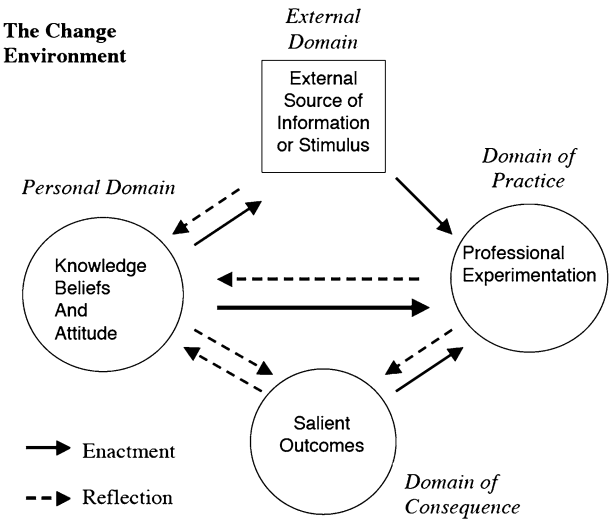


Fig. 1 Interconnected model of professional growth (Clarke and Hollingsworth 2002, p. 951)

The model connects one change domain to another with the teacher actions of *reflection* and/or *enaction*. This captures the notion of "active learner agency" through these two types of mediating actions—actions that a teacher may or may not to choose to employ in a given situation. The *change domain of consequence* conceptualises the notion of outcomes that are salient to an individual teacher—inferences that he/she draws from experiences and interprets in a particular way, related to his/her existing internal beliefs and also relevant to his/her particular change environment: the workplace or school context along with its constraints and affordances. Individual teachers may trial the same teaching approach in their classroom experimentation and interpret the same resultant phenomenon differently, which leads to differing conclusions about the strategy.

It is important to emphasise that the arrows of enactment, as depicted in Fig. 1, represent more than simply "acting". They imply a teacher's deliberate intent: the "putting into action of a new idea or a new belief or newly encountered practice" (Clarke and Hollingsworth 2002, p. 953). The term "reflection" (represented by dotted arrows in Fig. 1) is based on Dewey's (1910, cited in Clarke and Hollingsworth 2002) definition of "active, persistent, and careful consideration" (p. 6, as cited in Clarke and Hollingsworth 2002). These actions connect change in the personal domain (knowledge or beliefs or attitude) and change in the domain of practice in both directions. Enaction and reflection can be viewed as mediators in the possibly iterative and ongoing process of professional learning and changing teaching.

A more recent descriptive and interpretative model is meta-didactical transposition (MDT), considered in a review of collaborative mathematics teacher learning as likely to be an influential model for illuminating the process of teachers working and learning together (Robutti et al. 2016). It is based on the anthropological theory of didactics (Chevallard 1985) and was developed by Italian mathematics education researchers over several decades (Arzarello et al. 2014). MTD conceptualises the dynamics of teaching change using five features, presented in Table 1.

As with Clarke and Hollingsworth's (2002) model, MDT considers how teachers' practices are changed and how external and internal aspects might influence teachers' beliefs, preferences, and actions. MDT additionally focuses on the different types of constraints in teachers' institutional environments, for example, their time, space, and school culture, and expectations about textbook use, the prescribed national curriculum,

Table 1 Meta-didactical transposition model features (Arzarello et al. 2014)

1. *Institutional dimension* Aspects of the social context in which a project is situated that might influence or constrain change in knowledge/beliefs/attitude and teaching practice
2. *Meta-didactical praxeologies* The mathematical organisation of the tasks and approaches used with the teachers, and theoretical justifications that develop during the professional learning, which can lead to changes in teachers' and researchers' praxeologies
3. *Double dialectic* The development of teacher professional competences when researchers and teachers have contrasting interpretations (meta-didactic level) of the personal meanings that students in the classroom attach to a teaching situation (didactic level) that result in teachers changing their praxeologies to align with researchers
4. *Brokering processes* Actions that help transfer mathematical concepts from one community to another by brokers who belong to both communities (typically the researchers in the context of professional learning) and
5. *Internal and external domain status change* The process of internalisation by which external techniques (practices) or theory (knowledge) become part of a teacher's praxeology (and also a researcher's from encounters with a community of teachers)

and assessment (Arzarello et al. 2014). It also pays attention to the bidirectional nature of influences in the relationships between teachers and researchers in a project, and that each may learn from the other in collaborative professional learning projects.

MDT resonates with Clarke and Hollingsworth's (2002) mediating actions of enaction and reflection through its perspective of the "double dialectic", in which teachers (and researchers) experience student responses in the classroom (enaction) and then *reflect* on their own interpretations of what they noticed. Their interpretations may conflict and lead to discussions about their varying perspectives, whereby teachers (or researchers) might come to modify their praxeology, or *reasoning* about human actions, leading to change in their beliefs, attitude, or knowledge. Additionally, MDT highlights the "brokering processes" involved in professional learning, and the mutuality of influence when teachers and researchers interact, that *researchers* may also change their reasoning about teaching actions through their interactions with teachers in schools. The MDT model was also considered a useful complement to the Clarke and Hollingsworth (2002) model because it suggests specific aspects to examine when considering the effect of teachers' institutional contexts on their professional learning.

Research on mathematics teachers' personal domains influencing professional learning

"Opportunity to learn" is not the same as "being taught". "Opportunity to learn includes considerations of students' entry knowledge, the nature and purpose of the tasks and activities, the likelihood of engagement, and so on" (Hiebert and Grouws 2007, p. 379). This quote could also be applied to teachers' opportunity to learn. Attention to teachers' knowledge, beliefs, and attitude is necessary when studying the possible processes for professional learning. If teachers are viewed as active agents in their own learning, then an examination of their opportunity to learn needs to consider not only their access but also their engagement. The Clarke and Hollingsworth (2002) model highlights teacher knowledge, beliefs, and attitude in the personal domain as potential foci for change, as well as characteristics of the overall change environment, which might influence teachers' engagement.

There have been attempts over several years to build on Shulman's (1986) well-known conceptions of different types of knowledge for teaching. In mathematics education research circles, there is still no widespread consensus on a framework of types of knowledge specifically needed for teaching mathematics (Petrou and Goulding 2011). None of the prevalent models are considered easy to operationalise when analysing teachers' specific actions in classrooms (Wilkie 2016a; Zhang and Stephens 2013), but the model proposed by Ball et al. (2008) does highlight that there is much more to teachers' knowledge for teaching mathematics than merely subject-matter (content) knowledge. Even for that type of knowledge, they distinguished between three types: common content knowledge, specialized content knowledge, and knowledge at the mathematical horizon. They also proposed three kinds of pedagogical content knowledge: knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum. Fennema and Franke (1992) emphasised the importance of developing teachers' knowledge in their own context—through interactions with relevant mathematics content and with their students in class. They also conceptualised teachers' beliefs as having a bidirectional influence on their knowledge development, so that teachers' beliefs might facilitate or constrain their knowledge development and vice versa. In this study's design, attention was paid to the different types of knowledge suggested by the Ball et al. (2008) model so that teachers were offered opportunities to develop both content knowledge and pedagogical content

knowledge for teaching algebra using a functional approach in the lower secondary years. The study was conducted in the teachers' own school context and with opportunities for classroom experimentation.

