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Teachers' professional growth through engagement with lesson study

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Abstract Lesson study is highly regarded as a model for professional learning, yet remains under-theorised. This article examines the professional learning experiences of teachers and numeracy coaches from three schools in a local network of schools, participating in a lesson study project over two research cycles in 2012. It maps the interconnections between their experiences and their beliefs and practices, using Clarke and Hollingsworth's (Teach Educ 18(8):947–967, 2002) *Interconnected Model of Professional Growth*. Analysis of interview data and video-recordings of planning meetings, research lessons, and post-lesson discussions reveals the development of teachers' collaborative planning skills, increased attention to students' mathematical thinking, use of orchestrated whole-class discussion based on anticipated student solutions and focused questioning, and the enhancement of collaborative practices for teacher inquiry. Our findings illuminate the interplay between the *External Domain*, the *Personal Domain*, the *Domain of Practice*, and the *Domain of Consequence*, in the teaching and learning change environment, and the mediating processes of enactment and reflection. Changes in the domains across the period of the lesson study provide evidence of teachers' professional growth, with the iterative processes of enactment and reflection being critical in mediating this professional growth.

Keywords Lesson study · Mathematics teaching · Professional growth · Professional learning

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Introduction

While there is general agreement among researchers regarding the importance of teacher professional development in improving the quality of teaching and learning, and some consensus on what constitutes "effective" professional development, there has been much less attention paid to the mechanisms through which professional growth occurs.

Conceptualisations of professional development have moved from "deficit" and "workshop or training" models to models of "professional growth" where teachers engage actively in collaborative inquiry into their own practice to enhance their knowledge of content, pedagogy, and students (Borko 2004; Borko et al. 2014; Darling-Hammond and Richardson 2009; Matos et al. 2009).

Researchers in the field have identified features of high-quality professional development, including a focus on student learning; time for reflection and inquiry into practice; a focus on the development of teachers' content knowledge and pedagogical skills grounded in practice; support from school leadership; and the involvement of outside experts (Darling-Hammond and Richardson 2009; Guskey and Yoon 2009; Hunter and Back 2011; Joubert and Sutherland 2009; Ricks 2011). Moreover, it is now recognised that "learning tends to occur incrementally and iteratively" (Goldsmith et al. 2014, p. 20).

Since coming to worldwide attention through the *Third International Mathematics and Science Study* (TIMSS) video study (Stigler et al. 1999), lesson study has become highly regarded as a model for teacher professional learning. Lesson study can be seen as fulfilling Joubert and Sutherland's (2009) criteria for the best forms of continuing professional development to achieve teacher learning—namely that such models should address "all aspects of mathematical knowledge for teaching, engaging teachers in cycles of deep analysis of mathematics, detailed planning, predicting student responses and discussing actual student responses" (p. 8).

Adaptations of lesson study in Australian schools have revealed its potential to engage teachers in critical discussion with colleagues about mathematics and mathematics teaching practice (Hollingsworth and Oliver 2005; Pierce and Stacey 2011; Widjaja and Vale 2013; Groves et al. 2013). These studies identify valuable contributions of lesson study, such as an explicit attention to mathematics and students' mathematical learning, opportunities to provide and receive critical feedback, and opportunities to critically observe and analyse learning in classrooms.

However, despite its long-standing tradition in Japan, and its widespread adoption and adaptation elsewhere, lesson study is under-theorised, with Elliott (2012, p. 114) commenting that while "lesson study has helped to construct shared knowledge about how to teach, particularly in Japan, 'the pedagogical theories' that underpin such knowledge building are often implicit and unclear". Moreover, Xu and Pedder (2015) in their review of 67 research articles pertaining to lesson study found just five that investigated the process through which professional growth for teachers takes place, commenting that "there is still an absence of the kinds of theoretical work necessary for explaining how and why teachers learn both collectively and individually in LS contexts" (p. 48). This resonates with the findings from Goldsmith et al.'s (2014) synthesis of 106 research articles dealing with mathematics teacher learning, which found few studies focussing on the processes or mechanisms of teacher learning and called for "a more intentional and systematic focus on illuminating the black box of teachers' learning" (p. 21).

This article aims to investigate the professional growth of a group of teachers participating in two lesson study cycles in 2012 as part of a small-scale project, *Implementing* *structured problem-solving mathematics lessons through lesson study*. Our analysis will use Clarke and Hollingsworth's (2002) *Interconnected Model of Professional Growth* (IMPG) to identify "change sequences and growth networks" (p. 957) in order to explain the mechanisms through which this group of teachers' professional growth occurred.

Lesson study

Japanese Lesson Study is a professional learning activity whose origins can be traced back to over a century. In the school-based version, which has attracted most attention outside Japan, it has four components: (1) formulation of overarching school goals related to students' learning and long-term development; (2) group planning of a *research lesson* addressing these goals; (3) one team member teaching the research lesson while the planning group, and others, observe in order to gather evidence of student learning; and (4) the post-lesson discussion where the planning group and other observers (usually including an "outside expert") discuss and reflect on the evidence gathered during the lesson, using it to improve the lesson, the unit, and instruction more generally (Perry and Lewis 2008, p. 366).

In Japan, lesson study is widely viewed as a shared professional culture that provides a pathway for continuing improvement of teachers' pedagogical and content knowledge (Murata 2011; Shimizu 2013; Takahashi and Yoshida 2004). In lesson study, teachers take a central role in researching classroom practice and exploring ways to improve student learning, informed by students' work samples and observers' notes collected during the research lessons (Lewis and Tsuchida 1998; Takahashi and Yoshida 2004). Key elements involved in the planning process, such as choosing goals for the lesson, studying curriculum documents, finding and solving a suitable mathematical problem, and anticipating students' solution, are genuine research tasks. During the research lesson, observers act as researchers who collect evidence of student learning, and observe and document critical moments in the teaching and learning process. The post-lesson discussion provides a platform for teachers, researchers, and observers to examine and discuss their "evidence" of student learning and share ideas to improve the teaching and learning process (Takahashi and Yoshida 2004; Watanabe 2002). The focus of the post-lesson discussion is not on the teacher, nor on the student, but on the teaching and on students' learning. This is achieved by using students' work samples and teachers' interactions with students as the basis for observers' comments during the post-lesson discussion.

Extensive studies have documented the significance of collaborative inquiry involving teachers and researchers in lesson study (Knapp et al. 2011; Puchner and Taylor 2006; Takahashi et al. 2013). However, attempts to establish collaborative inquiry through lesson study in countries outside Japan, such as the USA and Australia, often face challenges due to teaching being viewed as a private activity, teachers' content knowledge, time constraints, leadership commitment, and lack of a common curriculum (Fernandez 2002; Groves and Doig 2010; Lewis et al. 2006).

Lewis et al. (2006, 2009) called for investment in repeated cycles of adaptations of lesson study to establish "a local proof path" (2006, p. 6) that explains learning pathways for teachers in different settings in order to create a stronger theoretical foundation for lesson study.

Teacher professional growth

According to Sprinthall et al. (1996), three main models have been used to explain teacher professional development: the craft model, where knowledge emerges from classroom experiences; the expert model, where teachers are taught what to do by experts; and the interactive model, where teachers are involved in active meaningful learning. Clarke and Hollingsworth (2002) argue that while multiple perspectives on teacher change are not necessarily mutually exclusive and "many are in fact interrelated", current professional development effort "most closely aligns with the 'change as growth or learning' perspective" (p. 948). Within this perspective teachers, as members of a community, are seen as active learners and change is accepted as a natural element of their professional activity.

