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Characterizing pivotal teaching moments in beginning mathematics teachers' practice

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Abstract Although skilled mathematics teachers and teacher educators often "know" when interruptions in the flow of a lesson provide an opportunity to modify instruction to improve students' mathematical understanding, others, particularly novice teachers, often fail to recognize or act on such moments. These pivotal teaching moments (PTMs), however, are key to instruction that builds on student thinking about mathematics. Video of beginning secondary school mathematics teachers' instruction was analyzed to identify and characterize PTMs in mathematics lessons and to examine the relationships among the PTMs, the teachers' decisions in response to them, and the likely impacts on student learning. These data were used to develop a preliminary framework for helping teachers learn to identify and respond to PTMs that occur during their instruction. The results of this exploratory study highlight the importance of teacher education preparing teachers to (a) understand the mathematical terrain their students are traversing, (b) notice high-leverage student mathematical thinking, and (c) productively act on that thinking. This preparation would improve beginning teachers' abilities to act in ways that would increase their students' mathematical understanding.

Mathematics teacher education · Beginning teachers · Student thinking · Teachable moments · Teacher decisions

Introduction

Accumulating evidence suggests that it is possible to identify instructional practices that lead to student achievement gains in mathematics (e.g., Fennema et al. 1996; Stein et al. 2000).

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The list of such practices includes engaging students in solving and discussing rich tasks, building on students' current thinking, and helping students connect mathematical ideas. For example, Fennema et al. (1996) documented the achievement gains of children in classrooms where the teachers focused on student thinking by learning about the Cognitively Guided Instruction (CGI) framework for student thinking, relating it to their own students' thinking, and adapting their instruction accordingly. QUASAR project researchers (e.g., Stein and Lane 1996) identified the critical role of mathematical tasks in students' learning of mathematics. They found a relationship between increased student mathematical abilities and tasks that encouraged "high-level thinking and reasoning and the use of multiple solution strategies, multiple connected representations, and mathematical explanations" (Silver and Stein 1996, p. 506).

The identification of instructional practices that enhance student learning has prompted the study of how teachers implement these ideas in their classroom. For example, Cengiz et al. (2011) explored in detail the teaching practice of *extending student thinking* and were able to identify individual instructional actions supportive of that practice. Researchers have also focused on what it takes for teachers to implement these ideas. Ball and Cohen (1999), for example, discuss the notion of teachers *learning in and from practice* that involves teachers "siz[ing] up a situation from moment to moment" (p. 11) and using what they learn to improve their practice. Van Es and Sherin (2008) use the term *noticing* to describe the way in which teachers attend to important elements of instruction that support student learning and then reason about them in order to make instructional decisions. An important consideration is how teachers can be supported in learning to focus their attention on the features of classroom practice that are most important to student learning—student ideas, evidence of student understanding, and the relation between student thinking and pedagogical decisions (Santagata et al. 2007; Sherin 2002; Stockero 2008; van Es and Sherin 2008). Research around mathematics teacher noticing has paid much attention to these issues in recent years, focusing on what teachers notice in classroom video, what teachers notice during instruction, how particular types of teacher learning experiences impact teacher noticing, and how teachers make sense of and respond to particular events that they notice (see, for example, Sherin et al. 2011). Although the specific language and methods researchers use to describe and study the practice of keying in on important moments during instruction varies, the idea is the same—teachers need to pay attention to students' ideas and consider how to use these ideas to advance student mathematical learning, and be supported in doing so.

A major difference between expert and novice teachers is the "form and structure of their attention" (Mason 1998, p. 243). Although skilled teachers and teacher educators are often able to recognize important mathematical moments that occur during a lesson and act on them in ways that support student learning, such moments frequently either go unnoticed or are not acted upon by others in the profession, particularly novices (Peterson and Leatham 2009). This raises the question of how teacher educators can help novice teachers recognize important mathematical moments during their instruction and use them to support student learning. Mason (1998, 2011) suggests that enhancing teachers' awareness may help to produce shifts in their attention—what they focus on in the act of teaching. Therefore, characterizing the circumstances likely to lead to important mathematical moments in a lesson is an important first step in making teachers aware of such moments and, thus, in becoming attentive to them during instruction.

This study focuses on understanding the types of instances that beginning teachers need to notice during instruction, how they currently respond to these instances, and how their responses potentially impact student learning. Specifically, we focus our attention on the fourth component of Rowland et al.'s (2005) knowledge quartet, *contingency*—a teacher's recognition of the value in students' unexpected ideas and their ability to deviate from the planned lesson, when appropriate, in response to these ideas. With the goal of providing a framework for helping teachers to notice opportune mathematical instances during instruction, we define a *pivotal teaching moment* (PTM) to be an instance in a classroom lesson in which an interruption in the flow of the lesson provides the teacher an opportunity to modify instruction in order to extend or change the nature of students' mathematical understanding. Thus, a PTM can be thought of as something that prompts the need for drawing on a teacher's *contingency* knowledge.

PTMs are also related to Remillard and Geist's (2002) idea of *openings in the curriculum* in the context of teacher professional development, which they characterize as moments in which teachers' questions, observations, or challenges require the facilitator to make a decision about how to incorporate into the discussion the mathematical or pedagogical issues that are raised. The nature of the facilitator's decision determines the extent to which the teachers' ideas advance the learning of the group.

Similar to what these other researchers have described, when a PTM occurs in a classroom lesson, a teacher first needs to recognize it as such and then make a decision about how to handle the interruption. Depending on the action taken by the teacher, the PTM may or may not enhance the development of students' mathematical understanding. Because teachers may or may not act on a PTM, determining whether an instance is such a moment is independent of the teacher's actual decision in response to the PTM. Consequently, an instance can be characterized as a PTM even if the teacher does not notice it. For the purposes of our work, the criteria used is that the instance must be noticed by someone (the teacher, other students, a researcher) who witnessed it, either by being present or by engaging with a record of the interactions. The distinction here is that, in identifying PTMs, we are not trying to capture whether the teacher *did* notice the instance and modify instruction to support student understanding, but, instead, whether an event provided an opportunity for the teacher to do so.

In the manner of Brousseau (1997), we are attempting to make sense of actions taken (or not taken) by teachers in classroom situations so that we can study *as a system* student thinking and teacher responses to it. Our goal in this exploratory study was to develop testable hypotheses about PTMs that can be used to improve the functioning of this system. Toward this end, we used videos of beginning teachers' classrooms to investigate the following questions:

a. What are characteristics of PTMs faced by beginning secondary school mathematics teachers during classroom instruction?
b. What types of decisions do beginning mathematics teachers make when a PTM occurs during their instruction?
c. What relationships exist among PTM characteristics, teachers' decisions, and the likely impact on student learning?

Developing a better understanding of PTMs has the potential to improve students' learning of mathematics by helping teachers notice and learn to build on high-leverage student ideas to enhance their mathematical understanding. PTMs can be used as a tool to improve the ways in which we educate teachers to use student thinking during instruction by providing a framework that can be used to help teachers learn to focus on mathematically rich moments that occur during instruction. Additionally, understanding the relationships among PTM components, teacher decisions, and the likely impact on student learning would allow teachers, teacher educators and researchers to know which moments or decisions might be more or less productive to focus on because of their likely potential to improve student learning. This information would focus researchers on analyzing critical aspects of teachers' noticing and use of student thinking during a lesson and would allow teacher educators and teachers to more accurately and efficiently make decisions that would maximize use of class time.