Later linear models of professional learning proposed that teachers are more likely to change their beliefs or attitudes after, rather than before, seeing the effectiveness of a particular practice for improving student achievement (Guskey 2002). Studies have found some teachers willing to try new teaching practices but because of their pre-existing beliefs, actually modify them in ways unintended by the programme, for example reducing a task's cognitive demand, believing that their students won't engage with it as is (e.g. Stein et al. 1996; Swan 2007). In a study of 22 elementary teachers, Wilson et al. (2014) found that although a programme helped teachers improve their expertise in making instructional decisions based on noticing students' mathematical thinking, it did not change teachers' pre-existing beliefs attributing students' successes or failures to non-mathematical stereotypical attributes such as innate ability, luck, effort, age, or out-of-school experience. It appears that "changing practices alone does not ensure belief change" (Gregoire 2003, p. 150).

Research on secondary teachers' practice and professional development in teaching algebra

Studies demonstrating secondary teachers changing their teaching approaches are much less prevalent than those involving elementary teachers (Keazer 2014). Even though algebra is an important domain in secondary school mathematics, Carraher and Schliemann (2007) suggested that research on teaching algebra effectively is still in its infancy. Algebra teaching and learning has been highlighted more recently as a major policy concern internationally (Hodgen et al. 2010). Kieran (2007) conducted an extensive review of research studies on algebra and emphasised that there was a need for observation and analysis frameworks for studying algebra teaching. She also asserted that there was a need for focused professional development in this area and for research efforts to focus simultaneously on algebra teaching and learning. Hiebert (2013), while recognising historically the resistance to change in mathematics teaching, emphasised that teacher learning takes time and practice, and that efforts to improve teaching need to anticipate this, also keeping in mind that learning is not the responsibility of teachers only, and that cultural contexts play an influential role. This study sought to understand more about the process of secondary mathematics teaching change and in the key domain of algebra, to study *how* things work—the different possibilities—rather than simply *what* might work mechanistically in the classroom (Sfard 2005).

A few studies in the literature specifically examined teachers' beliefs and algebra teaching practices. Nathan and Koedinger (2000) compared teachers' beliefs about how students learn algebra with their students' actual performance. They found that teachers believed that story problems and worded equation tasks would be harder for students than symbolic equation solving, yet the reverse was found to be true. A symbol-precedence view of algebra common in traditional textbooks seemed to propagate beliefs about how students ought to be taught algebra that conflict with student learning in reality. Chick (2009) examined Australian secondary mathematics teachers' approaches for teaching algebra and found pervasive use of unsound teaching strategies that reinforce students' misconceptions of the use of alphabetic letters. One common approach was the use of

"fruit salad algebra"[1] which inhibits students' understanding that letters stand for numbers, not objects. This approach was found in textbooks used by those teachers. Arzarello et al. (2014) highlighted textbooks as a possible constraint on teachers' development of teaching practice. There is a need for practising teachers to develop their knowledge of how students *actually* learn algebra and some textbooks may constrain their opportunities for doing so.

Recent research on student algebra learning and reform efforts in the USA have suggested an alternative to the traditional equations-based approach to teaching algebra: a functional approach incorporating pattern generalisation and multiple representations of functions (Kieran 2007). Studies have found that it supports students' conceptual development, particularly of functional relationships and the use of variables in algebra. Although contested by some mathematicians, this approach has been incorporated in the curriculum of several countries including Australia (Sutherland 2002). In a study of secondary teachers' development of knowledge for teaching a functional approach to algebra, three features of the programme were found to facilitate teaching change: a curricular focus relevant to the teachers' current programme; repeated revisiting of the mathematics concepts; and the teachers experiencing tasks as learners before experimenting with them in their classrooms (Steele et al. 2013). The student tasks developed for this study were based on this reform approach and were shared with the teachers before their own classroom experimentation (for sample tasks, please see Wilkie 2016b). The following section describes the design of the study to investigate the process of teachers' collaborative professional learning.

Research design

This study employed a design-based research methodology to focus on teacher learning and student learning as joint goals (Gravemeijer and van Eerde 2009)—a need particularly highlighted for research on new approaches to algebra teaching and learning (Kieran 2007). Teachers and researcher experience the project as a collective effort (Gravemeijer and van Eerde 2009) and enact continuous cycles of interaction among the instructional materials, teachers, and students, for the purpose of meaningful change in the context of practice (Baumgartner et al. 2003). The methodology has five key characteristics: the purposeful development of theories about learning and the means that are designed to support it; an interventionist nature; prospective and reflective aspects that can be enacted repeatedly; an iterative process; and theory development that is for a specific domain (Bakker and van Eerde 2015). This methodology resonates with the chosen theoretical models for professional learning (Arzarello et al. 2014; Clarke and Hollingsworth 2002) since it involves a processual collaboration of teachers and researchers for the purpose of studying teaching and learning in a particular domain (algebra), incorporates both enaction and reflection, and aims to improve both teaching practices and theoretical insights through cycles of experimenting and refining.

The interpretive accounts-of-practice methodological approach was used for analysis in this study, having been developed for examining how experienced mathematics teachers can be supported over time to change their teaching to align with current education principles (Simon and Tzur 1999). It also incorporates cognitive and situated perspectives on teacher learning and allows the researcher both to study and foster teacher professional

[1] "a is apples and b is bananas", and so for example, $3a + 5b + 6a - 2b$ is like 3 apples add 6 apples and 5 bananas subtract 2 bananas, and since you can't add apples and bananas we write $9a + 3b$. (This analogy does not work for $3a \times 5b$!).

learning. This is achieved through generating "accounts of practice" (p. 253), defined as "explaining the teacher's perspective from the researchers' perspectives" (p. 254). Including both teacher self-reports and researcher interpretations of them was considered important since in the Clarke and Hollingsworth (2002) model, the domain of consequence, refers to outcomes that are *salient to teachers* and inferred by them. In Arzarello et al. (2014) model, attention is also paid to the *double dialectic* where teachers and researchers might differ in their interpretations of a teaching situation but alignment might eventually occur through brokering. Simon and Tzur defined a teacher's practice as including not only their teaching actions but also their knowledge, beliefs, intuitions, values, and feelings, whereas the models developed by Clarke and Hollingsworth (2002) and Arzarello et al. (2014) distinguish between the internal (personal) and the external (practice) domains. Yet because the approach enables the researcher to pay explicit attention to teachers' self-reports and what the researcher interprets about them when viewed through the researcher's conceptual lens, it nevertheless supports analysis of the brokering processes, particularly between teacher and researcher, by which external techniques (practices) or theory (knowledge) might become part of a teacher's praxeology (Arzarello et al. 2014).

Study participants and school context

After initial consultation with the School's Head of Mathematics, six Year 7 mathematics teachers agreed to participate in the study over the course of 2 months. They had a wide range of mathematics teaching experience and all taught secondary mathematics across multiple year levels in one large suburban independent (non-government) K-12 school (middle to high SES) in Melbourne. (In the state of Victoria, more than 35% of students currently attend non-government schools.) Some of the Year 7 students had attended that same school at the primary levels on the junior campus nearby but most were from local government primary schools. In keeping with the school's streaming policy for mathematics in Years 7–10, the six classes of Year 7 students (11–12 years old) had already been assessed at the end of Term 1 that year and streamed into one advanced class, four mainstream classes, and one (typically smaller) support class. Table 2 gives some

Table 2 Teacher participant details

Pseudonym	Teaching career background	Years teaching mathematics	Year 7 class
Angie	Senior secondary mathematics specialist	>25 years	Advanced ($n = 19$)
Bridget	Secondary mathematics teacher	<5 years	Middle ($n = 21$)
Cath	Science teacher teaching mathematics out of field	5–15 years	Middle ($n = 18$)
Diana	Secondary mathematics teacher	5–15 years	Middle ($n = 17$)
Ella	Mid-career change to teaching and teaching mathematics out of field	<5 years	Middle ($n = 16$)
Fiona	Senior secondary mathematics specialist	5–15 years	Support ($n = 11$)

demographic information about the six teachers in the study and their Year 7 teaching allocation that year.