Clarke and Hollingsworth's (2002) *Interconnected Model of Professional Growth* (IMPG) (Fig. 1) has been used in many recent studies of teacher professional growth, including a number on lesson study (e.g. White et al. 2011). This interactive model has also been used as the conceptual framework for organising Goldsmith et al.'s (2014) synthesis of 106 research studies on practising teachers' learning, conducted between 1985 and 2008. The model has its origins in Guskey's (1986) linear model of teacher change, which posits that change in practice leads to change in student learning outcomes, which in turn leads to change in teachers' beliefs and attitudes. However, as Opfer and Pedder (2011) point out, simple cause and effect relationships are inadequate to model the complex process of teacher professional growth. They argue that much more sophisticated conceptualisations are required in order to take account of the "various dynamics at work in social behavior and [how] these interact and combine in different ways" (p. 378). Opfer and Pedder further "construe teacher learning as a complex system representing recursive interactions between systems and elements that coalesce in ways that are unpredictable but also highly patterned" (p. 379). They point to Clarke and Hollingsworth's IMPG as "being helpful in understanding why the correlational research on features of teacher professional development activity and change has been so disappointing… [by illustrating] the cyclic nature of the learning and change process" (p. 385).

Clarke and Hollingsworth's (2002) IMPG maps change in four domains of the *change environment*: the *External Domain* (which provides the external source of information or stimulus for change); the *Personal Domain* (teacher knowledge, beliefs, and attitudes); the

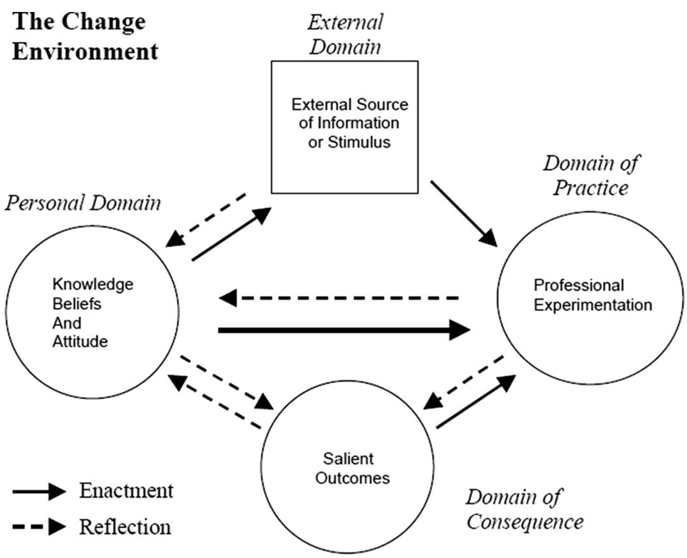

Fig. 1 *Interconnected Model of Professional Growth* (Clarke and Hollingsworth 2002, p. 951)

Domain of Practice (where professional experimentation takes place); and the *Domain of Consequence* (where teachers recognise salient outcomes inferred to be the result of experimentation). The External Domain is regarded as being located outside the teacher's personal world, while the remaining three domains are seen as constituting "the individual teacher's professional world of practice" (p. 951). This model posits that teacher change is a nonlinear process that occurs incrementally through the mediating processes of enactment or reflection, where the term enactment has been chosen to represent "the enactment of something a teacher knows, believes or has experienced" (p. 951).

The dynamic nature of the IMPG emphasises ways in which context and teachers' orientations to learning shape teachers' professional growth, with change that occurs in one domain not necessarily leading to change in another. When change in one or more domains does lead to change in another domain through the mediating process of enactment or reflection, *change sequences* can be identified through the analysis of empirical data to provide supporting evidence of causal connections. However, Clarke and Hollingsworth (2002) reserve the term *growth network* for more lasting change sequences that are taken to "signify professional growth" (p. 958).

In this article, we adopt Clarke and Hollingsworth's (2002) IMPG as our theoretical model for teacher professional growth and use it to identify growth networks for a group of teachers participating in a small-scale lesson study project.

Overview of the project

The *Implementing structured problem-solving mathematics lessons through lesson study* project worked with three schools from an Australian local school network. A cross-school model was chosen to enable a group of teachers to explore ways in which key elements of Japanese Lesson Study could be embedded into mathematics teaching and professional learning at more than one school, in a similar vein to the beginnings of the Chicago Lesson Study Group (Takahashi n.d.).

Ten participants including teachers and numeracy coaches[1] participated in the project. They took part in an initial professional learning day in June, after which they were divided into two cross-school teams, later self-named the Bobbies and Matomes. Each team consisted of 3 Year 3 or Year 4 teachers—one from each school—and two coaches.

Each team completed one lesson study cycle during each of Terms 3 and 4, planning their own research lesson, based on the same problems that were provided by the research team (see "Appendix 1" and "Appendix 3"). Participants planned each research lesson during four 2-h sessions. One member of each team taught the research lesson. All members of both planning teams, key staff at each school, together with all interested teachers who could be released from their classes, observed the research lessons and took part in the post-lesson discussions. In all between 20 and 30 people observed each research lesson and took part in the post-lesson discussions including members of the leadership teams from other schools, staff from the regional office, mathematics educators, and an outside expert. While including outside observers made these research lessons more like Japanese "open lessons" (Stigler and Hiebert 1999), it was consistent with the

[1] Numeracy coaches are teachers who were nominated by the school principal to undertake ongoing professional learning for mathematics curriculum leadership and coaching. The ongoing professional learning involves professional conversations with other coaches in the network of schools.

collaborations between schools in the network and enabled the teachers to showcase their work with others.

This article investigates the professional growth of the Bobbies team across the two lesson study cycles. For reasons of space, only one team's experience is discussed here. The Bobbies team was chosen for this article as the lead author was one of the researchers observing them during their planning meetings and was more familiar with their data. While the same detailed analysis has not been carried out for the Matomes, previous publications (Vale 2013; Widjaja and Vale 2013) relating to individual teachers from both teams indicated there would be similar findings for the Matomes.

The Bobbies team

The three teachers in the Bobbies team—Lynn,[2] Keith, and Henry—were trained generalist primary school teachers and had been teaching between 1 and 4 years. Megan had been the numeracy coach for 3 years, prior to which she had been a classroom teacher. Megan observed teachers and taught demonstration lessons, and conducted coaching conversations with classroom teachers. Paula, the network numeracy coach, was employed to provide leadership for teachers and numeracy coaches in the 22 primary and secondary schools in the network. Unlike the other participants, she was a qualified secondary mathematics teacher with many years of secondary mathematics teaching experience, as well as having worked for a number of years in her role as a numeracy coach in primary and secondary schools.

During the third planning meeting in Term 3, Lynn volunteered to teach the first of the Bobbies' research lessons in her class. In Term 4, Keith was chosen to teach the research lesson due to the perceived need for each school to have been the site for at least one research lesson.

Data collection

The study adopted a qualitative research design, based on intensive fieldwork and ethnographic content and event structure analysis (Miles and Huberman 1994). The study was conducted over 6 months and involved researchers' sustained engagement as participant observers during two cycles of lesson study planning meetings, research lessons, and post-lesson discussions. Throughout the study, the researchers kept field notes of planning meetings, research lessons, and post-lesson discussions. In addition, all sessions, including the professional learning day, were video-recorded. Planning meeting agenda, together with lesson plans and notes prepared by members of the planning teams was collected, as was students' written work from all research lessons. These data were supplemented by individual, audio-recorded, 30-min interviews with participants on three occasions: after the introductory professional learning day and after each of the post-lesson discussions. Interviews and post-lesson discussions were transcribed.

Data analysis

Clarke and Hollingsworth's (2002) IMPG provided the framework for the data analysis. Ethnographic methods involving open coding using a constant comparative method (Corbin and Strauss 2008) were used to analyse the data. Codes were grouped to form

[2] All names of participants are pseudonyms.

categories that were aligned to one of three of the domains in the IMPG. Relevant codes relating to the transcripts of teacher interviews (INT) were categorised as being aligned to beliefs about students, and beliefs and practices relating to planning and teaching, to form the basis for conclusions about teachers' *Personal Domain* (PD) at different times in the study—see Figs. 2, 3, 4, and 5. Some categories derived from the analysis of teacher interviews were used to identify teachers' reflections on the *External Domain* (ED).