Perspectives

The idea that there are important moments or events within a mathematics lesson that a teacher needs to notice and act upon is grounded in a particular vision of teaching. Consistent with reform recommendations for mathematics education (e.g., National Council of Teachers of Mathematics [NCTM] 2000), and supported by research on student learning (e.g., Fennema et al. 1996; Silver and Stein 1996), this vision is one in which teachers build on student thinking during instruction. This is accomplished through the teacher continually reflecting on the mathematical ideas that underlie students' comments or solutions and then using these comments or solutions in ways that help the class as a whole construct meaningful understandings of mathematical concepts.

Research has shown, however, that a teacher's intention to use student thinking during instruction does not mean that learning will necessarily be enhanced (Van Zoest et al. 2010; Walshaw and Anthony 2008). Inherent in this vision of teaching is a need for teachers to *reflect-in-action* (Schön 1983) in order to "[spot] the golden opportunities and wise points of entry that they can use for moving students toward more sophisticated and mathematically grounded understandings" (Walshaw and Anthony 2008, p. 539). Capitalizing on such moments requires teachers both to *notice* (Sherin et al. 2011; van Es and Sherin 2002, 2008) important mathematical moments when they arise and to have the *orientations, resources,* and *goals* (Schoenfeld 2011) necessary to act on them in ways that support student learning.

In their Learning to Notice Framework, van Es and Sherin (2002) highlight three main components of teacher noticing in the context of analyzing artifacts of classroom practice: (a) identifying important aspects of the situation, (b) reasoning about these aspects, and (c) connecting what is observed to more general ideas about teaching. Although related, we view the type of noticing that teachers engage in after instruction as quite different than that in which they must engage during instruction. During instruction, teachers rarely have the benefit either of time to systematically analyze student thinking and consider alternate interpretations of it, or of colleagues to help draw attention to important moments. Instead, teachers need to quickly draw on their own mathematical knowledge to recognize moments that may be mathematically important and then make decisions about how these moments could be capitalized on either to help address the goals of the lesson or to make connections to larger ideas in mathematics. Given the cognitive demand of this work and novice teachers' need to simultaneously pay attention to developing other more routine aspects of their practice (Berliner 2001), it is not surprising that they are often unable to recognize these important mathematical moments and, if recognized, have difficulty making pedagogically sound decisions about them.

An explanation for the difficulty teachers have making sound decisions about important mathematical moments is found in Schoenfeld's (2011) work on teacher decision-making. This work suggests that if teachers' orientations, resources, or goals are working at cross-purposes, it will be difficult for them to effectively act on student thinking. For example, a beginning teacher graduating from a teacher education program based on the ideas underlying the *Standards* (NCTM 2000) may be oriented toward using student thinking in his or her instruction but constrained by a lack of skill in doing so and/or by the goal of moving quickly through the curriculum to keep up with his or her colleagues.

Although capitalizing on important mathematical moments is likely to be difficult for a novice teacher, an important first step is recognizing that such moments exist. Without this awareness, teachers may experience *inattentional blindness* (Simons 2000)—a phenomenon described in the psychology literature as a failure to focus attention on unexpected events. In the context of teaching, a teacher's failure to recognize that a student's ill-formed idea may be mathematically significant may result from a lack of recognition that this could be the case. This is also related to the idea of *framing* (Levin et al. 2009)—the way in which a teacher makes sense of a classroom situation. From this perspective, whether teachers notice the value in student thinking depends on how they frame what is taking place during instruction. If, for example, a teacher views a student error as something that needs to be corrected, he or she is unlikely to consider the mathematical thinking behind the error or whether the error could be used to highlight a specific mathematical idea. On the other hand, a teacher who views an error as a site for learning is more likely to consider both the mathematics underlying the error and how it could be used to develop mathematical understanding. Although it is beyond the scope of this study to look at the reasons for the teachers' decisions, these perspectives motivate and inform our investigation of PTMs, the decisions teachers make in response to them, and their likely impact on student learning.

Methodology

In this exploratory study of PTMs, we focused on beginning teachers because we hypothesized that the decision-making process involved with PTMs may be more obvious with this group than with skilled teachers. That is, skilled teachers may recognize a PTM and make the decision to act so quickly and smoothly that the interruption would not be easily observable by someone who was not intricately familiar with the teacher's plan for the lesson. Furthermore, it is rare that a written lesson plan would have enough detail to note any but the most obvious changes in direction. Beginning teachers, on the other hand, were predicted to have a slower response time—slow enough that, although the thinking process itself would not be visible, the fact that they were making a decision would be. In the following, we describe the data that were used to study PTMs, as well as the analysis process.

Data collection

As part of a larger research project focused on examining teacher learning using practice-based teacher education materials (e.g., Van Zoest et al. 2011), over 45 h of video of classroom mathematics instruction was collected from six teachers with fewer than four years teaching experience. Each teacher was recorded for at least two class periods per day for three consecutive days in the middle of the fall 2008 semester. The class periods ranged from 50 to 90 min. All of the teachers were graduates of an NCTM (2000) *Standards*-based secondary mathematics teacher preparation program in the US that focused on teaching mathematics for student understanding. The participating teachers were a convenience sample of a group that had voluntarily returned to the university for a one-day professional development session that took place in the month prior to the start of the semester in which the video data were collected. The professional development session involved analysis of practice-based materials as part of activities that were similar to those they had participated in during their teacher preparation program. More details about the nature of these activities in the context of the teacher education program can be found in Van Zoest and Stockero (2008).

At the time of the data collection, the participating teachers were teaching mathematics in grades 8–12 (approximately ages 13–18) in a variety of school settings, including a rural school with a large Hispanic population, an urban school where approximately half the students were African-American, a suburban school with a predominately white population, and an alternative school for students who had not been successful in their local school. The topics taught included algebra, geometry, and trigonometry. Two of the teachers used investigative curricula, while the other four used transmission-based mathematics textbooks more typical of US schools. During videotaping, a microphone was placed on the teacher and a camera was focused on either the whole class or the area of the room in which the teacher was interacting. This captured interactions available to be noticed by the teacher.

Data analysis

The existing body of video data was examined for this study in order to develop a preliminary understanding of PTMs that occur during mathematics lessons. This analysis involved four phases: (a) reducing the video data to potential sites for PTMs; (b) identifying PTMs; (c) characterizing PTMs; and (d) examining the relationships among PTMs, teacher decisions, and their potential to support student learning.