The school's usual practice was to assign each mathematics teacher a mix of year levels and stream levels in their teaching allotment; in the year of this study, the advanced and support streams happened to be given to senior (Years 11 and 12) mathematics specialists. At each year level, the teachers would typically discuss and allocate planning to individuals who would then map out each topic's content and develop assessment tasks for the mainstream classes. The advanced and support stream teachers would then modify the plan and assessment tasks to suit the particular needs of their students. The Head of Mathematics had been focussing recently on modifying the School's assessment practices to complement the usual collation of summative test scores with criterion-based assessment and expressed particular interest in the teachers' use of the study's assessment tasks for this process.

In this study, the researcher (author) was well equipped to accept the role of "expert" and act as a source of information or stimulus in the teachers' external change domain, given nearly two decades of school teaching experience, familiarity with the relevant research literature, and previous research on functional approaches to algebra teaching and learning. The researcher provided an initial professional development session and a "set of exemplary instructional activities and materials" (Gravemeijer and van Eerde 2009, p. 512) sourced from the literature for teachers to experiment with in class. The materials included professional reading, a pattern generalisation learning progression, pre- and post-assessment tasks and several task handouts based on a functional approach that explicitly focuses on meanings for variables through connecting figural growing patterns, descriptive generalisations, symbolic equations, and graphs of functional relationships [see Wilkie (2016b) for sample tasks]. During the first session, the teachers completed their questionnaires. The researcher then overviewed the project and shared recent findings from the literature about the benefits of helping students learn conceptually about variables and symbolic equations through pattern generalisation and connecting multiple representations. The teachers attempted several tasks together using the hands-on materials (pattern blocks, matchsticks, counters) and were shown different levels of likely responses. They were given definitions of key terms, types of generalisation, and focus questions to use with students. The Head of Mathematics also attended the first half of the session and emphasised a particular interest in the teachers learning to assess the students' responses to the pre-assessment task. The teachers also developed their own joint topic plan incorporating the provided pre- and post-assessment tasks and five other provided tasks for the students in lessons.

Data collection

The teachers' involvement with each other and the researcher (author) during the study included: a questionnaire on prior experiences and beliefs about algebra teaching and learning; an initial professional learning session led by the researcher about functional approaches to algebra teaching; a joint planning meeting to incorporate the proposed tasks in the teaching programme; classroom experimentation with five tasks and comparison of student responses; co-teaching or researcher observation during one lesson and a post-lesson debrief; a mid-topic group discussion on progress, assessment meeting (teachers only); a final questionnaire; group interview; and individual interview. All but one of the teachers (Angie) also agreed to co-teach with the researcher for a lesson with one of the tasks. Table 3 presents an overview of the types of data collected at each stage of the study.

Table 3 Overview of the data collection process

Data collection	Research methods	Sources of data
Initial	Teacher survey	Questionnaires ($n = 6$) on prior experiences teaching algebra, beliefs about teaching approaches, student issues, suggestions for improving current approach
	Student survey	Questionnaires ($n = 102$) on a variety of algebra tasks including pattern generalisation, symbolic equations, graphical representation
Main	Teaching experiment (Year 7 Algebra topic) Professional development and Planning ↓ Classroom experimentation (teachers) and Lesson observations/co-teaching (researcher) ↓ Mid-topic team meeting ↓ Further classroom experimentation (including collaborative assessment of topic) ↓ Post-topic team meeting	Audio recordings of meetings Student work samples from 6 classes for 5 task lessons Photographs from lessons Researcher's journal
Final	Student survey	Questionnaires ($n = 102$) on variety of algebra tasks including pattern generalisation, symbolic equations, graphical representation
	Teacher survey	Questionnaires ($n = 6$) on involvement in teaching experiment, current beliefs about teaching approaches, student issues, suggestions for improving teaching approaches
	Individual teacher interview	Audio recordings of interviews

The semi-structured group and individual interviews were developed by the author to collect data on the teachers' expressions of any changes to their knowledge, beliefs, and/or practice. Sample questions are provided in "Appendix A". The individual interview transcripts were coded line by line with NVivo qualitative analysis software using an initial coding frame based on the Clarke and Hollingsworth (2002) model's change domains. The final hierarchy of codes is presented in "Appendix B".

Data analysis

This study employed an interpretive approach to qualitative data analysis that explicitly drew on the researcher's conceptual lens (Simon and Tzur 1999) about the process of changing in different domains through professional learning and the interplay of internal and external influences on teachers. A key tenet was the importance of "ongoing reflexive attention" to the researcher's role in interpreting the data (Yates 2003, p. 224) and perspective, as both an experienced secondary mathematics teacher and a mathematics education researcher. In this study, the data analysis involved exploring the teachers' self-

expressions and the researcher's observations and interpretations of the teachers' personal domains and domains of practice, including aspects which the teachers themselves may not raise. The Findings section differentiates between teachers' own expressed opinions and the researcher's observations. Two theoretical models (Arzarello et al. 2014; Clarke and Hollingsworth 2002) are employed to examine how the process of changing teaching in algebra might occur and the possible variety of individual teacher change sequences within the constraints and affordances of their environment. Possible mechanisms for such changes were analysed using the teachers' self-reports as well as the researcher's observations and interpretations. "Appendix C" presents details of the specific opportunities for professional learning given to teachers in each of four change domains (Clarke and Hollingsworth 2002).

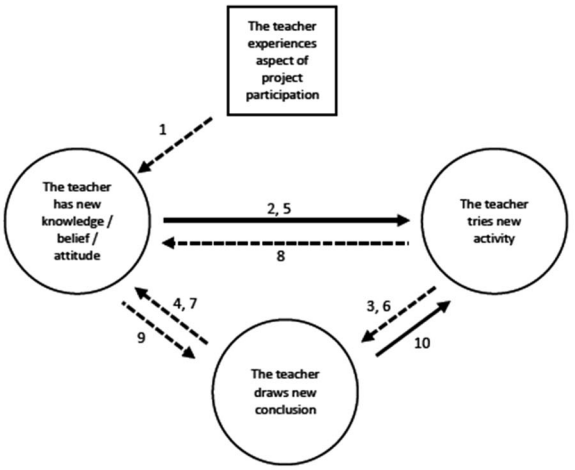
Findings

To address the first research question on the types and processes of change that were perceived during the teachers' participation, the following section draws on both the teachers' expressions (of their experiences, beliefs, and knowledge in questionnaires, conversations, and interviews) and the researcher's interpreted observations (in meetings, lessons, and debriefs) to discuss three different change sequences that were found among the six teachers. Clarke and Hollingsworth's (2002) model is used to create diagrams of each of these patterns. To address the second research question and take into account the institutional dimension of the teachers' contexts, an analysis of each teacher's change environment is included with each type of change sequence to explore how constraints and affordances might relate to the particular pattern noticed.

From change in the personal to change in practice

Ella had made a mid-career change to teaching and had only recently started teaching mathematics (out of field). She had previously undertaken other professional development courses in mathematics education. Figure 2 presents the change sequence in her professional learning experience as interpreted by the researcher (author).

Ella's focus during the initial professional learning session was noticeably on trying to understand the unfamiliar functional approach to teaching algebra. In her initial questionnaire, she had indicated that in her prior experience teaching Year 7 algebra, students had struggled with its abstract nature, and that it was "not concrete". Through the brokering process of seeing the concrete nature of the proposed tasks and hearing about current research on conceptual algebra learning, Ella became particularly interested in finding out about how she might teach with the new approach herself. She used the hands-on materials and attempted to solve the different tasks while discussing them with the other teachers. She demonstrated evidence of reflecting on how to use her new subject-matter knowledge (CK) to teach algebra differently (PCK) through the questions she asked during the initial session with teachers and researcher. She asked several questions during the session that showed that although as a learner she now understood the mathematics itself, she wanted to understand more about how her students might learn conceptually with the approach (her learning goal). This suggested internalisation of some knowledge of the new concepts of pattern generalisation with multiple representations, and associated teaching techniques, such as initial student hands-on exploration rather than teacher demonstration.



