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Zen and the art of *neriage*: Facilitating consensus building in mathematics inquiry lessons through lesson study

Noriyuki Inoue

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Abstract One danger of integrating inquiry-based problem-solving activities into mathematics lessons is that different strategies could be accepted without in-depth discussions on the cogency and efficiency of the strategies. To overcome this issue, Japanese teachers typically go through a series of lesson-study-based teacher learning sessions and learn how to help students build consensus on the best mathematical strategy and think deeply about problem solving (*neriage* in Japanese). Assuming that this can also be an effective model in other cultural contexts, a video-based lesson study was conducted for a group of US teachers to effectively incorporate consensus building discussions in their mathematical inquiry lessons. Through the lesson study, the teachers learned to release control of class discussions to their students and help them discuss and examine different strategies. This article concludes with various aspects that the teachers learned for effectively implementing *neriage* in their lessons.

Keywords *Neriage* · Lesson study · Consensus building · Mathematical inquiry lesson · Proportional reasoning

Introduction

It has been decades since mathematics educators embraced constructivism as a philosophy of learning that situates students as active agents who construct meaning (Cobb et al. 1991). Many techniques and ideas have been introduced to successfully implement this philosophy of learning in classroom teaching. One popular technique is to incorporate inquiry-based problem solving in mathematics lessons in which the students explore and examine different strategies to solve open-ended mathematical problems (Bloom 2007; Manouchehri 2007; Goos 2004). For instance, teachers might ask the students to find their own ways to calculate 2/3 of 12 liters of juice before teaching the

N. Inoue (✉)
School of Leadership and Education Sciences, University of San Diego, 5998 Alcala Park, San Diego, CA 92110, USA
e-mail: inoue@sandiego.edu

standard algorithm to solve the problem, and the students could construct their own strategies to generate their answers (e.g., 2 × 12 ÷ 3, 12 ÷ 3 × 2, etc.). However, this approach could create a pedagogical challenge for teachers: the strategies that students use in such an open-ended inquiry activity can be diverse and often inefficient or erroneous (Stafylidou and Vosniadou 2004; Steffe and Olive 1991). For instance, a student could respond to the aforementioned problem with an inefficient or inappropriate strategy such as "2/3 + 2/3 + … + 2/3", or "0.66 × 12". The question is how should the teacher respond to the students in such cases. Simply showing the right answer to the students could turn off the students' personal sense-making process and possibly reduce the mathematical inquiry lesson to a one-directional knowledge transmission model that elicits low student involvement (Turner et al. 1998). However, if the weakness of the strategy is not addressed, this might create an "everyone is right" atmosphere in which the students' problem solving is not critically examined or challenged.

Obviously, the answer does not lie in these extreme cases: in mathematical inquiry, a teacher's role should be to help students negotiate different mathematical meaning and assumptions with others and internalize new perspectives (Cobb et al. 1991). However, for teachers, interacting with each student and addressing all inefficient or wrong strategies can be time-consuming and challenging in classroom situations.

Researchers agree that instructional dialogues are characterized by various complex factors such as social dynamics of the classroom (Lienhardt and Greeno 1986; Leinhardt and Smith 1985), classroom culture (Cobb et al. 2001; Cobb and Bausersfeld 1995), and teachers' personal assumptions about the ways students construct their content knowledge (Inoue 2009). The ways teachers structure their lessons consist of multi-level considerations of the meaningfulness of the content, curriculum map, classroom situations, and the nature of understanding (Davis and Simmt 2006). While recognizing the complexity of instructional dialogues, researchers agree on the importance of teachers learning to orchestrate whole class discussions (Lampert 2001; Stein et al. 2008; Forman et al. 1998) and move beyond the "show and tell" model in which students simply take turns sharing their ideas and strategies in inquiry-based activities (Ball 2001). The key concern is how teachers can effectively facilitate classroom discussions in the ways that elicit negotiation of meaning and maximize the potential of mathematical inquiry activities.

