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Introducing a symbolic interactionist approach on teaching mathematics: the case of revoicing as an interactional strategy in the teaching of probability

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Abstract This study examines an interactional view on teaching mathematics, whereby meaning is co-produced with the students through a process of negotiation. Further, teaching is viewed from a symbolic interactionism perspective, allowing the analysis to focus on the teacher's role in the negotiation of meaning. Using methods inspired by grounded theory, patterns of teachers' interaction are categorized. The results show how teachers' actions, interpretations and intentions form interactional strategies that guide the negotiation of meaning in the classroom. The theoretical case of revoicing as a teacher action, together with interpretations of mathematical objects from probability theory, is used to exemplify conclusions from the proposed perspective. Data are generated from a lesson sequence with two teachers working with known and unknown constant sample spaces with their classes. In the lessons presented in this article, the focus is on negotiations of the meaning of chance. The analysis revealed how the teachers indicate their interpretations of mathematical objects and intentions to the students to different degrees and, by doing so, create opportunities for the students to ascribe meaning to these objects. The discussion contrasts the findings with possible interpretations from other perspectives on teaching.

Keywords Interactional teaching strategy · Teaching · Symbolic interactionism · Revoicing · Probability

Introduction

The teaching of mathematics and how instructional practices evolve have been studied from a multitude of perspectives (e.g. Ball and Bass 2000; Davis and Simmt 2006; Jaworski 1994; Ma 1999). The perspective that seems to have had one of the greatest

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impacts in research programmes worldwide is teacher knowledge and the resulting view on teaching. Since Shulman's (1986, 1987) work on the nature of teachers' knowledge, there has been keen interest in what type of knowledge a mathematics teacher needs to create opportunities for learning. Here, knowledge is an acquired form of a knowledge object, something that the teacher has acquired through education and experience and has thus become an expert. Shulman, whose work focuses on the type or form of knowledge, suggests that teachers must be able to transform their expert subject matter knowledge into something that a novice can comprehend. His conclusion is that the demands on mathematics teachers' subject matter knowledge were different from those on other practitioners such as mathematicians. Shulman called this special kind of expert knowledge pedagogical content knowledge. Since then, many have added to these research results with their various takes on it and have offered valuable insights into mathematics teachers' knowledge (e.g. Groth 2013; Hill et al. 2005; Rowland et al. 2005).

Our contribution to the mathematics teaching discourse is based on challenging the approach of explaining classroom mathematical practice based solely on teachers' knowledge by analysing classroom data from a symbolic interactionism approach. In other words, instead of accounting for teaching in relation to certain pre-defined and personal knowledge attributes, we will focus on how the teacher acts and interacts with his or her students in order to develop a mathematics practice in the classroom that provides the students with rich opportunities to enhance their ways of interpreting and ascribing meaning to mathematical situations and objects. We illustrate our analytical points by taking a close look at how two teachers interact with their students during probability lessons, which are based on data experiments. We start off by deepening the discussion of previous research related to our purposes. Then, we introduce and elaborate on the theoretical constructs of the study. The theoretical discussion ends with addressing the research question of the present study.

Previous research

There are several interesting current research programmes examining mathematics teachers' knowledge. These are used to contrast the views and findings in this article and to contribute to the discussion of how instructional practices progress. The focus of this review will be on the arguably most influential research programme in this area, led by Deborah Ball and her colleagues. Based on Shulman's conceptualization of teacher knowledge and an examination of mathematics teachers' practices, they have developed a framework of Mathematical Knowledge for Teaching (Ball et al. 2008). The framework consists of six forms of knowledge, organized under *Subject matter knowledge* (containing *Common content knowledge*, *Horizon content knowledge* and *Specialized content knowledge*) and *Pedagogical content knowledge* (containing *Knowledge of content and students*, *Knowledge of content and teaching* and *Knowledge of content and curriculum*). This conceptualization of teacher knowledge has been used by researchers in different ways, e.g. when Rowland et al. (2005) used it to explain how teachers' actions in the classroom are informed by their various aspects of knowledge, or when Petrou and Goulding (2011) conceptualized it by synthesizing results from several studies of teachers' knowledge. Speer et al. (2014) expanded the definitions of the knowledge categories by investigating what they would mean in the teaching of different levels of mathematics.

Since the empirical data of the present study were gathered from a probability lesson, we find it appropriate to reflect a bit on what research has to say about teachers' knowledge of probability and statistics. Burgess (2006) and Groth (2013) both added to the teacher knowledge discourse by studying what Mathematical Knowledge for Teaching statistics could entail and how such instructional practice evolves with the argument that there are fundamental differences between statistics and other mathematical topics. A key factor they identified was that mathematics could be viewed as a largely deterministic discipline, whereas statistics reasons under uncertainty. Random variation influences the inference in statistic reasoning; therefore, teachers' knowledge for teaching statistics should be considered separately. These two studies resulted in frameworks building on the tradition of pedagogical content knowledge (Shulman 1986), but included knowledge categories that took into account the special nature of teaching statistics.