Ella	
1	Reflecting on initial PD team session; reflecting on conversations with colleagues; reflecting on observation of co-teaching
2, 5	Enacting new knowledge about teaching algebra and how students learn conceptually with a functional approach
3, 6	Reflecting on outcomes: student engagement and student understanding
4, 7	Reflecting leading to further new knowledge / beliefs about algebra teaching / learning, new attitude to functional approach
8	Reflecting on own implementation of new practices in class
9	Reflecting on need to re-examine use of textbook for algebra after recognising problematic aspects
10	Enacting further experimentation with functional approach with Year 8 students

Fig. 2 Ella's change sequence as interpreted by the researcher

She said later, "When you were introducing this to us and you had the information and you were talking to us about that, I was like oh yeah that's just—I mean they're things that we can do because we're mathematicians, but to *teach* it… you need to understand… how to get on to the functional thinking".

Ella demonstrated evidence of enacting her new knowledge in subsequent classroom experimentation by teaching a sequence of lessons with each of the tasks as per the collaboratively developed topic plan. Her teaching actions in early classroom experimentation (as observed by the researcher) showed that she was applying her new knowledge of the approaches explored in the initial session. She later drew the conclusion that the new tasks and her teaching approaches were resulting in student engagement, which provided evidence of further reflection, this time on her students' responses in class:

> They were very keen, like when they finished one thing they were keen to go onto the next and they were very proud of themselves when they completed something and just the fun they were having with the sheets and just from the general noise amongst the group, compared to normal, you know doing from the book, notes from the board. (Ella, interview)

An additional external stimulus for Ella was co-teaching with the researcher in class early on in the lesson sequence. She later described reflecting on how the introduction to the lesson was structured by the researcher to engage the students and prepare them to tackle the task productively. This illustrates further brokering through the observation of another's teaching and the internalisation of new knowledge about a particular teaching technique. It also provides evidence of teachers paying attention to the engagement of their students in interpreting the effectiveness of a particular teaching practice (in this case used

by the researcher and observed by the teacher). Another external source of information was Ella's interaction with colleagues informally and during team meetings to compare experiences in the classroom. Ella mentioned that another teacher had described finding a particular task difficult for her class and so Ella had subsequently put extra thought into planning her introduction to that task for her class. This suggests that a school culture in which teachers informally seek each other out to compare notes on their teaching practice for a particular learning task may afford teachers with opportunities to adjust—"tinker with" (Huberman 1992, 1995)—their own approach. This affordance was made possible by the use of the same learning task in each class.

After a lesson, Ella would note down specific aspects of the students' responses that she then reflected on further to develop new knowledge based on her experimentation. One likely reason for her surprising ability to recall in detail what she had noticed from each lesson (during her final interview) was her comprehensive written approach to formative assessment. She kept a detailed checklist for each task (which she showed to the researcher), recording each student's correct/incorrect responses, and then explicitly followed these up in the next lesson. She had learnt these assessment practices from another professional learning programme and was now applying them in this project. An important aspect of her classroom experimentation seemed to be finding an effective way to assess the students' progress in learning with the new functional approach.

In her final interview, Ella described a salient outcome for her as "seeing almost all the students really improve". Another salient outcome was drawing the conclusion that some individual students did not respond to learning algebra this way as she had initially predicted, based on her prior knowledge of students: "a couple of them that I thought might find it harder, they really zoomed ahead" and "there were other students that have been performing really well who struggled more". This seemed to perturb her prior subject-specific beliefs about how students' might learn algebra and her prior assessment of particular students' understanding. Her interview responses gave evidence of her active reflection on what aspects her individual students found easier or more difficult with the new approach and reasons why they may not have matched her predictions. The researcher did not find evidence of actual changed beliefs from this salient outcome for Ella, but it is likely that in future experimentation with other classes in future (which she verbalised as something she intended) she might again pay particular attention to her predictions and then her students' responses to the tasks. Over time, this process might lead to an adjustment in her beliefs about how students learn algebra that takes into account pattern generalisation approaches.

Ella described her future intent to use hands-on materials for algebra learning in the lower secondary years and to trial similar tasks with higher year levels, but referred to a lack of timely access to concrete materials, which suggests that this is a constraint in her external environment that could interfere with her future experimentation. The researcher had provided the materials for the teaching experiment, and the teachers had discussed wanting the school to buy them for future use. Ella said, "We're so pushed for time finding all the materials, like I know in primary school it's a lot more hands-on, but we just—it's something we don't do enough of in secondary school. I really think we need to do more of it in Year 7 and Year 8".

A consequence of Ella's participation for both teacher and researcher was recognising the need to learn more about how teachers might assess students' functional thinking effectively in a school context. The teachers in the project had met together (without the researcher) to discuss how to use their students' post-questionnaires in their school-prescribed assessment programme and, according to Ella (and others), had experienced some

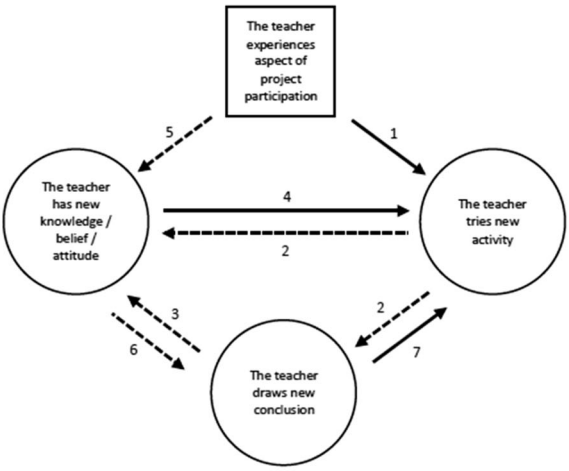
difficulties and conflicting opinions. This led to the researcher reflecting on how to address this to improve future professional learning projects, illustrating the bidirectional nature of influence, in this case of teachers on the researcher. Ella also reflected on her use of the prescribed textbook, mentioning that its sequencing and activities in algebra now seemed problematic to her for dealing with students' misconceptions about pronumerals and for giving students a context to "see a purpose" for algebra. It appears that Ella's praxeology had become more aligned with the researcher's and resulted in an ability to view the textbook more critically and with informed justifications developed during the professional learning. It also suggests that another salient outcome was a change in Ella's attitude towards the use of particular textbook activities for algebra teaching.

It appeared that Ella's openness to professional learning as a teacher quite new to the profession itself and to mathematics as a teaching domain initially facilitated her willingness to engage with the opportunity to acquire a different approach to teaching algebra. She also held the student learning goal of conceptual understanding and recognised problematic aspects of her previous teaching practice and the prescribed textbook. She seemed comfortable with recognising that there was more for her to learn about algebra teaching, as demonstrated by her questioning, reflecting, and openness to experimentation with new practices. She enacted new knowledge in experimenting in class with the tasks and drew the conclusion that the approach, although producing surprising results with individual students led overall to student engagement and worthwhile conceptual learning progress. Her active reflection throughout the project was evidenced in her engagement in team meetings, her detailed planning and implementation of the task lessons, and her comprehensive responses in the final interview. These actions, as interpreted by the researcher, suggested a change sequence that was initiated by reflection on an external stimulus, and connected initial change in the personal domain (new knowledge) to change in the practice domain through enactment of new knowledge, illustrating internalisation of external knowledge and techniques. It appeared to be facilitated by her prior noticing of student difficulties with algebra in her classes, and perhaps her belief that attributed this to something she could change—her own teaching practice.

