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# Characterizing a highly accomplished teacher's noticing of third-grade students' mathematical thinking

**Rukiye Didem Taylan**[1]

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**Abstract** This study investigated a highly accomplished third-grade teacher's noticing of students' mathematical thinking as she taught multiplication and division. Through an innovative method, which allowed for documenting in-the-moment teacher noticing, the author was able to explore teacher noticing and reflective practices in the context of classroom teaching as opposed to professional development environments. Noticing was conceptualized as both attending to different elements of classroom instruction and making sense of classroom events. The teacher paid most attention to student thinking and was able to offer a variety of rich interpretations of student thinking which were presented in an emergent framework. The results also indicated how the teacher's noticing might influence her instructional decisions. Implications for both research methods in studying noticing and teacher learning and practices are discussed.

**Keywords** Teacher noticing of children's mathematical thinking · A highly accomplished teacher · Attending to student thinking · Interpretation of student understanding · In-the-moment noticing of a teacher

## Introduction

One of the core principles of teaching mathematics for understanding and promoting mathematical inquiry is building teaching based on student understanding and thinking of mathematics (Ball et al. 2001; Fennema et al. 1993; National Research Council 2005). Research shows that attending to mathematical thinking of students in professional development programs can help improve both teaching quality and student achievement

✉ Rukiye Didem Taylan
tayland@mef.edu.tr

[1] Faculty of Education, MEF University, Ayazağa Cad. No: 4, 34396 Sarıyer, Istanbul, Turkey

(Carpenter et al. 1999; Hiebert and Wearne 1993; Jacobs et al. 2007; Kazemi and Franke 2004). Familiarity with students' prior knowledge and understanding of mathematics can be useful to teachers in different ways. In some cases, children's everyday (or informal) knowledge or their invented strategies help teachers in their introduction of formal or academic knowledge (Ginsburg 2009). Additionally, Mack (2001) argued, "building on informal knowledge in a manner that frequently returns to initial conceptions may be a viable way for students to develop understanding of complex content domains" (p. 293). On the other hand, when teachers anticipate student difficulties in a certain topic, they can plan accordingly such that instruction addresses a potential student misconception (Graeber 1999). Recent studies of professional development for teachers established that learning how to use student thinking helped teachers change their instructional practices in a way that enhanced student understanding of mathematics (Franke et al. 2001; Kazemi and Franke 2004).

Arguably one of the most influential research programs in mathematics education, Cognitively Guided Instruction (CGI), established that providing examples of student work in professional development programs in the form of video clips or written work helped teachers' understanding of student thinking. Through participation in the project, CGI teachers began to recognize important aspects of student thinking and how they could change their teaching based on the analysis of student thinking, instead of categorizing student answers as right or wrong. As teachers became more familiar with student conceptions of elementary mathematics topics and strategies for solving problems, they grew more adept at introducing mathematical tasks and problems that engage and extend each students' understanding of a topic (Fennema et al. 1996). Although these findings establish the importance of mathematics teachers' understanding of student thinking and their pedagogical decisions based on this understanding, "creating instruction that builds on children's thinking has proven challenging" (Jacobs et al. 2011, p. 98). Consequently, there is a need to understand this core practice of teaching and help teachers improve their practices.

A commonly used theoretical construct used to shed light on how teachers understand and use student thinking during instruction is teachers' knowledge about students. In a seminal work, Shulman (1986) identified knowledge related to students' mathematical understanding as one of the major components of pedagogical content knowledge (PCK) that teachers need in order to teach effectively. Drawing on Shulman's work, Hill et al. (2008) identified KCS (Knowledge of Content and Students) which included different domains of knowledge: common student errors, student understanding of the content, student developmental sequences, and common student computational strategies. Despite increased efforts and emphasis in improving teacher knowledge related to students, teachers' understanding of and knowledge of student thinking may or may not be enacted as they teach (Hill et al. 2008). Several examples from previous literature also indicate challenges of using knowledge of students as learners of mathematics in the process of teaching, even for teachers who have this knowledge (Ball 2001; Ball et al. 2001; Schoenfeld 1998).

In addition to importance of teacher knowledge (Shulman 1987) and beliefs (Thompson 1992), teacher noticing or attention-dependent skills may help to explain how teachers with expertise are able to recognize and build on student thinking in successful ways when compared to novices (Ainley and Luntley 2007). Different than the nature of teacher knowledge, teacher noticing is directly related to how teachers make instructional actions during the act of teaching (Mason 2002). Exploring the use of student thinking from a teacher noticing perspective rather than teacher knowledge perspective may provide a more helpful knowledge base in order to understand how teachers attend, interpret, and respond to student thinking (Sherin et al. 2011a).

> Schifter (2011) argued for importance of studying teacher noticing in the following way:
> …when mathematics teaching is focused on students' conceptual understanding and when classroom discussion is a major mechanism for learning, what teachers notice in their students' communication is essential. It is what teachers notice and how they respond that guide students' attention to what they are to learn (p. 218).

Noticing student thinking is a critical skill where teachers may experience difficulties, and it is important for the mathematics education field to know how teachers with expertise demonstrate such practices as they teach. The purpose of this study was to investigate the characteristics of a highly accomplished third-grade teacher's noticing of student mathematical thinking in the moment of teaching. I use the term highly accomplished teacher (Schoenfeld 2011a) instead of the term expert teacher because the latter has not been well defined (Russ and Sherin 2011). Schoenfeld (2011a) defines highly accomplished teachers as teachers who spend less time on classroom management issues and who allocate most of their time in engaging in student thinking or diagnostic teaching, "where teachers shape their lessons according to what they discover about their students" (Schoenfeld 2011b, p. 463). Because I investigate teacher noticing of students' mathematical thinking, I took into account Schoenfeld's definition of a highly accomplished teacher in order to select a participant who demonstrated expertise in teaching. Little is known about how teachers notice student thinking during the act of teaching (Luna et al. 2009) and the nature of their instructional actions as they notice student thinking (Even and Gottlib 2011). Therefore, a significant contribution of this study is a greater understanding of the nature of teachers' noticing of students' mathematical thinking by taking into account their instructional practices.