Groth (2013) points to the central role of probability in the teaching of statistics. However, research has found that teachers often find probability difficult to understand, which makes this an interesting topic to study in relation to students, who also might find it difficult. Liu and Thompson (2007) studied teachers' knowledge of probability by analysing their discussions. Nilsson and Lindström (2013) asked teachers to, individually, solve probability tasks presented in a questionnaire. Watson (2001) developed an instrument design to profile teachers' competence and confidence to teach probability. All three studies highlight, from an individual perspective, the difficulties teachers experience in understanding probability concepts and random processes. The studies highlight which forms of knowledge the teachers possess and which they do not. Although these studies provide insight into teachers' subject matter knowledge, questions can be raised as to how this type of knowledge relates to their practice of teaching since they all base their results on data collected outside the teachers' actual practice. In their overview chapter of educational probability research, Jones et al. (2007) conclude that the next step for studying teaching probability is classroom-based research that informs on teachers' instructional practices in probability. Focusing on teachers' instructional practices might provide alternative views on, and alternative insights into, teachers' roles in negotiating the meaning of probability concepts compared to studies, for example, situated in teacher education courses or based on the analysis of teachers' discussions and responses to questionnaires.

In response to the vast work being done on mathematics teachers' knowledge, Barwell (2013) raises the epistemological question about the meaning of *knowledge* in the Mathematical Knowledge for Teaching framework. Although there is a heavy emphasis on the concept of teachers' knowledge, very little is discussed about the nature of this concept. Barwell (2013) concludes that the epistemology of recent work on teachers' knowledge can be traced back to how Shulman viewed knowledge as being acquired and constructed by individuals. Steinbring (1998) adds that Shulman's conceptualization of teacher knowledge postulates a linear model of teaching whereby teachers take their scientific knowledge during lessons and transform and convey it to their students. By adapting to symbolic interactionism (Blumer 1986), we are provided a theoretical perspective that emphasizes the social and interactional nature of teaching and learning. In such a perspective, a teacher's actions are viewed within the frame of a negotiation process. The teacher participates in interaction and co-constructs knowledge with her students, rather than conveying knowledge to her students. If knowledge is viewed as being co-produced through interaction, the focus is on the processes and results of the interaction rather than on what may or may not exist as objects in the teachers' minds.

Theoretical frame

Voigt (1996) proposed an interactional approach to studying teaching, whereby the theoretical frame is influenced by symbolic interactionism (Blumer 1986). The idea is that meanings of objects are not fixed in classroom settings but are subject to interpretation; the meanings are then negotiated between the teacher and the students. Cobb et al. (2001) continued with the focus on symbolic interaction to uncover what they coined socio-mathematical norms, which govern teachers' and students' actions and are negotiated in instructional practices. In her recent study of students' small-group interactions on contentious mathematical issues, Gellert (2014) included a discussion of the teacher's role from a symbolic interactionism perspective. Although the teacher was not always successful in her attempts, her role was to focus and develop the negotiation further through different discursive techniques. In symbolic interactionism, meaning is viewed as being negotiated under the ongoing process of interaction. The notion of objects is central to interactionism. Participants in an interaction make sense of the world by interpreting how others act towards objects. The meaning of these objects is negotiated through social interaction (Blumer 1986). Objects are anything that can be referred to in social interaction; i.e., they can be physical, social or abstract. The ways people act towards objects are reflectively inter-connected with how they interpret objects of the social praxis of which the people and objects in question are part (Blumer 1986). Participants in an interaction do not merely react to others' actions towards objects but also interpret the meaning of these actions as they are indicated, before acting themselves and assigning meaning to the objects. As actions are interpreted and reactions are reinterpreted back and forth in a constant process, meaning becomes negotiated in a joint action involving teachers and students. Hence, by observing what we can see—teachers' actions, what they indicate through these actions and how their actions are taken up by the students—we can develop our understanding of teachers' strategies for supporting the negotiation process in actual mathematical classroom practices. In this article, we focus particularly on one common teaching strategy, namely revoicing (O'Connor and Michaels 1993). The mathematical objects in focus are different concepts of probability and teaching material used in the teaching of such concepts.

The discursive action called revoicing has the potential to direct students' participation in classroom discussions (O'Connor and Michaels 1993) and, together with probability theory, makes up the theoretical case we have chosen to exemplify our conclusions in this article. Revoicing involves the re-uttering of students' mathematical explanations; Forman et al. (1998) explain it as a strategy to "share the responsibility and authority for explaining and evaluating mathematical problems" (p. 313). In their study, they reported how teachers overlapped students' solutions the teachers did not concur with. The teachers revoiced the students' proposals and, over time, the students began employing the same solution strategies as the teachers. Forman et al. (1998) proposed that the purpose could be to align or create oppositions in students' arguments by highlighting certain aspects of their arguments through *expansions, rephrasings* or *pure repetition*. Adding words or sentences to the students' utterances is called expansion, while changing some of the words is called rephrasing. Repetition is when a teacher articulates exactly the same words as the student has. Herbel-Eisenmann et al. (2009) showed that the purpose of revoicing could be much more multifaceted from a teacher's point of view in the context of a lesson than research had previously reported. The purpose can be related to how teachers use revoicing as a way to influence classroom interaction. By highlighting certain aspects of students' arguments and sometimes clarifying them, teachers position students in relation to the content.

When a teacher interacts with students through revoicing, both the students' and the teacher's interpretations are in focus. When participants interpret each other's actions (in this article, mostly utterances), they interpret the meaning of the (mathematical) object in the focus of the interaction (Blumer 1986). While mathematical concepts are often viewed as objective truths, Voigt (1996) suggests that this is not the case in school mathematics.

