J Math Teacher Educ (2017) 20:207–230
DOI 10.1007/s10857-015-9329-4

Prospective elementary teachers' aesthetic experience and relationships to mathematics

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Published online: 24 October 2015

Abstract Previous research has adopted various approaches to examining teachers' and students' relationships to mathematics. The current study extended this line of research and investigated six prospective elementary school teachers' experiences in mathematics and how they saw themselves as learners of mathematics. One-on-one interviews with the participants were conducted, and their written reflections were collected. A grounded-theory approach and a framework for analyzing mathematics identities were adopted in data analysis. The findings showed that the participants' development of obligations-to-oneself was associated with not only their opportunity to exercise conceptual agency but also their aesthetic experience with mathematics. Their views on themselves as learners of mathematics had cognitive, affective, and *aesthetic* dimensions. The findings suggest that teachers and students can engage in a reflection on their aesthetic involvement in doing mathematics. There is a need for a local theory of aesthetics in K-12 mathematics.

Keywords Aesthetics · Affect · Agency · Identity · Relation to mathematics

Introduction

Many years ago a prospective teacher in a college algebra class once expressed his frustration in learning and said, "I feel like I'm learning other people's math. I don't see personal meanings here." I did not understand the significance of his feeling until I read about the ideas of *agency* and *identity* in mathematics (Boaler and Greeno 2000), and *mathematical intimacy* (DeBellis and Goldin 2006). Indeed, affective factors in the mathematics classroom cannot be overlooked. In the USA, the National Research

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Council's (2001) special report, *Adding It Up*, treats "productive disposition" as "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (p. 131). This definition involves the notion of *identity* in mathematics. A student with productive dispositions feels empowered to pursue mathematical knowledge in a way that is meaningful to him/her (*agency*); he or she establishes a close relationship with mathematics (*intimacy*). *Adding It Up* has helped to disseminate the message that dispositions *are* important. However, the definition focuses on the utilitarian aspect of mathematical knowledge and lacks in aesthetic and inspirational appreciation of mathematics. In this article, I will extend the line of research on prospective teachers' dispositions and identities in mathematics. I focus on six prospective teachers' experience in their past mathematics classes and their reflection on their relationship with mathematics. I argue that prospective teachers' utilitarian and aesthetic experience with mathematics influences their approach to learning mathematics and how they see themselves as learners of mathematics, which in turn influence their visions for teaching children mathematics.

My inquiry is a response to the call for more research on teachers' affect pertaining to the nature of mathematics and mathematics identities (e.g., Goldin 2002; McLeod 1992; Philipp 2007). These researchers consider affective factors (beliefs, emotions, attitudes, and values) as important as cognitive factors in a person's learning of mathematics and a teacher's approach to teaching, which is fundamentally related to how they see themselves as learners/teachers of mathematics. Indeed, affect and cognition are intertwined in learners' problem-solving process and their mathematics experience in general (Furinghetti and Morselli 2009; Mandler 1989; Schoenfeld 1988). Over the past two decades, the interconnected nature between cognition and affect has been explored in mathematics education research. While there are many aspects of affect, I focused on personal experiences and meanings as the six prospective teachers participated (or refused to participate) in mathematics activities. I asked the following questions: How did they see themselves as learners of mathematics? To what degree did they find personal meaning in learning mathematics? To what degree did they feel an emotional attachment to mathematics? During my investigation, the theme of aesthetic experience emerged, which adds another dimension to the cognition–affect duet.

Related research and theoretical framework

Learners of mathematics

Philipp (2007) reviewed research on teachers' affect and suggested that "Perhaps the most compelling finding is that the causes of negative affect toward mathematics or mathematics learning tend to go to prospective teachers' experiences as learners of mathematics" (p. 299). He called for more research on mathematics teachers' identities and orientations toward learning. There is also a need for vigorous analytical frameworks. I shall discuss a few frameworks.

First of all, a few researchers examine a broad concept of mathematics learners and factors influencing learners' views of mathematics. For example, Di Martino and Zan (2010) believed that a comprehensive model is needed to encompass various constructs related to learners' attitudes toward mathematics. The researchers proposed a threedimensional model for understanding how students describe their relationship to mathematics. The three dimensions are *emotion disposition toward mathematics* (e.g., like/dislike mathematics), *vision of mathematics* (e.g., beliefs about what needs to be done to be successful in mathematics), and *perceived competence*. This model implies the construct of agency, the extent learners can determine what is meaningful to them, and how they can participate in their educational experience.

Learner agency, meaning, and microanalysis

Many recent studies explicitly address mathematics learners' agency and autonomy (e.g., Warfield et al. 2005). Both constructs relate to the distribution of power and authority in a mathematics classroom. For example, Walter and Gerson (2007) used a "personal agency" framework to analyze a group of elementary school teachers' exploration of linear equations and slope. Personal choices afforded teachers to exercise their personal agency as they drew on their own intellectual power to investigate the concept of slope using an invented strategy, an alternative approach very different than the traditional idea of rise-over-run. The exercise of agency was done in a community of inquiry where a member's choices and actions impacted other group members. Boaler (2000) also emphasized the community and situated aspects of learning [based on Lave and Wenger's (1991) theory on community of practice] in her research on 76 students' (ages 12–16, in UK) views of the world of the school mathematics. Boaler (2000) argued that students do not just learn mathematics concepts and skills; they also learn to *be* mathematics learners. Students' interactions and relationships with their peers play a crucial role in shaping their agency, particularly in terms of participation and a sense of belonging and meaning. In short, Walter and Gerson (2007) and Boaler (2000) extend the research on teachers' and students' views of themselves as learners of mathematics to include the construct of agency as well as learning community and classroom context.

Cobb, Gresalfi, and Hodge (2009) extended this line of inquiry further by proposing a useful analytical framework with which to understand to what extent learners identify with the norms and practices in a mathematics classroom. A main construct in this framework is p*ersonal identity*. It concerns "the extent to which students identify with their classroom obligations, merely cooperate with the teacher, or resist engaging in classroom activities" (p. 47). Individual learners take on differently the norms and obligations established in a particular classroom. The term *obligations-to-others* describes the students who merely cooperate with the teacher and do not see personal purposes in the classroom. Students demonstrate *obligations-to-oneself* when they see personal values in learning and identify with classroom mathematical activity. These students believe they have the capacity and legitimacy to contribute to the collective learning of the class. The constructs of *obligations-to-others* versus *obligations-to-oneself* provide a valuable lens through which to examine learners' identities. In Cobb et al.'s analysis of two classes with different cultures, the same group of students showed obligations-to-others within the culture of one class while becoming obligations-to-oneself in the other class. The culture of the latter class featured exploration and justification of solution strategies.

Authority and agency are related notions for understanding learners' mathematics experience and identities. Authority means the degree to which learners are able to make decisions about the interpretation of tasks, the reasonableness of solution methods, and the legitimacy of solutions. "Authority is therefore about 'who's in charge' in terms of making mathematical contributions" (Cobb et al. 2009, p. 44). With respect to agency, learners with *conceptual agency* have opportunities to choose and justify solution strategies and make connections between concepts. On the other hand, *disciplinary agency* involves using established solution methods (e.g., applying a formula) instead of learners' invented strategies. For example, in Boylan's (2010) description of "usual school mathematics" (p. 63), students only exercise disciplinary agency.