Phase one: video reduction

Two graduate students who were experienced secondary school mathematics teachers completed the initial reduction of the video database. On their first pass through the video, they eliminated from the database any non-instructional activities, such as listening to school-wide announcements and completing individual assessments. On the second pass, they used the PTM definition to narrow the remaining video to episodes that had the potential to include PTMs. During this process, they were intentional in reducing the video so that each episode contained only what was necessary to make sense of the potential PTM. Each graduate student individually identified any episodes that they felt had potential to include PTMs and then the two conferred to verify that the episodes warranted further exploration. At the end of the reduction process, 65 episodes totaling 3 h and 57 min remained in the database. Each teacher had at least one episode included, with a teacher using an investigative curriculum having the most episodes and a teacher in an alternative school where students often worked independently having the fewest.

Phase two: identifying PTMs

The process of identifying PTMs involved a research team of the authors, one of the graduate students, and a fourth mathematics education researcher. All were experienced observers of mathematics teaching. The episodes were first watched by the team to determine whether they agreed that a PTM had occurred. If it was agreed that one had occurred, the episode was transcribed for further analysis. At the end of this stage of the process, 27 episodes totaling 2 h and 38 min remained in the database.

The four researchers next individually watched these episodes and marked on the transcripts the points in the dialogue at which PTMs occurred. Through a collective process of analyzing and discussing their results (i.e., Arafeh et al. 2001; Luft and Bemis 1970; Thompson et al. 2004), the research team identified a total of 39 PTMs. During this phase of analysis, the researchers came to think of the PTMs as triples that included the PTM, the corresponding teacher decision, and the likely impact on student learning. Although it was beyond the scope of this study to determine the actual student learning outcomes, drawing on literature about the mathematical quality of instruction (e.g., Horizon Research, Inc. 2000; Learning Mathematics for Teaching Project 2011; Stein et al. 2000) the researchers were able to examine evidence of the extent to which the PTM and corresponding teacher decision provided the opportunity for students to extend or change the nature of their mathematical understanding. The researchers identified where each element of the triple—PTM, decision, and likely impact—occurred in the transcript of each PTM episode. At weekly research meetings, these elements were discussed for each episode until agreement was reached.

Phase three: characterizing PTMs

After identifying where the PTMs, decisions, and likely impacts occurred in the transcript, each of these elements was tagged in the corresponding classroom video using the Studiocode video analysis software (SportsTec 2010). This process resulted in timelines that linked the coding to its corresponding video segment. The research team then used open coding (Strauss and Corbin 1990) to develop an initial characterization of PTM types, decision types, and likely impacts on student learning. To facilitate discussion during weekly research meetings, the researchers' individually coded timelines were merged to compare coding. This allowed the research team to more easily identify and discuss potential codes, refine codes and definitions, and come to agreement about the coding of each PTM episode.

The coding process resulted in five PTM types: extending, incorrect mathematics, sense-making, contradiction, and confusion. In addition, five teacher decision actions were identified: (a) extends mathematics and/or makes connections; (b) pursues student thinking; (c) emphasizes meaning of the mathematics; (d) acknowledges, but continues as planned; and (e) ignores or dismisses. Both the PTM types and teacher decision actions are discussed in detail later in the paper ("Pivotal teaching moment characteristics" and "Teacher decisions in response to PTMs" sections, respectively).

During the coding process, the researchers noticed that some PTMs seemed more powerful than others, and, furthermore, it was not only the decision that was important, but also the effectiveness of its implementation. This led the researchers to add additional coding to the PTMs and decisions. Each PTM was initially coded as having significant, moderate, or low potential to support student learning. The low potential PTMs were later eliminated from the data because it was agreed that instances with low potential were not likely to extend or change the nature of student learning and, thus, did not meet the definition of a PTM. Consistent with mathematics education literature (e.g., NCTM 2000; Stein and Lane 1996), PTMs coded as having significant potential involved rich mathematics and, frequently, connections among mathematical ideas. Often they provided a gateway to discussing important mathematical ideas that were not part of the planned lesson, but were an important part of the mathematical terrain the students were traversing. PTMs coded as having moderate potential often related to attributes of mathematics, such as its usefulness or coherency, or provided the opportunity to better understand procedures, definitions, or concepts.

Each teacher decision was coded as being implemented skillfully, moderately, or poorly. We drew on literature about the mathematical quality of instruction (e.g., Horizon Research, Inc. 2000; Learning Mathematics for Teaching Project 2011) to inform our coding of the implementation level. Often, decisions coded as poorly implemented included incorrect mathematics or directed the students' attention away from the mathematical goal. Those that were moderately implemented included correct mathematics, but fell short of supporting students to gain the intended mathematical understandings. Skillfully implemented decisions were those that effectively supported student learning of mathematics. As with the initial coding, each instance was coded individually and then discussed among the research team members until agreement was reached.

Using evidence from the videos, we assessed the impact each PTM and corresponding teacher decision and implementation level were likely to have on student learning. The categories that best described the differences among the instances were *high positive, medium positive, low positive, neutral*, and *negative*. Given the decisions and implementation levels among the beginning teachers in our study, we did not document any instances that were likely to have a high positive impact on student learning. There were, however, instances where a more skillful implementation of the decision would have introduced important mathematics beyond what was originally in the lesson and/or provided the students with a significantly richer mathematical experience, thus we included *high positive* in the framework. This kind of learning is what we see as the goal of work on PTMs—skillful teachers recognizing student thinking that is based on important mathematics and capitalizing on the pedagogical opportunity it provides to modify instruction in order to extend, or change the nature of, students' mathematical understanding.

The distinction between *medium positive* and *low positive* likely impact was primarily due to the type of learning that they supported. That is, some instances simply clarified a definition or procedure (*low positive*), while others focused on improving mathematical understanding of the ideas in the lesson (*medium positive*). Instances considered to have a *neutral* likely impact were those in which the students' learning was not affected one way or another. For example, a student might introduce a new idea or raise a question that requires unpacking by the teacher to get at the mathematics behind it. Failure to do so would be a lost opportunity, but as long as the teacher's response did not introduce misconceptions or send a negative message to the students, it would not have a negative effect on their learning. A *negative* likely impact occurred when unresolved confusion or misconceptions were introduced into the lesson either by the PTM itself or by the teacher's response to the PTM, leaving the students with an incorrect or more fragile understanding of the mathematics than if the PTM had not occurred.

As a result of this work, the research team was able to identify a preliminary coding scheme that was used to characterize (a) the type of PTM, (b) the potential it had for advancing students' mathematical understanding, (c) the teacher decision, (d) the way in which the decision was implemented, and (e) the likely impact the PTM had on student learning, given the decision made and its implementation. The framework that resulted from the coding and the data that informed it are discussed in the "Results" section.

Phase four: analyzing relationships

After the coding was complete, the researchers made multiple charts (see Table 2 for a summative example) to more closely examine the relationships among the five coded components of PTMs described above. This analysis involved determining whether there were patterns in the PTM triples (type, decision, likely impact), as well as looking for relationships among different pairs of coded components. For instance, charts were made to analyze whether some PTM types were more likely to have high potential, whether the participants were better able to respond to certain PTM types, which PTM types were most likely to lead to positive impacts on student learning, and how decision implementation affected the likely impacts of PTMs. Throughout the process, the researchers returned to the coded video to answer questions that arose.