## Literature review

Although noticing refers to general observations one makes in everyday life, professional noticing is defined as an area of expertise that enables members of a professional community to view complex situations in ways that enable them to process information more productively (Jacobs et al. 2010). Professional noticing has roots in the notion of *professional vision*, which is defined as "socially organized ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (Goodwin 1994, p. 606). Teacher noticing is a theoretical construct that has the potential to shed light on how to deal with complexities of teaching and make better instructional decisions as teachers build instruction by attending to particular classroom events, especially focusing on student thinking as recommended in reform documents. Sherin et al. (2011a) argued that, in general, researchers discuss teacher noticing as involving two main processes (or a subset of those processes):

- Attending to particular events in an instructional setting. To manage the complexity of the classroom, teachers must pay attention to some things and not to others. […]
- Making sense of events in an instructional setting. For those features to which teachers do attend, they are not simply passive observers. Instead, teachers necessarily interpret what they see relating observed events to abstract categories and characterizing what they see in terms of familiar instructional episodes. […] (p. 5).

In line with a focus on noticing in mathematics education research, Jacobs et al. (2011) stated that noticing children's mathematical thinking plays an important role in the moment of instructional decisions, especially when they need to respond to children's verbal or written explanations of mathematical work. Jacobs et al. (2011) conceptualized the construct of professional noticing of children's mathematical thinking as a set of three interrelated skills: (a) attending to children's strategies, (b) interpreting children's understandings, and (c) deciding how to respond on the basis of children's understandings.

A common way of researching teacher noticing is investigating how teachers react to video clips of their own classroom instruction, such as the context of a video club where a group of teachers get together and share their own clips of classroom instruction and comment on them collectively (Sherin and Han 2004; van Es and Sherin 2008). The findings of such studies suggest that most teachers are able to shift the focus of their reflections from classroom management issues or their own teaching to student understanding after participating in long-term professional development programs. Adding to what teacher educators know about teachers' reflections on classroom instruction, some studies have explored how teachers attend and respond to student thinking during professional development or teacher education programs by using video clips and written excerpts of student work (Goldsmith and Seago 2011; Jacobs et al. 2011; Son and Crespo 2009; Spitzer et al. 2011). Similar to the findings of the studies on video clubs, researchers observed that overtime and through participating in such programs, prospective and practicing teachers improved in their skills of productively attending to and making sense of student thinking after their initial difficulties. Teacher interpretation is considered *productive* when it is consistent with the details of specific student thinking (as seen in video clips) and research on student thinking (Jacobs et al. 2010; Mason 2002). Productive interpretations are *evidence-based*, as opposed to *snap-evaluations based on minimal evidence* (Jacobs et al. 2010, p. 173).

Previous research on teachers' reflections on videos suggests (Sherin and Han 2004; van Es 2011) that teachers generally struggle with making connections between general principles of teaching and learning and various interactions that take place in their classes. As a result of long-term interventions, some of these teachers are able to shift their reflections from a descriptive and evaluative stance to a more interpretive stance (van Es 2011). For instance, teachers who were in the final levels of noticing trajectory in van Es's study were able to provide evidence for their claims, view student thinking in connection to other elements they see in the video clips, make connections to principles of teaching and learning, and make alternative pedagogical decisions. Additionally, Goldsmith and Seago (2011) suggested that teachers who are able to productively interpret student thinking focus on the strengths or the potential of student understanding rather than deficits, and provide specifics of the mathematics content they analyzed.

Jacobs et al. (2011) investigated how teachers with different levels of experience differed in noticing students' mathematical thinking. One of the important findings of the study was that the experience of teaching did not necessarily result in advanced levels of noticing. Noticing is type of expertise that does not rely solely on innate ability or experience but can be enhanced by support and facilitation of teacher educators and colleagues. Although several factors such as teacher knowledge, beliefs, experience are hypothesized to play a role in teacher noticing, nature of video clips used for facilitation may play a role on the quality of teachers' reflections as well as what they see and interpret in professional development programs. For example, differentiating between viewing own practices versus other teachers' instruction, Seidel et al. (2011) observed that teachers who viewed their own teaching in video clips improved their noticing skills in better ways compared to teachers who watched other teachers' practices. More specifically, Miller (2011) argued that perspective of the videos that are used in noticing studies is important. According to Miller, video taken from the teacher's perspective provides a more complex and dynamic view of the classroom instruction due to movements of the teachers. Since teacher-perspective video is more similar to what teachers are accustomed to seeing in classrooms, these videos may facilitate a more detailed reflection from the teacher, thus allowing for an improved way of documenting teacher noticing.

Schifter (2011) noted that teacher noticing could not be fully captured in a professional development setting because "in the moment of teaching, noticing is guided by what the teacher is trying to teach and is followed by an action" (p. 218). Despite recommendations, only few studies documented teachers' noticing during teaching or took teaching practices into account while studying teacher noticing. Sherin et al. (2008) employed an innovative methodology to study noticing of thirteen teachers in the actual classroom contexts. This methodology required teachers to wear portable cameras. On the day the teachers wore the camera, the researchers instructed them to press the record button when there was an interesting instructional moment. The researchers deliberately did not define their criteria of interesting. Accordingly, teachers were then free to record as many different 30 s of video clips as they wanted. After each lesson, researchers interviewed teachers about the video clips they selected, asked their criteria for selecting the clips, and inquired about what teachers actually paid attention to.

Luna et al. (2009) and Colestock (2009) are two studies, which used the same data set by Sherin et al. (2008) and investigated teacher noticing using teacher perspective. Luna et al. (2009) investigated one high school biology teacher's selection of interesting events using a portable camera and her reflection on those events. Teachers reported selecting clips based on the following reasons: student thinking, student engagement, student characteristics, discourse, and task management. Similarly, Colestock (2009) investigated one mathematics' teachers' noticing. The analysis of the interviews revealed different types of student thinking that the teacher reflected on during the interviews: student justification of a solution, student thinking through a problem, student difficulty solving a problem, and students' insightful mathematical questions. An important implication of the study was that teachers' instructional actions during teaching were influenced by teacher noticing.

