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Using video analysis to support prospective K-8 teachers' noticing of students' multiple mathematical knowledge bases

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Abstract As part of a larger research project aimed at transforming preK-8 mathematics teacher preparation, the purpose of this study was to examine the extent to which prospective teachers notice children's competencies related to children's mathematical thinking, and children's community, cultural, and linguistic funds of knowledge or what we refer to as children's multiple mathematical knowledge bases. Teachers' noticing supports students' learning in deep and meaningful ways. Researchers designed and enacted a video analysis activity with prospective teachers in their mathematics methods

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course. The activity served as a decomposition of practice in order to support prospective teachers in engaging in an approximation of the practice of noticing. Our findings showed that prospective teachers evidenced noticing of mathematics teaching and learning as early as the mathematics methods course. We also found that the prompts and structure of the activity supported prospective teachers by increasing their depth of noticing and their foci in noticing, moving from attending primarily to teacher moves (and merely describing what they saw) to becoming aware of significant interactions (and interpreting effects of these interactions on learning). Implications for teacher educators interested in designing and enacting activities to support noticing are discussed.

Keywords Teacher education · Prospective teachers · Mathematics methods course · Children's mathematical thinking · Funds of knowledge · Diverse students · Noticing · Culture · Language · Community

Many teachers are underprepared to teach mathematics effectively in diverse classrooms. This is due to both a general lack of preparation to teach mathematics in ways that build on children's mathematical thinking and a more specific lack of preparation to teach mathematics to ethnically, linguistically, and socioeconomically diverse students (Howard 1999; Wiggins and Follo 1999). Teachers Empowered to Advance CHange in Mathematics (TEACH MATH) is a collaborative research project designed to address teachers' increased need to understand not only how to access and build on children's multiple ways of understanding mathematics and solving mathematical problems (e.g., *Cognitively Guided Instruction (CGI)*, Carpenter et al. 1998; Kazemi and Franke 2004), but also how to capitalize on students' diverse cultural, linguistic, and community knowledge in ways that support students' mathematics learning (Civil 2002; Ladson-Billings 1994; Turner et al. 2012). A goal of the project is to design, study, and refine instructional modules for K-8 mathematics methods courses that develop prospective teachers' (PSTs) instructional practices with a focus on children's mathematical thinking and children's community, cultural, and linguistic funds of knowledge, or what we refer to as children's *multiple mathematical knowledge bases* (MMKB). We see helping PSTs learn to draw upon these MMKB during instruction as critical to promoting equity and learning in mathematics classrooms.

Past research with PSTs and practicing teachers (Santagata et al. 2007; Sherin and Han 2004; Star and Strickland 2008; van Es 2011) indicates that teachers need support in learning to attend to, or notice, students' mathematical thinking and important classroom events and interactions—in other words, noticing is a practice that needs to be developed. The purpose of this study was to examine the extent to which a video analysis activity[1] during a methods course supported PSTs in learning to notice key moments in mathematics teaching and learning and how those connect to children's MMKB. Within this activity, PSTs used a set of four "lenses" (teaching, learning, task, and power and participation) to analyze teaching and learning practices. Within each lens, one prompt referred to "student resources" as a pedagogical representation of our construct of children's MMKB, as well as a way of focusing PSTs on this construct. These student resources included: prior

[1] This module is one of the three that have been developed by TEACH MATH for use in mathematics methods courses. All three of the modules were developed for elementary and middle school mathematics methods courses and are designed to support PSTs in developing competencies related to integrating children's MMKB in instruction. For a more detailed explanation of the activities within this or other TEACH MATH modules see Aguirre et al. (2012, 2013), Bartell et al. (2010, 2013), and Turner et al. 2012.

mathematics knowledge; cultural, community, family, and linguistic knowledge and experiences; student interests; and peers as supports for learning. This focus on student resources within each of the four lenses is based on our premise that if teachers are to support the mathematical learning of all students, particularly those who have been traditionally marginalized in schools, we need to support PSTs to identify and build on resources that children bring to the classroom.

Teaching as a developmental process

We consider learning to teach to be a developmental process that occurs within particular sociocultural contexts. The contexts in which PSTs learn to teach are varied and exist in the present as well as reach into the past, as they engage in and evaluate new activities and ideas against a backdrop of their own past schooling experiences (Lortie 1975). A sociocultural perspective leads us to investigate teachers' participation in various educational activities (e.g., viewing and discussing a classroom video) and to explore the ways in which teachers draw upon artifacts, tools, and others to make sense of their experiences and develop new understandings (Putnam and Borko 2000). Correspondingly, learning to teach entails not only developing knowledge for teaching but also developing teaching practices via repeated opportunities to enact practices or approximations of practices (Ball and Forzani 2009; Grossman et al. 2009). We do not see PST learning and development as just an individual phenomenon, nor do we assume that all PSTs begin their development with the same prior knowledge and practices. Instead, we see these practices developing dynamically as teachers participate in multiple communities of practice (e.g., as university students, as PSTs) across multiple spaces (e.g., university classrooms and elementary schools) overtime (Lerman 2001; Putnam and Borko 2000; Wenger 1998).

Developing noticing practices

An emerging body of research related to teachers' noticing supports us in studying how PSTs examine teaching and learning interactions (Hand 2012; Mason 2008, 2011; Santagata et al. (2007); Sherin et al. 2011; Star and Strickland 2008; van Es 2011). Noticing involves not only the attention that teachers give to classroom actions and interactions, but also their reflections, reasoning, and decisions based on this noticing. Mason's (2008) discussion of *attention* (i.e., *what* teachers attend to and notice) and *awareness* (i.e., how teachers *interpret* or *assign meaning* to what they notice) is helpful in considering differences in how PSTs focus on student resources. Indeed, Mason argues that "constructs such as attention … and awareness …are researchable using the discipline of noticing [and]…contribute to our appreciation of intricacies of learning and teaching mathematics" (Mason 2011, p. 35).

Noticing develops overtime and can be supported by teacher educators (Hand 2012; Jacobs et al. 2010; Mason 2008; Seidel et al. 2013; Star and Strickland 2008; van Es 2011; van Es and Sherin 2002, 2008). In van Es' (2011) work with practicing teachers in the context of a video-club, the group of teachers improved noticing over multiple experiences discussing video from their own teaching. The group's initial noticing focused on describing general impressions of teaching and progressed overtime to more detailed noticing with interpretations of teachers' pedagogy and students' thinking and learning.

In a study of PSTs in a middle level mathematics methods course, Star and Strickland (2008) found that by viewing video clips of mathematics lesson throughout the semester, PSTs' noticing improved. PSTs demonstrated limited observation skills at the beginning of the semester, attending primarily to management aspects. By the end of the semester, PSTs improved their noticing of classroom environment and tasks, and to a lesser extent mathematics content and communication. Although Star and Strickland did not study *how* the methods course may have led to these improvements in noticing, they speculated that their observation framework and research assessments supported PSTs in focusing their attention on more complex aspects of teaching and learning. They contended merely observing videos may not lead PSTs to notice what mathematics teacher educators (MTEs) intend PSTs to notice—PSTs needed to be prompted to observe specific features. They also noted that PSTs' prior beliefs and experiences may "interfere with their ability to observe" (p. 123). This conjecture aligns with what Mason (2011) referred to as *fragmented awareness*, or awareness that encompasses inconsistent orientations.

