Teachers' and researchers' collaboration in analysing mathematics teaching: A context for professional reflection and development

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Abstract   The study reported here concerns 4 years of collaboration between secondary school teachers and academic researchers, constituting an emerging community of inquiry into mathematics teaching. In the article, the process of the forming of this community is discussed, focusing on the reflective activity developed by its members as well as the tensions and conflicts that emerged in the shaping of an inquiry identity by them. Furthermore, shifts in the mode of participation in the community and thus learning gains of all members are identified, with the teachers gradually developing an awareness of teaching practice in general, and the researchers becoming progressively acquainted with the reflexive relationship between inquiring into teaching practice and teachers' professional development.

Keywords   Teachers' and researchers' collaboration · Community of inquiry · Co-learning · Mathematics teaching

Introduction

An increasing number of studies are focusing on the collaboration between teachers and academic researchers, as this is seen as a means of developing knowledge about mathematics teaching and learning. Recently, the theoretical construct of a "community of practice" (Wenger 1998) has been used to examine this collaboration. The relevant studies,

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conceptualising teaching as learning in practice, involved teachers working alongside university academics to investigate and hence develop mathematics teaching practice (for example, Goos and Bennison 2008; McGraw et al. 2003). Considering teacher change as a process of shifting participation in the emerging community, these studies have identified learning gains made by the teachers with respect to teaching mathematics through their involvement in a process of reflection.

In an attempt to make explicit the "practice" in which teachers and researchers participate when collaborating, Jaworski (2003) suggests shifting from the notion of community of practice to that of "community of inquiry", where teaching is seen as learning-to-develop-learning. In such a community, teachers and researchers both learn about teaching through inquiring into it. In the studies framed by this perspective, teachers are usually involved in collaborative action research, investigating aspects of their own practice (see, Edwards and Hensien 1999; Raymond and Leinebah 2000). They often write about their research in cooperation with the academic researchers, a particularly important process for teachers' professional development (Krainer 2006). Teacher educators' learning emerges only occasionally as an issue in the process of collaboration in these studies (for example, see, Goodwell 2006; Krainer 2008; Zaslavsky and Leikin 2004).

The case study reported here is based on a longitudinal collaboration initially between two and later three secondary school mathematics teachers and two academic educators/ researchers in mathematics education, examined from a community of inquiry perspective. The collaboration concerned an investigation into aspects of the teachers' mathematics teaching practice, with the ultimate goal of developing knowledge about it. In particular, the focus of the study is on the:

- content and form of the collaboration developed in the context of the emerging micro-community of inquiry into mathematics teaching practice;
- co-learning process taking place within this micro-community in relation to mathematics teaching.

Theoretical background

A community of practice is determined by three key structural elements (Wenger et al. 2002): domain (the topic of focus that brings members together), community (a group of people who interact, learn together, build relationships of trust and develop a sense of mutual engagement and belonging) and practice (the explicit and tacit knowledge the community develops, shares and maintains, as well as the relations between participants, the specific perceptions of the practice and the underlying understandings that frame aspects of the practice). For a practice to lead to the emergence of a community of practice, it should ensure coherence. Wenger (1998) identifies three features of practice that constitute sources of coherence: mutual engagement, joint enterprise and shared repertoire. Membership in the community is typically realised via the establishment of relations on the basis of mutuality. As participants deal with others' needs and competencies, they bring diversity to the practice, but they also work towards building homogeneity, through a process of mutual accountability and collective negotiation with respect to the enterprise being pursued. Moreover, in their practice, members develop resources (both physical and conceptual) and adjust their interpretations of actions and sayings, negotiating meanings which interrelate and make the emergence of a shared understanding of what it means to participate in such a community possible (Matos et al. 2009).