While Cobb et al. (2009) focused on the microculture of a mathematics classroom, Bishop (2012) zoomed her investigation further into how moment-to-moment discourses among students shaped their identities. She highlighted the interactions between two seventh-grade students (one is "smart" and the other "dumb") and theorized five types of "positioning acts" through which the two students jointly enacted their mathematics identities. Indeed, the microlevel analysis of structural patterns of the two students' discourse revealed how they treated each other, controlled discourse, and played their respective roles.

In summary, researchers have used a variety of theoretical and methodological approaches to studying the constructs related to teachers' and students' experiences in mathematics and their mathematical identities. Researchers have studied the cognitive, attitudinal, emotional, and sociopolitical dimensions (Wager and Stinson 2012). However, there is scant research on teachers' and students' aesthetic and inspirational experience with mathematics, which can provide an additional dimension for considering teacher education and for understanding learners' cognitive and affective development.

Aesthetic experience in learning mathematics

The nature of professional mathematicians' work is commonly misunderstood by students. Students find it easy to name three musicians or artists, but students have a hard time naming three professional mathematicians. Lockhart (2009) argued that this is so because the school mathematics curriculum does not provide opportunities for students to understand the history and philosophy of mathematics. Teachers do not incorporate the historical roots of mathematics either. Consequently, most students have a myth about the nature of mathematics and what professional mathematicians do (Boaler 2015).

Hardy (1992/1940) asserted that mathematics is an art and that mathematical activity is aesthetic in nature. Lockhart (2009) and many mathematicians in Burton's (2004) study shared the same sentiment. To both Hardy and Lockhart, mathematicians do not just create ideas; they also give aesthetic appraisal to ideas. An idea can be elegant, beautiful, and enjoyable. Another idea can be ugly and unpleasant. In Dreyfus' (2008) learning theory, "master" performance is also aesthetic in nature. He posited that learners' aesthetic experience is critically important for higher-order learning to occur.

Sinclair (2004) discussed three roles of the aesthetic in mathematical inquiry: *evaluative*, *generative*, and *motivational*. First, mathematicians use aesthetic criteria to evaluate proofs and theorems, judging their beauty, elegance, and significance. Second, aesthetic values provide guidance in mathematicians' pursuit of problems and proofs, often unconsciously (see Burton 1999). The aesthetic guide helps mathematicians generate new ideas and insights. Lastly, "the motivational role refers to the aesthetic responses that attract mathematicians to certain problems and even to certain fields of mathematics" (p. 264). Mathematicians' aesthetic tastes (e.g., sense of surprise and paradox and visual appeals of mathematical entities) and in some cases social interactions motivate them to select and work on particular types of problems.

Mathematics educators such as Papert (1978) and Sinclair (2001, 2002, 2004, 2006) have promoted the importance of aesthetics in mathematics education. Researchers have investigated teachers' aesthetic experience with mathematics. For example, Crespo and

Sinclair (2008) explored how prospective teachers used aesthetic criteria to judge the quality of mathematical problems that they posed for teaching and learning, showing the *evaluative* role of aesthetics. Hobbs (2012) examined four secondary teachers' "aesthetic understanding" of mathematics and science, which influenced their self-efficacy in teaching as well as their teaching practice. Hobbs argued that "a teacher's personal experience of, and response to, the subject [mathematics or science] is aesthetic in nature, meaning that the cognitive and affective are inextricably linked in both their experiences of the subject, and in the way current experiences provide parameters and expectations for future experiences" (p. 727). She further argued that the process of becoming a teacher of mathematics or science is a transformative experience, and thus, the process is essentially aesthetic in nature.

John Dewey's philosophy on aesthetic experience provides the theoretical framework for Hobbs (2012), Sinclair (2002), and Wickman (2006). These researchers draw on Dewey's work in *Experience and Education* (1938) and *Art as Experience* (1934) as the philosophical basis of their research. For an experience to be educational, Dewey (1938) posited that it needs to show *continuity*. In other words, previous experience is the basis of the present experience, which will in turn lead to further experience and growth. In addition, for Dewey, an educational experience occurs on the basis of the *interaction* between the learner and the objective conditions of learning. The two sides form a situation, in which the learner is emotionally involved as she/he sees meanings and purposes in the learning activity. Dewey (1934) also highlighted the role of aesthetic experience in human activities and life. Among many things, an intellectual activity can be aesthetic. For example, I think Dewey would agree that it is an aesthetic experience when a scientist or mathematician is emotionally participating in the production of the meaning of an object/idea. The aesthetic moment occurs when the scientist or mathematician is artistically engaged in the creative activity.

In summary, the account of research and theory given above shows the important role which aesthetic experience plays in terms of learning/doing mathematics and in the process of becoming a teacher. I argue that the addition of the aesthetic dimension can increase our understanding of how students and teachers see themselves as learners of mathematics. This line of inquiry will help us probe into inspiration or lack thereof in a mathematics teacher or learner.

Methods

Research questions

The review of related research reveals that further research is needed on teachers' and students' dispositions toward mathematics. The current study was conducted in response to this call. While there are many aspects of dispositions, I focused on the following questions:

1. How do the participants see themselves as learners of mathematics? To what degree do they develop agency in mathematics? Where are they in the spectrum between *obligations-to-others* and *obligations-to-oneself*?
2. What are the participants' relationships to mathematics?
3. To what degree do they find aesthetic value and personal meaning in mathematics?

Participants and context

The participants were six prospective teachers in two credential programs for prospective elementary school teachers in California, USA. In California, most teacher credential programs are post-baccalaureate, and prospective teachers spend 1 year taking credential courses as well as completing clinical practice. Four participants (22–28 years old) were in such a program. The other two participants (21–22 years old) were in a blended program, working simultaneous on their degrees in liberal studies and initial credentials for teaching in K-8 schools. Four participants appeared to be white, and the other two appeared to be Latino/a.

Data collection

This research is an ongoing investigation into prospective elementary school teachers' development of selves as learners and teachers of mathematics. The first phase of data collection consisted of autobiographies collected from the 51 prospective teachers in three classes of a mathematics methods course that I taught. The autobiographic data included drawings of what mathematics was to them, definitions of mathematicians, reflection papers on their schooling experience with mathematics, and assignments where they discussed mathematics teaching and learning. The data addressed the research questions. Only the work of the prospective teachers who volunteered to participate in the second phase (interviews) was included in the current report.

Once the classes had been completed, I sent an e-mail message to all prospective teachers and invited them to participate in one-on-one interviews. Six of them volunteered to be interviewed. I interviewed three participants once and the other three participants twice. The one-time interviews were approximately 70 minutes, while each of the two-time interviews was about 40 minutes. The interviews were semi-structured. The first set of questions was general. For example: What was your major in college and what math courses were required for your major? How do you use math? How did you decide that you wanted to be a teacher, and what influenced your decision? What is your vision of what it will be like to be a teacher? The next set of questions was designed to reflect the theoretical framework proposed by Cobb et al. (2009). I particularly asked questions about the norms and obligations in the mathematics classes the participants experienced and the classroom practices that might be related to the formation of the norms and obligations. To address the research questions, I asked about the ways the prospective teachers participated in classroom activities as a means to understand the nature of their affective involvement in mathematical activities. I also probed for the opportunities provided for learners to exercise their power in deciding what and how to pursue mathematical solutions and what counted as legitimate (that is, who was in charge?). Competency and self-efficacy were also addressed. These are issues pertaining to agency and personal meanings in doing mathematics. Lastly, I investigated how the participants established a relationship with mathematics. This part of conversation centered on the nature of mathematics, "What is math to you?" "How do you find meanings in mathematics?" "Describe a situation where you felt successful in mathematics." and "What do mathematicians do? Do you consider yourself as an amateur mathematician?" Overall, I probed for the participants' experiences, beliefs, and emotions as well as *how* they held such notions.