Pursuing the notion that frameworks may focus PST noticing, Santagata et al. (2007), Santagata and Angelici (2010) used a Lesson Analysis Framework that prompted PSTs to observe video-taped mathematics lessons for teachers' actions, students' learning and behavior, and mathematics content and learning goals, as well as analyze relationships among these aspects. They found that this framework supported PSTs in learning to elaborate on observations, propose alternative strategies for instruction, and analyze instruction (Santagata et al. 2007), Santagata and Angelici (2010). Similarly, Seidel et al. (2013) used two different instructional approaches with two groups of PSTs who analyzed the same video. They compared the groups and found that PSTs' learning differed based on the approach used. They argued that when engaging PSTs in video analysis, teacher educators should design instructional approaches to align with specific learning goals. In sum, research on video analysis with PSTs indicates that teacher educators need to deliberately design and structure activities for specific learning goal: simply using video may not lead to PSTs learning in ways intended.

In prior research on mathematics teacher noticing, the issue of supporting PSTs in focusing on equitable instruction for diverse students had received little attention. One exception is Hand's (2012) research with practicing teachers. She examined what teachers who are enacting equitable instructional practices notice while teaching and argued that learning to notice with a focus on equity supports the learning of diverse students. These teachers enacted equitable instructional practices in their classrooms by making connections to students' life experiences and promoting wide forms of learner participation (Hand 2012). We extend Hand's work by examining PSTs' learning to notice in ways that support equitable instructional practices, including a specific focus on PSTs' learning to notice the kinds of power and participation dynamics highlighted by Hand.

In this article, we focus on developing noticing in an activity situated in the mathematics methods classroom, one space in which PSTs learn practices that support their development as mathematics teachers. Video from K-8 classrooms acts as a bridge between school classrooms and the methods classroom by providing a context for *approximation* and *decomposition* of the teaching practice of noticing (Grossman et al. 2009). Attending to significant moments in classroom videos is an approximation of noticing in that during the video analysis activity, PSTs engage in noticing without confronting all of the complexities and distractions of actual classroom teaching. Practice is decomposed as the carefully designed methods class activity provides a space for analyzing a slice of classroom practice through videos purposefully selected to support noticing particular aspects of teaching and learning.

Building on research on teacher noticing, our study focused on examining how MTEs can support the development of PSTs' noticing key aspects of mathematics teaching and learning through a carefully constructed video analysis activity. Our work includes noticing of teacher moves, student thinking, and mathematics content and tasks that other researchers have attended to (e.g., Santagata and Angelici 2010; van Es 2011), and adds foci of power and participation and student resources, as a way to further promote noticing of equitable teaching practices that draw on students' strengths. Our study was guided by the following research question: How do PSTs in methods classes notice equitable practices in mathematics teaching and learning through repeated enactments of a video analysis activity?

Methods

Throughout the semester, PSTs from four mathematics methods classes (each at a different university) engaged in four or five video analyses of excerpts of mathematics lessons. Three videos were used at multiple project sites: *Marshmallow* (Annenberg Media 1995), *Questioning Data* (Annenberg Media 1995), and *Equality* (Carpenter et al. 2003). We examined data related to these videos.

Prospective teachers' used four lenses to analyze the videos. TEACH MATH researchers (who also served as course instructors) designed the lenses with the intention of supporting PSTs in developing noticing of mathematics teaching and learning with a focus on equitable instructional practices (Aguirre et al. 2012; Roth McDuffie et al. in press). Each lens focused on one of the four facets of a mathematics lesson: teaching, learning, task, or power and participation. All four lenses included an explicit focus on students' resources (e.g., mathematical, cultural, community, family, linguistic, student interests, and peers). See Table 1 for the prompts included in each lens.

Videos were selected for the opportunities each provided to support PSTs to notice teaching and learning through the use of our four lenses. See Table 2 for an overview of the three selected videos. Although the videos differed in which facets of mathematics teaching and learning were most prominent, researcher examination and analysis of the videos showed that each video afforded opportunities for noticing at the highest level (levels are described below) for each of the lenses. While it is beyond the scope of this paper to fully describe specific attributes and reasons for selecting each video, other manuscripts present our analysis in this regard (Aguirre et al. 2012; Roth McDuffie et al. in press).

Mathematics teacher educators prepared PSTs for the video analysis by assigning a reading that provided appropriate background for the focus of the activity (e.g., for the mathematics content or for foci on the lenses) and/or by engaging PSTs in a mathematics problem that they would later see students solving in the video. At all project sites, PSTs first analyzed the *Marshmallow* video using one question from each of the lenses as a way of introducing the four lenses. In subsequent video analysis activities, PSTs considered multiple prompts for one or more of the lenses (as described in Table 1). After watching the video, PSTs engaged in small group discussion guided by the prompts. The small groups included three to four PSTs, with group composition changing for each activity. Next, MTEs led a whole class discussion, wherein PSTs considered their noticing in relation to the thinking of other groups. Often a different lens had been assigned to various small groups, so the whole class discussion considered two or more of the four lenses. This scenario was repeated for each video analysis activity.

Table 1 Prompts provided for analyzing classroom video excerpts[a]

Lens	Prompts
Teaching lens	**How does the teacher elicit students' thinking and respond?** What opportunities does the teacher create for diverse learners to communicate their mathematical understanding—show what they know? How does the teacher implement the task in a way that maintains or changes the cognitive demand? What resources[b] and knowledge does the teacher use/draw upon to support students' math understanding?
Learning lens	**What specific math understandings and/or confusions are indicated in students' work, talk, and/or behavior?** How do students communicate what their understandings and sense making of others' thinking? In what ways does student engagement reflect conceptual and/or procedural learning? What resources[b] or knowledge do students draw upon to understand and solve the math task?
Task lens	What makes this a good and/or problematic task? How could it be improved? **What is/are the central math idea/s in this task?** How does the task make thinking visible? What resources[b] or knowledge does this task activate and/or connect to?
Power and participation lens	**Who participates? Does the classroom culture value and encourage most students to speak, only a few, or only the teacher? Where does the majority of the math "work" take place in the classroom?** Who holds authority for knowing mathematics? Do some students hold more status than others? What evidence indicates that differences in approaches and perspectives[b] are/are not respected and valued?

[a] In an initial experience of engagement with video, PSTs used a set of prompts that introduced the four lenses and included one question from each of the lenses. The one prompt was selected as a representation of the key idea of that lens. These prompts are indicated in bold

[b] The parenthetical description "(e.g., mathematical, cultural, community, family, linguistic, student interests, and peers)" was inserted here to remind PSTs to focus on specific resources

Participants

All PSTs in the methods courses were undergraduates working on certification to teach within K-8 grades. The university teacher education programs were generally three to four semesters, occurring during the latter half of the Bachelor's degree programs (with Site B's program spanning all 4 years). The methods courses were taken in the second or third semester of the final four semesters. The participants ($N = 73$) were predominately female ($N = 69$) and White ($N = 52$), with over half ($N = 41$) reporting that they spoke a language in addition to English (see Table 3 for additional participant demographics). All PSTs in each of the four methods classes were invited to participate in this study and 97 % consented.