From a community of practice perspective, teachers' learning and thus their professional development is seen as situated in forms of co-participation in joint activities, for which they have mutually held goals and to which they bring varying levels of expertise. As teachers become more experienced and move on to positions of greater responsibility within their professional community, they develop not only the required cognitive skills, but also the attitudes, motivations and values of those around them. In other words, they develop a sense of identity: "We conceive of identities as long-term, living relations between persons and their place and participation in communities of practice" (Lave and Wenger 1991, p. 53). Identity formation is a dual process of identification (investment of the self in relations of association and differentiation) and negotiability (control over the meanings that matter within a social configuration). The interplay of these two processes brings, unavoidably, issues of power to the fore, that is, issues related to the ability to act in line with the enterprises pursued and only secondarily in terms of competing interests:

> Rooted in our identities, power derives from belonging as well as from exercising control over what we belong to. It includes both conflictual and coalescing aspects – it requires or creates some form of consensus in order to become socially effective, but the meaning of the consensus is something whose ownership always remains open to negotiation. (Wenger 1998, p. 207)

Seen from a community of practice perspective, learning to be a teacher is a process of becoming a teacher within the community of the teaching practice by aligning with the norms of the practice (Wenger 1998). If alignment is critical with respect to its purposes and implications, then participants may "align with aspects of practice while critically questioning roles and purposes as a part of their participation for ongoing regeneration of the practice" (Jaworski 2006, p. 190). This places critical inquiry aiming at co-learning at the heart of the community's practice (Jaworski 2001), a shift requiring "a conscious level of awareness of the practice and a desire to achieve more than the practice, at a particular time, seems to make possible. The shift might be seen conceptually as one from learning within a community of practice to one forming a community of inquiry […] in which all participants are learners" (Jaworski 2006, p. 191–192). Thus, a community of inquiry differs from a community of practice with respect to the importance it places on the meta-knowing through reflection on the practice. Participants in such a community work continuously to reconstitute the community through critical reflection, resulting in critical alignment; consequently inquiry develops as one of the norms of practice and individual identity develops through reflective inquiry. In a community of inquiry, teachers develop inquiry approaches to their practice and together they use inquiry approaches to develop their practice. This points to a reflexive relationship between inquiry and development, which places emphasis on the collaboration between teachers but also between teachers and researchers in investigating ways of improving practice. Critically, questioning roles and purposes of teaching practice is at the heart of this collaboration, which aims, above all, to promote co-learning inquiry in relation to teaching.

We have, therefore, a dynamic framework for considering the collaboration between teachers and researchers within a professional development perspective, which aims at transforming the practices of both by involving them in a dialogue based on what Ruthven (2002) calls a 'mutual learning agreement'. In the context of this agreement, teacher and researcher are seen to be partners in an inquiry process of learning and teaching mathematics, holding separate but not incompatible roles. In particular, they are seen as insiders or outsiders to the teaching practice, both acting and reflecting on it, each informed by his/her own practice, both learning about teaching. This process, which is dominant in a

community of inquiry, determines the particular ways of recognising and understanding what is legitimate with respect to teaching, that is, the pedagogical identities that are shaped (Jaworski 2003). A study of the collaboration between teachers and researchers requires a focus on reflective activity as well as the pedagogical identities developed through the negotiation of the teaching practice among the members of the community of inquiry, which is, hence, constituted. The reflective activity of both teachers and researchers usually concerns aspects of teaching, while the process of identity formation often concentrates on the tensions and conflicts emerging in the shaping of identities. The study presented here constitutes an attempt to explore these two crucial elements in the teacher–researcher collaboration and in their co-learning with respect to teaching mathematics.

Methodology

The participants

The two teachers (teacher A, teacher B) have had more than 12 years of teaching experience, are friends, and have worked with the mathematics teacher educators (educator A, educator B) initially in the context of a 2 year Master's programme in Mathematics Education and then beyond it. The two educators, who are also mathematics education researchers with more than 20 years of experience, taught a Research Methods module in the Master's programme attended by the two teachers, and supervised their dissertations. The researchers/educators, who have also known each other for a long time and worked together on a number of occasions, developed a positive personal and professional relationship with the teachers from the outset.