Results

One important outcome of this exploratory research was a preliminary framework for understanding PTMs, the types of decisions teachers make in response to them, and their likely impact on student learning (Table 1). The coding and analysis that led to the development of the framework is discussed in detail in the "Data analysis" section. In the following, we focus on the findings from that analysis.

Table 2 summarizes the 39 PTMs that were identified in the data and is referred to throughout the remainder of our discussion. The numbers in parentheses denote the number of instances with the given attribute. Notice that there are 41 decisions and likely impacts; this is because two PTMs (an *incorrect mathematics* and a *contradiction*, both with moderate potential) were each followed by two separate decisions and likely impacts. For example, the *incorrect mathematics* PTM was first followed by a decision to pursue student thinking (moderately implemented) that resulted in a neutral impact. Perhaps because the teacher realized that her actions did not have the desired outcome, she then skillfully extended the discussion by making connections to the students' prior experiences. The result was a medium positive likely impact on student learning. To aid the reader, the two decisions, implementation levels, and likely impacts for each of these instances are noted in Table 2.

The following three sections, respectively, address the research questions: (a) What are characteristics of PTMs faced by beginning secondary school mathematics teachers during classroom instruction? (b) What types of decisions do beginning mathematics teachers make when a PTM occurs during their instruction? and (c) What relationships exist among PTM characteristics, teachers' decisions, and the likely impact on student learning?

Table 1 Initial framework for understanding pivotal teaching moments, corresponding teacher decisions, and the likely impact on student learning

<table>
<tr><th colspan="2">Pivotal Teaching Moment</th><th colspan="2">Teacher Decision</th><th rowspan="2">Likely Impact on Student Learning</th></tr>
<tr><th>Type</th><th>Potential</th><th>Action</th><th>Implementation</th></tr>
<tr><td>Extending</td><td rowspan="5">Significant
Moderate</td><td>Extends mathematics and/or makes connections</td><td rowspan="3">Skillfully
Moderately
Poorly</td><td>High positive</td></tr>
<tr><td>Incorrect mathematics</td><td>Pursues student thinking</td><td>Medium positive</td></tr>
<tr><td>Sense-making</td><td>Emphasizes meaning of the mathematics</td><td>Low positive</td></tr>
<tr><td>Contradiction</td><td>Acknowledges, but continues as planned</td><td rowspan="2"></td><td>Neutral</td></tr>
<tr><td>Confusion</td><td>Ignores or dismisses</td><td>Negative</td></tr>
</table>

Table 2 Summary of PTMs identified in the data

Pivotal Teaching Moment		Teacher Decision		Likely Impact
Type	*Potential*	*Action*	*Implementation*	
Extending (11)	Significant (10)	Extends/connections (4)	Moderately (3)	Medium (2)
				Neutral (1)
			Poorly (1)	Medium (1)
		Pursues (1)	Poorly (1)	Low (1)
		Emphasizes meaning (1)	Poorly (1)	Negative (1)
		Acknowledges but continues (3)		Neutral (3)
		Ignores or dismisses (1)		Neutral (1)
	Moderate (1)	Acknowledges but continues (1)		Low (1)
Incorrect mathematics (7)*	Significant (1)	Acknowledges but continues (1)		Negative (1)
	Moderate (6)*	Extends/connections (1)[a]	Skillfully (1)[a]	Medium (1)[a]
		Pursues (2)[a]	Moderately (2)[a]	Neutral (1)[a]
				Low (1)
		Emphasizes meaning (2)	Moderately (1)	Low (1)
			Poorly (1)	Neutral (1)
		Ignores or dismisses (2)		Neutral (1)
				Negative (1)
Sense-making (13)	Significant (3)	Pursues (1)	Poorly (1)	Negative (1)
		Acknowledges but continues (2)		Negative (1)
				Neutral (1)
	Moderate (10)	Pursues (3)	Moderately (3)	Medium (1)
				Neutral (2)
		Emphasizes meaning (2)	Moderately (2)	Medium (1)
				Low (1)
		Acknowledges but continues (3)		Negative (2)
				Neutral (1)
		Ignores or dismisses (2)		Neutral (2)
Contradiction (2)*	Significant (1)	Ignores or dismisses (1)		Neutral (1)
	Moderate (1)*	Extends/connections (1)[a]	Skillfully (1)[a]	Medium (1)[a]
		Emphasizes meaning (1)[a]	Moderately (1)[a]	Low (1)[a]
Confusion (6)	Moderate (6)	Pursues (3)	Moderately (2)	Medium (1)
				Low (1)
			Poorly (1)	Negative (1)
		Emphasizes meaning (2)	Moderately (1)	Medium (1)
			Poorly (1)	Low (1)
		Ignores or dismisses (1)		Neutral (1)

[a] Represents a PTM that had multiple decisions and likely impacts

Pivotal teaching moment characteristics

The following subsections elaborate on five different types of PTMs identified in our data—*extending*, *incorrect mathematics*, *sense-making*, *contradiction*, and *confusion*—and consider their potential to affect student learning.

Extending

One circumstance that seems to guarantee a PTM is when students make a comment or ask a question that is grounded in, but goes beyond, the mathematics that the teacher had planned to discuss. We identified 11 such circumstances in our data; all 11 were coded as PTMs, 10 of them with significant potential to positively affect student learning. For example, in one instance, the teacher was focused specifically on explaining how the “m” and “b” in the equation $y = \mathrm{m}x + \mathrm{b}$ can be found from the graph of a linear function. A student raised his hand and asked if it were possible to have more than one y-intercept. By doing this, the student opened up the possibility of extending the lesson to make a connection with the definition of a function. This was considered to have significant potential because the comment shifted the focus from a procedural explanation to one that was based on a deeper mathematical meaning.

Incorrect mathematics

PTMs often occur when incorrect mathematical thinking or an incorrect solution is made public. In some cases an error does not qualify as a PTM—as when it is based on an incorrect calculation or something else that is not likely to interfere with or provide opportunities to improve students' mathematical understanding. In other instances, however, the error is likely to affect what students take away from the lesson. There were seven PTMs in our data based on incorrect mathematics, with one having significant and six having moderate potential to affect student learning.

The significant potential instance occurred during a lesson focused on graphing and finding equations to model situations that could be represented by quadratic functions. When asked to construct a graph to model a situation in which a soccer ball was kicked into the air, one student's "distance-time graph" was a picture of someone kicking a ball in the air and the path of the ball as it returned to the ground, rather than a graph of points relating the distance of the ball above the ground to the time that had lapsed since it was kicked. The student's mathematically incorrect representation of the situation provided an opportunity to clarify the difference between a mathematical representation of a situation and a picture of the situation—a common idea with which algebra students struggle (Wagner and Parker 1993).