In summary, previous studies on teacher noticing were generally conducted in clinical settings, such as during professional development programs. This hindered understanding the nature of classroom environments, which influence teacher noticing. Due to nature of noticing and complexity of teaching, it was scarce to find studies such as Sherin et al. (2008), which investigated teacher noticing in the moment of instruction by means of observations of instructional actions and conducting interviews with the teachers on teacher-selected moments of instruction through a portable camera. Because my research question aims to investigate teacher noticing of students' mathematical thinking during instruction, methodology introduced by Sherin et al. (2008) is the most appropriate for my study. Using this method, teacher's selection of video clips during teaching and her reflections on the clips allow the researcher to gain insights on the teacher's mind during instruction. Since the teacher is not able to explain what she notices and teach at the same time, selecting moments she knows she noticed through using a portable camera may be a productive method to explore teacher noticing. Because teacher noticing investigated in this study is a highly accomplished teacher's noticing who engages in student thinking most of the time, using qualitative methods and exploring teacher noticing in natural settings has the potential to capture uniqueness and complexity of building instruction based on student thinking (Hatch 2002).

It is important to note that Sherin et al. focused “exclusively on a single component of noticing-that of attending to events” (Sherin et al. 2011b, p. 81), which excluded how teachers made sense of the events they attended to (i.e., their interpretation). Consequently, there is a need to investigate an important aspect of teacher noticing, namely interpretation, together with teacher attention. This is another aspect of my study, which has the potential to contribute to the literature on studying teacher noticing during teaching.

## Research methods

This study employed qualitative methods, specifically a case study research design (Bogdan and Biklen 1992; Hatch 2002). This study also investigated a teacher’s thinking from the perspective of the individual teacher, albeit paid attention to social perspectives at the same time by situating the teacher’s practices in her own social setting (Cobb et al. 2003).

### Participant

The participant was a highly accomplished teacher (Brooke). Brooke taught third grade for 6 years. She had 3 years of experience attending a contextualized and intensive professional development program within which she worked one-on-one with a nationally prominent teacher educator and educational researcher. As a part of a research team analyzing the influence of this professional development program on Brooke’s teaching, I had extensive opportunities to observe her classroom one year prior to this study. The research team recognized Brooke for her skills in teaching for understanding and using student thinking in her mathematics classes during the professional development program. As part of this program, Brooke was engaged in designing and adopting curriculum materials according to the needs of her students with the guidance of the teacher educator, and reflected on her own mathematics teaching practices through video clips selected by the research team.

### Data sources and data collection

Data sources in this study consisted of video records of three consecutive mathematics classes as captured by a whole-class camera, video records as captured by a portable camera worn by the teacher, student work and notes, field notes, and three 1-h-length videotaped interviews conducted with the teacher regarding her classroom instruction and video clips selected by the teacher from her portable camera. Three consecutive classes were scheduled to observe the teacher’s classroom. Interviews with the teacher took place shortly after each observed mathematics lesson and before the next day’s mathematics lesson such that the teacher was able to remember specifics of classroom instruction and why she selected “interesting” moments via her camera. During the data collection, the students worked on the concepts of multiplication and division.

## Data analysis

*Attending to student strategies and ideas*

The portion of teacher reflections when the teacher specifically provided explanation as to why she chose the video clip through the portable camera provided evidence that she attended to a particular event. Throughout the interview transcripts, teacher reasons for selecting the video clips were identified by open coding. The resulting codes were tested for reliability by comparing codes with another researcher's. Codes were refined until all the conflicts were resolved. Although I anticipated finding similar categories to previous research (Colestock 2009; Luna et al. 2009; Sherin et al. 2008), I began coding the transcripts with an open mind, in order to be guided by the data itself rather than by previous conceptual frameworks (Wallcott 2010). Because the purpose of my study was to analyze the way the teacher attended to third-grade students' mathematical thinking specifically, different than Luna et al. (2009) and Colestock (2009), I incorporated dimensions with regard to different types of students' mathematical thinking, which the teacher referred to during the interviews when she provided the rationale for selecting clips via the portable camera. It followed that specific dimensions of students' mathematical thinking that Brooke attended were *student strategies*, *student understanding*, *student difficulty*, *making connections*, and *providing explanations*. Additionally, I included details on classroom norms. The different classroom norms to which Brooke attended were *checking work*, *admitting/learning from mistakes*, *partnership (working in groups)*, *providing feedback for peers*. Figure 1 provides definitions of each code in analyzing how a teacher attended to particular elements of classroom instruction. It is important to note that rationale for selection of the clips are, in most cases, linked to multiple dimensions of the analytical framework.

**Student thinking**

- *Student strategies*: A student's strategy in solving a mathematics problem.
- *Student understanding*: Student understanding of a concept or a problem.
- *Student difficulty*: Student difficulty in understanding a concept or solving a problem or misconceptions.
- *Making connections*: A student makes a connection between two different problems, or two different concepts.
- *Providing explanation*: Student's explanation, which could be a valid or invalid explanation from the teacher's perspective.

**Classroom norms**

- *Checking work*: A student checks her answer in solving a mathematics problem or fails to check her answer.
- *Admitting\learning from mistakes:* A student realizes or admits making a mistake. In some cases it indicates student learned from a mistake.
- *Partnership*: Partnership and the way two students interact in solving mathematics problems.
- *Providing feedback to a peer:* A student provides feedback to a peer. The teacher typically refers to the quality of feedback.

**Student characteristics**

- Student characteristics such as students' personality or attributes as learners.

**Fig. 1** Analytical framework: analyzing what the teacher attends to

*Interpreting student thinking*

I explored how the teacher interpreted her students' mathematical thinking in this study by analyzing the interview transcripts. During the analysis of interpretation of student thinking, I paid particular attention to chunks of reflection related to student thinking. I primarily drew upon two of the analytical frameworks developed by van Es (2011) and Goldsmith and Seago (2011). Additionally, through open coding and grounded theory techniques (Glaser and Strauss 1967; Strauss and Corbin 1998), different patterns of elaboration of student thinking required attention. The resulting codes were tested for reliability by comparing codes with another researcher's. Codes were refined until all the conflicts were resolved.