The context of the collaboration

The collaboration was not conceived from the beginning (phase 1) in the context of a possible community of inquiry. This conception began to take shape towards the end of phase 1 through the researchers' own professional activity. The initial goal was to investigate aspects of mathematics teaching in the context of the two teachers' master's theses. To this end, both parties interest in learning more about teaching was discussed and agreed from the beginning. As a consequence, the strictly defined student–supervisor relationship gradually shifted to the closer one of collaborators.

The process of working together started 4 years ago and has continued to the present day. The collaboration was intensive during its three phases, where a research agenda was implemented, but irregular in the interim periods.

In phase 1, the collaboration between the two educators and each individual teacher was materialised in the context of the teachers' dissertations and lasted for about 12 months.

Teacher A studied his management of mathematics teaching dilemmas in the classroom through an action research approach. The research process included (a) the identification and categorisation of the teaching dilemmas that emerged in eight of his teaching sessions, (b) an analysis of the transcribed sessions accompanied by his comments related to (a), conducted independently by the two educators and (c) the negotiations between the three partners based on the different readings of the transcribed lessons, carried out in person or through e-mail. The data related to the collaboration between the educators and the teacher (of concern in action research and for the present study) included teacher A's diaries on the planning of lessons, his transcribed and commented upon lessons, the exchanged e-mail

messages with reflections (about 200) and the transcribed face-to-face group discussions on the issues under consideration.

Teacher B worked on the dialogue emerging in the mathematics classroom, focusing particularly on teacher questions, through a case study methodology. She analysed, among others, four lessons by teacher A, developing a typology of these questions. The two educators supported teacher B by analysing with her the transcribed lessons (working mainly within a student–supervisor relationship). This activity did not produce data related directly to the study reported here. However, it led to a realisation of the common interest on the part of all four in exploring mathematics teaching collaboratively. This was reinforced by an article written by the two educators and teacher A for a national conference on the negotiation of their different perspectives in interpreting teaching during A's action research.

In phase 2, the two teachers and the two educators/researchers started collaborating on inquiring into mathematics teaching, aiming at making sense of it at a deeper level. Following a suggestion by one of the researchers, it was agreed initially to analyse one of the three transcribed sessions of teacher A that teacher B had utilised in her study, exploiting the analytical tool that each teacher had independently developed in their original research. The resulting texts were exchanged, read and negotiated in a series of three group meetings, where the researchers' role was to provoke and maintain the inquiry by challenging the two teachers' understandings, interpretations and justifications of their teaching practices. The data gathered in this phase, which lasted for about 6 months, included the two independent analyses by the two teachers and the transcribed face-to-face group discussions.

In phase 3, lasting 3 months, the two teachers took the initiative to design a collaborative research study related to teaching mathematics. They invited another teacher (teacher C), a peer in the Master's programme and friend, to join the group. The teachers asked for the researchers' help in determining the focus of their study and finally agreed that this should be on how A and B manage students' errors. Teacher C visited their classrooms and observed, audio-recorded and transcribed three 7th grade (13 years old students) lessons by each (on the same topic). Each teacher initially analysed all the transcribed lessons independently with the goal of, first, identifying, characterising and categorising students' errors and, second, examining how these were dealt with in the classroom. The academic researchers read the initial analyses and encouraged the teachers in the group interactions to reflect on these analyses and explore their convergences and divergences. The data produced included the teachers' analyses and observations, the e-mail exchanges, the transcribed in-person group meetings and the participants' reflections on the whole experience. At the end of phases 2 and 3, an article was written by the researchers in collaboration with the teachers and presented at a national conference by the latter.

Data analysis

In analysing the data, first episodes where teaching issues were discussed in the data were identified. Then, the content (the kind of mathematics teaching issues raised by the teachers and researchers) and form of collaboration (tensions, conflicts, resolutions) in these episodes were examined. Finally, an attempt to locate shifts over time in these two aspects of collaboration was made, for both the teachers and researchers, looking for learning gains made by the participants with respect to mathematics teaching and its development.