During the interviews, my focus was not on getting the participants to answer the questions I had in mind. Rather, I wanted them to tell me their stories and what mattered to them. The participants had the opportunity to influence the direction of the conversation; we created a space in which the participants could share their mathematics journeys and express their emotions.

Data analysis

The interviews were transcribed verbatim. Participants' emotional expressions during the interviewing process were also noted. I coded the transcripts and reflection statements using two approaches. First, I followed Strauss and Corbin's (1998) approach to open coding. I did not create a priori codes but rather developed codes as I read and re-read the participants' written statements and interview transcripts. At the beginning, I coded the text sentence by sentence. I eventually examined a few sentences as a block, and I resumed microanalysis as needed. I used a concept to summarize the data expressed in each sentence or block of text. Examples of such concepts include: "teacher–student relationship," "instrumental relationship," "aesthetic appraisal," and "math inspiration." I also developed analytic memos focused on emerging themes. A theme itself makes little sense unless it is linked to others. Thus, I related each significant theme to others through these memos. These memos had helped me with my analysis, interpretation, and directions for further data gathering.

In a second iteration of text analysis, I utilized the analytical framework put forth by Cobb et al. (2009) to carefully read and re-read the texts (once again word by word) and used the key constructs (e.g., obligations, valuation, disciplinary vs. conceptual agency) to code the passages. I noted what opportunities were provided for participants to determine the legitimacy of their ideas, how they made sense of the mathematics activities, and how they identified with the classroom norms. Besides reading the transcripts, I listened to the recordings to recap the atmosphere of the interviews and to note participants' emotional expressions.

Findings

The purpose of the study was to extend the research on the affective domain of learning mathematics by means of investigating six prospective elementary teachers' relationship to mathematics, in response to the call for research on teachers' mathematics identities and orientations toward learning. The effort was also to explore potential theoretical perspectives for understanding teachers' experience of learning mathematics.

Three main themes emerged from the data analysis. First, the participating teachers developed different levels of obligations and autonomy, which were related to how they positioned themselves as learners of mathematics. Second, the participants had an instrumental relationship with mathematics and regarded mathematics as an objective, external world. Finally, aesthetic experience emerged from four of the six teachers' views of themselves as learners of mathematics.

The development of agency and obligations-to-oneself

The six participants developed different types of obligations and sense of agency toward learning mathematics. Jasmine, Julian, and Naomi (all names in the article are pseudonyms) showed signs of obligations-to-others, doing mathematics according to other people's agendas or just to cooperate with the teacher. Andrea also exhibited obligations-to-others, but her aesthetic relationship with mathematics distinguished herself from Jasmine, Julian, and Naomi. Lastly, Emma and Sally experienced a sense of obligations-to-oneself and derived personal meanings in learning mathematics.

A brief summary of the six participants' profiles is provided in Table 1.

It is important to explore how the participants developed such different types and degrees of obligations. The data showed that experience of conceptual agency was associated with the development of *obligations-to-oneself*. For example, in Sally's mathematics autobiography, she wrote:

> I am not saying that mathematic became easy at the university level because mathematics is still difficult. However, I enjoy coming up with my own way to finding a solution. I like justifying, explaining my work and discussing other ways to the same solution with my peers.

For many prospective elementary school teachers, mathematics is not their strong suit. Sally was not an exception. She found college-level mathematics challenging. The above statement incorporated Sally's sense of cognitive ability and enjoyment, which helped her tackle the challenges she encountered. Her ability to solve a problem in her way showed sign of power and agency, a feeling that she could influence her learning.

Emma reported many occasions where the classroom norms allowed her to exercise conceptual agency, sometimes individually and sometimes collectively with peers. She recalled her high school mathematics classes as follows:

Emma: It was a lot of problem-solving where the students could share how they got the answers, and then we all share with each other…there were multiple ways [to solve a problem]… most of my teachers would always make you not only solve the problem, but you'd have to show your work completely—how you solved it and then why the answer's right.

Interviewer: Did you need to follow the teacher's method, or you could use your own method?

Emma: Sometimes [the teacher] wanted us to practice a particular method. Other times we could solve it in our ways.

Indeed, Emma experienced conceptual agency and some autonomy in justifying the legitimacy of her solution strategies. Both Emma and Sally felt competent in mathematics. They were able to juggle between the established obligations and their own approaches to learning mathematics. They performed well in classes and found enjoyment in doing mathematics.

Jasmine, Julian, Naomi, and Andrea showed what Cobb et al. (2009) called *obligations-to-others*. Overall they did not identify with the mathematical activities, or they did not find personal values in doing mathematics even if they chose to abide by classroom norms and fulfill the obligations to various extents. In their classes, the teacher was the sole center of authority, and few opportunities were provided for students to come up with their strategies or to justify ideas. In other words, students were expected to copy the teacher's method; they exercised predominantly *disciplinary agency*, following established procedures and requirements. Andrea described a mathematics class as follows:

Table 1 Participants' profiles showing agency, obligation orientations, and aesthetic experience

	Conceptual agency (CA) versus disciplinary agency (DA)	Obligation orientation	Perceived self as a learner of math	Aesthetic experience
Emma	Both: While most classes were procedure-based, she had some opportunities to decide what worked for her.	Oneself	Considered herself as a non-professional mathematician.	Appreciated the beauty of math; Loved logic and reason in math thinking.
Sally	Both: College math classes were lecture-based; authority was on professors. But her high school math classes were more open to students' explorations; she had some power in deciding the legitimacy of her strategies.	Becoming oneself	Competent learner; found enjoyment in math.	Liked to play around numbers and symbols. Felt satisfied when getting a beautiful solution.
Andrea	DA: Math classes were lectured-based with a mind-set of covering the materials; teacher's way was the only way; she followed given methods.	Others	Competent learner in fundamental math but struggling learner in advanced math.	Appreciated logic and harmonious structure of math. Loved to make sense of the world with math.
Julian	Predominantly DA: He was quiet in math classes and identified with the classroom norms but was never an active participant. He was a follower. Authority was on the teachers/professors.	Others	Speculator; math was personal and not social for him.	NA
Jasmine	DA: She often had difficulty understanding concepts but identified with classroom norms/expectations and participated.	Others	Didn't think that she had the "math gene." Math was beyond her reach.	Negative appraisal of math. Experienced isolated fragments in math.
Naomi	DA: mostly lecture-based classes with only a little opportunity for exploration and group work. Teachers/professors and the textbook had the authority.	Others	Hard worker in math; good listener/receiver of math knowledge.	NA

> Most people were mostly there to just get the math done. I don't think there was very much of a class where you're exploring different methods [to do math], where like, you're open to that. It was just kind of like: do it, do it, do it! [pounding her fist].