Sense-making

Other PTMs occur when students are trying to make sense of the mathematics in the lesson. This was the most prevalent circumstance in our data—13 of the 39 instances—but, unlike *extending*, not every instance of *sense-making* qualified as a PTM. Verbalization of students' sense-making became a PTM when it provided opportunities to clarify or highlight critical mathematical components of the lesson. For example, a student who was trying to conceptually understand what was being presented as a purely procedural explanation raised a question about why the procedure works. The student's push for meaning was an indicator to the teacher that the lesson was not making important connections and a prompt to begin to do so. A variation of the *sense-making* circumstance occurred in lessons in which sense-making was the focus. In the process of trying to make sense of the mathematics, students sometimes over-generalized or misconstrued aspects of the mathematics that, when surfaced, provided learning opportunities for the entire class. *Sense-making* PTMs most often (10 of the 13 times) had moderate potential to affect student learning.

Mathematical contradiction

The occurrence of a mathematical contradiction also seems to guarantee a PTM. This can be as straightforward as two different answers to a problem that clearly should have only one answer, or as complex as two competing interpretations of a mathematical situation. Regardless, the contradiction creates an opportunity for the teacher to bring to the students' attention the nature of mathematics that makes such contradictions unacceptable. It also provides an opportunity to highlight critical aspects of the mathematics at hand that can help students determine which of the options holds up under scrutiny. The very process of this scrutiny—the justification needed to support the different options and the making of a decision—can create a powerful learning opportunity. Only two such observed instances made this the least common circumstance in our data, but both of those instances led to a PTM, one each of significant and moderate potential.

The *contradiction* PTM with significant potential occurred in a classroom where the teacher had posed the following task from the teacher's book: Use a table of values to graph the equation $x + 2 = -8$. After a brief whole-class construction of a table of values in which $x = -10$ was paired with a range of y-values, the teacher asked what the equation would look like if they were to graph it, ending with "Is this going to be a vertical line or a horizontal line?" The contradiction occurred when students shouted out both "vertical" and "horizontal" as their responses. This PTM had significant potential because it offered the opportunity to revisit several important mathematical ideas related to functions from earlier in the course.

The PTM with moderate potential occurred when a teacher was calculating the sine of an angle using an overhead calculator. The students' calculators gave a different answer than the teacher's because the teacher's calculator had inadvertently been set to use radians rather than degrees. This PTM provided the opportunity to discuss the importance of units of measurement and the internal consistency of mathematics.

Mathematical confusion

A student's expression of mathematical confusion can also lead to a PTM. It is important to distinguish general confusion—when students express that they do not know what is going on or that they cannot follow what someone has just explained—from mathematical confusion. Confusion seems to lead to a PTM when students can articulate mathematically what they are confused about. This occurred six times in our data, all with moderate potential to affect student learning. One example of this occurred when students were sharing their simplifications of expressions containing exponents (e.g., $(16x^2y^2)^{-1}(xy^2)^3$). As one student simplified an expression on the board, another student was able to point to the second step of the simplification as being her point of confusion. This gave a mathematical focus to the confusion—the properties of exponents that were used in that step—and created an opportunity for the teacher to refocus the students on the meaning behind the procedures they were applying to the problem.

Teacher decisions in response to PTMs

The decision a teacher makes about how to respond to a PTM—ranging from ignoring or dismissing it, to using it as an opportunity to extend or enhance the planned lesson—shapes whether the potential of the PTM is actualized. As described in the "Methodology" section, we realized early in the analysis that it often was not only the decision the teacher made that affected the outcome, but also the effectiveness with which he or she implemented that decision. Thus, in this analysis, we considered both whether the teacher actively responded to the PTM (coded as something other than *ignores or dismisses* or *acknowledges but continues*) and, in cases where an active response was made, the implementation level of the teacher's decision (skillfully, moderately, or poorly). For our purposes, we refer to skillfully or moderately implemented as effective implementation. Considering both the decision and implementation level allowed us to differentiate between ineffective decisions and decisions that had the potential to positively impact student learning, but failed to do so because of ineffective implementation. Note that we acknowledge that a teacher may have legitimate reasons for not actively responding to a PTM, such as knowing that the point will be returned to later or that it is an issue for only the student making the comment. However, we did not see evidence of this being the case in our data for this study.

Ignores or dismisses

One possible response to a PTM is to ignore it or dismiss it out of hand, for example, by saying, "We are not talking about that now." The teachers in this exploratory study either did not notice, ignored, or dismissed 7 of the 39 PTMs (18 %). As an example, consider further the *contradiction* PTM with significant potential described in the "Contradiction" section above. As the students were shouting out that the graph of the equation $x + 2 = -8$ would be both "vertical" and "horizontal," a student pointed out that the student textbook was different from the teacher's edition, giving the problem they were working on as $x + 2y = -8$. Rather than addressing the contradiction or using the two equations as an opportunity to explore the issues, the teacher told the students to forget about $x + 2 = -8$ and focus on $x + 2y = -8$ instead, which they then graphed as a class. The likely impact was neutral because the students were no worse off after the interchange than before, but the significant potential of the PTM was lost. Six times an *ignore or dismisses* decision led to a neutral likely impact and one time to a negative likely impact; it never resulted in a positive likely impact on student learning.

Acknowledges, but continues as planned

Another possible response is to acknowledge the PTM, but in a superficial way. The teachers in our study acknowledged, but did not capitalize on 10 of the 39 PTMs (26 %). Most often this occurred when the teacher would respond to the PTM by affirming that a student had posed a good question or comment and giving a brief response before moving on with the lesson as if the PTM had not occurred. In the illustration in the "Extending" section above, for example, the teacher responded to the student's question about the possibility of there being two dots on the *y*-axis—that is, for the linear function to have more than one *y*-intercept—by saying, "There won't be two dots on the *y*-axis. That's a good question though, you're thinking thick, you're thinking bigger. OK. So let's write the [equation of the] line [that] goes through the *y*-axis at plus one, so we add one to our equation." This led to a neutral likely impact because there were no misconceptions introduced and the students were encouraged to keep thinking and asking questions, but the teacher's failure to capitalize on the PTM by engaging the students in thinking about the mathematical reason that a linear equation could not have two *y*-intercepts kept it from reaching its significant potential. Overall, only one (10 %) of the decisions to *acknowledge but continue* led to a positive likely impact on student learning. Four of the decisions led to a negative impact and five had a neutral impact on learning.

Emphasizes meaning

One way in which a teacher can actively respond to a PTM is by emphasizing the mathematical meanings behind the issues it raises. For 8 of the 41 decisions in the study (20 %), the teachers focused on the meaning of the mathematics by highlighting a definition or the mathematics underlying a procedure. For example, in one instance students had been working in small groups on creating a graphical model of a rabbit jumping over a fence, given the parameters of a 3-foot-high fence and the rabbit leaving and returning to the ground 4 feet from either side of the fence. During the ensuing whole-class discussion, the students were engaged with the context and figuring out what was going on mathematically. A medium potential *sense-making* PTM occurred when a student made the statement, “All the values only gonna be on our line, though, right?” The teacher used this comment as an opportunity to engage the students in considering points not on the curve and to emphasize the meaning of domain. Her moderate implementation of this decision to emphasize meaning fulfilled the medium potential PTM and led to a medium positive impact on student learning. Six of the eight *emphasizes meaning* decisions (75 %) led to a positive likely impact. The only ones that did not lead to a positive likely impact were those that were poorly implemented.