Results

In the following, we present the results, first with respect to the content and form of the collaboration between the teachers and the researchers and then in relation to the participants' co-learning process. In doing so, we first discuss convergences and divergences in the teachers' and researchers' understanding of mathematics teaching and its practice, identified in the context of their collaboration. Second, we analyse the shifts located in both the teachers' and researchers' approach to inquiry into mathematics teaching and into teachers' professional development.

The content and form of the collaboration

In phase 1, collaboration was not yet defined in terms of a co-learning agenda; rather its focus was gradually moving from improving teaching practice to inquiring into it for the teacher, and from inquiring into teaching practice to an interest in teacher development for the researchers.

With respect to the content of the collaboration, the two educators and teacher A identified dilemmas related to epistemological issues, students' learning and classroom interaction. The epistemological characteristics that were identified by both the teacher and the educators/researchers were mainly related to the nature of mathematics and its representations. However, the way these issues were conceived by educators A and B and teacher A varied, allowing for negotiations and reflection to take place. For example, the comments made independently by the two educators on the teacher's view "mathematics is like a foreign language" were "A foreign language? So, what does it mean for someone who does not know this language?" (educator A) and "It is an epistemological issue but is this also one of your beliefs? Does this frame your teaching?" (educator B). The teacher responded by justifying his view and his teaching decisions and actions: "I do not consider mathematics as a foreign language but I stress this to my class… they have to know what they say as I don't want them to memorise rules, a practice that is adopted by many students under the encouragement of their parents".

In terms of student learning and classroom interaction, unlike the researchers, the teacher did not identify issues related to the students' mathematical development; he referred to their learning mainly in terms of performance, barely acknowledging the students' contribution: "He is a good student, I expect him to know the definition of the perimeter—on which I do not want to spend a lot of time … I cannot wait for others to share their ideas with the rest of the class". By contrast, the researchers' comments were about exploring students' contributions as a basis for the development of mathematical meaning: "You appear to try desperately to find what you are looking for … you do not seem to explore their contributions" (educator A), "It is not clear why you say it again so many times. Do you believe that repetition leads to understanding?" (educator B). Responding to these comments, the teacher justified his actions by appealing to a variety of factors that influence his decisions, such as the limited time, students' difficulties in remaining focused or decisions mostly related to curriculum demands. For example, "a lot of times I give them the correct answer because the teacher needs to validate whatever is said in the classroom … The students do not always pay attention to what is going on in the classroom. So, I need to repeat in between to maintain their attention". In general, in this phase, teacher A raised issues related to mathematical challenge and, partially, to students' emotions, while the researchers were more concerned with students' mathematical constructions.

The form of collaboration was greatly determined by the requirement of a Master's dissertation to be based on a systematic inquiry (that is, research). To this end, the researchers encouraged teacher A to exploit analytical/methodological tools to interrogate his practice and, in general, to function at a meta-level, while considering his teaching. The researchers independently commented on the teacher's thoughts and considerations; posing questions challenging his actions and views; suggesting the usage of academic resources (for example, research papers and tools) and seeking clarifications and justifications. There were occasions where the researchers' comments would be taken by the teacher as a criticism of his own teaching actions, especially in the beginning of their work together. The researchers attempted to deal with these moments by taking the stand that exploring different readings of one's teaching helps in acquiring a deeper understanding of teaching actions and their implications for learning: "… It is a path to self-awareness for all of us… and this is valuable" (educator A, e-mail message, 29th November 2004).