In order to achieve this goal, Japanese elementary school teachers typically go through a series of teacher learning sessions in which they learn to facilitate deep discussions of different strategies and to help students negotiate and co-determine the best mathematical strategy (Stigler and Hiebert 1999). In Japanese elementary schools, mathematical inquiry is typically structured with four key components: *hatsumon* (initial math question/problem that the teacher gives to initiate a rich conceptual discussion), *kikanjyushi* (students' individual or group-based problem solving as the teacher walks by their desks), *neriage* (whole class discussions) to compare and contrast different strategies and build consensus on the problem solving, and *matome* (summary) (Fernandez and Yoshida 2004; Shimizu 1999). Among the four components, the *neriage* stage is considered to be the most crucial stage in mathematical inquiry lessons. In the *neriage* stage, Japanese teachers encourage students to listen to other students' ideas carefully and consider the strengths and weaknesses of different problem-solving strategies. Then the teachers facilitate discussions to co-determine which strategy is the most reasonable and efficient one. According to Shimizu (1999),

> The term Neriage describes the dynamic and collaborative nature of the whole-class discussion during the lesson. In Japanese, the term Neriage means *kneading up* or *polishing up*. In the context of teaching, the term works as a metaphor for the process of polishing students' ideas and of developing an integrated mathematical idea through the whole-class discussion…Once students' ideas are presented on the chalkboard, they are compared and contrasted orally. The teacher's role is not to point out the best solution but to guide the discussion toward an integrated idea. (p. 110)

Japanese elementary school teachers employ other methods for teaching such as direct instruction and drilling, but when they attempt to establish firm conceptual understanding in classrooms, they tend to follow this particular structure (Fernandez and Yoshida 2004; Stigler et al. 1996). In a way, this lesson structure is not unique to the Japanese context, since the similar forms of lesson structure that emphasize social validation of inquiry strategies have been proposed and explored in the world (e.g., Brousseau 1997). However, in practice, mathematics instructions in Japanese classrooms are known to be distinctively characterized by this pattern of inquiry lessons when compared to US and German classrooms (Stigler and Hiebert 1999). It is also widely known that the Japanese teachers regularly engage in the lesson study with their colleagues and learn how to implement *neriage* effectively in their lessons as they discuss the lesson goals, develop lesson plans, observe each other's teaching, critique the observed lessons, and repeat the same cycle until their learning is saturated in their practice.

The ways Japanese teachers effectively teach mathematics inquiry lessons and conduct lesson study began receiving strong attention in the U.S. after TIMSS reported the advantages of mathematics education in Eastern Asian countries (Fernandez et al. 2003; Stigler and Hiebert 1999). However, very few studies have attempted to investigate how it would be possible to effectively incorporate the *neriage* stage of the inquiry lessons in non-Japanese contexts. This article introduces a video-based lesson study that explored how a group of U.S. teachers could successfully implement consensus building discussions (or *neriage*) in their mathematics classrooms.

Video-based lesson study

For this project, six 4th and 5th grade teachers were recruited from the San Diego area for a small compensation fee. All the teachers chose to participate in the lesson study because they were not confident about how to deliver inquiry lessons effectively. None of the teachers were familiar with the concept of *neriage*, or the structure of Japanese inquiry lessons, but all of the teachers were familiar with the concept of "lesson study" as a model for professional development. In fact, two of the six teachers had experienced some form of lesson study through district training, but they had never participated in a lesson study whose focus was on consensus building. The mean teaching experience of the US teachers was 6.4 years ranging from 3 to 14 years of teaching.

To ground our lesson study in the Japanese inquiry approach, three experienced Japanese teachers at a local Japanese school (*hosyuko*, Japanese weekend school located in the San Diego area) agreed to participate in the lesson study as advisors, because one of the missions of the school was to serve the local community. The Japanese teachers were trained in Japan and had gone through a number of lesson studies that focused on the *neriage* stage of inquiry lessons. They were mostly fluent in English, but occasionally the facilitator (author) needed to translate their ideas into English. The Japanese teachers were asked to play the role of advice-givers to the US teachers in the meetings in case they had questions or seemed to struggle with implementing *neriage* in their lessons.