*Using evidence* and *making connections to general principles of teaching and learning,* and *making alternative pedagogical suggestions* were shown to be indicators of productive reflections and a sign of expertise in interpreting student thinking in previous studies (Goldsmith and Seago 2011; van Es 2011). However, other patterns emerged from the analyses of teacher data: *making reference to future instruction*, *making connections between student thinking and teaching goals*, and *making connections between student thinking and another time of instruction*. This study argues that these additional dimensions add richness to a teacher's reflections by way of making several connections, including how a teacher views student thinking in relation to her future instruction and teaching goals as well as how she

**Table 1** Analytical framework: analyzing the teacher's interpretation of students' mathematical thinking

| Different types of interpretation | Definitions |
|---|---|
| Use of evidence in making claims about student thinking | Teacher refers to specific events or interactions as evidence when she makes a claim with regard to student thinking |
| Making connections between student thinking and teaching goals | Teacher makes connections between student thinking and teaching goals, whether student thinking meets or fails to meet these goals, including goals of curriculum |
| Making connections between student thinking and principles of teaching and learning | Teacher makes connections between student thinking and principles of teaching and learning. Teacher elaborates on student thinking in terms of their knowledge and understanding about teaching and learning |
| Making connections between student thinking and another moment of math instruction in her class | Teacher talks about student thinking by mentioning another moment of instruction, student thinking during previous lessons or previous concepts. Student thinking is not isolated from previous teaching moments |
| Making reference to future instruction | Teacher makes reference to future instruction based on their reflection on student thinking. Teacher refers to next lesson or future units. Again, student thinking is not considered in isolation |
| Making alternative pedagogical decisions | Teacher offers alternative pedagogical decisions to what they actually did during instruction. According to van Es (2011), this is the ultimate goal for teachers, to have them make alternative pedagogical decisions and implement them the next time they teach the same topic |

situates student thinking in relation to another time of instruction. These patterns found in a teacher's interpretations may be evidence of her expertise in building instruction based on student thinking while she constantly compares student thinking with her plans of future instruction, her teaching goals, and another moment of instruction. Table 1 illustrates different types of interpretation of student thinking as occurred in this study.

### Limitations of the study

One could argue that limited number of observations and interviews may hinder making of inferences from the study. Future studies need to investigate noticing of other highly accomplished teachers as they teach similar and/or different topics across various grades and determine patterns that could be helpful for teacher education research. Teacher noticing of student thinking may appear different in other grade levels and while teaching different mathematical topics. There are also methodological challenges associated with the task of assessing teachers' in the moment of noticing. For example, teacher-selected moments of classroom instruction constituted evidence of attending to a particular event in this study. While this method allowed me to understand what the teacher noticed during teaching, it is also likely that the teacher chose to ignore some instances that she had noticed. Therefore, not selecting a clip may not necessarily indicate that the teacher did not notice a particular moment. Instructions related to selecting moments of teaching through portable cameras were deliberately ambiguous; the teacher made her own decisions regarding the selection of clips. Future studies may experiment with giving more direct instructions to teachers regarding the criteria and the number of clips to be selected. Another limitation is regarding the data analysis process. This study investigated teacher noticing from the researcher's point of view. The classifications of attending to different events and interpretation of students' mathematical thinking could be different if the teacher analyzed her own actions. Future research may experiment with such methods of data analysis.

### Statement of human rights

The study has been approved by the appropriate Institutional and National Research Ethics Committee and has been performed in accordance with the ethical standards as laid down in the 1964 Declaration of Helsinki and its later amendments or comparable ethical standards.

## Results

What happens in the classroom and the classroom environment shapes teacher noticing together with teacher experience, knowledge and beliefs. Therefore, it is important to provide background information about Brooke's classroom environment and her instructional practices in order to have a better understanding of her noticing.

### Classroom environment and instructional practices

With regard to mathematics instruction in her classroom, Brooke aimed for her students to comfortably share their mathematical thinking and "think for themselves" (Brooke, the first interview) instead of seeking her approval. For the most part, she was happy that her students learned that it was okay to make mistakes, and that they needed to justify their answers instead of merely focusing on whether answers were right or wrong. Brooke explained that they switched curriculum in the district a few years prior and she was not as happy about the new curriculum materials, which led her to design activities with colleagues and other teacher educators during a professional development program. In the following paragraph, Brooke explained why she preferred the past curriculum materials, which revealed her emphasis on conceptual understanding rather than procedural knowledge as a teacher:

> Using a reform oriented curriculum, I feel kids were given power to do their own thinking and have authority in the classroom, up in front of a board talking to their peers about what they felt was an answer. And I feel there was an emphasis on a kid's journey to getting to an answer and not just 'is this correct or is this incorrect.' I felt there was less focus on algorithm and procedural thinking and we switched curriculums a few years ago to the new curriculum which was a lot of topics to cover but the way they organized it did not seem to make sense for a lot of my kids. Like one day it is one digit multiplication and second day it is two-digit multiplication, and the third day it is three-digit and it was not a lot of that hands-on thinking about this conceptually. I think there is definitely a place for working with algorithms and procedures but I don't think it is the second day or the third day that you learn something (Brooke, first interview).

The research project took place when Brooke's class was making a transition from learning multiplication to division. The activities that Brooke created and planned together with her colleagues involved different models of multiplication and division: students jumping on the number line taped on the floor to represent skip-counting situations, and cutting ribbons of equal length for wrapping gifts.

Brooke aimed to create meaningful experiences for her students so that they could make connections between the new topic of learning division and what they already knew in multiplication. The tasks Brooke introduced in each class and how she implemented them demonstrated her focus on mathematical inquiry and conceptual understanding of the concepts she was teaching related to multiplication and division. The classroom environment encouraged students to share their answers, comment on each other's thinking respectfully, and provide justification for their answers.

During the first day of data collection, students were introduced to the activity "jumping on the number line." Students jumped on the number line taped to the classroom carpet. Students were asked to figure out how many times a student should jump from 0 to 20 units on the number line if she always jumps 5 units (or is a plus 5 jumper). Additionally, students were asked to find how many units they should jump each time ("What kind of a jumper are you?") if they are to finish the number line by jumping 4, 5, and 10 times. Brooke asked students to discuss how students could translate the length and number of their jumps to division and multiplication sentences. Some students in the class had difficulties in understanding the difference between multiplication and division. Brooke also emphasized the meaning of each number in multiplication and division sentences. For instance, she emphasized that when a student was a +5 jumper (or her jumps are 5 units long), the multiplication sentence represented 4 groups of 5 units, which was equal to 20 ($4 \times 5 = 20$). Brooke discussed with students that 4 groups of 5 units or 5 groups of 4 units would help a student land at 20 (given that the student started at 0) but 4 groups of 5 units represented the situation better for a plus 5 jumper. Brooke also reminded students there were two types of division problems they worked on last week as a class. Brooke referred to *partitive* and *measurement* division problems (Tirosh and Graeber 1991) without identifying their formal names and emphasized the need to differentiate between finding the number of jumps one makes and how many units one jumps in the following way:

> Now I have two kinds of problems. I have the ones where you have to figure out how many jumps he makes and I also have these problems where you have to figure out what kind of a jumper somebody is. Do you remember from last week sometimes it was hard for you guys to figure out the number in each group or how many groups there were? (Brooke, 1st class, whole class discussion).