Data collection and analysis

We collected data for all instructor and PST activities related to the video analysis activities: PSTs' small group and whole class discussions after watching each video (audio

Table 2 Overview of selected videos

Title	Description of lesson	Classroom information	Timing of viewing in methods classes
Marsh-mallows (Annenberg Media, 1995 #10[a])	The class makes a bar graph based on data students collected at home. Prior to the lesson, the teacher gave her students the assignment of consulting with family members about how many marshmallows they would eat, and students use these responses to make a class bar graph. The class uses this graph to estimate the number of marshmallows each person in their class would eat on a camping trip. Students then determine how many people one bag of marshmallows would feed, and how many bags to take on the trip	Bilingual (English–Spanish) second-grade class	Viewed at all four sites at the beginning of the semester
Equality (Carpenter et al. 2003)	The class engages with the concept of equality. The teacher leads students through a series of true/false number sentences to help them understand the meaning of the equal sign and the relationship of expressions joined by an equal sign	Bilingual (English–Spanish) fourth-grade class	Viewed at all four sites at middle to end of the semester
Questioning data (Annenberg Media 1995 #32[a])	The lesson entails two primary tasks and begins first with a discussion of (a) a local newspaper article describing mall owners' desire to institute a dress code for shoppers and (b) a survey students conducted related to the article. Next, students work in small groups to interpret graphs from various newspaper articles, or to organize data from surveys they had earlier designed about topics of personal interest	Mixed grade (fourth-sixth) class	Viewed at three of the four sites

[a] Number refers to video number on the learner.org website

or video-taped and transcribed), as well as instructors' planning materials and reflective journals.

Our coding scheme began with van Es' (2011) "Framework for learning to notice" (p. 139) used in studying how a group of practicing teachers developed noticing with a focus on teacher pedagogy and students' mathematical thinking. The levels in van Es' framework provided our initial categories for axial coding (Strauss and Corbin 1998). For the primary categories, we used van Es' description for *how* teachers notice at each level (ranging from descriptions with general impressions and evaluative comments at Level 1 to analysis and interpretations of relationships between teaching strategies and students' thinking at Level 4). For each of these levels, we created sub-categories for *what* teachers notice. Expanding van Es' framework for more focus on equitable instructional practice, we included sub-categories of noticing students' resources and students' participation in addition to teacher pedagogy and students' thinking. Table 4 shows descriptions of each level, an example of PSTs' noticing at the level, and our analysis of the noticing.

Table 3 [a] Participants' data

	Site A	Site B	Site D	Site F
Number of participants	14	24	18	17
Male participants	0	2	0	0
Female participants	14	22	18	16
Speaks a language other than English	11	8	14	8
Ethnicity (self-identified)				
White	7	22	7	16
African American	0	2	1	0
Hispanic/Latino	5	1	5	1
Asian American	1	1	3	0
Native American	0	0	0	0
Mixed Ethnicity	1	0	2	0

[a] The larger TEACH MATH originally included six university sites. Data for this study were from four of those sites (Sites A, B, D, and F). For consistency among papers written about the project, we use those designations

The examples for Levels 3 and 4 in Table 4 may need additional explanation. For the Level 3 example, PSTs did not just provide evidence of what the teacher said (as with Level 2); PSTs also discussed teacher–student interactions, using evidence to demonstrate awareness of how the teacher worked from students' thinking in selecting problems. In reporting what the teacher and students said, however, we did not see the same depth of analysis as shown in the example for Level 4. Level 4 passages included deeper analyses of two or more foci with connections to and relationships among these foci, as shown in the Level 4 example in which a PST makes connections to teacher moves, the task, students' participation, and students' resources.

We coded transcripts of small group and whole class discussions using an iterative process, as described below, to systematically discern substance, depth, and complexity in noticing. As we analyzed data, we perceived the levels on a continuum and looked for similarities and differences within levels (e.g., we found instances of higher and lower Level 2 comments). For a first round of analysis, we used Hyper Research to code each *statement* (e.g., a phrase, sentence, or a few sentences focused on one topic) in a transcript. Following coding of statements, we found that group members often built on each other's ideas, provided evidence for each other's claims, and analyzed examples others provided. In a second round, we chunked statements that focused on a particular episode from the video clip or theme into *passages* and coded each passage as a whole. The passage-based coding aligned more naturally with exchanges of ideas through social learning experiences and analysis of the depth of ideas expressed in discussions. Throughout the second round, researchers wrote analytic memos for each activity within each site (Strauss and Corbin 1998). These memos included emerging patterns relative to each video, frequencies of statements and passages coded at each level, and examples of statements and passages from the data to illustrate patterns and discern differences among levels.

Review of these memos showed that although examining levels of statements and passages within group discussions were helpful to see initial patterns in noticing, consistent with van Es' (2011) analysis, we needed to shift our unit of analysis to group discussions as a whole. A *discussion* (small group or whole class) was defined as all that was said during small group discussion time (approximately 10–15 min per video) or during whole class

Table 4 Coding scheme (adapted from van Es 2011) with examples of passages from video analysis discussions

Level	Description	Example	Analysis
1 Baseline	Descriptions of events, pedagogy, classroom environment; general impressions; and/or evaluative statements. Vague, lacks detail and evidence	"She asked questions.[1] She gave them an opportunity to think on their own.[2]" (D414, *Equality*)	[1]General description of pedagogy [2]Vague reference to participation
2 Attention	Attention to teacher moves, questions, and task(s); may begin to attend to student mathematical thinking, resources and/or participation; begins to interpret and/or provide evidence; goes beyond impressions	"I think she had really different ways of getting kids to participate because there's always those kids that want to raise their hands and share and those that don't. So she used those post-its [for students] to go up to the board and while she was singing they were singing too….And she had so many different ways to participate, there wasn't a kid left out.[3]" (A308, *Marshmallow*)	[3]Begins to provide evidence for ways students participated while describing teacher moves
3[a] Awareness	Awareness with focused noticing, analysis and interpretation of particular students' mathematical thinking, resources, and/or participation; used evidence to support claims; and/or discerned details; discussed how and/or why events occurred	"For the problem $6 = 3 + 3$, the students said it wasn't true because you can't do the problem that way, so then the teacher said, "Well, $3 + 3=6$, so $6 = 6$.[4]" And then the students were able to make the connection that way, and it also helped them use prior knowledge because they knew that $6 = 6$[5], so then $6 = 3 + 3$ can be used as a like-problem[6]." (B319, *Equality*)	[4]Discerning details of students' thinking and corresponding teacher moves [5]Interpreting students' thinking with connections to students' resources (prior mathematics knowledge) [6]Interpreting why the teacher selected a problem relative to students' thinking and resources

Table 4 continued

Level	Description	Example	Analysis
4[a] Making connections	Analyzing connections to and relationships between or among: teaching moves; students' mathematical thinking, resources, and/or participation; task features; and/or student resources	"Students get to give their own opinion, but the teacher prompts the questions off of what the students say. And, the students …know what they are working for and what they are working toward. Because they have the power to choose the graph that they are working on, the power to collect the data, and they get to choose how it is organized. They also can partner with students. Students rely on each other for clarification.[7] The teacher monitors but in order for the students to show her what they know while the teacher helps expand their thinking. And, the organization of data reflects students own thinking and interpretation. And instead of the teacher telling the students what they should think about the data, she asks them what they think about their data. And so that they can move toward a decision.[8] And then, the teacher brings students together to teach each other. For example, Miguel's Venn Diagram is a solution for the Pepsi/Both/Diet Pepsi diagram [an example analyzed in detail for students' learning earlier in the conversation].[9] And then the students get to explain their thinking and results with everyone. And, the fact that she lets kids talk about what's being learned. The teacher shouldn't do all of the talking, questioning, and prompting…."[10](F416, *Questioning Data*)	[7]Analyzing connections between teacher moves, the task, and student participation, with references to students' power and students' use of resources (peer resources) [8]Analyzing teacher moves in relation to students' thinking [9]Analyzing connections between teacher moves and students' resources (peer resources) with specific evidence [10]Connecting use of student resources (peer resources) with students' thinking and teacher moves