During the lesson study meetings, the researcher played the role of facilitator, asked for questions and comments, made sure that everyone's voice was heard, asked for clarifications when there was ambiguity in the discussions, managed the time allotted for discussions, summarized the consensus that was built in each meeting, and occasionally translated back and forth between English and Japanese when there seemed to be communication errors or confusion between the US and Japanese teachers. The cross-cultural lesson study group met six times, biweekly.

The investigation employed the action research design and explored how the six elementary school teachers effectively incorporated *neriage* in their mathematical inquiry lessons. In the first meeting (orientation), the US teachers were given an article by Shimizu (1999) and learned about the four stages of Japanese inquiry lessons. Then they were asked to teach mathematical inquiry lessons by including *neriage* as much as possible and to record the amount of the time they used for Japanese inquiry lessons on each day. For the second meeting and thereafter, they were asked to bring a 15–20-min videotaped segment of consensus building discussions from their classrooms. The teachers were allowed to choose the content and frequency of the inquiry lessons based on the progress of their lessons and the students' needs.

In each of the lesson study meetings, each teacher first introduced the *hatsumon* (initial problem posing) that they gave to their students, and then played the 15–20-min videos that captured the consensus building discussions in their classrooms. Then the lesson study group discussed how the consensus building discussions could be better facilitated. The topics of the discussions were highly diverse, ranging from how to improve the teachers' guiding questions to how the teachers could improve the quality of *hatsumons*, the meaningfulness of targeted mathematical concepts, and the quality of interactions for consensus building in the classrooms. After the meeting, each teacher was asked to complete the reflection form that asked what they learned in the lesson study meeting and the amount of the time they spent for the Japanese inquiry lessons between the lesson study meetings (2 weeks). The progression of the videotaped lessons, content of the discussions, the teachers' written reflections, and the time they spent for the Japanese inquiry lessons served as the data for making inferences about the teacher development process. Due to the action research design of this study, the discussion in the data analysis section is inductive in nature.

Please note that lesson study meetings in Japanese schools typically involves recursive cycles of teachers developing a goal, planning a lesson together, observing the planned lesson in a classroom, critiquing the lesson, and getting ready for the next cycle. However, this lesson study did not involve planning a lesson together and physically observing the lesson in a classroom, since the teachers came from different schools and districts that had different levels of support for this type of professional development. Also, the teachers taught different material during the lesson study, which made the Japanese style lesson study impossible. However, this video-based study can be characterized as lesson study, since all the teachers agreed to implement *neriage* in their lessons as the overarching goal, went through several cycles of observing lessons in digital videos, and had meetings to critique the video-taped lessons to improve the lessons for the next cycle. In other words, this could be seen as a lesson study localized in the US context. The focus was placed on how consensus building discussions could be effectively implemented in US classroom settings and what it would take for the US teachers to improve their skill to facilitate consensus building discussions in their classrooms.

Teacher development

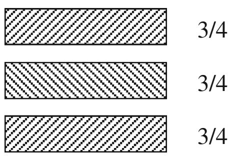
From the first session, the lesson study group engaged in dynamic discussions and active exchanges of perspectives, similar to the bi-national lesson study reported in Fernandez et al. (2003). The videotapes that US teachers presented demonstrated their attempts to incorporate consensus building (*neriage*) into the inquiry lessons, where the teachers: (1) first had the students work on an open-ended inquiry in groups, (2) then had the groups present their strategies in front of the whole class, and (3) finally had whole class discussions to evaluate the presented strategies. However, most of the videotapes that the teachers presented in earlier meetings did not truly incorporate essential characteristics of the *neriage* stage of mathematical inquiry lessons. The following is a representative sample of the videotaped segments in which one of the US teachers presented as her initial attempt to incorporate consensus building in her 4th grade classroom.

Videotaped classroom interaction #1

The teacher initiated the mathematical inquiry activity by giving her students a *hastumon*, "How can we multiply fractions by whole numbers? How can we calculate 3/4 × 3 using a model?" with the guiding instruction of "Draw a picture that shows how 3/4 × 3 would look like." The students had learned the standard algorithm (3/4 × 3 = 3/4 × 3/1) prior to this activity, but had not learned how to make sense of this calculational procedure.