The exchange (via e-mail) that follows is indicative of the conflicts and tensions emerging under these circumstances. It follows the complete cycle of analysis of one of teacher A's lessons by each individual participant:

> Teacher A: In general, I get positive comments about my teaching. Most of my students tend to agree that they learn mathematics… This is not compatible with your own analysis. Isn't there anything positive so that my teaching can be reinforced rather than being changed? (5th December 2004)
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> Educator A: You need to trust us. We don't criticise you in a rejecting way, we simply try to go as deeply as possible into what you are doing. To place the research lens over the points that prevent all of us from seeing how we operate…You are possibly right that, at least myself, I did not point out the positive aspects of your teaching. It is not because they do not exist, but because they are already there. (7th December 2004)
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> Educator B: You should not feel like this. It is true that, as researchers, we sometimes concentrate on the unsuccessful aspects of teaching… There are many positive elements in your teaching… Aiming at developing teaching, we look critically at your work and ask you to do the same. This is the torturing issue of reflection. (7th December 2004)

Despite the occasional moments of tension, the teacher was, on the whole, appreciative of the interaction with the researchers: "It is very interesting to view your teaching through somebody else's eyes… I like this very much and I think that this is what we want… Your evaluations are contributing to my professional development, I never considered them negatively".

On the basis of the above, the engaging and trustworthy climate within which the Master's study was implemented helped the three partners move beyond the typical student–supervisor relationship, towards struggling to form a shared agenda of developing a teaching situation. This process was not unproblematic, because of the constraints, tensions and dilemmas that the participants were facing in their attempt to negotiate their varying and often divergent perspectives towards defining common goals with respect to the practice of inquiring into teaching and, thus, to its development.

In phase 2, where the group of four worked together, one of the episodes that both teachers chose to analyse was a dialogue on a student's difficulty to find the intersection point of the graphs of two linear functions algebraically. The content of the collaboration was mainly determined by the different readings of the episode by each teacher, which were compatible with the research framework employed during their own postgraduate study. Teacher A conceived this example as a dilemma that he tried to resolve by rephrasing what the student had said, in order to focus his attention on the simultaneous

equations required. According to him, the dilemma was mainly related to the student's difficulty to relate different representations. Teacher B raised issues connected with classroom management and, in particular, with the student's limited opportunities to think about teacher A's questions. Teacher B considered that the task was highly demanding mathematically, but teacher A's numerous questions were too closed, thus trivialising the mathematical activity. She supported this claim by providing quantitative information (22 questions by teacher A in 3.5 min):

> This results in many students not being in a position to process the information needed in order to respond. Thus, the teacher gradually narrows down the focus of the questions, by partitioning the knowledge into small pieces, through questions of low demand, in order to secure his pre-determined mathematical product. (teacher B's analysis, 14st May, 2007)

On the whole, the teachers were mostly concerned with teaching practice issues: "The task could be more open but of lower cognitive demand… the pupils could have been given time to work together… my reservation is related to the repeated short questions" (teacher B). On the other hand, the researchers focused mainly on interpreting the local teaching practices within the wider perspective of mathematics education: "Why do you think that this is important for the student in terms of his overall mathematical experience?" (educator A).

In relation to the form of the collaboration, the two teachers can be located at its centre, with teacher A acting as an insider to the teaching practice (his own) and teacher B adopting an outsider-researcher's role to it. Their interaction gave rise to conflicts and tensions mostly related to their divergent readings of the teaching practice. Initially, teacher A strongly reacted to the analysis offered by teacher B, considering it: "a typical, dry text of those we are used to seeing in all research papers", and not taking into account the wider classroom and school contexts. The two researchers tried to encourage the teachers to regard these two ways of reading the same episode as complementary, but this seemed to be difficult initially. Teacher A argued that he was conscious about his teaching actions and decisions, the development of which was the result of his involvement in research and his professional growth with the researchers' support:

> I often did things spontaneously … I learned about certain things, why I did them the way I did, how I should do them, to discuss my lesson thoroughly – nobody knew about my lesson. I learned to justify students' behaviour, something that is very important to me. In the past, I used to say to myself: 'he does not understand, he does not know'. I would try to help him, but not much. I would abandon him from a point onwards. I don't do this now. (teacher A, 3rd group meeting, 10th June, 2007)