Another teacher, Diana, also experienced in mathematics teaching, appeared to follow Ella's change sequence in being open to developing new knowledge first and then using it in her experimenting. She was also observed as noticeably engaged in the initial session together and expressed her initial belief that algebra needed to be relevant and purposeful to students. This again suggests a pre-existing belief (attributing student learning difficulties with algebra to irrelevant or purposeless tasks) that promoted Diana's openness to a different teaching approach. In class, and unlike Ella, she appeared to focus more on watching initially what her students made of the tasks on their own, rather than on trying to teach explicitly with them, suggesting that for some teachers, learning a new mathematics teaching approach takes both time for tinkering (Huberman 1992, 1995) and increasing familiarity with the mathematical concepts.

From change in practice to change in the personal

Cath was an experienced lower secondary mathematics teacher, originally trained in science education, who started teaching mathematics to meet school staffing needs several years ago. She described her initial external stimulus from the first team session as not acquiring any new knowledge per se but "relearning" what she already knew about a functional approach to algebra. She was noticeably interested in the different tasks, compared them to other similar tasks she had encountered in the past, and was verbally



Cath	
1	Enacting incorporation of functional tasks in class using resources provided and as per topic plan
2	Reflecting on student engagement and on own implementation of tasks in class
3	Reflecting leading to new knowledge about usefulness of hands-on materials for lower secondary students learning algebra
4	Enacting further experimentation with tasks
5	Reflecting on colleagues' approaches to formative assessment, leading to new belief about assessing students' understanding
6	Reflecting on new attitude to planning with hands-on materials
7	Enacting plans to use tasks again in the future and explore assessment strategies

Fig. 3 Cath's change sequence as interpreted by the researcher

positive about trialling them in her class. Figure 3 presents the change sequence in Cath's professional learning experience as interpreted by the researcher from observations of her in class and team meetings and her responses in the final interview.

A repeated focus in Cath's initial questionnaire responses had been students' likely "anxiety" about algebra—being "confused early on", "thinking it is too hard", and that "algebra can be overwhelming". This suggested a belief in the importance of needing to overcome students' anxiety somehow and might explain why Cath was willing to enact the new tasks so readily—she was familiar with the approach of pattern generalisation but liked the additional possibility of using concrete materials (as she mentioned in the initial meeting). As with Ella, Cath later commented on her noticing the students' engagement when working on the tasks and appeared to interpret it as a sign of reduced anxiety: "their enthusiasm and their confidence in me. They really trust me, and so they're willing to go at it and they trust that I'll see them through the tough times". As with Ella, Cath appeared to have a student learning goal of understanding the mathematics, but to reduce their anxiety in particular. There was also evidence (from informal debrief after the co-taught lesson) that Cath reflected on her own implementation of the tasks and also on her students' positive affective responses. Her interpretation of the students' engagement and perceived sense of trust in her during the classroom experimentation appeared particularly salient to her, leading to her conclusion that the concrete tasks did produce the worthwhile consequence of reduced anxiety in line with her learning goal.

Cath also paid attention to her students' cognitive responses and their apparent progress in learning to understand algebra. She reflected, "I think doing those hands-on activities, gives them the ability to see it in a way that makes sense". This conclusion seemed to lead to a new or perhaps revitalised belief in the value of a functional approach to algebra

teaching, since although she had been familiar with the approach (she had shown me an old booklet of written pattern generalisation tasks), she had not subsequently changed her teaching practice to include it. Cath later reflected, "There's a greater depth of understanding of what algebra *is* if you do it this way".

A characteristic of the teachers' institutional context was the Head of Mathematics' expectation of including criterion-based assessment rather than only summative test scores (based on his own experiences of professional learning). The researcher had provided a learning progression that could be used as a scoring rubric for the tasks and Cath grappled with trying to use it for assessing the tasks with her colleagues (another type of experimentation). Afterwards, she expressed her inference that mathematics was difficult to assess this way: "Maths lends itself very much to a tick, cross, 1 mark, 2 marks.[2] And I think as soon as you do a task like this, which is very valuable, to be able to assign a grade or a points system, or whatever you use to it, is complex". This illustrates that external expectations on teachers for how they assess students influences their teaching practices; in this case, the Head of Mathematics was keen for the teachers to assess in a way that also aligned with the researcher's praxeology, but was problematic for some of the teachers including Cath. A salient outcome of her experimentation was drawing the conclusion that she needed to focus more in future on assessing the students' understanding during class time:

> I can look back and go, 'Gee I wish I would have taken notes more when they were actually doing it'. But, when they're doing it, you tend to be on your hands and knees explaining it to someone and not taking enough time to get out of that situation and actually observe them. (Cath, interview)

For the researcher, encountering the teachers' difficulties with assessment highlighted the need for more time and practice in learning to use such a rubric; otherwise, their perception of difficulty could act as a constraint on further use of the tasks after the project and limit their teaching change to the short term. Cath did express an interest in further assessing her students' progress in another algebra topic (linear graphing) later that year, suggesting that she wanted further evidence of the benefit of the new approach and that her experimentation would likely to lead to further "tinkering". According to the researcher's interpretation, Cath's change sequence appeared first to involve experimenting with changes in practice, and then, through reflection and drawing conclusions about their success in meeting her learning goals, to change in the personal with a new attitude to and awareness of conceptual teaching approaches. Her experimentation and struggles with some assessment processes, along with the other teachers, led to changes in the researcher's praxeology (reasoning) about the action of teacher assessment required in this particular institutional context. This illustrates that the brokering processes in collaborative professional learning can be bidirectional when communities of researchers and teachers interact. It was particularly noticeable with Cath that she was comfortable with making suggestions to the researcher about how to refine aspects of the project; this is perhaps something that could be fostered more by explicitly inviting teachers to suggest improvements.

Two of the other teachers appeared to follow a similar change sequence to Cath—experimenting first with a "wait-and-see" approach and then seeing students' engagement,

[2] Ticks and crosses are symbols typically written on student work to indicate correct and incorrect answers, respectively. Numerical marks are awarded according to the correctness of the answer and comprehensiveness of the solution.

which led to reflection and changes in knowledge and attitude. One was an early-career teacher, Bridget, who focussed on her new pedagogical content knowledge (from her observation of the researcher in a co-taught lesson) about the need to emphasise to students that in algebra letters represent numbers, not objects, and not to use the fruit salad analogy in her teaching practice anymore. She said, "I now make a point of describing an expression as 3a is 3 *bags* of apples not 3 apples. I won't be lazy like that anymore". The other teacher Fiona, an experienced senior secondary mathematics teacher, expressed a positive attitude towards a functional approach after her experimentation with the support class. Yet it appeared that the brokering process in this professional learning opportunity had not successfully disturbed her preference for teaching with the "fruit salad analogy" since she later mentioned that she found it useful for teaching. There was also some evidence in her assessment of student work of her misunderstanding of representing descriptive generalisations symbolically. This led to the researcher reflecting on ways to address the constraints on teachers of problematic teaching approaches in textbooks, which they seem to assume must be correct since they are published for teacher use.