This is clear evidence that Andrea merely practiced the mathematics that was handed over by the instructor, a learning agenda that was external to the students.

There was another basic difference among the experiences of the four prospective teachers who remained *obligations-to-others*. The difference lies in how they saw themselves as learners of mathematics and how they positioned themselves in the mathematics classroom. Jasmine's experience is a good case in point. She in particular did not identify with mathematics. She was similar to the students in Boaler's study (2000) that regarded mathematics as a foreign, meaningless subject. In fact, Jasmine took a deficiency position.

Interviewer: This is about the nature of mathematical knowledge. Do you think there is a mathematical world out there?

Jasmine: I definitely think there is a mathematical world out there where some people just might be more logical thinkers and really, you know, really geared toward math and understand how concepts and things kind of work together to make the problem correct, and then there's just some people, you know, like me personally, who can look at a problem and it's just a little more challenging.

Jasmine did not think she had the "mathematics gene." Other data indicated that she was alienated from the knowledge produced in her mathematics classroom. Although she participated in many classroom activities, she struggled to understand mathematics. When asked about an unsuccessful experience in learning mathematics, Jasmine said that "Oh—gee, it's probably my whole math history. I would just say, you know, within my math experience, just kind of never being at a proficient level. Math has kind of always been more of a disappointment for me." Despite low competency in mathematics, Jasmine felt obligated to attend classes. She said: "I had to be there, I think, for the grade. I passed it minimally, and that's all I needed out of the course." Obviously, her participation was for an external purpose instead of personal values. However, Jasmine described her passion for teaching: "I've always kind of had a passion to be a teacher… be able to be that person that helps structure their [children] lives, and help them advance in what they do. That was just always so important for me." She was committed to teaching children.

To summarize, the opportunity to exercise conceptual agency and the occasion of making sense of mathematics were associated with the development of obligations-to-oneself. Moreover, the manner the six teachers considered themselves as learners of mathematics, or the way they positioned themselves in the mathematics classroom was also a significant factor. It should be noted that, in all six teachers' cases, we see the interweaving of cognitive, affective, and identity-related factors in their mathematics history and self-concepts. Later in the article, I discuss an additional aspect—aesthetic experience. For now, I will turn my investigation to how the participating teachers constructed their relationship with mathematics.

An instrumental relationship to mathematics

Looking at the participants' drawings, autobiographies, and interview transcripts, I saw some of the elements of *instrumentality* emerging. It was taken for granted and thus did not receive a careful scrutiny among the participants. This instrumentality developed in reference to the relationship between the learner and mathematics, and the instrumental mindset found expression in various discourses and practices in mathematics, in which mathematics is treated as technology to serve human needs. In this article, technology is defined not only as a device but also as a broad term to refer to the apparatus or system for society's operation (see Feenberg 2002).

In the data the instrumental view of mathematics was apparent. The participants all talked about how they used mathematics for accounting, shopping, cooking, and so on. There was an overwhelming focus on the utilitarian aspect of mathematics. Mathematics is useful, practical, and beneficial. Figure 1 shows Naomi's notion that mathematics is a lifelong tool that is applied in many areas of the society. In Fig. 2, Andrea expressed both a utilitarian view and an existential view of mathematics (without mathematics, our world will not exist).

Jasmine put the instrumental view of mathematics in the context of need. She talked about when and who would use mathematics:

Interviewer: How much do you see personal meanings and purposes in learning mathematics?

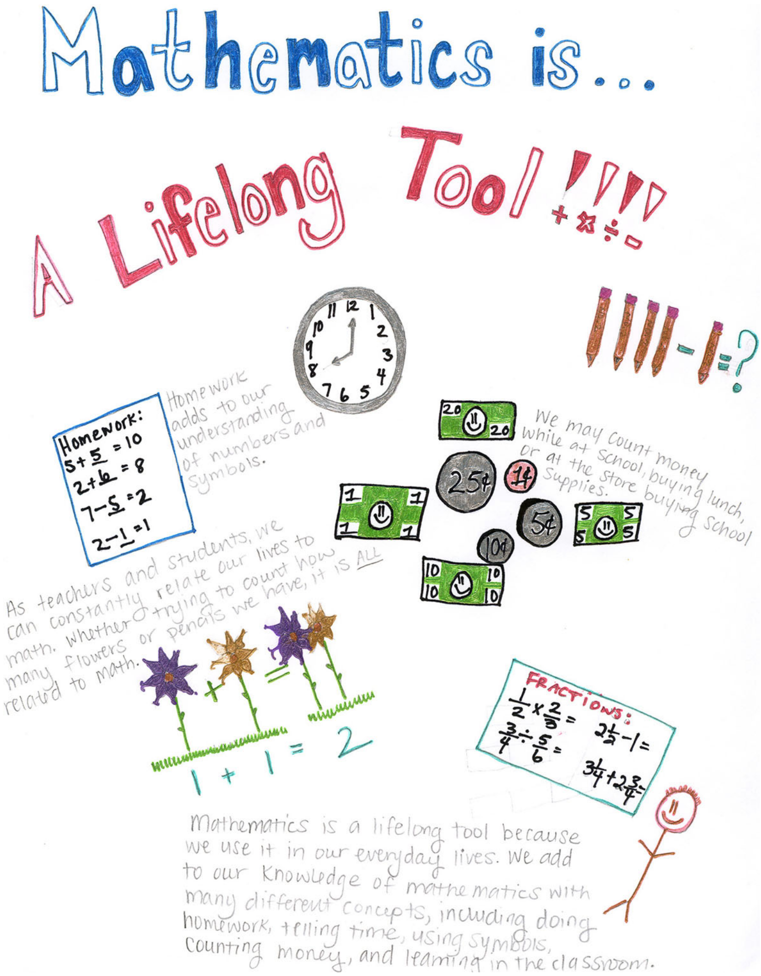

Fig. 1 Naomi's drawing of "what is math to you"

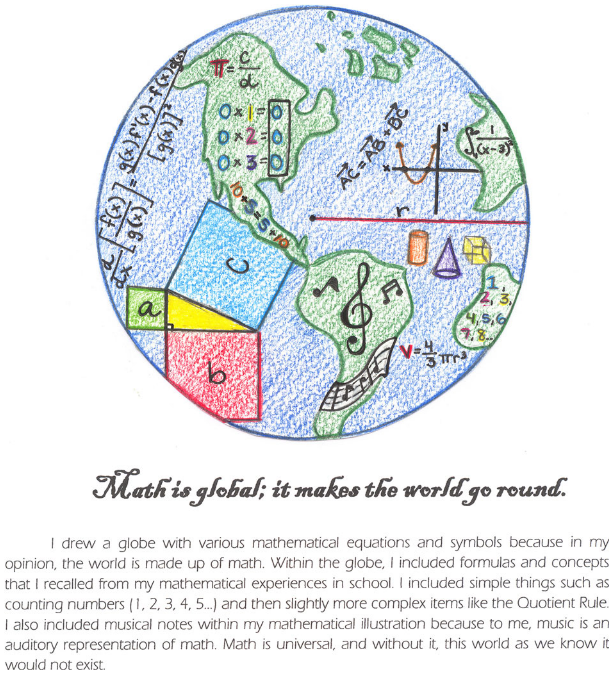

Fig. 2 Andrea's drawing of "what is math to you"

Jasmine: I don't think it's necessarily going to benefit you throughout life if it's [mathematics] something that you just don't really need, like, I'll need to know how to teach math to students, obviously. So in my profession, yes, I probably need it. Um, some other professions, maybe like firefighters or police, they might not need it, so maybe they just need to take the required courses. But I think it depends on the degree of what you're doing, that you may or may not need it.