After whole-class discussion, students were given a worksheet to model jumping situations with other group members. Final question of the worksheet was finding patterns in multiplication and division sentences. Brooke walked around the classroom to see how students worked in groups and their thinking about the questions. The students struggled to find patterns. Several misconceptions were revealed as Brooke walked around different groups. During the whole-group discussion at the end of the class, students shared their thinking and Brooke challenged misconceptions.

Brooke identified one student who demonstrated a better understanding of the problems during group work and asked her to come to the board and explain her thinking to the class (Fig. 2). Later in the interview, Brooke revealed that she preferred students sharing their thinking rather than her and, specifically, a student who could provide a meaningful explanation on the blackboard. During whole-group discussion, when Brooke invited students to comment on the one student's work and ask questions, another student commented on finding a pattern in multiplication sentences where numbers went from small to big ($5 \times 7 = 35$) and in division sentences numbers went from big to small ($35 \div 7 = 5$). Brooke provided a counterexample of multiplication and division sentences to challenge the student's misconception: $7 \times 5 = 35$ (where numbers do not grow from small to big) and $35 \div 5 = 7$ (where numbers do not get smaller) (Brooke wrote with a red marker in the picture on the smart board).

Brooke aimed for students meaning making, and she was puzzled that many students had this misconception:

> ….I want them to make sense of it and thinking about I have this group of things that I am either sharing or I have this linear example where I am splitting it. I want them

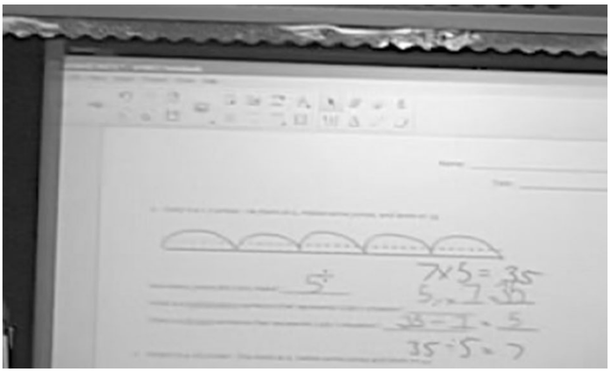


**Fig. 2** Finding patterns in multiplication and division sentences

> see that idea more than just I am plugging these numbers into this sentence. I am concerned that they have got these rules…. They are thinking about this rule that they have kind of thought when they were going through this pattern that they wanted to fit into so I am curious about how I am going to fix that (Brooke, interview after first observation).

Observing student difficulties on the first day, Brooke thought modeling the multiplication situation in different ways might help students further their understanding. On the second day of teaching, Brooke used a different situation that modeled multiplication and division. She brought boxes and ribbon and scissors to model a gift-wrapping scenario during the whole-class discussion. She asked students to share multiplication and division sentences, which modeled finding the length of equal pieces of ribbon given the total length of ribbon and the number of gifts to be wrapped as well as finding the number of gifts given the whole length of a ribbon and length of each equal piece. The following problems are examples of the worksheet problems:

"Mrs. Dicks has books to wrap for her students. She plans on tying a ribbon for everyone. Mrs. Dicks has 45 inches of ribbon. She is going to cut the ribbon into 9-inch strips. How many books can she wrap?"

"Miss Hatcher has 50 pencils as gifts for her students. She plans tying a ribbon around each one. She has a long piece of ribbon that is 200-inches long. She will cut it into equal sized pieces. How long will each piece of ribbon be? Write a number sentence to represent the problem."

Students worked on a worksheet with similar problems in groups of two making use of different types of manipulatives (number lines, meter sticks, cubes, etc.) to answer problems. Brooke walked around the groups of students and helped them understand the problems on the worksheet. Brooke had the flexibility of changing the lesson plans by adding tasks she considered to be necessary. For instance, on the second day of instruction, some students used multiplication sentences to model cutting a ribbon into equal pieces instead of division sentences. Clearly, students had a difficulty in understanding the difference between multiplication and division sentences. On the third day, Brooke presented the following task given in Fig. 3. Brooke later explained during the interview that she liked to present students with this type of "fake student work" which illustrated the same type of misconceptions her students had. This method allowed her students to comment on the method freely because they did not know whose work it was.

Most students who agreed with this misconception in the beginning of the lesson (more than half of the students in class) subsequently changed their thinking toward the end of the

> *Mrs. Paul has got this kid in her classroom named Sam. Here is the problem that she gave him:*
>
> *Mrs. Paul had 24 inches to wrap two presents. How long will each piece of ribbon be?*
>
> Alright so here is his work. Sam wrote 24 + 24 =48 and 24 X 2=48.
>
> If you agree with Sam's work you are going to go ahead and tell me why you agree. If you don't agree with Sam's work and his ideas, I want you to tell me why (Brooke, third lesson).

**Fig. 3** A supplemental task after coming across a misconception. Transcript based on the researcher-camera third class. Implementation of this task (Fig. 3) could be considered as responding to student thinking based on the teacher's noticing. The following results provide evidence of Brooke's attending to particular events by providing rationales for selecting video clips and her advanced levels of interpretation of student thinking that she focused her attention on.

## Characteristics of the highly accomplished teacher's noticing

### *Attending to student thinking and other elements of teaching across three classes*

Brooke picked eight clips during the first lesson, five in the second and eight during the third in class, for a total of 21 clips across three lessons. Brooke explained that she did not consider many instances of student thinking worthwhile to discuss because she did not want to choose similar types of thinking. She also noted: "maybe my standards for the conversations are fairly high" (Brooke, the third interview). Among all other elements of classroom teaching, Brooke referred most frequently to student thinking. The codes for attending to student thinking comprised of about 85 % of all codes applied to her reflections. More specifically, Brooke attended to student understanding and students' providing explanation for their answers. The other most frequently mentioned reason for documenting particular moments of instruction was student strategies.