School mathematics leaves room for individual interpretations of mathematical concepts based on, for example, students' previous experiences. The concept of chance serves as an example in this article, since it can be interpreted in many different ways, which offers opportunities for us to discuss the various negotiations taking place in the studied classrooms. Chance can be defined as an informal concept, whereby chance and randomness become synonymous. In Swedish (the native language of the informants in this study), chance and randomness are also expressed with the same word (Ordbok över svenska språket 1893), which might further complicate the definition of the concept. This informal concept describes chance as an effect or accident that we do not know the cause of or even whether or not there is a cause (Steinbring 1993). Chance events, with their unpredictable nature, are informally defined as the opposite of deterministic events and are used as a substitute for the deterministic explanation when it is deemed too complex. The fact that an outcome is predicted by a measure of probability, rather than by deterministic law, creates a link between the informal concept of chance and probability (Steinbring 1991). It marks the difference between reasoning in statistics compared to reasoning in other domains of mathematics (Groth 2013). Steinbring (1991) further claims that this link opens up for everyday definitions of chance being synonymous with improbable events and everyday ideas, such as "having bad luck". The formal concept of chance corresponds with parts of the informal definition. In the development of chance as a formal concept, a great deal of emphasis has been placed on the random sequence produced by a series of chance events (Steinbring 1993). The randomness of a sequence has been defined mathematically based on a general idea from the field of information theory (Shannon 1948). Steinbring (1993), opposing this formalized discussion in classroom contexts, states instead that the formal concept should be viewed as implicit and open, leaving room for interpretation in exploratory lesson settings. This openness to interpretation, even within the formal concept, provides an interesting setting for studying the joint enterprise of negotiating the meaning of chance in the classroom.

From a perspective of symbolic interactionism, teachers' knowing is considered from how it varies, emerges and is adapted when the teacher interacts with students. Such a perspective stresses the role of teachers' participation in terms of actions and interpretations in the negotiation of meaning in teacher–student interaction. As the meanings of probability concepts are negotiated in the classroom through interaction, teachers play an active role in guiding this negotiation by means of their own interaction. Patterns of their interactions can be interpreted as strategies, which lead to the formulation of the aim of this study.

Aim of study

The aim of this study is to investigate teachers' patterns of interaction as they negotiate the meaning of experimentally based concepts of probability in their practice. The study focuses on revoicing as a theoretical case of a teacher action based on interpretations of probability concepts in interaction with students. The research questions revolve around how components of an interactional teaching strategy influence the ongoing negotiation of meaning in the classroom. The inquiry is divided into the following two questions:

- What roles does the teacher action of revoicing play in supporting teachers' and students' negotiation of the meaning of probability concepts?
- What roles do teachers' interpretations of probability concepts play in negotiating the concepts in student interaction?

Methodology

This study has its roots in a constructing grounded theory approach, as we start our analysis in the processes of the social world (Charmaz 2006). In the present study, this means focusing on and studying teachers' symbolic interactions in the classroom. When comparing different interactions, one strives to let the "why?" emerge by addressing the "what?" and the "how?" (Charmaz 2008b). It is not only the negotiated meanings (the what) but also how the teachers interact when negotiating meaning that become a vital part of the analysis. A constructing grounded theory approach to teachers' patterns of interaction allows us to remain highly sensitive to analysing details of classroom processes. Teachers as individuals are studied as part of the practice they are participating in, but the focus is on neither their individual subject matter knowledge nor their knowledge of students but rather how they negotiate meaning through the interaction. The analytical process entails producing categories that capture and describe the observed interactions in a network of relationships. This study presents an example of how these intricate relations can manifest in the data and should be viewed as part of the process of developing theory.

The theoretical case of revoicing in teaching probability is used to exemplify relationships between teachers' interpretations, actions, in the process of negotiating meanings of mathematical objects in interaction with students. Revoicing serves as an example of a teacher action that emerged in the open coding, and previous research on revoicing is viewed as further data to clarify the example. Several probability concepts are also included to make up the case, by defining which objects are in focus in this analysis. The notion of chance is one such concept, but the law of large numbers, random variation and sample space are also discussed further in the article. This case is viewed to hold prototypical value (Flyvbjerg 2006) for our view on how a lesson progresses. The previous research presented is viewed as further data to enrich and deepen the analysis and discussion.

Design of the study

The data analysed in this study were generated from a series of lessons held at a public primary school in a medium-sized city in Sweden. The two class teachers carried out the lessons, and video recordings were made throughout each lesson.[1] Although the students were in the fifth and sixth grades, they had had no previous formal instruction in probability theory. The two teachers, Karen and Tilly, had several years of experience teaching math at primary level, but neither had any experience teaching probability theory.