The discourse about *need* and *utility* suggests the view that mathematics is a tool waiting for the right person to use it to serve his or her purpose. The passage also suggests the kind of learner–mathematics relationship in which the meaning of learning mathematics is external to the learner. Other data suggest that, for Jasmine, the purpose of learning mathematics was to get the credit and move on to the next stage of her education until there was no requirement of mathematics, and during her credential program, the purpose was to obtain the necessary knowledge and skills for teaching. All was based on need. Mathematics was seen as an instrument for a purpose external to Jasmine. For Andrea, mathematics was used to calculate life expenses: mileage, gas costs, balancing her checkbook, cooking, shopping, and so on. Andrea said that: "Most of it has to do with money." Mathematics was an essential tool for her to function in the society.

Here mathematics resembles technology with respect to people's utilitarian view of both as an external tool that serves human purposes. For example, as I asked about the nature of mathematics:

Sally: Math is applicable to just about any part of your day… math is what's going to get you through life and just help you out, make things a little bit easier.

Julian: I think there is a mathematics world out there, you know, all the numbers, equations, formulas, logic, and problems. I think that the mathematics or the physical representation of math, of measurement, of relationship, of quantity exists… Our job as a teacher is to introduce the [mathematics] world to children.

In Sally's passage, mathematics shares the defining characteristic of technology–efficiency. Mathematics, like technology, makes things easier and more convenient for humans. For Julian, mathematics has an external reality that is independent of the human mind. Julian's notion represents an objectivist view of mathematical knowledge. According to Tymoczko (1998), in this philosophy of mathematics, mathematical truths are believed to be absolutely certain, and rules are regarded as objective knowledge. Other participants had a similar view on the nature of mathematics.

In short, the six prospective teachers subscribed to an objectivist view of mathematics and adopted a utilitarian notion of the relationship between mathematics and the world. They treated mathematics as an instrument, much like technology, for serving humans' needs in their daily lives. Their ideas reflect the dominant discourse in mathematics education, that mathematics "literacy" is for people to function in society, and that mathematical knowledge is essential for a strong work force so that the USA can compete globally. I discuss the pedagogical implications of this objectivist view of mathematics in Sect. 5.2.

The instrumental and utilitarian view of mathematics is *correct* but not *true*. It is correct in that mathematics serves as a useful tool with which we develop technology and build modern societies. It is not true because people can also find meanings in mathematics in itself and have an aesthetic relationship to mathematics; the mathematics learner can appreciate the beauty of mathematics and feel fulfillment in doing mathematics. Indeed, the next section addresses the participants' encounter with the aesthetic aspect of learning mathematics. They held both *instrumental* and *aesthetic* views of mathematics.

Relationship to mathematics: aesthetic experience

The theme of aesthetic experience emerged in four of the six teachers' reflections on their relationship with mathematics. Emma and Sally had an aesthetic feeling toward mathematics. Jasmine and Andrea also gave aesthetic appraisal to mathematics. Recall that they showed signs of obligations-to-others, doing mathematics according to norms they did not internalize. However, Andrea's aesthetic relationship with mathematics distinguished her from Jasmine. Andrea appreciated human's capacity to use mathematics to model the world, while Jasmine offered a negative aesthetic judgment.

The participants' aesthetic experience with mathematics took two forms. First, the participants expressed aesthetic values in relation to the *process* of doing mathematics. For example:

Interviewer: Do you think you are a successful student?

Emma: I've always excelled in math. I've never done too poorly… I could have someone come to me and say, 'I don't understand this,' and I've never seen it before, and I can usually figure it out on my own and I feel good about having coming up with, you know, a good way to solve a problem and explain it to the person… It's like a self-gratitude.

In this experience, Emma felt *pleased* when she was able to find and share an ingenious solution. She appreciated her own insightful idea and also being able to help peers. So this experience was not merely about an intellectual solution to a problem; the experience also provided an inner source of pleasure and social value.

In Sect. 4.1, I described that Sally liked to play around numbers and patterns in her own way. She said that:

> Give me numbers and let me do it on my own. I need to figure things out on my own to learn… [describing a problem-solving experience] I struggled to solve it, I was frustrated, but once I was able to come up with a neat solution, I felt pretty satisfied.

This passage shows Sally's cognitive understanding of mathematics and emotions. Sally used a positive aesthetic judgment. The terms *neat* and *satisfaction* suggest aesthetic values in doing mathematics. A solution is not just a solution; it is a beautiful ("neat") one.

The second form of aesthetic experience concerns the learner–mathematics relationship and involves inspiration and identities. Sally and Emma developed an existential and aesthetic relationship with mathematics overall.

Interviewer:	So what is the relationship between you and math?
Sally:	You're like really making me think about my math philosophy. I think I have a good relationship with math, you know… I like to see connections… it's good to see how they [math ideas] come together.
Sally:	Math has become a part of my life that makes me feel good
Emma:	I think I'm a mathematician because I like math, I enjoy math, I appreciate math, I understand math, and I use it daily. So why would I not be a mathematician?

Both Sally and Emma found personal and existential meanings in mathematics. Sally appreciated the harmony among mathematical ideas and how they connected beautifully. She also experienced an integration of mathematics in her life that brought her pleasure. Sally's love for mathematics influenced how she envisioned what it would like to be a teacher. She stated, "I think teaching mathematics for me is going to be a lot easier [than other subjects] because I love math… I've always really loved teaching. I've always loved helping children." Sally's competence in mathematics transferred to a high level of self-efficacy in teaching mathematics.

Emma considered herself to be a (non-professional) mathematician that appreciated the beauty of mathematics. Elsewhere she showed her eager to share her pleasure in mathematics with students. She loved mathematical reasoning and thought it was wonderful to make sense of the world using mathematics. Perhaps it is not a coincidence that art and mathematics were Emma's favorite subjects in school. I wonder if she considered herself as an artist too. Emma's thought that she is a mathematician is a powerful idea. Besides the connotation of competency and agency, this idea implies that mathematics can be personal. Emma was not a virtuoso in "the" mathematics. However, in Emma's personal mathematics, she felt that she was a creative mathematician. This disposition should be valued.

Andrea showed the above two forms of aesthetic experience. Her aesthetic relationship with mathematics helped her to stay positive about learning and teaching mathematics. She had difficulty in college mathematics classes and felt out-of-place primarily because of the male-dominant classroom culture. One of her professors did not think mathematics was for her. These negative experiences shaped her mathematic identity, and she developed low self-efficacy in college-level mathematics classes. One would think that Andrea would not appreciate mathematics. Surprisingly, she actually had a positive view about mathematics and appreciated mathematical modeling in sciences and social issues. Andrea enthusiastically described her relationship with mathematics:

Interviewer: Tell me about the relationship between you and math.