During the group presentations, a member of a student group presented their strategy to solve 3/4 × 3 in terms of a repeated addition (i.e., 3/4 + 3/4 + 3/4) using a pictorial representation (Fig. 1). They further explained that the same answer could be obtained using the standard algorithm (3/4 × 3 = 3/4 × 3/1 = 9/4). After the presentation, the teacher initiated the following interactions.

Teacher	OK, so what you did very effectively is that you showed us this is a repeated addition. And as you think how to do it with the diagram, and then to figure out after you've divided it, you just figured out with an algorithm. Algorithm is how you would solve it without doing a diagram. OK, so the first thing is the pictorial representation figuring out that, and once you came out with the answer, you determined, OK, let's go backwards now, and see how you would solve this if we have just a problem, 3/4 × 3. OK. And did you guys notice, a lot of you, what they did was to change the whole number into what? … What did they change the whole number into, Sandra?
Sandra	…(silence)
Teacher	In ¾ × 3, the whole number is 3. What did they do with that?
Sandra	…(silence)



$$\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=\frac{9}{4}$$

$$\frac{3}{4}\times 3=\frac{3}{4}\times\frac{3}{1}=\frac{9}{4}$$

Fig. 1 A model for ¾ × 3

Teacher	OK. Who can tell Sandra what they did with the whole number? Who was listening, because they did explain it? What did they do with the whole number 3? OK, go ahead.
Tim	They simplified it.
Teacher	No, they didn't simplify it. Dennis?
Dennis	They changed the whole number to a fraction.
Teacher	OK. They changed the whole number 3 into a fraction so they could multiply a fraction times 3/4. And what did 3 look like as a fraction? Sandra, can you answer that question?
Sandra	…(silence)
Teacher	Who could answer (*so that*) Sandra understand what the fraction is, the whole number 3 look like as a fraction? Kathy?
Kathy	……
Teacher	What's 4/4 equal? One whole. Is one whole equal to three wholes? No. So that's not gonna work. I thought that they explained that.

In this interaction, the teacher attempted to help the students understand the presented idea (i.e., checking the result of the standard algorithm with repeated addition), but the way she interacted with her students in the lesson was not so different from the traditional instruction: she simply used the presented idea as another teaching material and initiated the interaction to check whether the students understood her perspective (i.e., "OK, so what they did very effectively is…", "No, they didn't simplify it.", etc.). She did so by endorsing the presented model as "very effective" before asking for other students' perspectives, and there was very little attempt to help her students co-determine the effectiveness of the presented idea or conceptualize how the pictorial representation of $3/4 \times 3$ as a repeated addition is related to the standard algorithm. In the video of her teaching, this was a consistent pattern: she attempted to initiate a consensus building discussion, but her attempt was to build consensus between her students and herself, who served as the authority, and other students simply served as the "listener" of the teacher–student interactions.

Though this is just a small sample of the interactions, the earlier videos of consensus building discussions that the teachers presented resembled this lesson: the teachers first had different groups of the students present their strategies in front of the other students, and the teachers gave comments on the effectiveness of the strategies (typically right or wrong) as they interacted with the students to see if they really understood their points. In the videotaped lessons, there was very little evidence of the teachers attempting to help the students consider strengths and weaknesses of multiple strategies. As mentioned before, they had read and learned about the *neriage* model of consensus building, but it is possible that they assimilated what they learned to the familiar schema, that is, classroom discussions are for making the students agree with the teacher.

Lesson study discussions

After watching the videos in each meeting, the lesson study group engaged in a series of discussions on how the video-taped discussions could be improved as well as how to overcome the discrepancy between the video-taped lessons and the *neriage* model of consensus building. During the discussion process, the facilitator attempted not to lead the discussion to any pre-determined direction, but instead focused on the role of facilitator who made sure that everyone had a voice and spoke up until consensus was built in the discussion, and that the discussion stayed on the topics related to how the *neriage* portion of the lesson could be improved, etc. During the lesson study discussions, both the US teachers and the Japanese teachers gave numerous comments on the presented lessons, but the Japanese teachers offered many essential and new ideas that the US teachers had not yet developed themselves or received during the earlier meetings.