The personal impeding experimentation in practice

Angie had taught mathematics for nearly 30 years and was an experienced senior secondary teacher. In her initial questionnaire and the first professional learning session, she expressed her subject-specific belief that Year 7 advanced students handled her traditional (equations-based) approach and "disciplined/formal manner" very well. She attributed the difficulties lower secondary students might experience with algebra to "poor comprehension" or "poor visualising skills". In her questionnaire, she wrote, "Year 7 students do not sometimes have the patience to master the skills and techniques associated with terminology". This gave insight into Angie's pre-existing beliefs about learning algebra (that it involved skills and techniques) and about student difficulties (it was related to something

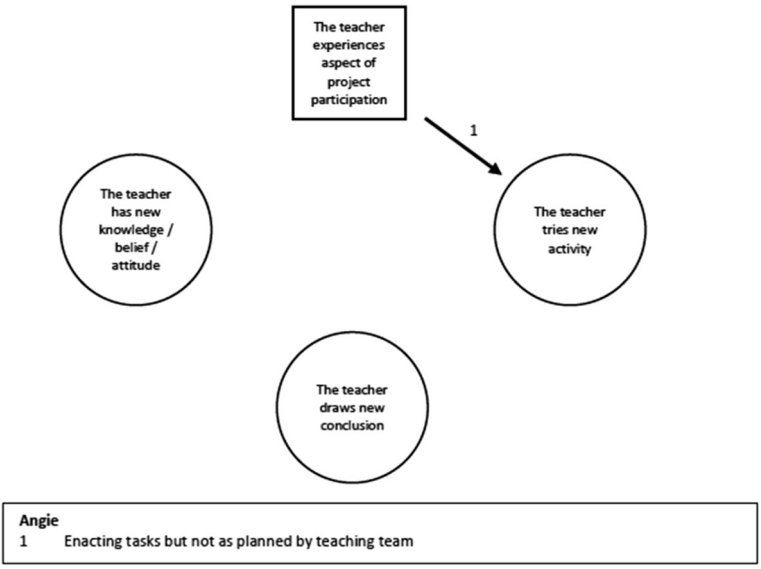


Fig. 4 Angie's change sequence as interpreted by the researcher

outside of her remit—an internal feature of her students). Figure 4 presents the apparent change sequence in Angie's professional learning experience as interpreted by the researcher, but with acknowledgement that there were limited opportunities to collect data for analysis in this particular scenario, out of respect for the teacher's choices for her level of participation.

Since Angie taught the only Year 7 advanced class, she was not obliged to follow the topic plan developed by the other Year 7 teachers but was free to choose her own content and assessment tasks to suit her class. During the initial professional learning session with the new tasks, she explained that she intended to fit them in somewhere but would continue with her own current teaching approach nonetheless. The other teachers did not question or respond to her verbalised decision during the session, but two mentioned later in the project that Angie had continued to express her disagreement with the new teaching approach to the other Year 7 teachers.

Angie subsequently gave her students all of the tasks together at the end of the topic and during her absence for a few lessons—enacting the tasks in a way but not for experimenting with a new teaching approach for herself. She did not assess any of her students' written responses to them, simply returning them to the researcher. She declined the opportunity to co-teach a lesson or have the researcher observe her students' responses in class. Other studies have suggested that low self-efficacy might contribute to a teacher being reluctant to trial new teaching practices (Gregoire 2003), but in this case Angie seemed to have high self-efficacy based on a long teaching career with advanced mathematics students at senior secondary levels.

It seemed that despite attending professional learning about the different teaching approach and being provided with new learning tasks to trial, Angie preferred from the outset not to change her teaching approach for algebra or to experiment with new practices. There appeared to be no changes in her personal domain or expressed conclusions or salient outcomes, perhaps since she had not actually observed her students' responses to the tasks in class. In her final questionnaire, when asked what she might change or improve about her current approach to teaching algebra, she wrote "nothing". Both verbally and in writing, Angie asserted numerous times her belief in the need for student discipline and formal approaches when learning algebra, for example:

> Terminology can be quite confusing if it is not taught in a disciplined/formal manner. (Angie, questionnaire)
>
> You have to have a very disciplined environment. I'm a firm believer in that because of all these experiences that I've had with the senior kids who are trying to do Specialist Maths. (Angie, interview)
>
> Algebra is something you really have to have the discipline in the classroom for; you can't learn algebra or master the techniques while chatting. (Angie, interview)
>
> So group discussions or small group work or things like that work for other things but for algebra you really need to have that discipline I think, that's why I like the formal approach of teaching algebra. (Angie, interview)

These expressions provide evidence that Angie's learning goal for her students was the mastery of algebra skills and techniques, matching her formal approach to teaching and her emphasis on silent individual work in class by students. Her satisfaction with these particular pre-existing beliefs, student learning goal, and current teaching practices seemed to preclude her willingness to learn about and experiment with a different teaching approach. And because the external environment did not provide an impetus to collaborate with the other teachers either for teaching the tasks or for assessing them (from being allocated the

sole advanced class), Angie was able to maintain her past teaching practices without disruption from the professional learning programme. Perhaps the outcome might have been different if she had been allocated one of the mainstream classes. When asked what she might have taken from her experiences in the project, she replied, "I don't know. To be honest with you, because of the model that I have to do, and also because I was absent for part of it I did not get involved in your activities as much as the others did… so I really can't comment on that". It was therefore quite surprising to the researcher when Angie chose to share at length in her final interview, her perceptions of her advanced senior secondary students' struggles with algebra:

> Some of the kids who have been in this advanced class throughout, they come to Year 10… where they can't get their head around it… they just cannot perform at that level after Year 9 or Year 10… some of the kids just failed miserably… It's unbelievable the amount of questions they had… A lot of them who came to me for extra help those days, they just couldn't understand, they just, they don't connect… They cannot sort of somehow get their head around what to do, what the question is asking them to do. (Angie, interview)

Angie's detailed description of her frustrating experiences with senior mathematics students struggling to understand algebra even though they were from the advanced stream implied that this was a dilemma for her. Yet this perturbation appeared not to have been enough of an influence to motivate her to engage with the teaching experiment. This was puzzling. Why had the brokering process been unsuccessful with this teacher? She had attended the initial professional learning session and been exposed to the new teaching approach: using figural pattern generalisation to develop symbolic equations and connecting multiple representations. It seems possible that Angie's belief in attributing the source of the problem to the students' lack of discipline, did not relate the students' difficulties to her teaching approaches and this may have precluded her from seeing the potential value of trialling a different teaching approach to improve student understanding. This apparent lack of a change sequence illustrates that a teacher's internal influences, such as particular learning goals for students or attributions about the cause of student difficulties, may affect his/her utilisation of opportunities to learn by interfering with the process of internalisation of new knowledge or experimentation with alternative teaching practices. The additional lack of impetus for collaborating with the other teachers during the teaching experiment, because of the institutional context of streaming, also appeared to act as a constraint on the potential for changing teaching practices.

Discussion

To address the first research question on the types and sequences of changes that were perceived during the teaching experiment, the study drew on Clarke and Hollingsworth's (2002) processual model of four change domains that can be linked in multiple ways by teachers' enactment or reflection. The analysis included both the teachers' expressions (of their beliefs, knowledge, experiences, and conclusions) and the researcher's interpreted observations and inferences and found three noticeable change sequence patterns among the group of six teachers. These patterns indicated that the teachers, experiencing the same professional learning programme for algebra, nevertheless responded in different ways and to different aspects of it, creating varying pathways for change (or no change) in their

practice. For some teachers, their reflection on an initial professional learning session led to new knowledge, which they then enacted in classroom experimentation. Others took the familiar "wait-and-see" stance (Guskey 2002) to see if the tasks or strategies were worthwhile from their point of view. One of the teachers did not demonstrate any changes in the personal or practice domains. The teachers who did demonstrate changes but along different pathways nevertheless described some similar salient outcomes in drawing positive conclusions about their students' engagement and conceptual learning during their classroom experimentation.