Table 2 provides examples of coding what Brooke attended to. Although Brooke mostly referred to student thinking, she also chose clips because they illustrated norms of her classroom teaching such as admitting and learning from mistakes and providing feedback to a peer in an appropriate way. Additionally, she referred to student characteristics. An interesting pattern in Brooke's sequence of lessons is that Brooke referred to student characteristics more frequently during the third lesson when there was less focus on whole-class instruction and more focus on one-to-one interaction with students. As seen by the example in Table 2, Brooke did not mention student characteristics in an evaluative way, rather she utilized what she knew about student characteristics in order to make sense of what happened in her classroom.

The clips Brooke chose can be grouped into three categories in terms of the content of student thinking: clips about student strategies and misconceptions in finding the pattern between multiplication and division sentences, student thinking in figuring out wrong student work presented in Fig. 3, and student strategies and difficulties in finding the length of equal length ribbon pieces or the number of gifts to be wrapped. In the next section, I present the details of Brooke's interpretation of student thinking.

### *Interpreting student thinking*

Brooke not only described classroom events in relation to student thinking but also made several types of productive interpretations of student thinking. Specifically, she provided evidence of her claims about student thinking and made connections between what she observed in the video clips with general principles of teaching and learning. Brooke also referred to future instruction and shared instructional goals and potential to-be-implemented instructional strategies. Moreover, Brooke made connections between what she saw in the video and other moments of instruction (e.g., last class, last week's instruction, or times she taught addition and subtraction). On the other hand, making alternative pedagogical decisions was not observed as frequently as the other codes (only a few times throughout the interviews) and consequently was dropped from the results section. The

**Table 2** Evidence of Brooke's attention to particular classroom elements

| Dimensions | Examples |
|---|---|
| *Student thinking* | |
| Student strategies | "I thought that she had a really efficient strategy for doing this because a lot of my students wanted to skip count but they wanted to check every number and skip count 50 times to 200 and so that takes a really long time and they are not gonna be accurate so she decided well I am just gonna figure how many 50s in 200."(Brooke, the third interview) |
| Student understanding | "When I saw what she had done, she actually was making sense of what she was doing and also her picture I mean it was really pretty efficient. I mean she was using 50s. 4 groups of 50s. She knew that represented 4 inches. I thought that was really sophisticated." (Brooke, the third interview) |
| Student difficulty | "I chose this one because Daniel had that misconception that he had done 36 times 3 [instead of 36 divided by 3]." (Brooke, the third interview) |
| Making connections | "I chose this clip because I thought Daniel's thinking, I thought he did some really good connections between what he was doing with multiplication and addition to figure out this division problem." (Brooke, the second interview) |
| Providing explanation | "I just chose it because I felt like I saw this he had the same thinking as some of the other kids I had talked to but I felt like in his explanation he was a little bit further along in his thought process why it would make sense."(Brooke, the first interview) |
| *Classroom norms* | |
| Admitting\learning from mistakes | "And the at the end he talks to me and he is like "look I did it right, this is what I have got" and so I felt like it was a kind of good moment of showing a moment of "oh maybe I have made a mistake in this but I am gonna change my thinking."(Brooke, the third interview) |
| Partnership | "I thought it was a really sophisticated way of partnership to work that it is okay to have two different answers and they both could speak to why they had figured it out in that way to me." (Brooke, the second interview) |
| *Student characteristics* | "I just chose it because it was interesting to see he had really good explanation for what he had done but he is a quieter kid so I wanted to choose him because I think also because he wants me to tell him. I wanted to show where kids are at the spectrum he definitely wants that approval of things he says." (Brooke, the first interview) |

section below contains examples of patterns in Brooke's interpretations of student thinking as expressed through video clip reflection and description.

*Using evidence in making claims of student thinking* Brooke frequently used evidence when making claims about student thinking. In fact, it was the most commonly observed code throughout the analysis of her reflections. The following video clip reflection is an example of Brooke using evidence in making claims about student thinking. In this video clip, the student described why he wrote division sentences in the way he did ($35 \div 5 = 7$ and $35 \div 7 = 5$). Unlike other students who plugged in numbers in the multiplication and division sentences in a coincidental way, this student demonstrated numbers sense. He stated that 7 divided by 5 would be about 1 and 5 divided by 7 would be "negative." Brooke acknowledged that the student did not have a complete understanding of division sentences, and he was confused about fractions versus negative numbers. However, Brooke also appreciated that the student had an intuitive idea about why his answer was correct.

The italicized statements indicate where Brooke made a claim and used evidence to support her claim. The brackets following the italicized statements show how I coded the statements.

> I feel he had thought it through a little bit more thorough [than other students], "maybe 5 should go there, oh it does not make sense" or "maybe 7 should go there that does not make sense" so I feel like he kind of hashed out some of the things that other kids had not really thought about yet. And so that is something that I will probably draw upon later on if I notice kids still having just plugging numbers in. *His explanation where he was talking about giving an example "if I had 7 and shared with 5 it would be about", that would make sense. I think him going through examples and going through some choices* [providing evidence that the student is demonstrating more advanced levels of thinking when compared to peers who were plugging in numbers without understanding] *would be probably a little further along in his thinking than others* [making a claim that a student's thinking is more advanced than others in a specific problem]. (Brooke, the second interview, Lesson 2, Teacher-selected Clip).

In the above excerpt, Brooke claimed the student had an understanding more advanced than his peers. She also explained the student's thinking in detail, and in a way that supported her claim about student thinking.

*Making connections between student thinking and general principles of teaching and learning* Another common pattern observed in Brooke's interpretation of student thinking was the connections she made between student thinking and general principles of teaching and learning. Making this type of connection may be natural for a teacher familiar with research and with experiences in both teaching and professional development. In the following example, Brooke explained the difficulties of assessing student understanding when students worked in pairs. Brooke was unable to assess the particular student's thinking because he was working with a partner:

> I don't know if he was relying a lot on Rita's work or not so that's why I need to talk to him tomorrow. *Partner work I like but it is really difficult to figure out whose work you are really looking at, who understands, who does not understand.* So I don't really use that as a gauge to pull groups for certain skills because it is really hard for me to know [Make connections between student thinking to general principles of teaching and learning- it is difficult to gauge individual student understanding by only paying attention to group work] (Brooke, the third interview, Lesson 3, Teacher-selected Clip).

Some of the other general principles with regard to mathematics teaching and learning referred to by Brooke during the three interviews included taking into account students' characteristics in group work, and the need to use a variety of models of multiplication and division to support students' understanding.