Event structure analysis, involving memoing and coding of events identified in researchers' field notes, participants' planning documents, and transcripts of video-recordings of planning meetings (PM), research lessons (RL), and post-lesson discussions (PLD), was used to map teachers' *Domain of Practice* (DP)—that is, the experiments conducted concerning the planning, teaching, and observation of research lessons, together with the post-lesson discussions, across the two cycles. Event structure analysis was used to identify reflection on actions and salient outcomes observed regarding student learning and learning behaviours, and teacher actions or practices, to form the basis for the *Domain of Consequence* (DC)—see Figs. 2, 3, and 4. Further event structure analysis enabled

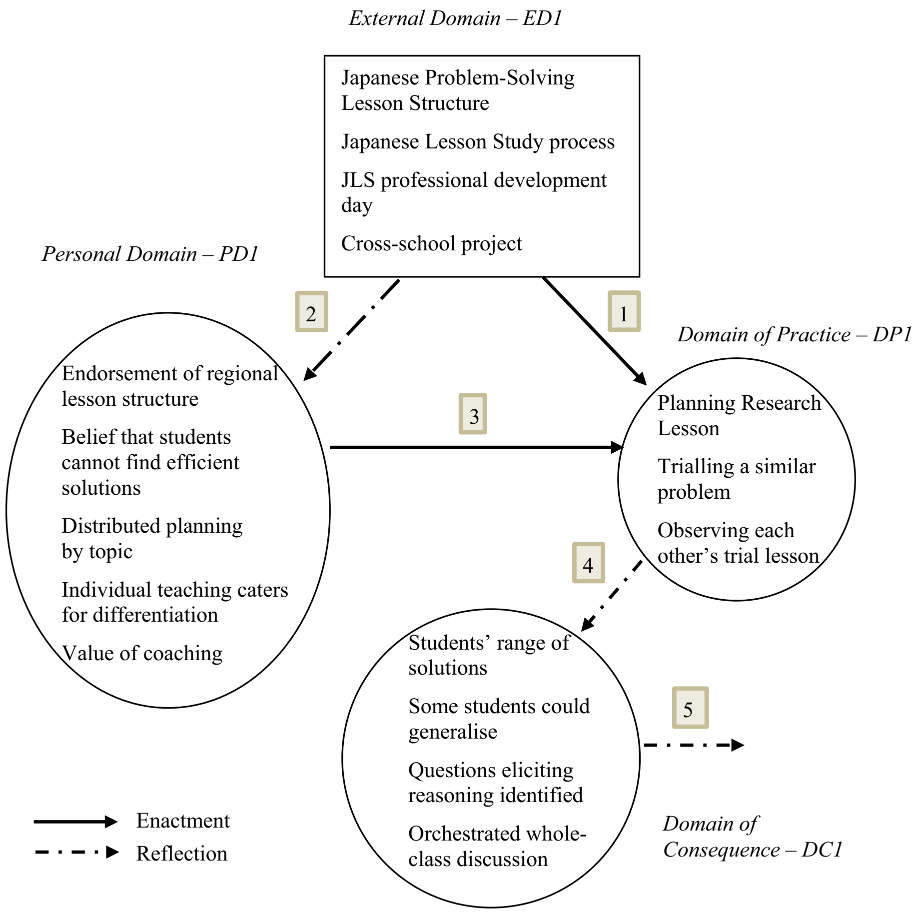

Fig. 2 The change environment during *Cycle 1 Planning*

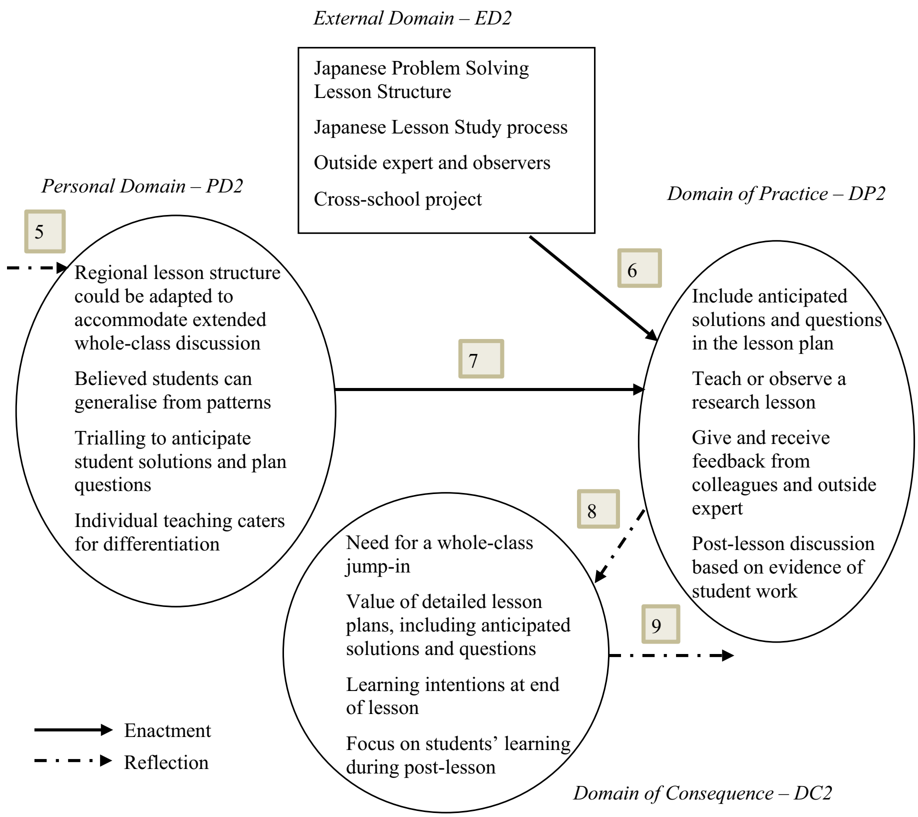

Fig. 3 The change environment during *Cycle 1 Research Lesson and Post-lesson Discussion*

identification of causal connections between domains where change in one domain led to change in another domain—that is, the change sequences. These change sequences are illustrated by the arrows in Figs. 2, 3, 4, 5, and 6.

While change is often incremental and not all members of the Bobbies team experienced the same changes at the same time, analysis of data collected during research revealed two critical phases in terms of the change sequences that occurred—the first of these we will refer to here as *Cycle 1 Planning* and the second as *Cycle 1 Research Lesson and Post-Lesson Discussion* (see Figs. 2, 3, respectively). Analysis of the *Cycle 2* data suggested a single phase, referred to here simply as *Cycle 2*, was sufficient to discuss the change sequences identified (see Fig. 4). The final outcomes in terms of changes to the participants' *Personal Domains* (Fig. 5) are reported in the final section of the findings.

Cycle 1 Planning

Figure 2 shows a summary of the results of the analysis of the data for *Cycle 1 Planning*. The content of each of the domains (the text within the shapes) and the ways in which connections between these were mediated by participants' enactment and reflection (the arrows) are discussed below.