Andrea: I love math! Even when it gave me a hard time, I still love it. I love numbers; they're just so logical and just so rational. I feel like it's so essential for everything and anything can be broken down into numbers. Math is so structured and organized, and I appreciate that… I feel a sense of satisfaction when I can make sense in mathematics… I would break it [a situation] down into numbers, symbols, and relationships.

Andrea appreciated the harmonious structure of mathematics. She loved the logical connectedness in mathematics. For Andrea, it was a pleasure to be able to make sense of the world through mathematics. The phrase "make sense" appears 30 times in her interview transcript, referring to understanding in some instances and both cognitive and aesthetic evaluation in other instances. In her drawing of "what is math to you?" (see Fig. 2), Andrea drew a globe with numbers, shapes, equations, graphs, theorems, and musical notes. In her explanation of the drawing, she wrote, "without [math], this world as we know it would not exist." She appreciated that mathematics helps humans create a world that makes sense. Therefore, despite her negative experience in recent mathematics classes, Andrea's thought on the aesthetic and anthropological aspects of mathematics helped her sustain her interest in mathematics and her commitment to teaching children mathematics.

While the above three teachers had positive aesthetic experience with mathematics, Jasmine had negative aesthetic experience. She encountered "nasty" mathematics that one component did not agree with another.

Jasmine: We learned this and that piece of math… they were disjoined. I didn't see how they were related.

Interviewer: How much did you participate in classroom activities?

Jasmine: I usually didn't say much. I didn't think my ideas were *nice* and *worth* sharing (emphasis added).

Not seeing the patterns in mathematics, Jasmine found mathematics unpleasant. She did not appreciate her ideas in mathematics. Her language referred to the aesthetic *value* of mathematical ideas. Unfortunately, Jasmine's perception did not express a positive tone. In her case, it is not just a lack of understanding; it is also a lack of opportunity to see the coherent structure of mathematics.

The theme of aesthetic experience did not emerge from Julian's or Naomi's data. It does not mean that they did not have any aesthetic moment in mathematics. Naomi did mention the role of music and art in her teaching of mathematics. She opposed the No Child Left Behind Act because she thought that students were taught to the test.

Interviewer: So then what would you do differently?

Naomi: I would try to find other ways to teach the concept to make it more interesting. So, if I'm trying to teach, um, multiplication, I would try and find ways to incorporate song or to incorporate art, because I feel like a lot of that type of stuff is being removed from public education, and I feel like we need to find ways to push for that.

This vision of teaching does not necessarily mean that Naomi would teach mathematics as an art (she might use art to sugarcoat mathematics). But she showed she would be receptive to a pedagogy that integrates art with mathematics. I wonder, if I had asked questions specifically to solicit her aesthetic experience with mathematics, what might have she said? What might be the impact of aesthetic experience on her vision of teaching children mathematics?

In summary, four of the six participants had aesthetic experience of mathematics. It was integrated into specific mathematical activities as well as occurring after years of learning mathematics. The first form arose in the process of doing mathematics. The second form pertained to the relationship between the learner and mathematics, and it was fundamentally related to mathematics inspiration and identities. This inspirational relationship helped to unify the various aspects of the four participants' study and life, bringing meanings and purposes for learning and teaching mathematics. For Andrea, Sally, and Emma, mathematics was not just a subject of study and a subject that they would teach. They also appreciated mathematics and wanted to pass their experience to children. The data did not show Julian's or Naomi's aesthetic relationship to mathematics. They might have some aesthetic experience if I had probed. I will discuss a few methodological strategies in Sect. 5.1.

Limitations, discussions, and implications

Limitations and connections with previous research

In this research project, I interviewed six prospective elementary teachers and analyzed their drawings, reflections, and autobiographies. The study has a few methodological limitations. First, the data collection relied heavily on participants' (selected) memory and self-report. I did not have observational data on what exactly happened in the participants' mathematics classrooms. I did not reveal the detailed nuances of individual teachers' experiences. Second, the data collection is based on participants' ways of knowing. A person with procedure-based approach to learning mathematics might not see how he or she could exercise conceptual agency.

Despite the limitations, the study confirms and extends previous research on mathematics teachers and learners. First, the findings are in agreement with previous research (e.g., Boaler and Humphreys 2005) that concluded that students' exercising conceptual agency is associated with active participation in mathematical activities and positive views of mathematics and of themselves. The challenge for mathematics educators is that in many mathematics classrooms students are still not given adequate opportunities to exercise their power and agency to make sense of mathematics, despite the growing awareness of the role of cognitive and affective factors in learning mathematics (Hiebert et al. 1997). One of the key issues pertains to the *nature of mathematics*, which I will discuss later.

The current study extends the scope of analysis of prospective teachers' experience by bridging a gap in previous research. Specifically, I used constructs such as identity, agency, authority, and the analytical framework proposed by Cobb et al. (2009) to examine how the prospective teachers saw themselves as learners of mathematics. The existing research literature provides a set of useful constructs to describe the identities these teachers developed. Constructs such as *norms*, *procedural versus conceptual agency*, *distribution of authority*, *obligations-to-others versus obligations-to-oneself* helped me understand and describe the teachers' experiences and the relationships among these constructs. However, a limitation of Cobb et al.'s framework is the omission of teacher–student relationship and mathematics learners' inspirational experience with mathematics. Indeed, Cobb et al. were aware that their framework is just one of the many perspectives that one may adopt to understand mathematics learners' identification (or non-identification) with mathematics. I contend that researching learners' existential relationship with mathematics can extend our understanding of mathematics learners. I propose an additional dimension to capture the aesthetic experience of teachers and students.

What are some methodological strategies for understanding teachers' and students' aesthetic experiences in mathematics? It can be a futile attempt to directly ask them to articulate their aesthetic experience. Indirect methods are needed. In the current study, I asked prospective teachers to draw and explain what mathematics was to them. I also interviewed six teachers. In particular, the interviews with Sally and Emma were conducted at the same time, and the interaction prompted reflections from each other. Therefore, a group interview is a viable method. In addition, researchers may ask teachers and students to discuss their own and peers' work and to discuss criteria for judging a good question or conjecture (Lehrer et al. 2013). Lastly, since a person's aesthetic experience is usually connected to their affective responses, it is helpful to use reflection prompts that probe for learners' emotions and feelings in mathematical inquiry and their overall relationships to mathematics.

The dialectic of instrumental and aesthetic relationships with mathematics

The prospective teachers in the study subscribed to an instrumental view of mathematics. Mathematics was thought of as a useful tool for serving human needs. The teachers' notions reflect the dominant discourse about the role of mathematics; it is a means to the ends of performance, success in the job market, and economic competitiveness. Teachers and parents tell children that mathematics is key to entering lucrative careers in business, science, medicine, and engineering. Children learn to treat mathematics as an instrument for fulfilling individual and national dreams. Indeed, these purposes are external to the mathematics learner. With such an extrinsically motivated justification for mathematics education, it is difficult to instill intrinsic motivation to learn mathematics. Children's agency can be lost in the instrumental view of mathematics, as they are processed for advanced study or the job market. This phenomenon is a threat to humanity. It is similar to sociologist Jacques Ellul's (1964) and philosopher Heidegger's (1977) observations of a technological system. Heidegger argues that we are involved in a transformation process of the entire world into "standing reserves," which is to say that everything is reduced to being raw materials. Our engagement in this process would make us subject to a transformation which enables objectification, where all, including humans, could become objects. Indeed, the philosophers concern humans' instrumental relationship to the world.