Superscript numerals are used to link statements within passages to the analysis of those statements

[a] Higher levels of noticing often continued to discuss teaching moves and tasks, but in addition, focused more on specific aspects of students' thinking, participation, and resources

discussion time (approximately 10 min, following small group discussions). Analyzing PSTs' group discussions was important given our orientation toward learning as a sociocultural activity (Lave and Wenger 1991; Lerman 2001; Wenger 1998). Moreover, we found that parsing statements or passages within a discussion did not necessarily reflect the level of the overall discussion. In some cases, a passage in the last few minutes of a discussion referenced and built on statements from the first minutes of the discussion—the exchanges of ideas were not linear. Similarly, we recognized that it was not possible to track an individual PST's noticing across activities in that a PST could be learning to notice at higher levels by listening to peers and not necessarily verbalizing or evidencing this noticing at every moment. In coding discussions, we applied the same levels shown in Table 4. In determining the level of the discussion, we maintained a conservative stance. That is, one instance of a brief, higher-level passage (e.g., a Level 3 passage from a single group member) was not sufficient for the discussion to be coded at Level 3. Our intention was that a higher-level code was not assigned until clear evidence of that level of noticing was demonstrated and sustained. The levels assigned to these discussions provided a more summative indicator of the levels of groups' noticing in class activities at different points during the semester (see Table 5).

Throughout this iterative process involving multiple rounds of data analysis, the research team met to discuss use of codes and continually review data for consistent interpretations and to reach intercoder agreement (Saldaña 2013). Initially, data were double-coded using and refining a code book to establish a consistent use of codes within and among researchers. Next, one researcher coded all data associated with a video activity and wrote an analytic memo for that data set (as described above). A second researcher reviewed the coding by examining data by level and by sub-categories within levels. Examining data both within a full transcript and by levels helped to reveal patterns within levels, discern differences between levels, and provided another way to check for consistency. Again, discrepancies between researchers and within/between levels were discussed by the research team until consensus was reached. In generating findings, we moved among statement-based coding, passage-based coding, and group-discussion-based coding to examine patterns, looking for similarities and differences in how and what PSTs were noticing, and verifying consistent use of the levels. Throughout each pass through the data, we met to ensure intercoder agreement.

Findings

We discuss ways PSTs noticed mathematics teaching and learning focused on equitable instructional practices through the repeated enactments of video analysis activities by presenting findings at each of the levels of noticing. As an overview, we found that discussions were predominately at Levels 2 and 3, with no discussions at Level 1 and only one that attained Level 4 (see Table 5). Although we found Level 1 passages within discussions, these passages did not dominate, and thus, we do not focus on Level 1 in our findings. For Levels 2 through 4, we found that patterns of noticing shifted to higher levels as the semester progressed. Almost all discussions (20 of 22) evidenced Level 2 noticing at the beginning of the semester, and more than half (15 of 27) evidenced Levels 3 or 4 at the end of the semester (see Table 5). We explore these shifts from the beginning (with the *Marshmallow* video) to the latter part of the semester (with the *Equality* and *Questioning Data* videos).

Table 5 Levels attained by video during small group and whole class discussions and by video, data source, and time in the semester (Total number of discussions analyzed = 49[a])

Video/ Data source/ Time in semester	Level 1 frequency	Level 2 frequency	Level 3 frequency	Level 4 frequency
Beginning				
Marshmallows				
Small group	0	19	0	0
Whole class[b]	0	1	2	0
Total (beginning)	0	20 (90.9 %)	2 (9.1 %)	0
Middle/end				
Equality				
Small group	0	6	7	1
Whole class	0	1	2	0
Questioning date[c]				
Small group	0	5	3	0
Whole class	0	0	2	0
Total (middle/end)	0	12 (44.4 %)	14 (51.9 %)	1 (3.7 %)
Total by level	0	32 (65.3 %)	16 (32.6 %)	1 (2 %)

[a] The number of groups analyzed varied by activity due to attendance and issues with recording equipment (as noted below). Overall, at each site, four to six groups participated in each activity and activities occurred approximately once each month over the semester

[b] One site did not record the whole class discussion for this video, but other data (e.g., PSTs' notes and instructor reflections) indicate findings consistent with sites that recorded

[c] One site did not record small group or whole class discussions for this video, but other data (e.g., PSTs' notes and instructor reflections) indicate findings consistent with recorded sites

Level 2 noticing (attention)

Prospective teachers' noticing aligned with Level 2 more than any other level with just over 65 % of all discussions coded at Level 2 (see Table 5). Typically, these discussions described particular teaching moves but included little to no interpretation for these moves, as well as little attention to students' moves. When we investigated patterns as the course progressed, we found three shifts in regard to Level 2 noticing. First, within Level 2, by the end of the semester PSTs included more detail and examples in passages within group discussions. Second, in regard to PSTs' noticing of students' resources, PSTs focused primarily on the use of students' home language during mathematics instruction. Conflicted and uncertain perspectives regarding language expressed early in the semester were not evident at the end of the semester. The shift toward including more detail and the shift away from conflicted perspectives are presented below.

Beginning of semester

Level 2 comments were often brief and with minimal evidence, such as D415's[2] statement, "Even if the students gave the wrong answer, [the teacher] would be like, 'Oh why do you think that?' And then she accepted every answer. It wasn't like she [said], 'Oh that is not

[2] To protect the identities of PSTs, they all are identified by a code. The first letter represents the PST's site (A, B, D, F), and the number represents a particular PST at this site.

right'" (*Marshmallow*[3]). Moreover, if PSTs mentioned foci other than teaching, these passages tended to contain brief examples. The following statements are representative of PSTs' noticing relating to other foci, as indicated for each example below:

> "I think the only confusion was that in the beginning they weren't really like, 'Oh this is less than this.' So some of them were still confused about what does less mean or more" (F417, *Marshmallow,* noticing learning).

> "It's a good problematic task because, like you said, it's a real life [context] and they're in control. They're excited for it. It's not just a hypothetical question that they're talking about. They're really going camping and they're all you know excited for it" (D404, *Marshmallow,* noticing task).

> "[For] power and participation, I thought that overall the class participation was pretty equal.… Each student contributed an estimate because they all did their own guess of how many marshmallows each person could eat" (B314, *Marshmallow,* noticing power and participation).