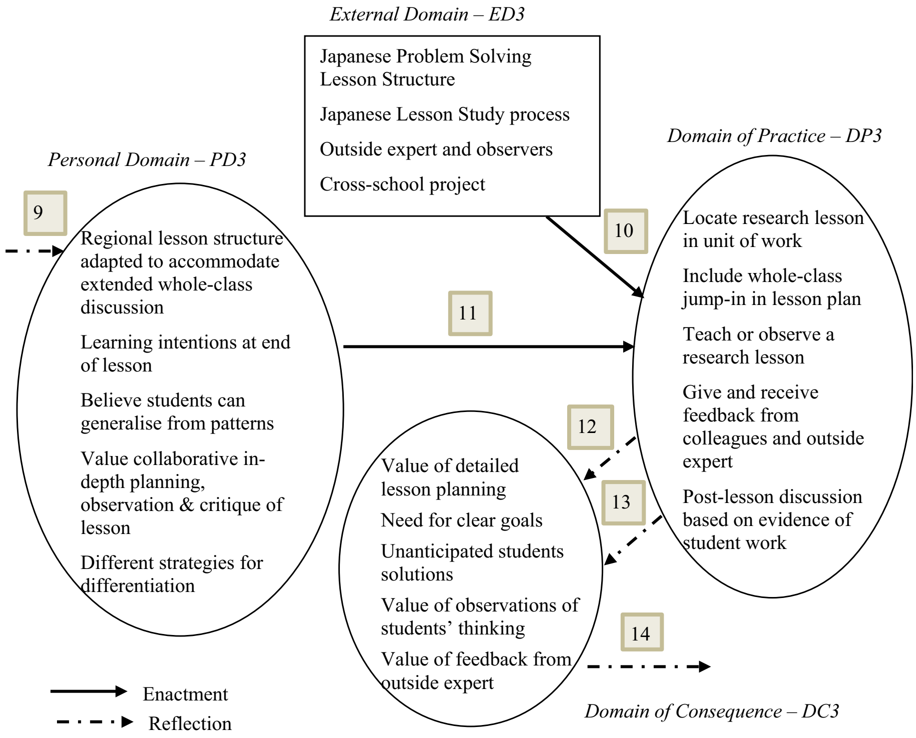

Fig. 4 The change environment during *Cycle 2*

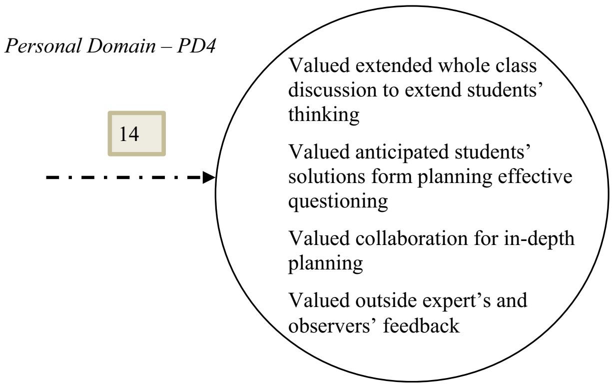

Fig. 5 *Personal Domain—PD4* of the Bobbies planning team at the end of *Cycle 2*

External Domain: ED1

The June professional learning day began with an overview of Japanese Lesson Study and an overview of the project. This was followed by a "mock" research lesson, during which the teachers tackled an algebra generalisation problem in order to experience the problem-

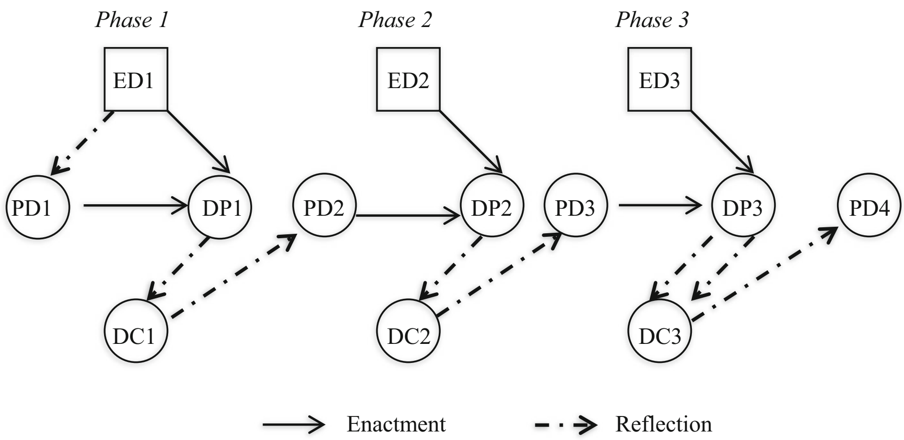

Fig. 6 The *growth network* for the Bobbies team across the two lesson study cycles

solving approach and the typical Japanese research lesson structure (Takahashi 2006). The "lesson" included an introduction to the problem (5 min), individual work (15 min), whole-class orchestrated discussion (20 min), teacher summary of main points (5 min), and written student reflection (2 min). Participants were given a copy of the six-page lesson plan, developed by two of the researchers, which included five possible solutions explained in detail. Everyone took part in the post-lesson discussion. The day ended with a discussion of the role of observers and the format for post-lesson discussions. Readings about and protocols for conducting lesson study were distributed.

The teachers in the Bobbies team worked at three different schools. Apart from the network numeracy coach, they had no prior experience of planning, teaching, or observing lessons with teachers from other schools. For this reason, the practices of other schools regarding curriculum, planning, and teaching constituted an element of the External Domain.

Personal Domain: PD1

We identified five key beliefs and practices held by teachers prior to, or early in *Cycle 1 Planning*. Firstly, as a way of addressing poor mathematics outcomes for disadvantaged schools, the school region had developed a lesson structure for mathematics, namely warm-up (5–15 min), introduction (5–10 min), student activity (20–45 min), and sharing and reflection (5–15 min). Some schools, including the project schools, insisted that teachers explain the "learning intentions" to students at the start of each lesson. Teachers believed the purpose of whole-class sharing and reflection was to allow students to report their learning or share their strategy, rather than as an opportunity for further learning. For example, Lynn usually rephrased students' responses rather than expecting other students to rephrase the response. It was readily accepted that teachers often ran out of time to share.

There were diverse beliefs and practices regarding problem solving. Lynn typically used worded real-life problems; Henry described open-ended problems he had used; and Megan focussed on developing problem-solving heuristics. In general they believed their students could not find efficient solutions. Rather problem-solving lessons enabled differentiated learning as students could choose their solution strategies.

> So you need to be aware of the students in your class and how they best learn. That's a big issue – having a problem where all students can work and be challenged. (Henry, INT1[3])

The teachers believed that individualised teaching caters for differentiation. Lynn adapted her year-level planning team's unit plans based on assessment data, while Henry adapted the lesson during the lesson, as he did not believe you could anticipate students' solutions.

Planning was distributed among teachers in different ways. At Megan and Lynn's school, teachers at each year level collaborated to plan the sequence of topics. At Henry's school, teachers also planned and sequenced lessons for each topic, using formative assessment data. Teachers at Keith's school were expected to plan for differentiated learning.

Finally, the teachers valued coaching for professional learning, commenting that they valued opportunities to observe and discuss one another's teaching and demonstration lessons by the network or school coach. For example, Henry stated that:

> I feel I get the most out of seeing someone else teach and I've also had my leading teacher coming in and run a lesson in my class and then we've discussed what was good, what was bad, and then I've had a go at a lesson and she has given me feedback. (Henry, INT1)

Domain of Practice: DP1

In *Cycle 1 Planning*, the Bobbies team participated in four 2-h meetings to plan the research lesson. These meetings included identifying the goals for the lesson, selecting the wording of the task (the *Hatsumon*), anticipating student solutions, as well as investigating teaching materials and studying the curriculum (*Kyozaikenkyu*). As illustrated in Fig. 2, their experimentation was a direct result of their participation in the project—that is their *enactment* of what they understood to be the critical features of planning a structured problem-solving lesson. However, their experimentation was also mediated by their *reflection* on these critical features, which did not necessarily align with the personal beliefs, the *enactment* of which also shaped their experimentation as discussed below.