For instance, after the group watched the videotaped lesson described earlier (i.e., making sense of $3/4 \times 3$), one of the US teachers pointed out the importance of encouraging students to critically discuss presented strategies, referring back to evidence in the video of the lack of a whole class discussion to evaluate the presented strategy. Then other teachers expressed a sense of reluctance to the idea of the whole class evaluating the strategies that the students presented. Those teachers were not sure about the importance of considering the superiority of particular approaches over others, because they wanted to value and honor the diversity of their students' approaches over the whole class consensus building discussions. In response, one of the Japanese teachers pointed out that it is important to honor each student's approach, but if the students do not understand the strength and weakness of each presented approach by examining the mathematical concept from different angles (i.e., why $3/4 \times 3 = 3/4 \times 3/1$), the students could encounter stumbling blocks in their future math learning. Then the group discussed various classroom experiences and that some students are unable to perform at the grade level because they continue to use ineffective or wrong approaches that they learned in earlier lessons about mathematical problem solving. After this discussion, all the teachers came to agree that it is important to facilitate the consensus building discussions among the students so that students could examine different strategies and co-determine what they should do (or should not do) next time they encounter a similar problem. The group also agreed that the main purpose of consensus building is to give students the opportunity to think deeply about different ways to approach the problem and then construct an integrated idea, rather than just listening to their teacher evaluate which strategy is right or wrong. All the teachers, including the one who presented the earlier lesson, agreed to make more efforts to encourage students to listen to others carefully, interact with each other, and compare and contrast different strategies for building consensus *among themselves*, rather than between the teacher and the students. The first lessons study meeting concluded with consensus on this point.

However, in the second lesson study meeting 2 weeks later, not all the teachers were successful in the attempt to facilitate consensus building discussions among the students in their classrooms. In fact, most of the teachers expressed that they did not feel confident enough to implement *neriage* in their lessons. In most of the videotaped lessons presented in the second meeting, the teachers struggled to elicit whole class discussions in mathematically and pedagogically meaningful ways. For instance, one of the teachers presented their video-taped lesson with the following *hatsumon*.

Hatsumon: How do you know if a number is a prime or composite number?

In the video, the teacher first reviewed what prime and composite numbers are, and then the students started their explorations. During the group-based problem-solving time, the students randomly chose many different numbers such as 32, 14, 25, 100, 46, etc. for their explorations, and did not use any systematic strategy in the problem solving (e.g., dividing the number with a number that they randomly chose, factoring the number in many different ways, etc.). In the video, the strategies that the students presented during the whole class discussion (*neriage*) were so diverse that the teacher struggled to facilitate the consensus building discussion in a mathematically and pedagogically meaningful way. The teacher who presented the video reflected,

> I tried to get the students to have more of a discussion with one another. I also asked students to pick a favorite strategy and share. However, consensus was not built because each student felt comfortable with different strategies.