The development of the lesson sequence was based on the principles of the design research cycle (Cobb et al. 2001), whereby we viewed the teaching sequence as an ongoing, and iterative, process that should be improved based on the outcome of the lessons. This developmental process took place during group meetings between each lesson. The developmental group consisted of the two teachers and the two researchers (the authors of this article). The overall instructional design originated from a teaching experiment by Brousseau et al. (2001) whereby the students was asked to investigate a chance event with an unknown sample space. We used an opaque soda bottle containing an

[1] Anonymization of the participants has been achieved by changing the names in the research reports, which was also a condition of the informed consent every participant provided. Although complete anonymization is difficult to achieve since there might be people who remember our involvement at the school and read this report (Miles and Huberman 1994), careful steps have been taken to ensure the participants' integrity.

unknown amount of small coloured balls (neither the students nor the teachers knew the content of the bottles) during the first lesson. When the bottle was turned over, the colour of one ball was revealed while it was kept inside the bottle. This created a constant but unknown sample space. The activity was presented as a race in the first lesson, with three contestants on a six-step track. As one of the three colours was observed on a bottle turn, this colour advanced one step down the track. The students were asked to guess which colour would be the first to have six observations during each race. Based on the topics discussed by the students in the first lesson, the upcoming three lessons revolved around a transparent bottle with a visible sample space. Here, the students were asked to discuss chance, random variation, sample space, sampling and the law of large numbers. The importance of the sample space was highlighted in the second lesson because of ideas discussed in the first lesson. The students discussed whether each outcome was equally likely and had problems picturing how an uneven sample space could have influenced the outcome, as discussed by Nilsson (2007). The students returned to the opaque bottle in the last lesson(s) and, again, in an organized manner, investigated the unknown sample space from the first lesson using the law of large numbers. By producing a large enough sample, they could reason about the sample space in the opaque bottle by translating the relative frequency of each outcome into the probability of that outcome. Overall, one class needed a total of five lessons and the other class six lessons to reach agreement about the unknown sample space in the opaque bottle.

Questions can surely be raised about the influence we as researchers had on the teachers' interactions with the students. Our presence in the classroom, as well as our participation in the discussion between each lesson, might have had an effect. Since Karen and Tilly had never taught probability before, it is reasonable to believe that they would not have taught it without our participation; it is also reasonable to believe that they were more mindful of their wording about probability since they knew we were listening. This was not regarded as a weakness, however, since the idea of the analysis was not to assess their knowledge but to focus on their actions as they influenced the progression of the lesson. It could mean, though, that they were more mindful of their actions and, although they claimed that they "forgot" about our presence, this is something that has been considered during the analytical process.

Method of analysis

The data were analysed through the grounded theory-inspired techniques of open coding and constant comparison (Strauss and Corbin 1997). Transcripts containing teacher–student interaction from the lessons were coded without predetermined analytical categories. Instead, individual sections, lines or words were designated a code sprung from the data. As the data were coded, different sections were compared within the data as well as different codes to find similarities and differences. During this process, categories were formed by joining similar codes together and defining what was interpreted as patterns in the teachers' interactions.

Both symbolic interactionism and constructivist grounded theory rely on the principles of open (interpretative) inquiry (Blumer 1986; Charmaz 2006). As the research process proceeds, there is an emphasis on interacting with the data and remaining open to the twists and turns of the analysis. This openness can be both a blessing and a curse: it provides an opportunity to experiment with any idea and keeps the researcher close to the data, but it also leaves the researcher very vulnerable in the beginning of the process as it offers little guidance. Symbolic interactionism offers a solution to this, by bringing sensitizing concepts into open inquiry (Charmaz 2008a). Sensitizing concepts offer places to begin exploring the data, by allowing previous research to act as a guide in the beginning of the analysis without constricting it, and then allowing the previous research to resurface at the end of the analytical process to deepen the results. By engaging our theoretical assumptions of symbolic interactionism and revoicing with the empirical world and the research discourse, we actively interact in a theorizing practice.

To ensure the credibility of this work, as a measure of trustworthiness (Lincoln and Guba 1985) raw data are presented in the upcoming section intertwined with the results of the analysis. This provides the opportunity to compare the newly created abstract world of categories containing patterns of interactions with the raw data in order to judge how good the fit is. As researchers in this theoretical perspective, we account for *our* interpretation of the teachers' actions and the interpretations they indicate. We do our best to account for this process, but in the end each reader is invited to judge the trustworthiness of the study for himself or herself.

Results

The analysis presented below is focused on the interactions from the teachers' first two lessons. It shows patterns in which both Karen and Tilly use revoicing to negotiate the meaning of chance with the students in relation to how they interpret chance in various situations. Karen and Tilly modify the negotiation of meaning by either inactively or actively revoicing the students' contributions. Inactive revoicing takes the form of word-by-word revoicing, without indicating the teacher's intention or interpretation. When revoicing in a more active manner, the teacher makes minor alterations when revoicing a student's utterance, thereby indicating her intention and interpretation of the utterance in the process of negotiation. The results show how these two patterns of revoicing are connected to how the teachers interpret the probability concepts emerging in the situation. The analysis and categorizations were made from a large quantity of teacher interactions, and the results are exemplified in the attached transcripts. All transcripts are the authors' own translations from Swedish into English; it bears repeating that Swedish has only one word for chance, which could also mean randomness, so the use of chance and randomness in the transcripts reflects the authors' interpretations of the utterances.