Towards the end of the group interaction, teacher A accepted that he often restricted the mathematical communication and talked about his attempts to leave more space to the students to think: "What I changed this year was that, when I gave them a worksheet to work on, I gave them 5 min to think about it before starting discussing it in the classroom". Thus, teacher A appears to shift from defending his teaching decisions and actions to considering small modifications in his actual practice. Teacher B also started looking at her own practice critically through the discussion with her colleague, as indicated by the following extract:

> My own objection is on the short, frequent questions. I think that they are not needed. Something that I, myself, have reduced a lot, [is that] I am not so 'suffocating'

anymore, after … my dissertation … I tell them to think in pairs … I allow them enough time to think. (teacher B, 3rd group meeting, 10th June, 2007)

The researchers tried to contribute to the collaborative effort via encouraging the teachers to justify their positions by exploiting the analytical tools they had acquired academically and to challenge their assumptions with respect to teaching and its development: "How is your interpretation incompatible with that of teacher B?" (educator A) and "You seem to think much about how the student feels. However, you appear to lead her to paths you consider mathematically important. Why is it you leave so little space to her? (educator B)".

In phase 3, when teacher C joined the group, collaboration was initiated by the teachers—its content framed by the researchers' academic interest in exploring the meaning of students' errors within a socio-cultural perspective. The research agenda finally adopted focused on epistemological, cognitive and managerial issues related to students' errors. To this end, the teachers identified, initially independently and gradually interactively, categories of errors (for example, conceptual, procedural, interpretive or divergent from teacher's goals) and patterns of error management (such as, simplification of the task, re-phrasing of a student's response, posing clarifying questions). The researchers' role in this process was to help the teachers devise coherent analytical schemes justified on the basis of the relevant literature, a role also adopted by teacher C, who, however, was more concerned with how to use these schemes with the data.

In reflecting on their way of dealing with students' errors, teachers A and B seemed to be able to gradually 'distance' themselves from their own practice and to identify constraints:

I admit that for errors beyond my teaching goal, I direct them very closely … I pose questions that represent my own path of approaching …, leading unavoidably to one-word answers and, finally, to what I wish to hear. However, this way, the right answer comes from a student, not from me. Thus, the student is not abandoned, but s/he receives the help needed for cognitive conflict to emerge, leading to the awareness of his/her mistake. (teacher A's written account of his dealing with errors, 19th May, 2009)
On some occasions I do not explore deeply what the student actually means, although part of his response can be right … It seems that I do not focus properly on the student's answer, which results in my looking for a way to avoid a discussion about what I see being beyond what I have in mind. (teacher B's written account of his dealing with errors, 15th May, 2009)

The form of the collaboration dominating this phase was determined by the interaction between the three teachers, who were leading the implementation of the research agenda. Barely any tensions occurred at this stage, as the group seemed to function at a meta-level, considering predominately global rather than local teaching issues. This could be attributed to the strengthening of the emerging community membership (a growing through inquiring into teaching practice) and to the history of the group, resulting in all participants gradually, and sometimes hesitantly, aligning themselves to the norms of the inquiry practice—the researchers acting as co-researchers and not as evaluators of the teaching practice, and the teachers inquiring into teaching practice in general rather than defending their own teaching actions.

The co-learning process and the participants' development

Teachers A and B developed a deeper awareness of mathematics teaching by being engaged in an analysis of mathematics lessons; comparing their analyses to those of the

other teachers and the researchers; identifying and interpreting important learning and teaching issues, and by communicating their work in public. This development did not occur automatically but was a rather long and occasionally stressful process, marked by the interaction of divergent and convergent trajectories of participation, and the teachers' efforts to identify and also negotiate the meanings that mattered in these contexts. Teacher A's reflection on phase 1 below indicates aspects of this process:

> After the analysis of the first lesson, a lot of ideas emerged that were negotiated and transferred to actions. However, in that phase I could not follow your thinking, because my goals were quite different from yours. I was considering things in the borders of my classroom. I had to face my students… Later on I started reflecting on my own teaching actions, I considered things about my teaching that I had never thought about before and your contribution to this was tremendous… (teacher A's reflections, 30th of June, 2005)

The two reflective accounts that follow indicate how both teacher A and B conceived their own trajectory of participation in the emerging community, their struggle to reconcile local with broader ways of belonging and how they negotiated new meanings within this community by the end of phase 3:

> Before, I believed that strong mathematical knowledge by a teacher and students' involvement were the key factors for effective teaching. My strong mathematical background allowed me to lead my students where I wanted … It was enough for me to get a 'one word' response and nothing more. I attributed students' misconceptions to their responsibility and not to my teaching … In my attempt to be sensitive to their needs, I was giving myself the answers instead of waiting for theirs or I was asking them closed questions, in order to feel that they can do something. During the collaboration, I started to reflect on my teaching on an everyday basis, to look for more open teaching tools, to inquire into my teaching actions and practices and how these were related to students' learning and to think about issues of communication and management. My students started to talk more … to explain their ideas and thoughts. Now, I try to allow more space for my students to think, although I still struggle with the issue of sensitivity that makes me occasionally too direct with them. I have also not overcome the tortuous dilemmas of time and fulfilling my teaching goals. (teacher A's overall reflection, 26th June, 2009)
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> There are changes in my teaching. First, I reduced my talking time … I discovered that the time that a teacher talks in the classroom compared to students' talking is enormous. I also observed what you (the researchers) were doing in our collaboration, you were posing an issue for discussion and then you were listening to our opinions. I liked this very much and it helped me a lot. I also noticed that this helped my students as well. I additionally learned … that we cannot ask students too many questions. … By observing teacher A, I was thinking what I was doing and I was trying to improve it. For example, I now try to control my intervening in students' thinking. Furthermore, by examining students' mistakes, I learned that there is something deeper there; it is not only that a student made a mistake, the type of mistake is what matters … Through our collaboration, I started to understand my goals and the reasons I do something, something that I also try to do with my students. (teacher B's reflection, 26th June, 2009)

On the other hand, the researchers worked at two different levels of inquiry, one related to mathematics teaching and the other to teacher development. In terms of the former, through their collaboration with the teachers, they started becoming aware of the need to view mathematics teaching through the teachers' eyes, that is, in the context within which it really operates. To this end, they had to identify and operationalise theoretical issues such as the socio-cultural interpretation of students' errors and an epistemological consideration of learning and teaching mathematics. At the same time, they had to negotiate and achieve some consensus with respect to the meaning of these issues within the emerging community. As educator A commented in the e-mail exchanges of phase 1, "this interaction gives us the opportunity to relate our readings to the reality of the classroom". With respect to the second level of inquiry explored by the researchers, a shift in their trajectories of participation is noticeable, as they gradually moved from evaluating the teachers' teaching actions to allowing more space for them to take charge of and responsibility for promoting their learning about teaching, that is, for their development. Within these contexts, the researchers appear also to have gained access to the complexity of building up an inquiry community and to their role in this laborious process. The following two extracts from the researchers' reflections exemplify some of this process:

> I realised that … teachers often act on behalf of mathematical knowledge … and not in favour of students' learning… (They) follow a pre-determined agenda of work … As to their professional development, I started thinking that their learning and teaching development do not necessarily follow parallel paths. Teachers … need time and continuous support to start acknowledging what is really happening in the classroom. Finally, the collaboration between teachers and researchers… In the beginning, there was suspicion … ambiguity and ambivalence …difficulty in distinguishing between challenge and provocation or imposition … [resulting] in the emergence of conflicts and tensions. These … led the researchers to become more careful observers and listeners … gradually … [allowing] more reflection to take place. (educator A's reflection, 26th June 2009)
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> During these years of collaboration I was facing the difficult question of how to bridge research and teaching practice. This proved to be a very difficult task … I had to learn many things in this process. I had to be involved in critical inquiry of teaching but at the same time to be supportive of the classroom teacher. This is crucial in co-learning agreements as the teachers and researchers have an active role … at all stages of the work. I also had to consider teaching beyond classroom borders and to identify other social factors that were framing it. I had to balance the varying roles and responsibilities I have, such as supporting teachers to become researchers, researching teacher development or researching mathematics teaching in the context of collaboration with another colleague and the three teacher-researchers. (educator B's reflection, 26th June, 2009)