Clarke and Hollingsworth (2002) asserted that professional learning ought to be offered in a variety of forms and across different change domains to suit individual teachers' inclinations. This also resonates with Huberman's (1995) emphasis on individual teachers benefiting from ongoing "tinkering" in their own classrooms with support from colleagues and inputs from external sources. Although this study also found that teachers evidenced *individual* inclinations for particular aspects of the programme, an additional feature related to *collective* participation. The teachers had been provided with the same set of student learning tasks to try in class, and because the school context required them to follow the same topic plan (particularly the three mainstream classes), they had been experimenting with each task during a similar time period. Additionally, they seemed socially comfortable, regularly seeking each other out to debrief about their experiences with a new task. This collective aspect seemed to afford additional and timely opportunities for the mediating processes of reflection and enaction. Because the tasks were the same, they could "compare notes" on their approaches and the students' responses and, as expressed by some of the teachers, this culture of informal sharing seemed to stimulate further reflection and enactment for them individually.

This study confirmed findings from other studies in the literature that having a range of external stimuli (such as professional development sessions, professional reading, lesson tasks to trial, co-teaching with the researcher, and formal and informal interactions with colleagues) alongside experimenting with teaching approaches in their classrooms can facilitate different individual teachers' inclinations to develop new knowledge or beliefs or attitudes and try new practices. The teachers seemed to benefit from provision of a variety of "opportunities to learn in a fashion that each teacher finds most useful" (Clarke and Hollingsworth 2002, p. 965). Yet unlike Clarke and Hollingsworth's (2002) assertion "teachers change *inevitably* through professional activity" (Clarke and Hollingsworth 2002, p. 948, italics mine), this study showed that for some teachers, professional activity might not lead to change, as one teacher implemented the tasks (although not as intended), and reported no changes. To understand this further, the study addressed a further research question about internal and external influences on teachers' professional learning. Although Clarke and Hollingsworth (2002) did conceptualise constraints and affordances as being within the overall change environment (in which the four change domains reside), the study's use of their model for diagramming the teachers' change processes was not found to support the depiction of influences within each change sequence pattern.

An additional model for professional learning—Arzarello et al.' (2014) theory of meta-didactical transposition (MDT)—was used to address the second research question since it explicitly describes different types of influences: the personal and institutional dimension of teachers' contexts, which might constrain or afford their learning; potential conflicting interpretations of experiences; and bidirectional influences between teachers and researcher. In this study, it was found that in the process of sharing the student tasks and demonstrating specific teaching approaches in classrooms, the researcher brokered the process of stimulating mathematical reasoning about pattern generalisation and multiple

representations for algebra learning, with the community of teachers. Some teachers reflected on their interactions with the researcher and the tasks to develop new knowledge, which they then enacted in classroom experimentation for themselves. Other teachers drew new conclusions from their evaluation of their students' responses to the tasks during classroom experimentation and appeared then to modify their beliefs about algebra teaching and learning. The reverse brokering process was apparent when teachers experimented with assessing the new tasks and encountered difficulties and disagreement with each other. Their expressions of frustration and concern to the researcher led to changes in the researcher's reasoning and to reflecting on how to better support their institutional environment's assessment requirements.

If, as Hiebert (2013) suggested, teaching is hard to change because of a lack of consensus on clearly defined learning goals for students, then it makes sense that if a teacher initially evaluates a professional learning opportunity as unaligned with his/her learning goals for students, he/she less likely to engage from the start. One teacher expressed a student learning goal for algebra (mastery of skills and techniques) different from the other teachers' goals (conceptual understanding and less anxiety) and at odds with the teaching approach the researcher had introduced. It is difficult to ascertain if more explicit exploration and discussion of these differing goals and beliefs, in the initial stages of the programme, would have brokered that teacher's willingness to experiment with the new approach. Such a design feature is worth exploring in future research. There is more to understand about how brokering processes can be used throughout professional learning programmes to support personal domain change, particularly conflicting learning goals and beliefs that might constrain teachers' participation.

The notion of a *double dialectic* was found to be bidirectional between researcher and teachers as theorised by MDT, since both teachers and researcher were found to modify their praxeologies through their interactions over time. This seems a valuable feature of school-based collaboration in design-based projects between researchers and teachers, since it has the potential to promote everyone's professional learning by addressing differing perspectives through a joint effort to improve student achievement. Additionally, however, this study highlighted the need for this notion to be more closely considered in terms of teachers' bidirectional interactions *with each other* for influencing change (or not). This unexpected finding was demonstrated in the teachers' disagreements about assessing the tasks and also in another context: the perceived awkwardness of one teacher expressing contrasting beliefs and intentions to those of the other teachers, observed first-hand by the researcher in meetings together, and also reported by the teachers as occurring informally (without the researcher present). The teachers had not responded to that teacher's assertions during the meetings, which was a little puzzling since as a group they appeared quite vocal and comfortable with sharing their views. The researcher's presence and role may have been a factor. It is speculated that perceptions of that teacher's authority as a highly experienced senior specialist may also have acted as a constraint on the other teachers' willingness to engage in the dialectic, particularly those with less teaching experience or less self-efficacy. There is more to understand about the notion of a double dialectic in professional learning, not only between researcher and teacher but also between teacher and teacher. In communal situations, individual teachers' change pathways seem likely to affect or modify each other's, to afford or constrain ongoing change.

Cobb et al. (1990) suggested that "beliefs are expressed in practice, and problems or surprises encountered in practice give rise to opportunities to reorganise beliefs" (p. 145). Yet it seemed in this study that if a teacher's attribution of encountered problems ("my students struggle to understand algebra") does not infer the *need* to change teaching ("the

students' lack of discipline is the issue, not my teaching"), it constrains his/her professional learning. The problems or surprises encountered seem, then, to require their attribution to something that the teacher feels motivated or empowered to change. Yet Wilson et al. (2014) warned that non-mathematical and often stereotypical attributions of student success or failure in learning may not be easily reorganised in a professional learning programme. If teachers choose not to engage in classroom experimentation, then opportunities for the double dialectic and belief reorganisation may not even occur. One teacher in the study decided not to experiment with a different teaching approach, thereby preventing an opportunity for her to revisit her reasoning about algebra teaching and learning and her attribution of difficulties. Efforts were made to stimulate her participation with a variety of aspects to the study's design: sharing research-based evidence on student difficulties and the potential of a new approach, providing tasks to trial, and offering to co-teach classes. Is there a way to determine and then target effectively the underlying reasons for a teacher's resistance to professional learning?

In this study, the institutional context appeared to have also played a role in constraining teaching change for one teacher. In this school context, there was a lack of impetus for her to participate alongside the others since she taught a separate advanced class and was able to follow her own teaching plan. Had there been a requirement for joint planning, as with other teachers, the opportunity to observe the students' responses to the new tasks in class might have occurred. It appears that a combination of misaligned learning goals, conflicting attributive beliefs about student difficulties, social context, and the institutional dimension, lessened this teacher's motivation to engage in acquiring new knowledge or in classroom experimentation, thus limiting the process of internalisation of new teaching practices.

Conclusion

This study examined how the process of changing teaching might occur during an algebra teaching experiment with secondary mathematics teachers in efforts to understand more about the affording and constraining influences on teachers' professional learning. Two theoretical models developed by mathematics education researchers (Arzarello et al. 2014; Clarke and Hollingsworth 2002) were used to analyse possible change pathways, dynamic interactions between praxeologies, brokering processes, the double dialectic between researcher and teachers, and various internal and external features of the teachers' change environments that might influence the process of professional learning.