Revoicing while negotiating meaning

Tilly and Karen showed two distinct patterns of interaction when revoicing students' contributions, *active* and *inactive* revoicings. Inactive revoicings are repetitions, and their main attribute is that they do not indicate the teacher's interpretation; at least this is not apparent in the action. Therefore, the teacher does not add any further information to what the students have provided to the situation themselves. Even though the teacher makes an interpretation of the task and the students' contributions, the content of how the teacher understands the situation is not brought into the communication in an inactive action. An inactive action is also to great extent a neutral act, in terms of not making explicit the teacher's intention in choosing to highlight that particular contribution. Active revoicings include expansions and rephrasings and often involve, contrary to inactive revoicings, making alterations when revoicing a student's utterance. Active revoicing entails balancing two processes of interpretations. On the one hand, the teacher interprets the content of the student's contribution; on the other hand, however, this interpretation is made against the background of the teacher's own interpretation and meaning-making of the concept in question. The following transcript is from Tilly's first lesson, in which she asks about the students' predictions for the upcoming race. The class has already run one race in which blue had come first, white second and red last. It is our interpretation that Tilly intended to steer the negotiation process towards making sense of the connection between the sample space composition and the observations. She challenges the students to justify their answers by reflecting on their experiences and the outcomes of the first race. Surprisingly, many of them vote for red to be the next winner, which could be interpreted as counter-intuitive by Tilly, according to sample information. Depending on what type of contributions the students make, Tilly interacts through both inactive and active revoicings:

Tilly:	Those of you who chose red, why do you think that? What is it that makes you believe red is going to win? Can someone who made that choice tell me—Sara?
Sara:	I like it.
Tilly:	You like that colour. Yes, Gustaf?
Gustaf:	Good odds, and…
Tilly:	Good…how do you mean?
Gustaf:	A lot of people voted for it.
Tilly:	A lot of people voted for it. Yes, Steve?
Steve:	I think it's rather ehm…it only got one in the last race so now I believe, a little, that it's going to win.
Tilly:	Ok, since it performed so–so the last time, it's red's turn. Yes, Derek?
Derek:	Liverpool.
Tilly:	It has to do with Liverpool? In what way can Liverpool influence red?
Derek:	Since they're the greatest.
Tilly:	Mm. Ehm, blue, how does "blue" reason? Edgar?
Edgar:	Some people said there were like five before and we can't change that now since we see you all the time so we would see if you changed it.
Tilly:	Okay, I can't add more and change the conditions, it's the same conditions. Okay, do I understand you correctly that you're saying there are more blue balls in it? Or hmm, what did you say?
Edgar:	The blue are the most numerous.
Tilly:	Do you believe blue were…okay, yes, anyone else choosing blue, how did you reason, Jake?
Jake:	First of all, I never let blue down.
Tilly:	Okay, blue's your favourite colour.
Jake:	Yes, and I agree with Edgar.
Tilly:	And you agree with Edgar. Okay, white then, how did you reason?
Aaron:	Well I thought just because red got, well, they did the worst in the last race. Then came white, they were pretty, well not very close but they did appear a couple of times near the end. So I think they might win here again.
Tilly:	Okay, white has a chance of winning this time?
Aaron:	And it might be that there's only one red [inaudible]
Tilly:	Stop stop stop, Aaron had the floor; Frits, please take a seat, thank you. I didn't hear the last part.
Aaron:	Well there could be only one red. Then you could reason that there's only one red and perhaps a couple more white and a couple more blue.
Tilly:	You've started to reason about how it looks inside. Okay.

Tilly gives her response inactively revoicing what Gustaf and Jake had said in their follow-ups. Gustaf had made an interesting contribution about odds, which Tilly asks him to elaborate on. But as Gustaf does this Tilly chooses not to follow up on it, with the consequence that she terminates the negotiation of the meaning of odds through an inactive revoicing of his elaboration. In his second contribution, Jake simply states that he agrees with another student's argument, which is also answered with an inactive revoicing by Tilly. Neither Gustaf's nor Jake's contribution seems to have been interpreted as productive by Tilly, due to an intention only she knows. An alternative interpretation of why Gustaf's contribution about odds was not actively revoiced, in terms of adding information to the meaning-making process, could be that Tilly did not have sufficient subject matter knowledge to properly assess it. However, reviewing the complete set of data, it is our interpretation that the inactive responses are the result of Tilly interpreting that Gustaf's and Jake's contributions did not fit into the negotiation she was trying to develop in this particular episode.

Looking at Tilly's active revoicings in the episode above, we note how her action on one occasion seems to influence the negotiation of meaning, while on another occasion it does not. Tilly's revoicing of Sara's utterance and Jake's first utterance is based on her interpretation that the students are influenced by colour attributes when reflecting on the chance of different outcomes. It could be that Tilly intends to engage in a discussion about the subjectivity of probability estimations. Subjective probability estimations are dependent on the amount of information the observer holds. If Sara or Jake has additional information about the experiment compared to their classmates, their probability estimations would be more accurate from their point of view. But, most likely, one's favourite colour is a personal attachment that Tilly finds easy to acknowledge by actively revoicing this without risking it having a strong influence on the negotiation trajectory she is trying to establish. The other active revoicing strategy appears when she reacts to Steve's justification for favouring red, Aaron's idea that since white was close to winning before it might win the next time, and Edgar's idea about the composition of the sample space. It is our interpretation that since Steve, Edgar and Aaron base their reasoning on the observations from the first run, Tilly sees this as productive for her intentions for the interaction and thus revoices it actively. She revoices Steve's utterance actively by emphasizing a fairness factor: if the distribution is even, as is the case with many other probability distributions the students have encountered before, the results should even out in the long run. Edgar bases his contribution on the argument that the result in the first trials appeared due to an abundance of blue balls in the bottle. He points to an uneven sample space, which he considers will influence the outcome in the following runs. Tilly revoices Edgar's utterance actively by emphasizing the predetermined condition of the sample space: the number of blue balls in the bottle has not changed, and neither has its influence on the results. Both cases, Steve's and Edgar's, serve to negotiate the importance of sample space in random events from either a fairness perspective or an uneven distribution perspective. In the case of Aaron, who already seems to have recognized the importance of the disposition of the sample space, Tilly feeds into her revoicing aspects of unpredictability by using the word *chance*. By indicating that even though we know, or at least have an idea of the sample space, she provides information to the class that, despite this knowledge, there is always a level of uncertainty in predicting random events.