Arrow 1: Enactment of the lesson study planning process

During the first planning meeting, one of the researchers introduced and modelled the matchsticks problem (see "Appendix 1"), which had been selected by the researchers as the problem for the first research lesson. Completing the problem enabled the teachers to derive a set of anticipated student solutions for their lesson. Participants were also provided with a lesson plan template to assist their planning. They were asked to focus on articulating an overarching goal and the goals for the lesson, and on how the lesson fitted with the Australian curriculum. Participants struggled with developing these goals, not only in the first but also in later planning sessions. They also began to discuss the wording of the problem and whether or not it should be introduced through a real-world context.

[3] INT1 refers to Interview 1. Similar notation is used for Planning Meeting 1 (PM1), *Cycle 1* (C1), etc.

Arrow 2: Reflection on the problem-solving lesson structure and learning goals

During the second planning meeting, the team continued their discussion of the wording of the problem and its introduction, the lesson structure, and possible use of manipulatives such as matchsticks. This discussion caused them to reflect on the mismatch between the problem-solving structure of the research lesson and their established lesson structure. This discussion highlighted their understanding of problem solving and generalisation, as well as their belief that their students would use inefficient problem-solving strategies and would not be able to participate in an extended whole-class discussion. Keith voiced his concern, saying "I don't know if they could sit for a reflection for, I think 20 min. [This] might be pushing it for some". Henry agreed.

Participants' current practices for differentiated teaching were challenged when they discussed the number of squares to use in the problem and their learning goals. Megan suggested they initially use "5" to make it accessible for all students and then differentiation could occur by choosing another number after a whole-class discussion of the efficiency of different strategies (see "Appendix 2"). They debated whether the learning goal was to use problem-solving strategies to find a number of solutions (Henry and Lynn), to find an efficient solution (Keith), or to find a general solution (Megan and Paula). They grappled to find a wording of the question that would lead students to generalise and recalled their own strategies.

Arrow 3: Enactment of beliefs as a catalyst for conducting trial lessons

To resolve teachers' uncertainties regarding their students' ability to solve the matchstick problem using strategies other than counting and their ability to participate in extended whole-class discussion, the researchers recommended that everyone trial a problem similar to the matchstick problem in their class, record students' responses, and share their findings and reflections at the next meeting. Lynn worked with Megan and Trevor[4] from her school, trialling different problems in their own classrooms. Lynn and Trevor observed each other's lessons, while Megan supported them both in her role as numeracy coach. They video-recorded the trial lessons and discussed their observations. The first problem they trialled was booze buses (Downton et al. 2006), a context-based problem, while the second was a growing pattern problem based on a triangular number pattern. Keith and Henry trialled a similar problem, supported by their school numeracy coaches.

Arrow 4: Reflection on lesson trials using similar problems

Participants' reflections on the outcomes of their trial lesson were evident in the third planning meeting when Lynn and Megan reported the trial of the two problems in Lynn and Trevor's classrooms and Keith reported his trial of an orchestrated whole-class discussion for a lesson that had used differentiated tasks. Lynn and Megan found that students were able to find a solution to each problem and were more successful with persisting and searching for a generalised solution for the triangular pattern problem than expected, although Lynn conceded that finding a general solution for this was much more challenging:

[4] Trevor was a member of the Matomes team.

> This one was it had three … [in the triangle] and it [was] just add the row each time. It was really hard for them to make the generalisations. Algebraic equations took me a good hour to figure out myself, but the kids could see that each time it added … (Lynn, PM3, C1)

Keith found that students could learn from an orchestrated whole-class discussion and were able to identify and adopt a more efficient strategy for a subsequent task:

> So students who just counted one by one, and next building up to the more effective strategy and I got them to explain to the whole class and because there is still time left, I said "Okay, now that you have seen different strategies, go back to your desk and if you want to try different [problem] and see which works best for you and share from your desk after they use different strategies… Almost everyone found that the more efficient strategy worked a lot better and … so that was positive that they acknowledged more efficient strategies that someone else came up with and it was OK to use it. (Keith, PM3, C1)

The lesson trial also enabled teachers to observe the questions used by other project participants to elicit students' reasoning and challenge them to justify their solutions.

> Megan: … like if they were saying "right, is that wrong?" so then it is going to a stage when "how do you know it is right?", "how can you prove to me that it is right?", "can you prove to me that is wrong? No, so if you can't prove to me that it is wrong then it is right", "Oh OK", but you can see their faces were like that [they wanted] the reassurance from you "right, yeah that is right" which in some circumstances is I think appropriate, but for this you [want to] understand their thinking more…
>
> Lynn: and just the right question to ask to get them talk about their strategies because as teachers, and I know as myself because I am guilty of accepting what they said the first time or putting this is what you did rather than [them] telling us what they did. (PM3, C1)

Domain of Consequence: DC1

As discussed above, participants' reflections on the trialling and observation of lessons involving similar problems lead them to note a number of salient outcomes as shown in Fig. 2, including acknowledgement of the range of solution strategies used by students, some students' ability to generalise, examples of appropriate questions to use to elicit students' thinking, and evidence that orchestrated whole-class discussion could be used in their classrooms.

Arrow 5[5]: Reflection on the consequences of experimenting with orchestrated whole-class discussion

The outcomes from the trial of similar lessons lead teachers to revise their beliefs about extended whole-class discussion and students' ability to problem solve and generalise. The

[5] Arrow 5 links the two phases *Cycle 1 Planning* and *Cycle 1 Research Lesson and Post-Lesson Discussion* and therefore appears in both Figs. 2 and 3.

trial lessons challenged their existing practice concerning questioning, lesson structure, and whole-class discussion.

Trialling the lesson structure in collaborative situations enabled participants to see that the regional lesson structure could be adapted to include extended, orchestrated whole-class discussion. Keith observed whole-class discussion supported students to use more efficient strategies, while Megan commented on strategies for ordering student sharing:

> 'cos Trevor and myself and Lynn were all in the same room doing the lesson at the same time so it was good to sort of bounce off each other and say yep that makes sense for that students' work to be spoken about first, second, third, fourth and so on (Megan, PM3, C1)

Trialling the lesson structure provided Lynn with opportunities to observe her students' mathematical thinking, finding some students could attempt to make generalisations:

> So a lot of them were trying to make generalisations. So one of my boys found what 10 was and he doubled them all, times them by 10, which gives him 100, which did not give him the right answer. But he was trying to make a generalisation, which is good. (Lynn, PM3, C1)

Furthermore, trialling the same problem in different classes allowed participants to find a wider range of anticipated solutions to incorporate in planning. They also valued this as helping their questioning.

> I think, mapping out the questions, how you're going to question the student, because I guess something that we don't typically do is always think about the student responses… I think we plan a lesson. Oh yeah this is what I want the students to do, but I don't think we really think about student responses. But also what possible student misconceptions there would be that we would have to think about in a lesson. (Megan, INT2)

These changes in teachers' beliefs and practices are shown in the *Personal Domain—PD2* in Fig. 3.

Cycle 1 Research Lesson and Post-Lesson Discussion

Figure 3 shows a summary of the results of the data analysis for *Cycle 1 Research Lesson and Post-Lesson Discussion*.

Personal Domain: PD2

Reflections on the *Domain of Consequence* in *Cycle 1 Planning* lead participants to believe the regional lesson structure could be adapted to include extended, orchestrated whole-class discussion, and that some students could generalise from patterns. They also learnt to value planning for anticipated solutions and focusing on questions to probe student understanding.

External Domain: ED2

Other elements of the Japanese Lesson Study process impacting on this lesson study were the involvement of outside observers and the outside expert in the research lesson and post-lesson discussion.