Pursues student thinking

Another active response is to find out more about what the student(s) who initiated the PTM was thinking. The teachers in our study chose to pursue student thinking 10 times (24 % of the decisions). In these instances, the teachers focused on understanding the meaning of what a student had said, generally by asking the student to provide more information about their thinking. One instance in which this occurred was during a teacher’s review of the relationship between the lengths of the sides in the 45-45-90 reference triangle for the unit circle. After it had been determined that the length of each leg of the triangle was $1/\sqrt{2}$, the teacher asked the question, “Does anyone remember how to get rid of square root of two in the bottom?” A significant potential *sense-making* PTM occurred when, during the ensuing discussion, a student said, “I know how to do it, but I still don’t understand why. I just don’t see the logic in it.” The teacher then asked the student a series of questions to try to pinpoint where the breakdown in the student’s understanding occurred, but in a way that did not support the student’s sense-making attempts. For example, the teacher asked whether the student knew how to eliminate the radical in the denominator of the fraction and whether he could find the length of the leg of a 45-45-90 triangle given the length of the hypotenuse. None of the teacher’s questions helped the student make sense of why the relationship among the sides of the triangle was true. The teacher concluded the interchange by saying, “You know, what you [have to] be able to do with this is start with the pattern and use it. The why part is the harder part. The why part is tough and the why part will come with time and practice.”

In effect, the teacher’s poor implementation of the decision to pursue student thinking sent a strong message to the students that they were not supposed to make sense of the mathematics, resulting in a significant potential PTM having a negative likely impact on student learning. Seven of the ten (70 %) *pursues* decisions were moderately implemented and three (30 %) were poorly implemented. Five *pursues* decisions led to positive likely impacts (50 %), three neutral (30 %) and two negative (20 %), with both of the negative impacts on student learning occurring with poor implementation.

Extends/makes connections

This type of decision occurs when teachers go beyond the topic that students are working on in the lesson to revisit and make connections to past learning or to foreshadow or lay a foundation for future learning. The study teachers made decisions to extend the mathematics and/or make connections six times (15 % of the decisions), four of which occurred in response to extending PTMs. Five of them had medium positive likely impacts (83 %) and one a neutral impact. *Extends/makes connections* was the only decision type that had no likely negative impacts on student learning.

In one instance of an *extends/makes connections* decision, the class had been investigating horizontal and vertical translations of quadratic equations. During group presentations, the class came to the conclusion that the graph of $y = x^2$ "moves up" when a positive number is added to the x^2 and down when it is subtracted. At this point, a student asked, "Yeah, but what if it was like $x^2 + 3x$?" This comment was immediately followed by a protest from another student that the first student was making things too complicated. The teacher validated that the student's example was a quadratic equation and, although right now they were going to focus on just adding a constant, her idea was connected to a broader generalization and would be looked at in future lessons. The way in which the teacher did this sent the message that thinking beyond the immediate task and asking extending questions was an important part of mathematics and a welcome form of participation in the class.

Summary

In summary, the data suggest that the decisions a teacher makes in response to PTMs significantly affect the way in which a PTM is likely to impact student learning. Active responses have a higher likelihood of having a positive impact, while inactive responses are more likely to lead to a neutral or negative likely impact. Additionally, implementation appears to affect the impact, with poorly implemented decisions less likely to have a positive impact on student learning. These findings are further elaborated in the following section, particularly in Hypotheses 2 and 3.

Relationships among PTM framework components

Our interest in PTMs extends beyond the development of an identification framework, to learning enough about PTMs that the construct can be used as a tool to support teacher learning. Thus, we were particularly interested in identifying which types of PTMs and teacher decisions might be more likely to lead to improved student learning opportunities. This information would allow teacher educators to help teachers learn to change the focus of their attention (Mason 1998) to key in on high-leverage classroom instances that are likely to improve student learning (Walshaw and Anthony 2008). It may also be that there are types of PTMs that are not likely to do so. If this is the case, knowing so would allow teacher educators and teachers to more accurately and efficiently make decisions that would minimize unproductive use of class time.

To determine whether we could develop hypotheses about how specific PTM components or combinations of them might affect student learning, we analyzed the data in Table 2, developing additional charts and returning to the original video data when warranted for further investigation of potential relationships. In the following, we discuss five hypotheses about PTMs that were drawn from the data in this study. These are hypotheses due to the exploratory nature of the study and the limited data set. Our intent is to provide a foundation so that future research can test and expand these hypotheses to provide more definitive results.

Hypothesis 1 The type of PTM does not affect teachers' ability to respond to it effectively.

The decision and implementation data showed that there was no one PTM type that the beginning teachers were able to respond to better than the others. In this analysis, we considered whether the teacher actively responded to the PTM and, if so, the level at which the decision was implemented.

Confusion PTMs were associated with active teacher responses the most often (83 % of the time), but only 60 % of these responses were implemented effectively. This implies that this type of PTM may be easier for beginning teachers to recognize and respond to in some way, but that implementing the decision well is still difficult. *Extending* PTMs were actively responded to by the beginning teachers 55 % of the time, but had the lowest effective implementation rate, with only 50 % of those responded to associated with a moderately, and none with a skillfully, implemented decision. *Sense-making* PTMs were actively responded to 46 % of the time, with all but one of those responses (83 %) being moderately implemented.

Incorrect mathematics and *contradiction* were the only PTM types in the study that were associated with a skillful implementation—once for each type. Overall, however, these PTM types were not generally associated with more active decisions than the other PTM types. *Incorrect mathematics* PTMs were actively responded to 63 % of the time (five of the eight decisions), with 80 % effective implementation (one skillfully, three moderately). The two *contradiction* PTMs were actively responded to 67 % of the time (two of the three decisions) and had a 100 % effective implementation rate (one skillfully, one moderately), but the small number of instances of this PTM type makes it difficult to draw any conclusions.

In summary, the active response rates for the five PTM types ranged from 46 to 83 %, with effective implementation rates for active decisions ranging from 50 and 100 %. Although it may appear that these rates could imply a difference in the beginning teachers' ability to respond to certain PTM types, there was no clear relationship between percent of active responses and percent of positive implementation. In other words, the PTMs that the beginning teachers were most easily able to identify and act upon were not those associated with the most productive implementation. Furthermore, with the exception of the *contradiction* PTMs, where the sample size was very small, no PTM type was associated with active, effectively implemented decisions more than half of the time. These findings are not surprising given that previous research has documented novice teachers' difficulty responding to student comments during a lesson (Berliner 2001; Davies and Walker 2005; Jacobs et al. 2010; Peterson and Leatham 2009). Our findings, however, provide additional perspective on the problem, as they suggest specifics about novice teachers' difficulties responding to all PTM types.

Hypothesis 2 Teachers' inability to notice when important mathematical opportunities take place has a negative affect on students' opportunities to learn.