In Tilly's active revoicings, her own interpretations of the mathematical concepts become apparent. The crucial aspect of active revoicing is the teacher's interpretations of the content of the students' utterances, and the teacher's own interpretations of the meaning of the mathematical objects. By utilizing the students' different interpretations through actions as active and inactive revoicings, Tilly can guide the negotiations of meanings and create opportunities for the students to ascribe meaning to the mathematical objects in focus. This guiding is done through a process of altering the negotiation through active revoicings or confirming already negotiated meanings through inactive revoicings during the lesson to align with the teachers' overall intention for the lesson.

Interpreting probability concepts in classroom interactions

The following is an example of Karen's actions based on interpretations of chance in an interaction. It takes place during a whole-class discussion during the first lesson, after a race in which blue came in first, white second and red third. The discussion is based on this result. Karen's actions indicate that her interpretation of chance alters slightly during the interaction, but stays within the bounds of what we interpret as a colloquial interpretation. Moreover, we see how her actions, focusing on certain aspects of the experiment, encourage the negotiation to progress in a direction she might not have anticipated. After the following interaction, a long discussion developed about how someone might have tampered with the experiment and that Karen had in some way influenced the outcome:

Brian: Now I understand why we discussed those words. Statistics are what we're working with. Probability, what's the probability that blue will win? And chance, it's a bit random what can happen, what happens.

Karen: Could you repeat that? That was a good summarization.

Brian: Statistics is the actual thing we're doing. Probability is, for example, what the most probable to happen or the least probable to happen is. Blue comes up the most and perhaps red or white the least. And chance, it's a bit random what comes up.

Karen: What comes up … AH! Now red came up, was that chance? White…blue

Nils: But can't we open the bottle?

Karen: No, we're not allowed to. Now red came up.

Nils: Perhaps it's stuck.

Karen: Perhaps it's stuck. White, white, do you think differently now when I'm doing this?

Robert: Or wait, can you do like this?

Karen: Does it matter how I do it?

Miriam: That's why you did that!

Karen: No, absolutely not. That was also just chance.

Karen makes use of Brian's summarization of chance. Her upcoming action, the revoicing of "what comes up", indicates that she interprets Brian's utterance as being based on a result-oriented interpretation of chance. She then continues to contribute to the negotiation of the meaning of chance by asking "was that chance?" We can interpret this articulation as guiding the student to a notion of chance as being a force influencing the outcome of a race. Some of the significance is lost in the translation: in Swedish, she uses the definite form and chance as a noun. As the class negotiates the meaning of chance, Karen's actions seem to favour the idea of chance in this particular case as a deterministic factor that can be manipulated. She refutes this at the end of the transcript when she is challenged by a student about the way she turned the bottle. The student exclaims, "That's why you did that!" Karen explains her action, "That was also just chance", as being a result of mere chance. She sticks to her colloquial, "force-oriented" interpretation that chance *influences* the results, albeit unpredictably. We find it difficult to infer a clear intention from Karen's actions. It seems as if she is having problems deciding what kind of reasoning she is trying to involve the students in. Her actions indicate that she has not been able to predict her influence on the negotiation in this interaction, when she tries to refute the students' conclusion that her way of turning the bottle influences the outcome. Based on interpreting chance as something that can be manipulated, Karen initiates a new discourse, which triggers the students to discuss different ways the bottle may have been manipulated. In this discussion, the concept of chance and other mathematical concepts receives limited attention.

During the meeting between Lessons 1 and 2, Karen expressed frustration that the discussions had taken this unexpected turn, primarily focusing on different ways the experiment could have been manipulated. She was determined not to repeat the same thing in Lesson 2 when working with the clear bottle and the known sample space. In the following extract, from about 15 min into Lesson 2, we note that this time Karen expresses herself in a more conscious and mathematical way:

Farak: I think blue will come first and then white but… It's hard to know.
Karen: It's hard to know. Can you know?
Farak: No.
Karen: No, what can we know?
Peter: That *a* colour will appear.
Karen: That *a* colour will appear. And do you know what? You've already reached the next [question], if you know that you can see it in the table here, but we'll get to that later. Then we know that the outcome of this investigation that we're going to perform is white, blue or red. But we don't know what order they'll appear in, since that's ch…?
Class: Chance.
Karen: Wow, you're smart, that's right.

Karen negotiates with a more conscious goal of guiding the negotiation of the meaning of chance towards one with more mathematical substance than was seen in the previous extract. She revoices "it's hard to know" inactively, but follows up on this by making more explicit attributes of uncertainty and Kolmogorov's second axiom by asking "can you know?" and "what can we know?" Kolmogorov's second axiom tells us that *what we can know,* for sure, is that some outcome of the sample space will be the result of a random experiment. We interpret Karen's revoicing and added questions as a result of interpreting chance as a concept with attributes in line with the formal mathematical definition. The randomness of the produced sequence is also attributed to the random variable of this investigation. Karen's summarization of the negotiation of meaning up to this point at the end of the transcript also contributes to the subsequent interaction. This contribution can be interpreted as indicating a notion of chance as being a concept that holds the properties of a variable that produce random sequences (Steinbring 1993). However, since Karen continues to discuss another topic after this sequence, we have no opportunity to further study how her contribution influenced the negotiation of chance in this situation.