Arrow 6: Enactment of the research lesson and post-lesson discussion

Lynn taught the first Bobbies research lesson, with both teams participating as observers and participating in the post-lesson discussions, together with the outside expert, key staff, and teachers at each school and invited professionals, including mathematics educators, numeracy coaches, and leadership teams from other schools. The post-lesson discussion followed the protocol of Japanese Lesson Study (Chokshi et al. 2001) with the teacher and planning team reflecting first, followed by discussion, and final comments from the outside expert.

Arrow 7: Enactment of orchestrated whole-class discussion in research lesson

The trial of a similar problem convinced teachers that students could participate in and learn from orchestrated whole-class discussion of solutions. This helped them prepare for facilitating this in the research lesson. The Bobbies team adapted the regional lesson structure to accommodate an orchestrated whole-class discussion and reflection.

Domain of Practice: DP2

The Bobbies teams' experimentation with the lesson study process was mediated by their enactment of their changed beliefs regarding the role of the whole-class discussion and their focus on questioning based on students' anticipated solutions. Their lesson plan included eight anticipated solutions and questions such as "How can we be sure that our number sentence or strategy will work for a higher number of squares?" The post-lesson discussion was based on evidence from observers' notes and copies of student work, which were available to everyone before the discussion commenced.

Arrow 8: Reflection on the research lesson and post-lesson discussion

As part of the post-lesson discussion, Lynn reflected that the time spent on individual teaching limited her opportunities to extend other students' thinking.

> I feel like I didn't get around to all the kids … I spent a lot of time with Emma who came up with great generalisations … but not every single kid can be pushed to 8 so we were talking about maybe in our lesson to have a time of restock and extend them all to 8 rather than doing it individually so they all have that opportunity. (Lynn, PLD1, C1)

Participants' reflections also prompted some reconsideration of the choice of numbers in the problem.

> Should we have introduced the 8 squares? Should it have been like after a certain time or after students had a certain number of sheets and whether the kids that had done 6 or 7 sheets if that got exhausted that strategy for 5 squares whether they needed to be pushed on with 8 squares? (Megan, PLD1, C1)

The outside expert, who had extensive experience with both mathematics and lesson study, suggested including a whole-class "jump-in"—i.e. stopping the class working and talking to the whole class—5 min into the lesson to address difficulties and extend the problem.

The teachers valued the opportunity to observe just a few students during the research lesson. Keith realised this enabled him to find evidence of anticipated solutions given in the lesson plan; Lynn and Paula noticed unanticipated solutions, especially from high-achieving students who did not attempt to generalise; Henry valued it to help plan questioning.

> You might have had 6 anticipated responses, and… seeing… other students also thinking the same way… was quite interesting. … Also the way that I observed, focussing on just 2 students, made me think. (Keith, INT2)

> I know that it's not until… the team actually trialled and saw it today that they had that "aha" moment… because they could see that their students were giving different responses to what they had anticipated. And it was very interesting today that Lynn said that her … high flyer in the classroom… the boy who gave us the half result [did not provide a general solution]. (Paula, INT2)

> But I liked how [in] this [post-lesson discussion] one looked more at the actual learning, so what the students were learning and the strategies and trying to figure out what they were going to say to help guide those teacher prompted questions. (Henry, INT2)

Megan and Henry highlighted the value of trials for developing questions for Lynn to use to elicit students' thinking and promote student learning, with Megan attributing the success of the research lesson to detailed planning.

> I think, mapping out the questions, how you're going to question the student, because I guess something that we don't typically do is always think about the student responses … but also what possible student misconceptions there would be that we would have to think about in a lesson. So, I think that deeper understanding of what we want to achieve from a lesson, I think is really important. (Megan, INT2)

A critical feature of a Japanese structured problem-solving lesson is the summary that follows the sharing of solutions. It is here that the teacher's learning intentions are revealed to the students. For example, in the Bobbies' lesson plan they planned to "Verbally sum up main learning point of lesson based on strategies students have used". This violated the mandated explanation of learning intentions at the start of lessons.

> I think… we've really pushed a lesson structure that has said articulate the lesson intention at the beginning of the lesson. And I think that was a real surprise to teachers… and even the principal, that the team didn't do that at the beginning. But… there was good reason for that and the students reached the learning intention by the end. (Paula, INT2)

The feedback from the outside expert and other observers during the post-lesson discussion was seen as valuable, with the focus not on evaluating the teacher but on improving students' learning.

> So it's while I've got that hundred eyes approach with everyone watching, having people bring up things that we discussed in the discussion. Showed that you know we're all on the same page, we've all got different viewpoints. But we're all thinking for the kids, we're all thinking you know for their benefit. (Keith, INT2)

Domain of Consequence: DC2

Participants' reflections on the research lesson and post-lesson discussion, including the outside expert's comments, lead them to note a number of salient outcomes as shown in Fig. 3. These included the value of including a whole-class jump-in 5 min into the lesson to address student difficulties and extend the problem; detailed lesson planning, including the inclusion of anticipated student solutions as well as preparation for unanticipated solutions; explicating learning intentions at the end rather than the beginning of a lesson; and focusing on student learning during the post-lesson discussion.

Arrow 9: Reflection on the consequences of the research lesson and post-lesson discussion

As a result of participants' reflections on the salient outcomes they noted in the *Domain of Consequence*, changes to their *Personal Domain* in *Cycle 1 Planning* were consolidated in *Cycle 1 Research Lesson and Post-Lesson Discussion* and adopted by other members of the planning team. For example, Henry expressed his changed views about the role of extended discussion of solutions after observing the research lesson.

> Definitely do use reflection and share in my lessons… And I think this has started to make me change … giving my students 15 to 10 min at the end of the lesson for them to share their work. Because there's a lot of that time in that share and reflect where the other students are making the connections. (Henry, INT2)

Participants also began to question the mandated explanation of learning intentions at the beginning of lessons, as opposed to these arising through the teachers' summary after discussion of shared solutions.

As a consequence of their positive experience of the post-lesson discussion, teachers valued collaboration for planning and observation and reflection on teaching. Analysis of data from the interviews and planning meetings revealed that participants' views regarding the planning process had changed, and that they had gained increased confidence in planning and teaching structured problem-solving lessons. Initially participants were overwhelmed by the level of detail and amount of time devoted to planning a single research lesson. However, while participants agreed it would be hard to change their schools' planning process, Megan highlighted the importance of planning as a group, rather than distributing planning:

> I think we plan as a group, but I don't think we necessarily plan specific lessons as a whole group. So, getting conversations happening about one particular lesson, rather than one person doing all the thinking behind it… I think is really important. (Megan, INT2)

The Bobbies team regarded collaborative, cross-school, in-depth planning as beneficial in producing a quality lesson because it allowed opportunities for detailed, ongoing professional exchanges about the lesson, including teacher questions to elicit students' thinking.

> Definitely a benefit I think was … a mixture of people from different schools. So they still could have… ongoing professional discussions in the fortnight between the team's meeting … so then they had a chance to reflect and think about where they stood and then they could come back to the planning group with that. (Paula, INT2)

Keith valued the collective effort of planning a lesson as a team and wanted to see this practice embedded in his school.

> I've learned that, it's three teachers working on one lesson, or one unit is a great thing. And that, you can only hope that, we can take it to our schools and implement it within one team or two, and then to show to the rest of the school that, it's not as time consuming as you may think and it's really beneficial for the teaching and learning that happens within a school. (Keith, INT2)

Teachers began to adapt their differentiation practices. Both Keith and Henry acknowledged the value of the outside expert's advice to have a whole-class jump-in point in order to check if students understood the problem and were ready to move to the next level. Keith reported that he incorporated the outside expert's advice in his mathematics lessons by stopping halfway to check students' understanding.