Consistent with Walshaw and Anthony's (2008) conclusion from their synthesis of research on mathematics classroom discourse, the data in this study strongly suggest that a teachers' ability to notice and act on important mathematical opportunities that occur during instruction is critical to improving students' opportunities to learn mathematics. In the study, the beginning teachers' active response to such opportunities in some way was associated with a medium or low positive likely impact on student learning 67 % of the time—83 % for *extends/connections* decisions, 50 % for decisions to *pursue student thinking*, and 75 % for *emphasizes meaning* decisions. Teachers' inactive responses to such opportunities (coded as *acknowledges but continues*, or *ignores or dismisses*) were associated with a positive impact only 6 % of the time—10 % for *acknowledges but continues* and 0 % for *ignores or dismisses*. Furthermore, a teacher's failure to actively respond had a 29 % chance of being associated with a likely negative impact on student learning.

Consider the "distance-time graph" example from the "Incorrect mathematics" section where the student's mathematically incorrect representation of the situation provided an opportunity to clarify the difference between a mathematical representation of a situation and a picture of the situation. The teacher's failure to take advantage of this opportunity created a situation that could introduce or reinforce a common misconception, thus resulting in a negative likely impact on student learning.

The one case where a minimal teacher response was associated with a likely positive impact was an instance in which a student proposed to write a solution as $y = (2^x)/2$. The teacher acknowledged this was correct, but did not discuss it and proceeded to push for the equation that he wanted, $y = (1/2) \cdot 2^x$. In this case, even though the teacher did not spend time on the student's idea, the students' exposure to the idea that there might be more than one way to write the solution may have had a low positive impact on their learning.

Other studies (e.g., Davies and Walker 2005; Peterson and Leatham 2009) have documented similar difficulties associated with novice teacher noticing. These studies have highlighted how a lack of content knowledge limits novice teachers' abilities to both notice important mathematical moments during instruction and effectively respond to them. In addition, Peterson and Leatham also highlighted other factors that affect novice teacher noticing, including an inability to simultaneously attend to student thinking and other aspects of teaching, and a tendency to make assumptions that students understand the mathematics even though their statements about it are incorrect. Although it is beyond the scope of this study to know *why* the participants did not actively respond to student comments, it is clear from this and other studies that noticing and actively responding are important to supporting student learning, and thus, must be a focus of teacher education programs.

Hypothesis 3 Improving teachers' abilities to implement productive decisions will reduce the number of negative impacts on student learning.

We documented nine PTM episodes that were associated with a medium positive likely impact on student learning—the highest likely impact documented in the study. Of the nine, two were related to decisions that were skillfully implemented, six to decisions that were moderately implemented, and only one to a decision that was poorly implemented. We also documented eight instances in which the likely impact on student learning was negative. Of these, four were coded as *acknowledges but continues* and one as *ignores or dismisses*. The remaining three resulted from poorly implemented decisions. Interestingly, four of these negative impact instances were associated with significant potential PTMs—two in which a decision was poorly implemented and two where the teacher acknowledged the student comment, but continued as planned.

Looked at another way, when a decision was skillfully implemented, it always had a medium positive likely impact, and when moderately implemented, it was likely to have a low (33 % of the time) or medium (40 % of the time) positive likely impact. When a decision was poorly implemented, however, it was more likely to have a negative (43 %) or neutral (14 %) impact. In sum, skillful or moderate implementation was associated with a positive likely impact on student learning 76 % of the time, while poor implementation of a productive decision was associated with a positive likely impact only 43 % of the time.

An example of a productive decision (*emphasizes meaning*) that resulted in a likely negative impact due to poor implementation occurred in an instance in which a class was discussing the graph of the square root of x. As a student group was sharing their findings about horizontal and vertical translations of the graph, another student interrupted to ask, "Are those asymptotes?" referring to whether the graph had a horizontal asymptote. This comment provided an opportunity to extend beyond the planned lesson to discuss this important idea related to graphs. The teacher responded by saying, “[L]et’s think about the equation. It is a square root, right? So as I go out, is my square root getting bigger, or will it eventually level off? Like what’s the square root of a hundred?” After looking at one other value, 144, the discussion concluded with the teacher saying, “As I go, don’t I keep getting bigger? So I don’t think there will be an asymptote. I think that as we get bigger it will keep getting bigger.” In this case, the teacher attempted to emphasize the meaning of an asymptote, but looked at only a couple of sample values and failed to highlight the critical aspects of an asymptote. It is likely that this explanation left students with misconceptions about what it means for a graph to have an asymptote.

Taken together, the data suggest that implementation matters. Negative effects on learning tend to be associated with decisions that are poorly implemented or do not address the PTM, while effective decision implementation tends to be associated with a positive impact on student learning. This implies that teachers need not only to learn to recognize PTMs and make appropriate decisions, but also to develop the skills and expertise needed to effectively implement those decisions. Jacobs et al. (2010) found that although expertise in responding to student thinking does not naturally develop through teaching experience alone, teachers’ decision-making skills can be enhanced through targeted teacher education activities. For example, they documented improvements in novice teacher decision-making following sustained professional development focused on children’s mathematical thinking. Other researchers are also engaged in ongoing investigations of ways to support novice teachers’ reasoning about student thinking and decision-making. Peterson and Leatham (2009) are studying the effects of a restructured student teaching experience that focuses on eliciting and building on students’ thinking. Stockero (e.g., Stockero and Thomas, in press) is engaging prospective mathematics teachers in research-like analysis of unedited classroom video to help them learn to notice and respond to mathematically important moments that occur during a lesson. Santagata and colleagues (Santagata and Guarino 2011; Santagata and van Es 2010) are using frameworks to guide prospective teachers’ analysis of classroom lessons by focusing on the use of evidence of student learning to guide decision-making. Although the results of these efforts are not yet fully documented, initial findings regarding their effectiveness in supporting novice teachers’ decision-making are encouraging.

Hypothesis 4 Focusing on mathematical trajectories and connections will produce more positive impacts on student learning.

The decision to extend or make connections was found to be the most productive teacher decision in our study data. In the study, a teacher’s decision to extend or make connections was never associated with a negative likely impact, regardless of the implementation level. This was not the case with any other decision type. In fact, five of the six instances in which this decision was made were associated with a medium positive likely impact on student learning—one as the result of a poor implementation—and the last had a neutral likely impact. This makes the decision to extend or make connections the one most likely to have a positive impact on learning (83 % of the time in this study), regardless of the PTM type that the decision was in response to.

Although this decision type held the most promise in terms of supporting student learning, the data also suggest that it may have been difficult for the beginning teachers to recognize PTMs for which this decision was appropriate. We had anticipated that most *extending* PTMs would be paired with an *extends/connections* decision, but only 4 of the 11 *extending* PTMs were followed by this decision type. In fact, 5 of the *extending* PTMs did not elicit active decisions by the teachers. We hypothesize that the beginning teachers may have made the decision to extend or make connections more often if they had a better understanding of the trajectory along which mathematical ideas develop and connections among mathematical ideas (Ball et al. 2008).

Hypothesis 5 PTMs with significant potential to improve student learning may be able to do so on their own, whereas those with moderate potential require a well-implemented decision in order to positively impact student learning.

The highest likely impacts on student learning that were documented in the study—medium positive—were all associated with one of the following: (a) moderate potential PTMs that had well-implemented decisions, or (b) significant potential PTMs, regardless of decision implementation level. In total, nine PTM episodes were coded as having a medium positive likely impact on student learning.