In regard to noticing student resources at the beginning of the semester, PSTs attended to language as a resource in a range of ways including: (a) attending to two languages spoken in the video and (b) indicating uncertainty or conflicted views [i.e., fragmented awareness (Mason 2011)] about whether it is appropriate to allow two languages to be used in the classroom. For example, although PSTs noticed culture as important, they also voiced perspectives against using languages other than English while teaching and learning mathematics. As an example of the first form of language noticing, this passage shows how PSTs noticed students using Spanish in the lesson:

B305 Another big thing I noticed was she mentioned that she was bilingual, and there is a point where one of the kids had written in Spanish
B318 In Spanish, I was very surprised by that
B305 And she just explained it in English. I don't know, I thought that especially in that culture, they were in Tucson, Arizona, there's a big Hispanic community there, I think that's very important. (*Marshmallow*)

B305 began to interpret language as an "important" cultural resource, demonstrating Level 2 noticing. However, the group did not continue to discuss why it might be important or how using home languages could influence learning, and so the passage did not clearly evidence awareness (Level 3).

In a different group at this same site (showing that differences occur within a class), B311 (with B319 agreeing) seemed uncertain and even conflicted about using home languages: on the one hand, B311 expresses that it is "fine" for the student to write in Spanish, leveraging the linguistic knowledge she brings to the classroom to communicate her reasoning, and on the other hand, B311 positions English as the only language to use in discussions, as shown below (italics are added to emphasize conflicting thoughts within the passage):

B311 I don't, I think that the girl that wrote it all in Spanish I maybe would have had her say it in English just because like,
B319 Yeah, that's what [I thought]

[3] In addition to the speaker, the video that was being discussed is provided for direct quotations. Recall that the *Marshmallow* video was shown at the beginning of the semester, and the *Equality* and *Questioning Data* videos were shown near the end of the semester.

B311 I mean I understand it is ESL or whatever, but *she is there to learn how to speak English*. So I think that *it's fine that she wrote it in Spanish because that's what she knows*, but maybe [the teacher] could [say], "*Could you say it in English so that everyone else knows what you're saying*?" So the whole group could understand it, and then, she could practice speaking her English. Because I think writing in English is harder than speaking it

B324 Okay, … Everyone in the classroom, including the teacher, participated in the activity. The children had the marshmallows, and the teacher would ask questions and record the information on the board, so there was participation from both parties…. And overall I thought it was a positive classroom culture where the students were not afraid to speak. (*Marshmallow*)

B324 did not indicate whether she agreed or disagreed with this view, and instead dropped this thread of the discussion and moved on to discussing participation, noting that the "students were not afraid to speak." This statement might have been a response to B311, as a way to point out the value of using students' home languages. The passage revealed that B311 (and seemingly B319) found it problematic that the teacher used students' home language as a resource to support learning while others recognized their value.

End of semester

We continued to find that discussions focused primarily on teacher moves, regardless of the lens used; however, Level 2 passages included more detail and examples than at the beginning of the semester. For example, for the *Equality* video, the group below started by discussing teaching approaches and then identified a specific confusion for students, describing details that were not typically provided at the beginning of the semester. Some evaluative statements remain in the passage (e.g., "I liked…," "That's my favorite…")—Level 2 passages often included Level 1 statements, yet detailed attention to teacher moves merited Level 2 coding and evidenced a shift toward more detail over the semester:

B323 I liked that she made the students so much more comfortable with the sum being on the left-hand side. She kept going over that, and they really got it

B315 The other students said [the equation] was backwards [when it was written as $5 = 4 + 1$]

B322 That's my favorite, seven doesn't equal seven, but five equals five, six equals six that was funny

We also found that PSTs began to interpret the impact of specific teacher moves in terms of students' learning:

> I noticed … the teacher asked the students to give answers to the equation, … and then she encourages the students to share with another like a buddy. And that encourages everyone to participate instead of just sitting there listening to the teacher talk. (F403, Equality)

Although the interpretations were not supported with evidence (as shown for Level 3 later), PSTs demonstrated that they were beginning to consider interactions in ways that went beyond general impressions and summary evaluations.

Prospective teachers' also attended to teachers' interactions with particular students, such as the interaction with Miguel in the *Questioning Data* video. In the video, Miguel used a Venn diagram to show survey results related to wining by luck (yes, they won by

luck, no, they did not win by luck, and don't know). Given that the choices did not involve an overlapping category, a Venn diagram was not appropriate. The teacher questioned Miguel and asked him to reconsider his data display, but later came back to Miguel and asked him to work with a group for whom a Venn diagram was an appropriate representation for data involving an overlapping category (Favorite drink: Pepsi, Diet Pepsi, and both). PSTs described the interaction in detail, recognizing the importance of positioning Miguel as a resource for his peers. For instance, in referring to the teacher's interactions with the group struggling to represent their favorite drink data, F412 said,

> [The teacher said,] "So you have Diet Pepsi and Pepsi. What if I came up to you and said I sometimes drink regular and sometimes I drink diet?—You [would need to] check both." [Later, the teacher] sent Miguel over to this group [who were struggling to organize their data] to show [them] how to do a Venn Diagram, which is [an] appropriate [use of the Venn diagram] for this group.

Notice that although this comment represented Level 2 noticing based on describing (without substantive analysis), the PST provided details about the teacher's specific comments, question, and moves, and began to interpret the teacher's moves with the phrase "which is appropriate."

In addition to noticing interactions in detail, PSTs provided evidence of students participating and learning in the lesson, as in the following exchange:

A305 They [students in the video] verbally answer the teacher's questions and gave their own opinions. And with that they responded with their thoughts and their own personal examples in the work that they had been coming up with. They did a lot of showing their own work, either with classmates or with groups. And as a class, they just shared their data with each other

A312 When [students] made their own survey,… they asked each other questions—so if one of them didn't understand something they would ask. Like those two boys, he was like "Wait, how did you find that, where did you get that from?" And the teacher would also bring other students from other groups to teach them so that they could build on what the other student was learning. (*Questioning Data*)

Although passages such as these did not delve into analysis and interpretation of actions (as we see in Level 3), PSTs attended with more detail to important aspects of teaching and participation structures that support students' learning.

By the end of the semester, we also found that PSTs asked each other to defend their interpretations about students' thinking and understandings, similar to the way instructors pressed PSTs for explanations. For instance, F416 asked F414 to explain her observation and then added to it:

F414 [Students] showed that they understand how to interpret the graphs … they like indicated their confusion by the questions they came up with based on the graphs…

F416 How did they [students] show [that they understand] how to interpret graphs?

F414 By analyzing and discussing their data

F416 … And then, they talked about it as a class

This kind of pressing for examples was not evident in discussions at the beginning of the semester.

In regard to students' resources, at the end of the semester PSTs continued to focus on home language as a resource, but we did not find instances of PSTs expressing conflicted

perspectives, as we had at the beginning of the semester. Indeed, PSTs commented on students' home languages serving as a resource for learning math. For example, B309 explained,

> I thought [the teacher] was promoting their confidence, she really got their confidence up because they were able to speak to her in Spanish and she stepped back just in case the kids didn't really know how to say it in English or maybe they just couldn't think of the right words. They were able to say that in Spanish. And she was able to understand that. And then respond back to them. (*Equality*)

Throughout groups' discussions, PSTs noticed positive effects of using two languages on students' confidence and engagement in the lesson and demonstrated that they were beginning to interpret the value of encouraging students to speak in home languages.