> Well since participating and including [discussion] time… [I have also been] stopping halfway checking on the students as a whole and getting them to share their strategies, then returning to their work… and then sharing again at the end to see if their strategies and work had changed as a result of sharing at the first break. (Keith, INT2)

These changes in teachers' beliefs and practices are shown in the *Personal Domain—PD3* in Fig. 4.

Cycle 2

Figure 4 shows a summary of the results of the analysis of the data for *Cycle 2*.

Personal Domain: PD3

Reflections on the *Domain of Consequence* in *Cycle 2* lead participants to believe that the regional lesson structure could be adapted to explicate the learning intentions at the end, rather than the beginning, of lessons. Participants expressed their belief in the value of cross-school collaborative planning, observation, and critique of lessons, as well as becoming aware of different strategies to cater for individual differences.

External Domain: ED3

During *Cycle 2*, the *External Domain* remained constant, except that members of the two planning teams decided on the topic (multiplication) for the unit of work in which the research lesson was embedded. The outside expert was a Japanese professor of mathematics education with extensive experience of lesson study.

Arrow 10: Enactment of the lesson study process

Keith taught the research lesson in his Year 3 class. The research lesson was embedded in a unit of work on multiplication. While participants from the two planning teams selected the topic, the researchers sourced the problem for the research lesson—see "Appendix 3" for the problem 23×3 (Tall 2008).

Arrow 11: Enactment of different strategies for differentiation

One of the main issues raised by the outside expert and some observers during the *Cycle 1* post-lesson discussion was the teacher spending too much time working with individual students, preventing them from noticing a wide range of students' strategies. In *Cycle 2*, Megan, Lynn, and Trevor trialled the multiplication problem to collect possible student solutions. They also acted on the advice of the *Cycle 1* outside expert, who advised teachers to spend the first 5 min after introducing the problem roving—i.e. walking around the room—in order to note a range of students' strategies and clarify the problem if necessary. During the last *Cycle 2* planning meeting, Megan and Henry reiterated this point.

> Megan: Henry and I are just saying for initial student activity time, Lynn got a bit caught straight away. You are going to spend that five minute time pretty much just roving, looking for any student who might not have made a start…
>
> Henry: Like Lynn said before, what kids you would choose to go to first? Like in my class two boys and a girl that probably wouldn't start in the first five minutes … so you know that you'd go to those kids first.
>
> Megan: In all the times we have tried this, is there anyone in your class who did not make a start doing this activity?…
>
> Henry: Not in my class.
>
> Megan: Only really seen one, so the chance of [it] happening is probably quite small but at least you are prepared if it does. (PM4, C2)

Domain of Practice: DP3

In *Cycle 2*, the Bobbies teams' experimentation with the lesson study process was again mediated by their enactment of their changed beliefs—this time regarding the use of different strategies for catering for individual differences and the value of planning for a whole-class "jump-in".

Arrow 12: Reflection on detailed planning

During Planning Meeting 2, Megan and Lynn reported trialling the problem. Unlike their experiences in *Cycle 1*, they found orchestrating the whole-class discussion at the end of the lesson very challenging as the team had not yet established lesson goals.

> The first thing was that it was really hard to do it… [because] we went in there without establishing a goal. … So what is exactly the thing that we are looking for here? So that would be the first thing that we found difficult as opposed to when we do it last time'cos we already established our goals so we knew what we were looking for. (Megan, PM2, C2)

Keith, who taught the *Cycle 2* research lesson, valued the detailed planning of questions to use in the whole-class discussion. Even though they did not work in the same school, Megan observed one of Keith's trial lessons that he conducted with a different class, providing feedback on his questions and choice of student work for sharing.

> Well one thing I've been focussing on the whole time was the sharing part at the end of the questioning. And I think after today's lesson and having all the preparation behind me, the questions that I used helped the students and also helped me. Knowing what kind of things to ask and what questions worked best. (Keith, INT3)

The Bobbies' detailed planning made them ready to receive and respond to feedback during the post-lesson discussion.

> I want to try what [the outside expert] said about having a solution, the equation written and asking students to interpret that on the diagram… as 23 plus 23 plus 23. That's in giving students the diagram asking them to interpret what that means. (Lynn, INT3)

Arrow 13: Reflection on teaching, observing, and discussing research lesson

Teaching, observing, and discussing the second research lesson revealed the consequences of the research lesson. Opportunities to observe students allowed teachers to become aware of the progress and changes in students' ideas. Megan noticed that while students were engaged with the problem during the research lesson, the time for individual work was too long. Henry commented on the value of observers:

> Everyone has got perspectives on student's thinking and noticing a lot of different things that go over my head or you miss out on, so it's good to have … that extra set of eyes in there because it really helps to give the teacher a good understanding of how some of their students are thinking. (Henry, INT3)

Having observers collect specific evidence of students' thinking enabled others to notice things the teacher might miss. For instance, during the short preparation time for the post-lesson discussion, Keith said he had not seen any students count by ones, but Paula and Megan observed some students who had done so.

> It was interesting because we talked about, when we came together after the planning component before we went to the discussion, that Keith observed that he didn't think any children counted by ones, but then because of all the eyes you could actually see there were. (Paula, INT3)

Members of the Bobbies team continued to be surprised by unanticipated responses from students. Keith remarked on how Sarah progressed in her thinking during the lesson and the way in which she used diagrams to represent her strategies. While Sarah did not arrive at the correct answer, Keith appreciated how far she had progressed:

> The students surprised me overall because… their diagrams [were] better than I anticipated … in particular linking their… equations with the groups that they drew on their dots… Sarah … really surprised me … [Pointing to her work sample] This is the first way where she had groups, two groups of three and the rest in sixes and she began adding them up, just using addition, $12 + 12 = 24$ and she arrived at an answer of 59, which is incorrect by that stage, but she then changed her thinking and she did something similar to Mary. (Keith, PLD, C2)

Domain of Consequence: DC3

Participants' reflections on their planning, teaching, and observation of the research lesson and post-lesson discussion lead them to note the salient outcomes shown in Fig. 4. These included the value of detailed lesson planning, including the need for clear goals; the fact that despite their attention to anticipated solutions, unanticipated solutions still arose; the opportunities provided to observe students' thinking processes; and the value of the outside expert's comments.

Arrow 14: Reflection on the consequences of the Cycle 2 research lesson and post-lesson discussion

The impact of teachers' reflections on the consequences of their participation in *Cycle 2* was identified through interviews at the end of the study. As no data were collected in teachers' "regular" classrooms, these self-reports of change are described as being in the *Personal Domain*.

Personal Domain: PD4

The teachers changed their views regarding the regional lesson structure and came to value an extended, orchestrated whole-class discussion. Lynn saw this as a major change in her teaching practice. She claimed that "there's been a lot more sharing of solutions rather than the show-and-tell type of share time" and highlighted the importance of "letting them work on the problem and then discussing and doing a lot of teaching at the end rather than at the start". Keith and Henry identified the extended sharing of students' solutions as a practice they wanted to continue.

> And I have started using student responses and putting those up on the board in just normal maths lessons and outside of lesson studies. That's something that I've done. (Keith, INT3)

> I can see that the kids learn a lot through that structured reflection, and I know that we're meant to have between 5 or 10 min for our reflections and that's what mine is usually—but I think now after being involved in it, I will probably stretch them out to 10 to 15, and I like the process in which they showed more than one response. (Henry, INT3)

The value of anticipating students' solutions was acknowledged as a key learning point for all participants. Lynn, Henry, and Keith reported the use of anticipated solutions in mathematics lessons and other subject areas. Henry clearly articulated his new appreciation of anticipating student responses to plan for effective questions that allowed him to elicit students' deeper thinking:

> I know that previous lessons I haven't really planned for the anticipated responses and I wait till I get them and then you come up with the question on the spot as to ask the student. But I think when you've got the anticipated responses already it gives you a good opportunity to have ready the questions that you're going to ask the students to guide their thinking deeper. (Henry, INT3)

In line with valuing extended discussion, Lynn acknowledged the value of getting her students to articulate their thinking and how this changed her beliefs and knowledge about her students' capacity.