Expanding on the two episodes

It is our interpretation of Karen and Tilly's revoicing and other teacher actions that they interpret chance both colloquially and mathematically when interacting with their students. We view these interpretations as something co-produced in the interaction rather than their acquired teacher knowledge. Following Steinbring (1991), we refer to colloquial interpretations when chance is viewed as an unknown or unpredictable force that governs the outcome of an event. Such a view is associated with an informal use of the concept and is grounded in an everyday language. In mathematical interpretations, chance is viewed as a random variable (Steinbring 1993). Furthermore, it is then considered unpredictable in small samples and predictable in large samples (Stohl and Tarr 2002). Karen and Tilly interpret chance in both ways during the lessons, and their different interpretations appear to modify the negotiation processes. The teachers' actions are based on how they perceive the interaction and interpret the meaning of the concepts involved. Based on these interpretations and subsequent actions, the teachers' contribution to the dialogue creates opportunities for the students to react to and modify the way they talk about and ascribe meaning to the concepts of chance and probability. Hence, Karen and Tilly act and thereby influence the negotiation of meaning in different directions in relation to the meaning-making processes they intend to involve the students in.

The comparison of the transcripts from Karen's two teaching episodes can be considered an example of how teachers' different interpretations of subject matter can impede or support the joint enterprise of meaning-making through negotiation. When several unclear intentions rather than a focus on just one, and her colloquial interpretation of chance, are indicated through her actions, the negotiations result in something that does not encourage the further development of probability concepts. This became obvious both in the classroom discussions and in her own reflections expressed at the group meetings. When Karen's articulations consciously indicated a more formal definition of chance when interacting with Farak and Peter, we believe she was in a better position to lead the negotiation process towards the intended meaning-making process, which includes attributes that hold explanatory power over the experiment. From the position of the students, for the same reasons we can also infer that Karen's actions during this episode offer them better opportunities to take into account her contributions and act on and ascribe meaning to the objects in focus in a mathematically sound way. The same pattern of interaction can be observed in Tilly's lessons. When her actions indicate interpretations of chance consistent with mathematical convention, the negotiated meanings tend to highlight attributes such as variability in the samples. This can be observed, for example, in her whole-class discussion in "Revoicing while negotiating meaning" section, when the students discuss and justify predictions. A difference between Tilly's and Karen's patterns of interaction is that Karen seems to more often switch between a colloquial and the mathematical interpretation, whereas Tilly more consistently indicates that she interprets chance in a mathematical fashion. This difference can be ascribed a higher degree of sensitivity when Karen interprets her students' actions, and an understanding that it is a fruitful strategy to engage students in a discussion about their preconceptions. Alternatively, the difference could be credited to an insecurity in her subject matter knowledge, impeding her in determining the implications of the difference between the two interpretations. Reviewing the first two lessons in both classes as a whole, one can see that Tilly's approach led to fewer ideas that the experiment had been manipulated than did Karen's and that Karen had to refute such ideas to a greater extent than Tilly did in both Lessons 1 and 2.

Conclusions

The two teachers in the study revoiced the students' utterances in two fundamentally different ways that influenced the negotiation of meaning in the interaction in respect of function. Active revoicings provided the students with more information about the object under investigation, by indicating the teachers' interpretations. Active revoicings also indicated the teachers' intentions more clearly for the students to interpret. This in turn had a strong influence in shaping the negotiation of meaning and revealing the interactive nature of interpretations and intentions. Whether an active revoicing was based on a colloquial or mathematical interpretation of chance, it played an important role in the continuation of the discussions. Inactive revoicings, on the other hand, did not indicate the teachers' interpretations or intentions to the students. This had the effect of shutting down some of the students' ideas, which in turn limited their opportunities to express alternative views. But it could also signal that the teacher and the students had similar interpretations of the mathematical object under consideration. It could also leave the continuation of the negotiation unguided, as it was left to the students to continue negotiating the meaning of the mathematical objects and develop more mathematical interpretations.

Discussion

The study presented here is part of a larger project with the aim of investigating the interactive nature of a mathematical teaching practice. Contributing to the discourse on teaching mathematics, the present study focused particularly on the interactive role of teachers' interpretations of probability concepts rather than on a pre-defined disposition of teacher knowledge. By means of focusing on how teachers use revoicings as a discursive tool in teaching, the present study points to the social and interactive nature of meaning-making in the mathematical classroom. Moreover, by considering a teacher's ability to teach from the social and interactive context within which the teacher is acting, we are provided certain opportunities to understand how and why teachers apply the strategies they do. We also gain an understanding of what possibilities and consequences the teaching strategies may have for students' opportunities to learn from these interactions. The current mathematics teaching discourse is dominated by Shulman's legacy of pedagogical content knowledge, with its individual perspective on knowledge (Barwell 2013). This tradition focuses on what knowledge the teacher does or does not possess, and the result in the classroom can be interpreted as missed opportunities for learning (Burgess 2008) due to deficits in teacher knowledge. We structure our discussion around contrasting such a perspective with our proposed symbolic interactionist perspective. Adopting a symbolic interactionist approach enabled us to provide alternative explanations for the teachers' actions in classroom interaction, rather than simply distinguishing between sufficient and insufficient teacher knowledge. Symbolic interactionism also enabled us to view teaching as a process in a system of interactions, instead of a linear model whereby expert knowledge is transformed and then transferred as indicated by Shulmans' view of teacher knowledge (Steinbring 1998). It allowed us to uncover the interactional character of teaching strategies in terms of being dynamic and dependent on the situation. The progression of the negotiation of meaning emerged in the analysis, as did the teachers' own interpretations and intentions.