Summary of level 2

The structure of the video analysis activity that required PSTs to view videos from four lenses (with each lens including a focus on student resources) seemed to support PSTs in attending to specific aspects of equitable instructional practices such that PSTs evidenced noticing above Level 1 from the beginning of the semester. This finding is in contrast to findings that teachers attended only to forming general impressions or focusing only on management, as found in other research, wherein specific prompts for noticing were not provided (Star and Strickland 2008; van Es 2011). In regard to noticing student resources as part of equitable instruction, Level 2 noticing tended to focus on students' home languages, and for some PSTs, their comments at the beginning of the semester indicated confusions as evidenced, for example, by the example from Site B. We cannot claim that PSTs who voiced conflicted perspectives truly had shifted their views over the course of the semester—we can only state that we did not find evidence that these views were expressed. The lack of passages suggesting conflicted views regarding home languages at the end of the semester, however, indicated that class activities, discussions, and readings which were designed to support PSTs in developing strength-based perspectives of students' resources may have increased PSTs' attention to ways home languages can support learning. An alternate explanation is that PSTs learned that instructors would challenge these views, therefore learning not to voice them.

Level 3 noticing (awareness)

Level 3 noticing went beyond attention to details in teaching and learning to include substantive analysis and interpretation of observed events that included evidence for claims, indicating an awareness of how and why teaching and/or learning unfolded in certain ways (see Table 4). We found two shifts in regard to noticing at Level 3. First, Level 3 noticing increased over the semester. At the beginning of the semester, only 2 of 22 discussions were coded at Level 3, and both of these discussions occurred with an instructor and the whole class. By the end of the semester, slightly over half of the discussions (14 of 27) evidenced Level 3 noticing (see Table 5). Second, in regard to students' resources, by the end of the semester, PSTs analyzed and interpreted the role of incorporating home languages in instruction (going beyond merely attending to language) and began to notice ways in which students' resources other than language can support learning (e.g., prior mathematics knowledge, peers, and family involvement).

Beginning of semester

For passages within discussions that were coded at Level 3, PSTs typically began by focusing on particular teaching moves and then noticed students' mathematical thinking. The passage below illustrates how a PST identified specific questions the teacher used and then analyzed how that teaching move affected students' learning, providing evidence, and explanations for her claims and interpretations.

> When the teacher asks …what the graph should start with, the kids weren't really sure of how the graph should be, and the kids responded by saying, "Ten." But then when she restated her question to ask, "What was the smallest number?", and the kids knew [the correct answer] was two. So by restating the question and giving them another way of understanding it…the children were able to accurately place the numbers on the graph in the correct way. They were able to tell the [most] frequent number of the marshmallows on the graph by looking at it, by just looking at the pictures but [also] by being able to count the numbers of how many sticky posts were on the graph. (B325, *Marshmallow*)

The comment showed an awareness of how interactions between teacher and students (the teacher restating and rephrasing a question) can support students' thinking, along with a description of what students understood and how they were using the graph.

In regard to student resources, Level 3 passages evidenced that some PSTs were aware of ways that home language use can support learning. For example, in the passage below, a PST shared her interpretation regarding the role of students' home language in supporting student participation and feelings in school:

> Bringing in their native language and encouraging them to respond or answer a question in their native language, that's what they were more comfortable in. That really boosts participation. Because without that, they are going to be more nervous to talk and write their results. (F416, *Marshmallow*)

Recall that Level 2 noticing about student resources typically focused on students' home languages, but when PSTs analyzed and interpreted teaching and learning at Level 3, they began evidencing an awareness of additional resources (e.g., family). For instance, D414 offered her observations and interpretation for the reasons to include families in the *Marshmallow* task, implying that families can be a resource for learning. As she explained:

> The students were able to go home and actually experiment with the activity first. [The teacher] told them to go home and think about how many marshmallows you could eat with their parents. So it allowed them to think and be somewhat prepared for the activity. So they already knew that this was what was going to be expected of them so they knew how many marshmallows they could eat. (D414)

For all instances of Level 3 noticing in small group discussions at the beginning of the semester, the passages were isolated and not taken up by the group for further discussion, indicating that most PSTs were not yet noticing at this level and those that did exhibit Level 3 noticing were just beginning to become aware.

In addition to evidence of Level 3 noticing in occasional passages within small group discussions, two of the whole class discussions evidenced Level 3 noticing at the beginning of the semester. In both of these cases, the instructor supported PSTs in improving noticing from Level 2 to Level 3. The excerpt below from a whole class discussion illustrates how the instructor pushed noticing to a higher level. Note that the first PST's comment showed

attention to specific aspects of the task (Level 2), but after the instructor focused PSTs' attention on involving family, the discussion shifted to Level 3 interpretations for ways families support learning.

Instructor	And about the task? … Were these good tasks for second graders? Not good tasks?
A310	It was relatable to them, [children] like to eat marshmallows, "How many would I like to eat?" and "How many do you want to eat?" It was relatable to them and something that they were eager to work with. So it was hands-on for marshmallows
Instructor	So what do you all think then that she had them not just come up with a reasonable estimate in class? … She said they had to go home and talk to their family about what a reasonable number of marshmallows might be. Why do you think she might have done that? Do you think that was a good move on her part? Or not so good move?
A302	Maybe kids don't know what a "reasonable [number]" meant and so by asking their parents, they would say, "Oh you can probably … eat eight," because they would think …, "Oh I want 50 marshmallows," but they don't know how many [a child] could really eat…
Instructor	…Other reasons why you think she might have wanted to do that?
A307	It allows parents to get involved too. And to have the student be involved with the parent in their math lesson. Even though it is something really small, their parents can help them out and they can bring that to the classroom
A301	… They are not just remembering it for the class period or practice within the class. It's something that they learned at home and something that they have to remember all night and the next morning

Prior to these instructor questions, PSTs' noticing remained at Level 2 and did not reference the role of families, and yet with questions to focus their attention, PSTs demonstrated awareness for ways that involving families can support learning. Although discussions such as these were not common at the beginning of the semester, they provided evidence that some PSTs entered methods classes with understandings and perspectives to analyze teaching and learning at Level 3, particularly with support from instructors.

End of semester

In addition to increases in noticing at Level 3, we found three patterns at the end of the semester within Level 3 passages: teaching continued to be the dominant focus but other foci were also included (with deeper interpretation of the importance of students' power and participation), PSTs' small group interactions supported deeper analysis and interpretation, and passages focusing on student resources often included discussion of resources beyond language.

Although analysis of teacher moves continued to dominate Level 3 passages, at the end of the semester PSTs typically noticed teaching moves in relation to other foci such as students' authority, status, and competence (aspects of the power and participation lens). For instance, in a group discussion analyzing particular teaching moves, F414 described her analysis of classroom culture with regard to authority and status.

> The students hold the authority because when [the teacher] put 12 in the box because the student said 12, she didn't say, "No that's wrong I'm not going to put that there."

> [Instead she] said, "Is that the answer you all agree on?" …Then she put it in there. And then each example, it was all about them and their responses, and then they were all working in groups …. So it wasn't like one student was picked out or called upon most often. … You couldn't tell who the high kids or the low kids were; it was just a big classroom. They were all working together. (*Equality*)

As another example, in a group's analysis of the teacher's interactions with Miguel in the *Questioning Data* video, a PST noticed that the teacher's actions were likely to support Miguel in feeling competent in mathematics, in spite of his initial confusion during the lesson:

> I liked it when [the teacher] was talking to the boy who started with the Venn Diagram, and it wasn't really working out with what he was doing. … I like how she incorporated him with the group [of students using] the tally marks [since this group did struggle to organize their data, and a Venn diagram would be appropriate for their data]. So it made him feel like, "Okay, maybe this didn't work for me, but maybe this can work for another group." (A305, *Questioning Data*).