*Making connections to another (previous) moment of instruction, referring to future instruction and connecting student thinking with teaching goals* Brooke believed sharing her students' strategies or misconceptions with other students was helpful for furthering student understanding of a topic. Consequently, she chose video clips, which were examples of student thinking that she could share with other students in the following lessons. In doing this, she kept comparing her goals of instruction and student thinking she observed. For instance, when students had difficulties in solving a problem, she looked for different strategies and types of thinking, some of which she planned to share with other students. She caught a variety of student thinking that demonstrated students' strengths and weaknesses in making meaning of multiplication and division sentences. In general, Brooke chose video clips of students' thinking based on their strategies and how she could use a particular student's thinking in building future instruction, or how she made sense of the situation by remembering past situations. She was able to remember details about student learning in past weeks, even past months, and made connections to what she saw in the video clip. These qualities of Brooke's noticing may enable her to adapt her teaching based on student understanding.

Below is an excerpt from Brooke's reflections on using the video camera. In it she described how she made connections across different moments of instruction, her goals, and future instruction by way of discussing a particular student's misconception in using multiplication and division sentences. From this excerpt, it is clear that Brooke made instructional decisions based on her noticing of what she saw in the video clips. The brackets following the italicized statements show how I coded the statements.

> *I had like noticings with Rita yesterday I was making the connection that she was having this misconception about this being multiplication situation* (*referring to task presented in* Fig. 3) *and seeing that she had that misconception in our big group and then she continued that in her work and then when we came back she was also having those same misconceptions yesterday and then when I saw the video I noticed that she had started off the day with that* [Making connections to another moment of instruction] so then today I was able to look…*I wanted to make sure that she was a kid that changed her ideas* [Making connections between student thinking and instructional goals] so then I was able to look at that and think "*oh today I am definitely gonna talk to her before she goes back and does her independent work"* [Referring to future instruction] and so I was able to change what I was going to do today based on some of the things I saw in the video yesterday (Brooke, third interview).

When Brooke reflected on 30-s video clips she selected, she was able to provide productive and rich interpretations of student thinking which most often included different types of connections she made to student thinking: specifically, connections to instructional goals, future plans of instruction, and previous moments of instruction. Because these types of connections appeared to be closely tied to Brooke's instructional actions, and they frequently occurred, I provide more examples in the following paragraphs, which illustrate integration of such connections. In the following excerpt, Brooke described how a student worked on solving a problem, which required dividing 36 inches of ribbon to 3 equal pieces. Brooke focused on student thinking by making connections to the same student' strategies she observed during the previous week. She compared students' understanding at the beginning of the lesson to understanding at the end of the lesson. Additionally, she made connections to instructional goals. The italicized statements indicate where this type of interpretation occurred throughout Brooke's reflection. The brackets after the italicized statements show how her statements were coded and provide detail on the problem Brooke referred to.

> I chose this one because Daniel had that misconception that he had done 36 times 3 and so then after he listened to Adam's explanation of "maybe I need to deal these 3

> out and do this strategy" *I might have chosen a clip after this that talks about him saying "oh I am gonna deal" which is a strategy that he had used last week which I thought was really smart* [Make connections to another moment of instruction-student strategies she observed last week] which is different that he was making those connections between the sharing situations and this model too. For this clip he had that moment of "oh this does not make sense" (referring to Fig. 4) and so *I was happy to see that he was ready to look over it again and realize that* "*you know what I actually did this 36 times 3 instead of dividing it up*." *And the at the end he talks to me and he is like* "*look I did it right, this is what I have got*" *and so I felt it was showing a moment of* "*oh maybe I have made a mistake in this but I am gonna change my thinking*." [Make connections between student thinking and instructional goals, one of the classroom norms Brooke aims to implement is students becoming aware of their own mistakes].

The following excerpt from Brooke's reflections on teacher-selected clips lends insight into how Brooke made connections across different moments of instruction (both previous instruction and future plans) and particular students' thinking and attended to specific details of student thinking in the moment of her teaching. In the video clip Brooke reflected on, a student was working on a problem, which asked students to find the length of ribbon pieces needed to wrap each of 50 pencils by cutting the total of 200 inches of ribbon equally. The student wrote the division sentence of 50 ÷ 200 and justified his division sentence saying that 50 appeared first in the problem and 200 second. The student was not making sense of the problem. Brooke tried to help the student by suggesting skip counting together and using tally marks for each skip count. This method did not help the student. In the following excerpt, Brooke interpreted student difficulties by making connections to previous moments of instruction and proposed alternative strategies she may implement for future instruction. The italicized sentence identifies where Brooke made a reference to future instruction. The bracket labels the coding.

> …I think with the right strategy he is perfectly capable of solving the problems but he needs to find what works for him. *I might go back to discrete model and have him work with actual models for example money because I know he is interested in money that might be helpful to do 50 cents or 25 cents* [Referring to future instructional strategies Brooke may implement to help a particular student]. Number line does not make any sense to him and even in multiplication number lines for him did not make a lot of sense either because he counts where you start, he counts not the jumps, he counts the lines [Make connections to another time of instruction, Brooke remembers specific details of a particular student's misconceptions about a related topic]. So that did not work for him in multiplication and it is not working for him in division either.

**Fig. 4** Daniel's work where he drew a picture representing 36 × 3 instead of 36 ÷ 3 in answering a division problem which involved cutting a 36-inch ribbon equally to wrap 3 presents

*But making pictures worked for multiplication, so we can go back to that* [Suggesting another instructional strategy she may implement].

Another interesting pattern evident in the above excerpt was that Brooke focused on the potential of student understanding in her evaluation of student thinking. In cases where the student did not demonstrate understanding, Brooke made suggestions regarding how future instruction using a different strategy might help that student.

## Discussion

With regard to attending to different elements of instruction, Brooke, the highly accomplished teacher, paid the most attention to student thinking. She also attended to classroom norms, such as providing respectful and mathematical feedback to peers, working well within partnerships, and admitting and learning from mistakes. These norms supported the goal of establishing a classroom environment in which students were encouraged to share their thinking. Consistent with previous research (Colestock 2009), in this study, what happened in each class influenced what the teacher paid attention to. For instance, Brooke attended to student characteristics more often when she worked one-on-one with students.