Six of the medium positive impact instances were associated with a moderate potential PTM with a decision that was implemented either skillfully (two instances) or moderately (four instances). These instances were associated with four different PTM types (*incorrect mathematics*, *sense-making*, *contradiction*, and *confusion*) and with all three types of active decisions (*extends/makes connections*, *pursues*, and *emphasizes meaning*). Thus, the PTM or decision type does not appear to affect whether a moderate potential PTM episode has a medium positive impact on learning, but rather it is the teacher's ability to implement the decision that makes the difference. The remaining three medium positive likely impact instances were associated with an *extending* PTM that had significant potential to support student learning; in each case, the teacher made a decision to extend or make connections. This decision was moderately implemented in two cases, and poorly implemented in the third.

Taken together, the data suggest that significant potential PTMs may be able to carry themselves, whereas moderate potential PTMs need to have well-implemented decisions in order to achieve their potential to positively affect student learning. This raises interesting questions about what moments might be most productive to focus on in teacher education, which are discussed in the following section.

Implications and conclusion

PTMs seem most likely to occur when students are actively engaged in the mathematics lesson—regardless of the nature of the classroom instruction or the curriculum used. That is, they can occur in classrooms where students are doing mathematics themselves and sharing their thinking with their classmates, as well as in classrooms where students are listening to the teacher present information and asking questions about what they are hearing. In our study, classrooms in which the teacher was focused on making student thinking public had higher numbers of PTMs, but PTMs existed in even the most transmission-based classrooms. This points to the importance of helping all teachers to improve their ability to notice and act on PTMs in their classroom instruction and of the need for additional research into the most effective ways of doing this.

The results of our work suggest that it is possible to categorize the circumstances that lead to PTMs—an important first step in helping teachers learn to notice them during instruction. In our exploratory study, we identified five circumstances that led to PTMs: (a) when students made a comment or asked a question that was grounded in, but went beyond, the mathematics that the teacher had planned to discuss; (b) when students were trying to make sense of the mathematics in the lesson; (c) when students expressed incorrect mathematical thinking or an incorrect solution; (d) when a mathematical contradiction occurred in the public space; and (e) when students expressed mathematical confusion. Although we do not claim that our list is exhaustive, the fact that all of the PTMs in our data fell into a small number of categories is promising in that it gives teachers a manageable set of circumstances in which to look for PTMs during instruction.

Similarly, we were able to classify a set of three productive teacher decisions: (a) extend the mathematics and/or make connections among mathematical ideas, (b) pursue student mathematical thinking, and (c) emphasize the meaning of the mathematics. These decisions provide a starting point for helping teachers learn to use student thinking in ways that support the development of students' mathematical understanding—a second key step in capitalizing on student thinking during instruction. In general, the initial PTM framework that has resulted from this work has the potential to be used as a tool to help teachers focus on mathematically rich moments that occur during instruction and to inform teacher educators as they develop activities to support both teacher noticing and teacher decision-making.

This study provides insight into, but also raises a number of questions related to, which components of PTMs might be most productive to focus on in teacher education. For example, it was found that teachers' inability to notice PTMs generally had a negative impact on student learning. This supports the notion that noticing should be an important focus in teacher education (e.g., Sherin et al. 2011) to reduce the number of important moments that are missed during instruction. The data also suggest, however, that all moments may not impact student learning in the same way. Although it was found that the teachers in this study did not respond to any one PTM type better than others, a relationship was noticed between likely impacts on student learning and PTM potential. Recall that the data suggested that PTMs with significant potential to improve student learning may be able to do so if the teacher merely creates an opening for them to be considered, whereas those with moderate potential require a well-implemented decision in order to positively impact student learning. Thus, helping beginning teachers recognize significant potential PTMs could be productive, since these PTM instances are likely to positively impact learning even if beginning teachers cannot carry out their decisions well.

In addition, it seems critical to focus on helping beginning teachers implement good decisions in response to moderate potential PTMs, since these types of PTMs occurred more often in the classrooms of the beginning teachers in our study and their success was more dependent on effective decision implementation. In fact, focusing on improving teachers' abilities to implement productive decisions overall might be an effective approach, since it might decrease the number of negative likely impacts on student learning resulting from poor decision implementation. Considering the PTM potential and decision data together, it may be optimal to use a two-pronged approach that focuses on both identifying PTMs and thinking about how the teacher might best respond in order to optimize student learning.

The data also suggest that focusing on mathematical trajectories and connections might produce more positive likely impacts on student learning. This speaks to the need to focus on developing knowledge of the mathematical terrain in teacher education coursework—including what Ball and colleagues refer to as *horizon knowledge*—and highlights the more general issue that strong *mathematical knowledge for teaching* (Ball et al. 2008) is key to responding to PTMs in ways that enhance students' mathematical understanding. For example, to productively respond to a PTM, a teacher needs to decipher the mathematics underlying a student's response and consider how it fits with the goals of the lesson or broader goals of the course (Leatham et al. 2011). Work conceptualizing the *mathematical knowledge for teaching* required to act on these moments productively is needed to provide important insights into the process of becoming the kind of mathematics teacher advocated by organizations such as NCTM (2000).

Fully understanding PTMs and teacher decisions in response to them requires expansion of the exploratory study in two important ways. First, other groups of teachers, including those who are more experienced and those who may have participated in learning experiences aimed to promote noticing and capitalizing on PTMs during instruction need to be included in the data set. Although the participants in this study engaged in activities related to understanding student thinking in their teacher education coursework, they did not focus specifically on what student thinking might be most productive to build on during instruction. Second, teacher decisions require further investigation. This study classified teacher decisions in relation to what could be observed in the classroom video. We acknowledge, however, that teachers' decisions may be mitigated by a host of contextual and teacher-specific factors (e.g., Jaworski 2004; Schoenfeld 2011). Thus, additional research should include interviews with teachers that will provide more complete information about not only whether they noticed a PTM during instruction, but the thinking behind their decisions to either act upon or ignore PTMs.

The intent of this exploratory work was to provide an initial understanding of high-leverage instances of student thinking that occur as interruptions in the flow of a classroom lesson. The resulting framework can be used by researchers to engage in detailed analysis of teachers' noticing and use of student thinking during a lesson. For example, it provides researchers with a tool for analyzing *what types* of student thinking a teacher is able or unable to recognize and effectively respond to, rather than simply analyzing *whether* a teacher responds to student thinking. The framework can also be used by teacher educators to help teachers capitalize on PTMs during classroom instruction. Teachers are faced with a myriad of decisions on a moment-by-moment basis in their classrooms. We argue that PTMs, by their very nature, prompt teacher decisions that can have a high impact on the learning that goes on in a classroom. As such, they are worthy of analysis so that PTMs and effective responses to them can be identified and shared with teachers. This is particularly useful in the case of novice teachers who have less experience and knowledge to draw on as they make their decisions. Being able to recognize PTMs and effective responses to them would improve beginning teachers' abilities to act in ways that would increase their students' mathematical understanding.

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