This increase in noticing beyond teacher moves and questions suggests that as PSTs gained more knowledge and experience observing videos of classrooms using the four lenses, they began to understand more complex factors of equitable instruction that could influence teaching and learning such as how students participate in learning. Correspondingly, PSTs noticed over a broader range of foci *while* achieving higher levels of noticing.

Consistent with patterns from the beginning of the semester, when passages evidenced Level 3 noticing, it often seemed to be a result of interactions among PSTs in their small group. As is shown in the passage below, once again referring to the episode with Miguel, each of the four group members contributed to the passage, and collectively analyzed, interpreted, and supported claims with evidence.

F416 I love how she brought Miguel over to talk to the other group about, I mean using peers as resources, I think that's just so wonderful. Something that didn't work for him, but he kind of knew the concept, and he was able to teach to this other group [who needed help]…

F414 She goes and gets him and brings him to the table and says, "Look, help them do this….You had an idea that didn't work but I think it would be really great for their data."

F405 Look at how it built his self-esteem!

F412 I like how it showed him the right application of the Venn diagram [and] about what was wrong

F416 And it puts him in the teacher role. (*Questioning Data*)

Each of the PSTs contributed unique aspects to this analysis to generate noticing at Level 3. While this pattern of PSTs supporting each other to notice at higher levels was identified at the beginning of the semester, this pattern became even more apparent at the end of the semester with the emergence of more Level 3 passages.

Passages focused on student resources were common in discussions at the end of the semester. PSTs' analysis continued to focus on students' home languages as a resource; however, they also considered ways teachers supported English Language Learners (ELLs). For example, the following group noticed how a teacher facilitated ELLs' learning by using visual representations, letting students solve the problem, encouraging students to talk about the problem, and expecting students to explain their reasoning:

B321 [Writing students' responses on the board] helps the students that don't speak English to see it in a number form and actually understand it that way. That way [students] can look at and say, "Twelve plus whatever is not right." I like how [the teacher] built on it too because I think that really helps [students] to clear up their misconceptions. Like [the teacher] started with a problem and just let [students] solve it. And then they had to talk it out and figure out what the equals sign actually meant before they returned to the first problem and expanded on it. Which, especially when English is a weakness, I think it's important to keep reiterating and just building on what they already understand and [addressing] those misconceptions

B312 Yah, I like how [the teacher]… with the whole 6 = 6 and 5 = … and [the teacher said], "Does this equal this? Does this equal this."… She's kept asking "Why?"—which is really good.… Obviously [the students] had to think about it, and it would allow them to give their answers. She [always] asked why. So if 5 = 5 and they said, "Yes," she would never ignore that, like she would always ask them [why], which is good. (*Equality*)

Although this response included detailed noticing and analysis, we found it interesting that these moves were observed by one PST as key supports for ELLs in particular; indeed, they align with pedagogical moves that we promote in the courses as important for all learners. In this case and in other groups' conversations, it seemed that considering needs for ELLs prompted PSTs to look more closely at pedagogical moves that support learning; however, it was not clear that PSTs were connecting these moves to all learners.

In regard to PSTs focusing on resources beyond language, at the end of the semester, PSTs evidenced awareness of students' resources in the form of experiences in the community, interests, use of peers as resources, and prior mathematics knowledge. The following example illustrates how a small group identified and analyzed resources employed by students in *Questioning Data*. This passage also is another example of PSTs' building on and questioning each other's noticing:

A300 [Students related the task to] their own personal experience

A302 Experience of being at the mall and knowing what people dress like and people wearing bandanas or whatever

A301 Using articles or graphs clipped from the newspaper. So like show them that other people are interested in learning about this

A302 Do you have any other knowledge [that you noticed in the video]?

A300 I said for mathematical knowledge the girl that said, "Well we usually go up to 100 %, and it adds up to 118." So they knew [that] percentage… usually adds up to 100

A301 And when they did their own things [conducted their own surveys], they asked their family and community members

Prospective teachers demonstrated their awareness for how connecting tasks to students' communities, personal experiences, and their prior mathematical knowledge supported students' engagement in and learning from the lesson.

Summary of level 3

Level 3 noticing was not common at the beginning of the semester, and when it did occur, the instructor played an important role in supporting higher-level noticing in whole class

discussion. Yet when considering Level 3 whole class discussions in combination with the occurrence of Level 3 passages within group discussions, we found that Level 3 noticing is not beyond PSTs, even early in the methods course. By the end of the semester, Level 3 noticing was common. Not only did PSTs regularly analyze and interpret what they attended to, they expanded their range of foci: they expanded noticing to become aware of the importance of students' authority and sense of competence in learning (going beyond just noticing involvement and engagement, as we saw at Level 2), and their noticing of resources extended beyond language to include interpretations for the role of students' families, community experiences, interests, prior math knowledge, and peers in teaching and learning.

Level 4 noticing (making connections)

At the beginning of the semester, no passages demonstrated Level 4 noticing for any of the sites, and correspondingly, no discussions were coded at Level 4. Thus, Level 4 findings are presented only for the end of the semester, where we found occasional passages at Level 4 for both the *Equality* and the *Questioning Data* video analysis activities. Moreover, we found one small group discussion that evidenced Level 4 noticing (see Table 5). Although this discussion was not representative of other discussions, we examine this unique example below.

End of semester

For this small group discussion, all four group members participated by building off of each other's claims and evidence, discussing relationships among the foci, and contributing examples to support connections.

F413 I don't think there was a single time when [the teacher] said, "This is the answer," like gave him the right answer…. She led them where they can find it themselves. … She would take them back to something easier, like $6 = 6$ …

F416 Yes, it was backward [to him]; he said you can't write it like that

F413 …Once they found that $6 = 6$… they proved that themselves, and then she kind of worked that from there

F416 And she used multiple examples to move up from there. "Okay, you got that right, now let's move on to the first part."

F415 She used really basic [equations] too

F414 $5 = 5$, will this $5 = 4 + 1$? Does $6 = 3 + 3$?

F413 Right and then even after they got the problem, they went back and they got it wrong and fixed it. She did another example for them to practice

F415 And she went back to the original one… After they viewed that answer, they just went to $15 + 4 = 19$ to make sure they got that concept

F416 And I liked how she was clarifying that 19, 19 goes in the box, and they were like "Oh no, that's not what we meant." Nineteen was the answer … and she emphasized that they needed to be specific because $15 + 4 = 19$, but $19 + 11$ doesn't equal 19

F413 And that helped them realize their thinking to point out what's going on with this. (*Equality*)

In the discussion, the group explored: (a) the teacher's questioning approach (F414 and F413), (b) students' authority and participation in that they needed to "find it themselves"

(F413), (c) the teacher beginning with students' perspectives and understandings about the equations (F413 and F416), (d) the students' reasoning about the mathematics (F413 and F416), and (e) the overall sequence of moves in the lesson to support learning (F414, F413, and F415). The PSTs included interpretations for students' thinking and reasoning [e.g., "it was backward (to him); he said you can't write it like that" (F416)] and interpretations for teacher's moves and questions [e.g., "Even after they got the problem, they went back and they got it wrong and fixed it. She did another example for them to practice." (F414)]. Although details about teaching moves continued to be more prevalent than other foci in this discussion, PSTs analyzed multiple aspects of the episode and interpreted relationships and connections between the teacher's moves and students' thinking and responses, delving deeply into understanding multiple dimensions of teaching *and* learning. Unlike Level 3 passages, this passage included analysis of the relationships and connections between teaching and learning. Although Level 3 passages contained interpretations for one of these aspects, they did not analyze, deconstruct, and connect multiple aspects.