The results that illustrate how Brooke attended to different classroom events were generally in line with previous studies that utilized methodologies similar to this study. Specifically, categories of attending to student thinking were similar to what Colestock (2009) found in his study. However, Brooke did not focus on the instruction elements such as “task management,” or “student engagement” as did the teacher in the study by Luna et al. (2009). This might be because Brooke, a highly accomplished teacher, did not experience classroom management or task management problems in her class. Instead, Brooke believed that understanding and discussing students’ mathematical thinking in the classroom were more important than other classroom events. Additionally, her focus on attending to classroom norms in selecting video clips was an extension to previous research that utilized the same methodologies (Colestock 2009; Luna et al. 2009).

The highly accomplished teacher in this study interpreted students’ mathematical thinking in productive ways as characterized by previous research (Goldsmith and Seago 2011; van Es 2011): She provided evidence when making claims about student thinking, and she not only described events in the video clips but also made connections between student thinking and larger issues of teaching and learning. In fact, providing evidence in making claims of student thinking was the most commonly observed pattern throughout her reflections. The connections she made to general principles of teaching and learning may be expected due to her experience in professional development and teaching. Similar to what Goldsmith and Seago suggested as a quality of productive interpretation of student thinking, Brooke also focused on students’ potential in understanding and on the positive aspects of their learning rather than student deficits in understanding. On the other hand, making alternative pedagogical decisions did not occur frequently in this study. Based on previous research, even though this type of interpretation is considered difficult for teachers (van Es 2011), one might expect it to occur more frequently throughout a highly accomplished teacher’s reflections. This may result from the fact that the teacher in my study was not prompted to offer alternative pedagogical decisions during the interviews.

Moreover, open coding of teacher reflections revealed three types of interpretations, which may be further evidence of noticing students’ mathematical thinking and extend previously developed frameworks: (1) making connections between student thinking and teaching goals; (2) making connections between student thinking and another moment of classroom instruction; (3) making references to future instruction based on student thinking. These practices seemed to help the teacher build instruction on student thinking, which had the potential to improve student learning (Wiliam 2007). As a highly accomplished teacher, Brooke paid most of her attention to student thinking. The teacher's interpretations are indicative of her capacity to provide very specific details on how students understood different concepts at various times in class. This is similar to what Colestock (2009) found in his study where he documented how a teacher's noticing influenced his teaching. Different than previous studies mentioned, in my study the teacher not only provided evidence of attending to student thinking but also interpreted a variety of student strategies and different levels of understanding by making connections to different moments of instruction, instructional goals, and principles of learning and teaching. The teacher's rich interpretation of student thinking provided insights on how she might incorporate such connections in her teaching and arrange instruction according to the needs of her students in concrete ways. The participant in this study reflected on her own practice rather than other teachers' practices. Previous research indicates that this may create differences in noticing (Miller 2011; Seidel et al. 2011). Similarly, the rich interpretation of students' mathematical thinking in this study may also be the result of viewing one's own teaching practices.

## Conclusion

The triangulation of interviews, observations, and video clips of instruction purposefully chosen by the teacher provides a rich description of how one teacher makes sense of significant moments of instruction from her perspective. Consequently, one of the unique contributions of this study is to provide insights on how one teacher makes instructional decisions in using student thinking in a complex environment. Because the teacher has to make conscious choices during the instruction about selecting video clips, this study allows the readers to enter the teacher's mind during instruction, a perspective difficult to capture by employing other methodologies (Sherin et al. 2011b). Another important contribution of this study is the details regarding how one teacher's noticing of student thinking shapes her instructional actions and how these actions help student learning in better ways. The participant in this study, who attended most frequently to student thinking compared to other elements of instruction, was able to make connections between student thinking and different times of instruction, principles of teaching and learning, and her expectations for each student. These practices seemed to help her build instruction based on what her students knew and needed, such as using a previously encountered student misconception as a way to help students understand. Unpacking specific practices related to the expertise of noticing student thinking has the potential to benefit other teachers who could learn from such practices.

Investigating teachers' noticing of student thinking and how this noticing may shape teaching practices is an important research agenda for teacher learning. Future research needs to investigate trajectories of noticing practices for practicing teachers and the most productive methodologies in undertaking such studies (Schoenfeld 2011a; Sherin et al. 2011a). Although there is a need for the accumulation and analysis of myriad studies in order to have a grasp of the nature of noticing skills in teachers and how it could be improved, the results of this study could help further future research efforts on teacher noticing and teacher learning. For example, multiple-case studies of teachers who are at different levels in their careers may help future research in studying trajectories of development related to noticing expertise. The methodology used in this study has the potential to offer insights for other researchers exploring productive ways to study teacher noticing. This study also contributes to the mathematics education field through the analytical frameworks utilized to analyze teachers' noticing. These frameworks extend previous research findings by providing details in understanding teacher's interpretation of student thinking. Specifically, additional aspects of productive teacher interpretation revealed in this study were (1) making connections between student thinking and teaching goals; (2) making connections between student thinking and another moment of classroom instruction; (3) making references to future instruction based on student thinking. With regard to attending to student thinking, this study extends the previously developed frameworks by inclusion of the teacher's focus on classroom norms. Future researchers of teacher noticing may benefit from the frameworks developed for this study.

Previous research (Jacobs et al. 2011; van Es 2011) identified characteristics of teacher noticing of children's mathematical thinking. However, because teacher noticing in these studies tended to exclude authentic classroom contexts and teachers' own instructional practices, they may not fully explore and characterize teacher noticing (Schifter 2011). This study contributes to the field by investigating teacher noticing in genuine classroom contexts, thus yielding a more authentic characterization of noticing. Understanding how highly accomplished teachers notice and build instruction on student thinking may help teacher educators in supporting teachers' learning. Just as teachers strive to build instruction on student thinking, teacher educators need to find ways to build on teachers' already existing abilities of noticing to help them become better in such practices (Jacobs et al. 2011). For instance, by using this methodology of asking teachers to select video clips of instruction during instruction and reflecting on those clips in a long-term professional development, teacher educators could help teachers practice productive ways of noticing revealed in this study: making connections across different moments of instruction and student thinking, making connections between instructional goals and student thinking, making connections between principles of teaching and learning and student thinking, and referring to future instruction. Engaging in such noticing practices seemed to support the teacher's instruction as she strove to adapt her instruction based on her students' knowledge and needs in this study. Unpacking such expertise may help teacher educators and teachers in building instruction based on student thinking.

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