With respect to mathematics education, the data suggest that the concern of an instrumental relationship to mathematics can be alleviated if educators provide an opportunity for learners to explore the aesthetic dimension in doing mathematics. Recall that three of the six participants did not lose their love for mathematics even though they had an instrumental view of mathematics. Andrea's case is even more encouraging. Although she encountered difficulty in learning, her aesthetic appraisal of mathematics helped her carry on with her interest in mathematics and aspirations for teaching. However, the data also show that the participants' instrumental view of mathematics is more noticeable and much stronger than the aesthetic view. I will discuss how to provide opportunities for teachers and students to reflect on the aesthetic aspect of doing mathematics. For now I shall return to the issues of agency and identity that I outlined in the introduction and review of literature.

The findings show that for teachers who have little sense of developing students' obligations-to-oneself as a learner of mathematics, they will find it hard to allow a classroom culture that encourages a wide distribution of authority and agency. Jasmine's description of her teaching is a case in point.

> I started with that [a jar of candies for 5th grade students to estimate the quantity], and then I did a little excerpt in their books, and then I went over a few problems with them, and I just did guided practice with them and showed them what instruction they needed to do, showed them kind of how to do a few problems and then basically just had them do a few problems on their own.

Many researchers (e.g., Hiebert et al. 2005) find this practice ineffective. The episode illustrates the challenge for teachers like Jasmine, whose orientation was obligations-to-others, to reconceptualize teaching for developing students' autonomy.

To help teachers overcome this challenge, educators can provide an opportunity for teachers to (re)learn mathematics in an environment where they can develop and apply conceptual agency. The autobiographies I collected from 51 prospective elementary teachers show that by and large they had more personal involvement in doing mathematics before secondary school. This observation suggests that we need ways for teachers to re-appropriate their personal involvement with mathematics. Duckworth (2006) provided a theoretical account and practical strategies for teacher education. I have also found that engaging teachers in exploring carefully selected/designed problems such as the "sums of consecutive integers" problem (modified from Parker 1998) can invite them to have a personal feeling for *doing* mathematics. In addition, teachers can have vicarious experience by observing and reflecting on how students exert agency and autonomy in such a classroom in Cobb et al. (2009).

Second, it will be challenging for teachers to promote agency and autonomy without *seriously* addressing the nature of mathematics. This study reveals the presence of both an instrumental and an aesthetic nature of mathematics. There is no doubt that it is important to apply mathematics as a tool to model a scenario and to solve a real-world problem. However, teaching practice within the instrumental nature of mathematics can limit learners' experience such that they will not use a more inspirational and aesthetics-based approach to doing mathematics. Recall that three of the six teachers described a positive aesthetic appraisal of mathematics (at a novice's level). Their experience in making sense of mathematics brought them pleasure and made them feel good about mathematics and about themselves. This attitude toward the aesthetic nature of mathematics will likely be carried on in their teaching.

Moreover, these teachers' aesthetic experience with mathematics intertwined with their level of mathematical understanding (cognitive factor), self-efficacy and attitudes (affective factor), and views on themselves as learners of mathematics (identities). Therefore, I suggest that the aesthetic dimension be added to the cognitive, affective, and identity-related dimensions of the teaching and learning of mathematics (see Fig. 3).

When it comes to aesthetics in mathematics, the popular discourse focuses on elegant proofs and beautiful theories, usually the work of great mathematicians. Hardy (1992/1940) praised Euclid for his "simple" and elegant proof of the existence of an infinity of prime numbers. Hardy considered mathematics as an art and the mathematician a creative artist. Pacey (2001) described that Kepler had difficulty when he worked on the laws of

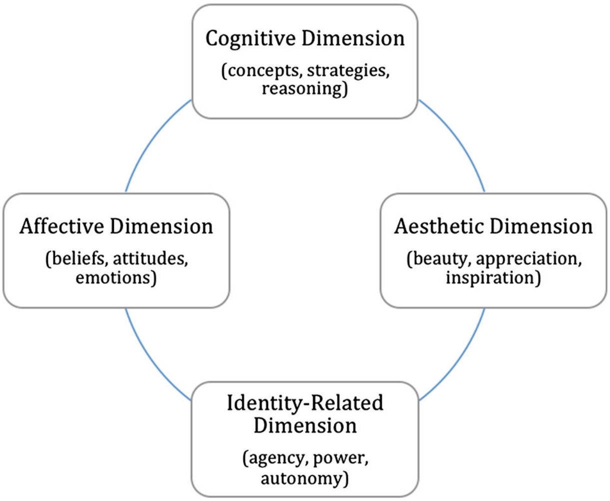

Fig. 3 Cognitive, affective, identity-related, and aesthetic dimensions of mathematics teaching and learning

planetary motion because the data just did not fit the beautiful mathematical model he thought was correct. According to Pacey, Kepler invented his patterns and eventually imposed them on nature. "His insights were poetic but his mathematics seems contrived" (p. 29). Indeed, mathematicians can experience the world through mathematics much like an artist who sees the poetic beauty of the world.

Aesthetic experience with mathematics is not proprietary to professional mathematicians. The findings of this study show that ordinary teachers can have some aesthetic experience with mathematics. This is evident in previous research as well. For example, prospective teachers developed aesthetic criteria to judge the mathematical quality of the problems they posed for students (Crespo and Sinclair 2008). Middle school students generated criteria for judging the aesthetic qualities of mathematical questions and conjectures (Lehrer et al. 2013). These two studies show the *evaluative* role of aesthetics, which is also found in the current study. In addition, four of the six participants developed a general aesthetic relationship to mathematics. In particular, Andrea's anthropological account of mathematics reflects a broad human–mathematics relationship. Witz and Lee (2009) found that some teachers developed a "higher aspect" of science and attached the subject with metaphysical, moral, or aesthetic values, which influenced how they responded to science education policy and reform. Indeed, the current study found that prospective teachers' aesthetic inspiration about mathematics influenced how they related to mathematics, which was a part of a larger understanding of their mathematics histories and journeys of becoming teachers.

It is important to explore the different aesthetics between professional mathematicians and ordinary learners. As described in Sect. 2.3, Sinclair (2004) discussed the evaluative, generative, and motivational roles of aesthetic. Her analysis is mostly based on professional mathematicians' experiences. Similarly, Burton (2004) used this tripartite model in her case studies of research mathematicians. The current study extends this line of research to include prospective teachers' aesthetic encounters in mathematics. The findings suggest that the aesthetic played the *evaluative* and *motivational* roles but not the *generative* role.

Four of the participating teachers had aesthetic evaluation of their mathematical ideas. The aesthetic aspect also motivated three of them but discouraged the other teacher in mathematics. It is worth noting that the nature of the motivational role in these four cases is different than what Sinclair (2004) discussed for professional mathematicians. For Sinclair, aesthetics motivates mathematicians to pursue certain disciplines and problems. This choice is not feasible in schools because teachers and students often are given a set of standards and a curriculum to follow; they do not choose areas of investigation. Therefore, Sinclair's three roles will likely to be different in schools.