Summary of level 4

We were not surprised that Level 4 noticing rarely occurred for PSTs at this point in their development. However, the existence of passages and one group discussion evidencing Level 4 noticing indicated that at least some PSTs can begin to engage in deeper noticing—even early in their professional development. Within Level 4 passages, we did not find any direct references to student resources related to students' home languages, culture, family, or community. However, given that many forms of student resources were a focus of discussions at lower levels, we have established that PSTs can attend to and be aware of effects of student resources in teaching and learning. Although it is possible that noticing connections to home languages, culture, family, and community is more challenging than other connections, we do not believe we have adequate data for such a finding at this point.

Discussion and implications

A growing body of research reveals how teachers' noticing is an important part of practice that supports students' mathematical learning in deep and meaningful ways (Hand 2012; Sherin et al. 2011; Star and Strickland 2008; van Es 2011). Correspondingly, we explored PSTs' noticing of equitable instructional practices and children's MMKB over the course of the mathematics methods semester. To support PSTs' learning in this regard, we decomposed and approximated practice (Grossman et al. 2009) by designing the video analysis activity. Consistent with our work across the larger TEACH MATH project, we endeavored to respect PSTs as developing professionals with prior experiences and knowledge that informed what they noticed, while resisting a stance of attempting to identify "what is wrong" with PSTs (Bartell et al. 2013).

Our findings indicate that PSTs evidenced noticing of mathematics teaching and learning during the mathematics methods course. In comparison with the practicing teachers in van Es' (2011) research, these PSTs demonstrated higher levels of noticing during their first experiences with video analysis. Van Es found that practicing teachers evidenced Level 1 noticing in their initial analyses of classroom video, while PSTs in this study began at Level 2. In contrast to our study, the teachers in van Es' study were analyzing video of their own classrooms. Furthermore, van Es did not design specific prompts to use repeatedly. Instead, the teachers in her study considered a general prompt

(i.e., "What did you notice?", p. 137) and then moved to prompts that focused on specific aspects of each video. In our case, we deliberately designed prompts and selected videos to focus PST noticing on targeted aspects of teaching and learning. Consistent with implications from related research that recommends providing prompts or a framework for viewing video (e.g., Star and Strickland 2008; Seidel et al. 2013), these scaffolds seemed to support PSTs' noticing at higher levels. Our findings suggest that the repeated use of our lenses focused PSTs' attention on key aspects of teaching and learning mathematics, allowing them to consider the impact of teachers' and students' decisions and actions, and in some instances begin to make connections between teacher moves and students' mathematical thinking, students' participation, and students' resources.

Similar to van Es' (2011) results, we found that PSTs tended to hold a teacher-centric perspective when demonstrating lower levels of noticing: noticing at Level 2 they attended predominately to teaching. In regard to student resources, PSTs tended to attend only to students' home languages at Level 2 and evidenced noticing of other resources (e.g., family, community, and mathematics knowledge) as they began to analyze intentions and impacts of teaching and learning (Level 3). This may be attributed at least in part to the fact that two of the three videos discussed in this article come from bilingual classrooms and afforded the opportunity to focus on language.

In regard to participation, PSTs tended to limit attention to students being involved or engaged in a lesson at Level 2 and evidenced noticing of the importance of authority, status, and competence at Level 3. Correspondingly, if we want PSTs to notice multiple aspects of students' resources and participation early in their development, MTEs need to provide ways to prompt PSTs to focus on these aspects of teaching and learning, including selecting videos that feature a range of student resources (including language, culture, and community). Van Es' framework that focused levels of noticing with regard to teacher pedagogy and students' mathematical thinking provided a strong foundation for our work. Given our focus on equitable instructional practices and children' MMKB, we built on this framework by adding foci for student resources and student participation. Our findings indicated that PSTs can learn to notice across multiple foci as early as during the methods course. We thus see that focusing on equitable instructional practices (including teacher moves directed at students' mathematical thinking, as well as other resources) does not overcomplicate or overwhelm PSTs' learning (i.e., we do not need to take on these multiple aspects of practice as isolated parts). Furthermore, we found that when provided support for noticing in these areas, PSTs can begin to notice a range of student resources and to analyze how and why these resources enhance learning. Furthermore, they can notice how students participate and are provided opportunities to demonstrate authority and competence in learning.

With our deliberate efforts to focus attention on equitable instructional practice through repeated enactments of noticing, PSTs demonstrated that they can develop awareness for multiple aspects of teaching and learning and notice at deeper levels; however, our prompts and video choices, along with opportunities to repeat the activity, seemed critical components of inducing and developing this noticing. As Ball and Forzani (2009) argued, "repeated opportunities for novices to practice carrying out the interactive work of teaching" (p. 503) should be a focus of teacher education. As we research practice-focused approaches to teacher education, the role of repeated enactments of practice merits further study. For example, what number of enactments is both necessary and sufficient? And how might results from these enactments be influenced by other experiences such as field observations in the practicum classroom? These are important considerations when we take into account the small number of weeks that

PSTs spend in mathematics methods courses and the number of experiences that MTEs would like them to have.

Consistent with sociocultural theories of learning (Lerman 2001; Lave and Wenger 1991; Wenger 1998), as PSTs interacted in small groups and then in the whole class, we found that they built on each other's ideas, perspectives, and observations such that higher levels of noticing were attained. Future research also might explore ways to deliberately plan for interactions to support PSTs' learning and to study how understandings improve in social contexts. Moreover, we recognize that the activity as we constructed it provides specific affordances and constraints in regard to what is observable in the videos and how PSTs respond to the video through consideration of the prompts we provide. Future research might explore how specific prompts supported or limited noticing. In addition, future research might examine elements of teaching and learning that were not featured in these particular videos, as well as MTEs' practices in facilitating activities such as these.

We also endeavor to understand PSTs' past experiences that led them to perspectives that evidenced fragmented awareness toward children and their home languages, so that we can disrupt these views. Although passages such as those from the *Marshmallow* video at the beginning of the semester indicated that some PSTs were struggling with whether students' should use their home languages while learning math, we appreciated that the PSTs were willing to express their confusions and perspectives as they began to learn about how home languages can support learning. As we also conclude in a previous TEACH MATH study (Bartell, et al., 2013), when views remain hidden, then the class as a community learning together cannot grapple with difficult and uncomfortable issues. Yet we found that the views expressed in group discussions at the beginning of the semester were not static; PSTs engaged with each other and their instructor to consider the interaction among teaching, students' perspectives, and students' MMKB. This finding suggests that PSTs need multiple opportunities to expose and identify their fragmented awareness and to develop more informed and considered perspectives. Discussion with peers as well as input from instructors can help PSTs move toward a greater understanding of the resources available to and used by students.

This study provides some understandings of PSTs' learning through a particular form of approximation and decomposition of practice (Grossman et al. 2009) by developing PSTs' noticing through video analysis. As we continue to follow these PSTs in the TEACH MATH, we are interested in studying how activities such as these support PSTs in composing their future practices as teachers.

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