Karen's and Tilly's choice of using active or inactive revoicings could be accredited to insufficient teacher knowledge for teaching probability. This analysis would depend on whether they, in our opinion, missed an opportunity to build on their students' arguments (Burgess 2008). But lack of knowledge—in the sense of conceptual and procedural knowledge as conceptualized in, for example, the Mathematical Knowledge for Teaching framework (Ball et al. 2008)—does not sufficiently account for why the teachers act differently in different interactions. In the complete set of data, there were several accounts of Karen and Tilly revoicing students' thoughts and arguments both actively and inactively. The analysis revealed how revoicings, based on interpretations made by Tilly and Karen, formed an interaction strategy that directed the negotiation of meaning. To assess the actions, each revoicing has to be considered in respect to the interaction of which it is part. In every instance of action, the teachers had to interpret the students' actions as well as the negotiated meaning emerging in the interaction to form their own actions.

Comparing active and inactive revoicings with previous results from Forman et al. (1998) in which revoicings are divided into expansions, rephrasings and pure repetitions, we find both similarities and differences. Both expansions and rephrasings are of the nature of active revoicings, but, modelling teachers' and students' interpretation as reflexively dependent on the interaction in which the interpretations are made, we argue for the need to add another dimension to revoicing. By viewing it as an interactional phenomenon, the action becomes more active or inactive depending on how much of the teachers' interpretations and intentions are indicated to and perceived by the students. After close inspection, pure repetition can also be considered active depending on how clearly a teacher's intention is indicated to the students through the selection of passages to revoice. In a similar fashion, an expansion or a rephrasing can become less active depending on how substantial a teacher's contributions are and how well these indicate the teacher's interpretations to the students. We believe that the categorization of active and inactive revoicing becomes dynamic by taking into account both parties in the interaction, which is helpful when analysing interaction and understanding teaching as an interactive process with both teachers and students as actors.

We see no generalizable way of deeming one revoicing better than the other because they are to be understood as a strategy, emerging in a social and interactive context. Mathematics is often understood as a deterministic discipline, and in such a context teachers' interpretations could be understood as higher/lower quality depending on objective criteria. Steinbring (1991) argued that the case of probability differs from other mathematical topics in how its concepts are justified and the nature of random events, which opens up for the use of informal concepts. We find evidence in our data that ratifies such claims, when Karen and Tilly indicate colloquial interpretations to support the negotiation of meaning. It is not apparent in our data that the indication of colloquial interpretations of chance is necessarily unbeneficial in the negotiation, or that a mathematical interpretation is exclusively beneficial. The use of different interpretations must be evaluated in relation to the interaction and the teachers' intention with the interaction.

Decisively determining teachers' intentions in interaction is complicated. Their intentions are indicated by their actions, but we cannot see inside their minds. By comparing a multitude of teacher–student interactions, however, we can analyse patterns in these interactions to gain insight into the processes. In some of the interactions, we interpret that Karen and Tilly act according to different interpretations and with a clear intention. With this argumentation, we provide an alternative interpretation of the concept of missed opportunities depending on lack of teacher knowledge of statistics stressed by Burgess (2008), and therefore contribute to the discourse on teacher knowledge. If active or inactive teacher actions are used as a strategy coupled with an interpretation appropriate to that particular interaction, with an intended outcome of the negotiation of meaning, learning opportunities should be viewed as skipped rather than missed. The skipping of an opportunity then becomes a result of teacher professionalism. We offer an alternative interpretation, rather than deeming Karen and Tilly less knowledgeable, when they do not follow up on every utterance by the students. Active and inactive teacher actions can be understood as strategies with intentions of directing the negotiation of meaning towards a goal they as professionals intend.

Teaching implications

Practical implications of this study are mainly directed at teacher educators and teachers who wish to improve their practice. Instead of talking about teacher knowledge as a condition for teaching, we suggest that we talk about teaching as interaction, whereby meaning is situated in this interaction. Revoicing can be studied as a separate strategy for orchestrating classroom discussion (O'Connor and Michaels 1993), but also as part of a complete interactional strategy whereby the teacher's role is to direct the negotiation of meaning according to his or her own intentions. As Karen and Tilly make use of revoicing, they do this as part of their classroom interaction strategy by indicating to the students their own interpretations of probability concepts, their interpretations of the students' actions, and their intentions for the negotiation of meaning. The proposed view of teaching offers further incentive to situate teacher training close to classroom practice, where student teachers are faced with the complexity that practising teachers deal with and where they can learn to use interactional strategies such as revoicing in real teaching situations (Putnam and Borko 2000).

Based on the outcome of the present study, we encourage future research to further examine the meaning and role of teachers' interactional teaching strategies supporting learning processes of mathematics in general and probability in particular. Moreover, supported by the theory of symbolic interactionism, the present study suggests the need to further study teachers' roles in mathematics teaching by identifying and exploring the components of interactional teaching strategies that are crucial in guiding the negotiation of mathematical concepts in real classroom practices.

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