Megan, the numeracy coach, reflected on how she came to value the detailed planning, although the lesson study process initially put her out of her comfort zone.

> I was probably put a bit out of my comfort zone, that the planning was about eight hours all up, and it's not very often that teachers would spend that amount of time on one lesson, planning for one lesson, but that it was actually really valuable. So highlighting that importance of planning and establishing common goals in a team. (Megan, INT3)

Her comments indicate the importance of experimenting with professional practice, with lesson study protocols challenging participants' practices and beliefs.

These teachers' beliefs and practices at the end of *Cycle 2* are shown in the *Personal Domain—PD4* in Fig. 5.

Conclusion

The use of Clarke and Hollingsworth's (2002) IMPG has illuminated the interplay between the various domains in the change environment and the mediating processes of enactment and reflection across the three phases of the study. Clarke and Hollingsworth reserve the term *professional growth* for change that is more than momentary, with such more lasting change being associated with change sequences that are referred to as *growth networks*. We contend that the changes evident in the Personal Domain for the Bobbies planning team constituted professional growth, with Fig. 6 showing the corresponding growth network.

In mapping professional growth of members of the Bobbies team, it was evident that everyone came with different experiences, which were reflected in the varying pace and degree of growth observed. For example, while Lynn and Megan changed their views about the regional lesson structure and the value of including an extended, orchestrated whole-class discussion after the first few *Cycle 1* trials, Henry, the least experienced teacher, was still pessimistic about this in the second *Cycle 2* planning meeting, but later came to value the shared discussion. This concurs with findings from previous studies regarding the influence of prior knowledge and experience on teacher learning (e.g. Knapp et al. 2011; Clarke and Hollingsworth 2002; Goldsmith et al. 2014; Opfer and Pedder 2011).

Ongoing collaborative enactment and reflection was critical throughout the two lesson study cycles, with a number of critical elements identified. Firstly, a key focus of our project was the Japanese problem-solving lesson structure. Takahashi et al. (2013) identify three key supports for Japanese teachers who did not themselves learn mathematics through problem-solving approaches: the availability of "well-tested problems" incorporated into textbooks as each new key concept is introduced; the "shared knowledge about the mathematics and student thinking related to each problem base; [and]… a well-articulated set of instructional practices [that] has grown up around the Japanese [problem-solving] approach… [including] teacher's questioning strategies" (p. 240). In their enactment of this lesson structure as part of their planning for research lessons, the Bobbies trialled a similar problem. While trialling similar problems is not usually part of the Japanese Lesson Study process (Fujii 2014), these trials were instrumental in helping teams develop their confidence and skills in planning research lessons due to the unfamiliarity of the Japanese problem-solving lesson structure.

Secondly, the lesson study process foregrounded the key role of the *Domain of Consequence* for impacting on teachers' belief and practice. Data from the two lesson studies provided evidence of teachers' growing appreciation of anticipating students' solutions and the need to plan their questioning in order to elicit students' thinking. In line with Remillard's (2000) study, we found that participants felt challenged by unanticipated student responses. However, reflection on these responses prompted them to change their beliefs about their students' problem-solving abilities and their own ability to conduct extended whole-class discussions.

Thirdly, having a mixture of people from different schools allowed a broader range of views and experiences to be shared and afforded critical inquiry and collegial collaboration. The involvement of the regional and school-based numeracy coach in the cross-school planning team was instrumental in supporting teachers within and beyond the school, ensuring that growth translated into practice. Moreover, the presence of the outside expert and other observers was acknowledged by participants as crucial to advancing their professional growth, particularly for learning more about their students, lesson planning and for the design of the problem. For instance, the feedback from the *Cycle 1* outside expert about the need for "a jump-in point" after 5 min was incorporated in the planning of the *Cycle 2* research lesson. Similarly, the *Cycle 2* outside expert's feedback regarding the lesson goals and the choice of numbers prompted the planning team to reflect on the connection between the goal of the lesson and the problem. This is in line with Guskey and Yoon's (2009) critique of the notion that professional development should build strictly "on the combined expertise of in-house members" (p. 496). They contend that, in their research synthesis of what works in professional development, they found that the "efforts that brought improvements in student learning focused principally on ideas gained through the involvement of outside experts" (p. 496).

Consistent with Clarke and Hollingsworth's (2002) and Opfer and Pedder's (2011) contention that teacher professional growth is a nonlinear and iterative process, we have attempted to show the patterns of connectedness between the various domains and the dynamic influence of the mediating processes of enactment and reflection.

Da Ponte (2013) points out that few studies of mathematics teachers' knowledge, practice, and development make use of theories and calls for more consistent use of theory and frameworks "to deepen our collective understanding" of pertinent issues in these areas (p. 319). Similarly, Goldsmith et al. (2014) found many studies focussing on personal and practice domains while "few studies examine the mechanisms of teachers' learning" (p. 21). This aim of this article is to provide a thorough and systematic description of the lesson study process, using Clarke and Hollingsworth's (2002) model to map participants' professional growth. Like Opfer and Pedder (2011), our goal was "verstehen—to reach a point at which we have teased out the interconnected and overlapping processes that makes the learning of teachers plausible" (p. 382).

Our findings suggest that investing in in-depth, quality planning, with a focus on advancing students' thinking and building teachers' capacity for implementing structured problem-solving lessons through lesson study, leads to teachers' professional growth (Hargreaves and Fink 2003). In order to foster teacher professional growth, we aim to examine further critical factors for sustainable implementation of lesson study in the Australian context.

A limitation of this study was the fact that, other than through self-reporting, the data did not include evidence of how either structured problem solving or elements of the lesson study planning process were incorporated into participants' regular practice. In future research, we plan to include data that capture directly the impact of lesson study on such regular practice.

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Appendix 1

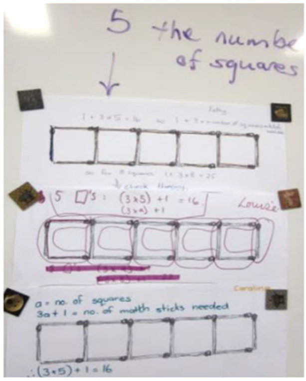
The matchstick problem introduced by the researchers for *Cycle* 1 *Research Lessons* with some solutions generated by the teachers.

Matchsticks are used to make squares that are joined together in a row as shown in these [animated] pictures.

How many matchsticks will we need to make eight squares?
What if we want to make 100 squares?

Let's think about five squares first.

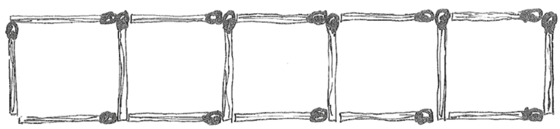
Use the diagram to find a way of working out the number of matchsticks needed to make five squares.
Show your method using the diagram and write it down.
Find as many different ways as you can.



Appendix 2

The matchstick problem used by the Bobbies in *Cycle 1*.

Here is a picture made up of some matchsticks.
How many matchsticks are there altogether?
How can you show your thinking using the diagram?



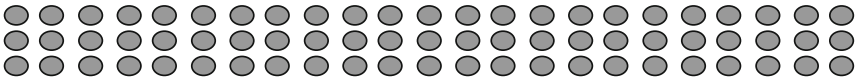
Appendix 3

The 23 × 3 multiplication problem as presented by the researchers for *Cycle 2 Research Lessons* and used by the Bobbies team.

Here is a diagram of some dots.

Can you work out how many dots are in the diagram without counting them one by one?

Please make sure that you're showing your thinking in the space provided.



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