This study contributes to the picture emerging from empirical research of a more unpredictable and idiosyncratic process for mathematics teacher professional learning than perhaps theorised by earlier models, and even in participative contexts such as school teaching teams. The findings suggest that even within one school context teachers experience differing external and internal affordances but also constraints in a collaborative professional learning programme, which are not easily addressed even with a variety of opportunities to suit individual inclinations. The six teachers, who were part of the same school context, taught the same year level, and were given access to the same aspects of an opportunity to learn about functional approaches to teaching algebra, were found to interpret events and respond individualistically. It seems that each teacher experiences a unique combination of internal and external aspects in a particular institutional context, which influences the extent to which he/she develops new knowledge, experiments with

approaches in class, or changes teaching practices. External aspects found in this study included: the requirement of shared topic planning leading to the teachers' informal debriefs on experimenting with the same tasks, one teacher *not* being required to plan with the others, problematic textbook approaches for algebra (recognised by one teacher but not another), and the Head of Mathematics' focus on alternative assessment practices. Internal aspects included: distinctive (and conflicting) student learning goals for algebra and differing attributions of previously experienced student difficulties in understanding algebra.

Although Hiebert (2013) suggested that a focus on changing teaching rather than teachers would be more likely to succeed, this study found that efforts to support teachers to change their teaching in a collaborative context may yet be impeded by an individual teacher's lack of impetus. Individually held student learning goals and attributive beliefs about students' difficulties in algebra were found to promote five teachers' engagement and hinder one. It was evident that disagreement with the proposed teaching approach, as well as non-mathematical and stereotypical attributions of students' difficulties with algebra (Wilson et al. 2014), contributed to one teacher's decision not to experiment with changing teaching. Such influential aspects of the social, institutional, and personal contexts of each teacher in a project might require more nuanced awareness by researchers of brokering processes and the double dialectic, not only between researchers and teachers (Arzarello et al. 2014), but also *among* the teachers themselves—an additional dialectic found in this study. This seems particularly important for certain areas of mathematics, such as algebra, where there is likely to be disagreement among teachers about the best approaches for teaching and learning them (Kieran 2007). An initial lack of teacher engagement at the beginning stages of a programme, because of a mismatch in beliefs about how to teach particular areas of mathematics, may lead to ongoing resistance if left unaddressed. Yet there is also the potential for positive influence and change, as seen with the teachers choosing to reflect together informally about their experiences with the different approach for algebra and to enact further experimentation with their teaching.

Five out of the six teachers described outcomes salient to them from the teaching experiment, and it appears that either their initial interest in improving their own knowledge for teaching algebra or a later positive evaluation of their students' engagement and/or learning during classroom experimentation supported the process of knowledge development and different teaching approaches becoming internalised. One limitation of this study was that it examined observable short-term change and teacher-described intentions for future change in practices or experimentation rather than on observed long-term growth over time as envisaged by the Clarke and Hollingsworth (2002) model. The teachers perceived a lack of school resources as a possible constraint on future "tinkering" with the approach, since the researcher had supplied the hands-on materials needed in class. Data were not collected on whether or not this would be overcome by the teachers after their participation. In future research, it would be worthwhile to gauge the longer-term growth of teachers with longitudinal research that explores how different mathematics topics, particular change pathways, and a varying institutional context (such as different teaching allotments each year or newly prescribed assessment approaches by school mathematics leaders) might relate to the extent to which secondary teachers continue to enact new teaching practices over time.

There is more to understand about the double dialectic among teachers and researchers since it is likely to be inherent in this type of school-based collective professional learning context and with certain areas of mathematics. It would be worth exploring it in relation to influential aspects such as teachers' identities, self-efficacy, teaching experience, and career stage, the social "climate", and the institutional nature of the context. A future

direction for research might be investigating brokering approaches that address misaligned or conflicting beliefs and praxeologies, to increase teacher participation in joint classroom experimentation, reflection, and enaction. A starting point might be investigating ways to create teacher buy-into agreed-upon learning goals for students (Hiebert 2013) in a particular area of mathematics that can then shape and direct subsequent professional learning directions. It would also be valuable to research ways to address or adjust mathematics teachers' attributions of student difficulties in that area, as these appear to influence their decision-making and engagement. Finding ways to support secondary mathematics teachers effectively for their professional learning is an ongoing focus of research efforts.

Appendix A: Teacher interview schedule

1. What was the highlight for you in observing your students' learning of algebra during this teaching experiment?
   - 1.1 What was the greatest conceptual difficulty for your students?
   - 1.2 Did anything surprise you in relation to their responses or learning?
2. How did you find assessing your students' learning (during lessons, looking at their written work afterwards, discussing with colleagues)?
3. What is the single greatest thing you have learned from this teaching experiment about algebra?
4. Has participation in the teaching experiment changed your perceptions or knowledge or future teaching practice in the area of algebra?
   - 4.1 If yes, what specific aspects have changed?
5. How did you find the meetings with other teachers in the experiment?
6. Is there anything you might consider doing differently in your teaching of algebra as a result of participation in this teaching experiment in Year 7?
   - 6.1 At other year levels?
   - 6.2 Is there anything you might consider doing differently in your mathematics teaching practice *generally*?
7. What advice would you give to another teacher who is about to teach Year 7 algebra for the first time?
8. Are there any other issues, suggestions or information you would like to mention?

Appendix B: Coding hierarchy for teacher interview analysis

Code
External domain
Researcher in team meetings
Interaction with other teachers
The tasks provided
The concrete materials
Researcher in class

- Personal domain
  - Knowledge for teaching mathematics
  - Misconception in algebra
  - Beliefs
  - Attitude
    - Positive
    - Negative
- Domain of practice
  - About the students
    - Student affect or engagement
    - Student cognition or learning
  - About mathematics concepts
  - About teaching the mathematics
  - About assessing
- Domain of consequence
  - Awareness of new needs or interest
  - Change in knowledge
  - Change in attitude
  - Change in beliefs
  - Change in future practice
  - No change
- Constraints in the change environment
  - Time
  - Procedures
  - Resources
  - Other

Appendix C: Features of the professional learning programme

- *External domain* The teachers were provided with external sources of information, stimulus, and support in the form of: professional reading; a series of exemplary student tasks and materials as documented in the research literature, including a student pre- and post-questionnaire; initial discussion of the tasks and the key concepts developed in each of them; concrete materials for use in lessons; the opportunity to co-teach with a researcher (author); and iterative facilitated discussions with other teacher participants. A learning progression for assessing students' responses to the algebra tasks (Wilkie 2014; adapted from Markworth 2010) was provided to teachers as a specific theoretical tool (Robutti et al. 2016) to guide discussions of student learning and subsequent lesson preparation.
- *Domain of practice* The teachers had the opportunity to experiment in class with a series of five tasks incorporated in their existing unit on algebra; they could choose the option of including the researcher (author) as a co-teacher or observer during one of the five lessons according to their own preference. This domain encompasses change in all forms of professional experimentation, not just classroom experimentation (Clarke and Hollingsworth 2002) and so the teachers' use of student pre- and post-questionnaires

for trialling new formative and summative assessment strategies is also included here.

- *Personal domain* The teachers had opportunity to reflect on their initial knowledge, beliefs, and attitudes in a pre-questionnaire and were asked to explore changes to these in a post-questionnaire, debriefs with the researcher and other teachers, and final group and individual interviews. The researcher (author) also inferred and interpreted changes to knowledge, beliefs, or attitudes from observations of the teachers' engagement, actions, and responses in the classroom and with each other.
- *Domain of consequence* In their final individual interviews, the teachers were invited to reflect on their involvement in the teaching experiment. Outcomes (positive and negative) were considered as salient to each teacher individually, and changes in each teacher's domain of consequence were also analysed and interpreted through comparison of their pre- and post-questionnaires and from recordings of team meetings.

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