I have focused on mathematical aesthetics. I shall not lose sight of the cognitive and affective dimensions of a person's mathematics identity. Distinguishing the aesthetic from the cognitive and affective will only give a fragmental picture. Witz (2007) posited that mathematics learners' abilities, interests, attitudes, personalities, and so on formed a great unity, and their whole involvement and aesthetic inspiration in mathematics must be understood as only a part of this unity. This understanding is in agreement with the argument put forth in this article, that the aesthetic dimension should be considered in relation to cognitive and attitudinal dimensions when we explore mathematics teachers' identities and professional orientations.

Implications

A clear implication for research is a need for a range of theoretical accounts of the aesthetic dimension, particular for multicultural education. As I discussed in the above, researchers apply John Dewey's philosophy of aesthetics to understand students' and teachers' experience with mathematics. Indeed, Dewey's philosophy on experience and aesthetics provides a theoretical lens through which to analyze learners' involvement in mathematical activities and their relationship to mathematics; however, he did not address multicultural education (Noddings 2012). Dewey would have little to suggest for dealing with pressing issues pertaining to culture, race, poverty, and power in mathematics education. How do culturally bound perspectives on the role of mathematics in teachers' lives shape their aesthetic relation to mathematics? How does the distribution of power in classroom influence learners' aesthetic understanding? Additional theoretical lenses are needed in order to understand diverse learners' aesthetic experiences.

Another implication pertains to the particular form of mathematics practiced in schools. The prospective teachers in the current study had a different aesthetic experience than professional mathematicians. Willingham (2009) argues that it is unrealistic to expect mathematics learners to think and feel like a mathematician because of the different cognitive structures between novices and experts. Therefore, future research can explore the distinct roles of mathematics aesthetics in *schools* and how teachers can feasibly enact these specific roles in their classrooms. There is a need for more research and a localized theory of aesthetics that is situated in the K-12 setting. A unique aesthetics in schools may avoid prescribing an abstract "aesthetic learner" formulated in an alien context. Critics (e.g., Shutkin 1998; Valero 2004) have questioned the taken-for-granted assumption of the Cartesian, cognitive learner in mathematics education research and practice. By the same token, discourses about the aesthetic nature of mathematics, particularly among professional mathematicians, may constitute educational subjects alienated from teachers' and students' lived experience. If we acknowledge the diverse histories of teachers and students, their psychological and sociopolitical identities, and their different inspirations for doing and feeling mathematics, then we will appreciate different styles of aesthetic relationships to mathematics than those of professional mathematicians. How do these factors shape teachers' and students' notions of which aspects of mathematics as beautiful, ugly, delightful, or scary? How do individual styles of aesthetics reflect their participation and agency in the mathematics classroom? These questions are worth pursuing.

The findings of the study suggest a need to engage teachers in a reflection on the nature of mathematics and what it means to do mathematics. Buerk's (1982) experience with adult learners provides useful suggestions for teacher education. Buerk challenged the five women in her study to re-conceptualize the nature of mathematics from a narrowed, dualistic view that mathematics is a collection of right answers and correct procedures to a view that they could be creative and could have their unique approach to doing mathematics. These women's understanding that mathematical thinking could take on many "styles" reflects an aesthetic experience of mathematics. Buerk's "secret" lies in carefully designed mathematical tasks and reflection prompts for adult learners. First of all, mathematics problems that have some ambiguity and can be approached in diverse ways (rather than calling for a specific formula) are used. This way, teachers can bring their own experience and meaning to the problem. Inquiry should focus on the meaning of the problem rather than its answer. Problems that may generate a conflict between teachers' intuition and the logic/data are helpful because they can lead teachers to take the responsibility for making commitments to their world views. It is also important to examine supportive evidence and argument, evaluating the effectiveness and elegance of ideas. Moreover, opportunities should be given to teachers to create their own problems and select options. Lastly, teachers can reflect on the nature of mathematical knowledge, their relationship to it, and what it means to do mathematics.

As for students, Duckworth (2006) stated that "The having of *wonderful ideas* is what I consider the essence of intellectual development. And I consider it the essence of pedagogy to give Kevin [a student] the occasion to have his *wonderful* ideas and to let him *feel good* about himself for having them" (p. 1, emphasis added). The phrases "wonderful ideas" and "feel good" reflect exactly the aesthetic experience in learning. One of Naomi's teachers encouraged students to share their good ideas. Most teachers stop here. If they can further provide reflective prompts for aesthetic appraisal of good ideas, then students will have the opportunity to experience the aesthetic nature of mathematics and inquiry. For example, the middle school students in Lehrer et al. (2013) discussed about: "What makes a good question or conjecture? Are all of these good questions and conjectures? Or, are which ones are *really good* and why?" (p. 370). Other examples are: Is that a beautiful idea or what? Look at these two solution strategies. Which one is more elegant? How did you feel when you came up with that idea? Why did you choose to use (or not to use) this method? Do you think/feel it is neat or awkward? In addition, teachers can engage students in developing their own aesthetic criteria in doing mathematics. On an ongoing basis, students can contemplate what it means to do mathematics and their relationship to mathematics. Indeed, mathematics can be taught as an art besides as a science of pattern.

Promoting aesthetics in mathematics education can become a double-edged sword. On the one hand, Sinclair (2009) worries that the discourses about the aesthetic nature of mathematics may reinforce the dominant belief that such aesthetics resides only in a few intellectual elites. According to Sinclair, this belief, together with other subtle educational discourses and practices such as the marginalization of geometry in the school curriculum, may function to maintain the current unjust power relations in mathematics education (in the Foucaultian sense of *power*, see Walshaw 2004). On the other hand, "the attention to aesthetic considerations in research—especially in terms of power dynamics—may also have more transformative influence, helping to suggest new possibilities, draw attention to problems that have not been recognized, or whose solutions have been taken for granted"

(Sinclair 2009, p. 58). One new possibility is for teachers to conceptualize empowerment not only in the sense of providing students with access to powerful mathematical ideas but also a way to fashion themselves as pleasant doers of mathematics. This may not be adequate to shake up the existing power relations in mathematics education, but it will likely make teachers and other educators aware and become sensitive to students' inner inspirations and creative ideas, a prerequisite for promoting agency and autonomy.

To conclude, I believe that teachers and students can broaden their views of mathematics and reconsider their relation to mathematics. Even for ordinary learners, mathematics needs not be merely an instrumental reality; mathematics can be immanent in one's lifeworld. Teachers should consider the aesthetic dimension of mathematics along with the cognitive, affective, and identity-related dimensions. Learners can develop aesthetic meanings and an emotional affiliation with mathematics, a condition that Dreyfus (2008) regards as necessary for a novice learner to become an expert. To this end, teachers can provide students with the opportunity to experience and reflect on the aesthetic value of mathematics and see how mathematics can be part of a larger understanding of oneself. The aesthetic aspect of mathematics sees mathematics as an expression of human thoughts and a means to appraise such thoughts. Although such issues are being examined, they are rarely presented in mathematics methods courses and K-12 classrooms. If the process of preparing mathematics teachers starts when students enter kindergarten and first begin to learn mathematics in school (Philipp 2007), should we not provide children with broad linguistic and aesthetic frameworks for them to think about mathematics and about themselves?

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