The teachers and the researchers, acting either as insiders or outsiders to the research as well as the teaching practice, developed knowledge not only about the various parameters that frame mathematics teaching but also about their interrelations; at the same time, they expanded their awareness of their own teaching actions and decisions. Teachers A and B seemed to gradually adopt a reflective stance on their teaching, abandoning local in favour of broader styles and discourses, and aligning to not-necessarily-convergent perspectives related to it. As for the researchers, they appeared, on the whole, to progressively shift their role from providers of "academic feedback" to that of "nurturers of disturbing the

'normal' teaching practice", encouraging the teachers to continuously challenge their teaching achievements by inquiring into their specific teaching decisions and actions, thus making inquiry an emergent practice of the formed community (Jaworski 2008).

Concluding remarks

An important issue, demonstrated in the context of the present study, is the complexity of the collaboration between teachers and educators/researchers.

During the 4 years of working together, the collaboration between the teachers and researchers involved in the study changed in content (shifting from focusing on local practices to tackling broader issues of mathematics teaching) and in form (being transformed from a student–supervisor relationship based on a common interest in systematically inquiring into mathematics teaching to one of co-inquirers of its development). These changes can be attributed to all the participants' commitment to explore each other's perspectives in relation to teaching mathematics, with the ultimate goal of arriving at a deeper understanding of it. This brought forward issues of 'belonging' and 'becoming', that is, of identity, and thus of identification with existing meanings and negotiation of new ones. As a consequence, tensions and disputes emerged, particularly at the beginning, usually related to the interplay between local mathematics teaching practice and its global context. However, at the same time, coalitions and alignments were enabled, forced by the requirement for some form of consensus in order for the whole process to be socially effective. As a result, the whole process gradually turned into an apprenticeship of all participants in inquiring learning to teach mathematics, that is, it gave rise to a community of inquiring into mathematics teaching and its development.

In the context of the above process, both teachers and researchers went through a succession of forms of participation in the emerging community, which provided them with opportunities for self-understanding and a re-conceptualisation of mathematics teaching and professional development (that is, with opportunities to learn). In particular, the teachers moved from a defensive attitude to one that involved trying to make sense of others' perspectives utilising, in the process, research resources to exemplify important aspects of teaching, and adjusting interpretations to varied readings as well as questioning their teaching actions. This process bears features similar to those identified in the process of critically aligning to the "normal desirable state" of teaching practice (Jaworski 2008). As for the educators/researchers, they became more conscious of the nature of their contributions and of their effect on teachers' engagement in a reflective examination of their own teaching practice. Thus, as time went by, they moved away from an evaluative to a co-inquiring attitude, providing more space to the teachers to distance themselves and think critically about their own teaching actions, that is, to take over the role of inquirers. This could be seen as a process of the researchers gradually allowing the ownership of teaching knowledge to become open to negotiations within the community (Wenger 1998).

In summary, the collaboration between the teachers and researchers on the basis of exploring mathematics teaching practices for professional reasons gave rise to an emerging community of inquiry, within which learning gains were made by all members with respect to teaching and teacher development. More specifically, the teachers started to take initiatives in systematically inquiring into teaching, shifting from insiders to outsiders to the practice of mathematics teaching, as the researchers gradually moved to looking critically at their role in teachers learning, switching from outsiders to insiders in the development of mathematics teachers.

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