After watching the video and hearing the teacher's reflection, the group discussed this issue by exchanging various perspectives. During the discussion, one of the Japanese teachers asked the teacher what the goal of the lesson was. The teacher simply stated that the goal was that students give their own answers to the *hatsumon*. None of the US teachers questioned this assertion. (This was the most common response from the US teachers when they were asked about the instructional goal in the early lesson study meetings.) Then another Japanese teacher responded that simply letting the students come up with an answer is not a goal for mathematical learning. (She gave her comments in Japanese, and the facilitator immediately translated her comments into English). She pointed out that the goal of the lesson must be what the teacher wants her students to come to realize in the lesson (*kizuki* in Japanese). She added that simply posing a problem without envisioning *kizuki* among the students is seen as problematic in Japanese schools. Indeed, it is the responsibility and pride (*hokori*) as an educator to help everyone in the classroom become ready for the next lesson by eliciting essential *kizuki* among everyone in the classroom for the next lesson. The whole group discussed this point by referring to different classroom experiences. All the teachers including the one who presented the lesson agreed that the goal (or *kizuki*) of the lesson was not well-defined or thought through in the presented lesson. (In fact, we learned that some of the teachers did not even solve the inquiry problems by themselves and simply gave the problems to the students.) The group agreed that the source of the problem was the quality of *hatsumon*—that is, the *hatsumon* was too complex and allowed the students to use too many mathematical skills and concepts that were difficult to capture in one lesson. By sharing their experiences and exchanging perspectives, the group agreed that it is important to deeply examine the educational goal of the lesson prior to the lesson. They further agreed it is important to be ready to start an inquiry lesson with a clearly defined educational goal (i.e., *kizuki* for students) in relationship to the curriculum map, rather than to simply throw a challenging open-ended problem at the students. The group also built consensus that at the end of each inquiry lesson, it is important to generalize the mathematical principles on which the students built consensus, to write them down with mathematically accurate terms on the board, and, finally, to invite the students to apply the principles to other problems at the end of the lesson.

Then one of the US teachers asked what teachers should do if the students' answers are wrong or so confusing to everyone that it is difficult to facilitate a whole class discussion in a meaningful way. One of the Japanese teachers responded that before the lesson, teachers need to anticipate the students' responses and develop a lesson plan with the section on anticipated student responses, and during the lesson, teachers need to arrange the order of the students' presentations to make it easy for the students to see the key points that distinguish effective and ineffective strategies. He added that it is not necessarily bad to make students confused by a wrong answer, but the teacher needs to intervene and clarify the source of the confusion if the students seem to become too confused or frustrated, which could motivate students to avoid the wrong or confusing approaches in the future. Some of the US teachers shared their experiences and supported the Japanese teacher's comment. After exchanging their perspectives on this issue, all the teachers came to agree that in such cases, teachers need to give "traffic control" in the consensus building discussions. The teachers' reflection forms captured their important insights and intentions:

> I need to clarify for myself ahead of time what the focus is, what I want students to come away with.
>
> I really learned the importance of building consensus through comparing and contrasting and also by deciding on the strategy that is most efficient. Another key point would be on the teacher understanding the math concept and having a specific outcome/goal for the lesson. Teacher must anticipate trouble-spots and possible student responses in order to plan/prepare accordingly.
>
> Teachers need to step in before students get too confused. Students were able to understand that there was a relationship between diameters and circumference because they were able to discover it and see that their answers were different.
>
> I need to make sure *hatsumon* is open-ended, yet elicits expected students responses.
>
> I need to refine question to elicit responses desired and allow for more interaction between students as they explain their thinking.

As we went through the lesson study meetings, all of the teachers agreed that one of the largest obstacles in initiating meaningful student-to-student interactions was that many of their students struggled to explain their thinking clearly in front of the whole class. One of the teachers who presented more successful video-taped consensus building discussions pointed out that in such cases, she repeats or rephrases their explanations so that the whole group can grasp the idea that the student had presented. Then one of the teachers who was learning about questioning techniques in the district-based professional development program pointed out that using a set of guiding questions (e.g., "Tell me why you disagree", "What's different about the answer?", etc.) were useful in such situations. The group agreed to share the questions that they found to be effective in consensus building discussions in the future meetings.

Interestingly, as we engaged in more discussions on *neriage*, the discussions we had during the lesson study meetings began to resemble one of our aims for the elementary classroom, that is, listening to each other carefully, thinking about others' ideas deeply, and building consensus among ourselves. This "cultural shift" became increasingly salient to the group as it moved through the lesson study meetings.

During our lesson study meetings, each teacher made different contributions to the discussions. As we could see in the earlier examples, the lesson study discussions were highly interactive and complex, and no linear description could capture the overall dynamics or process of them. The whole group discussed various issues until it reached an agreement on each issue. As a result, each lesson study meeting went far beyond the scheduled time (2.5 h). Many different sets of teachers' tacit knowledge ("zen and the art of *neriage*") were discussed as the facilitator took notes. A more detailed summary of what teachers agreed in the lesson study discussions will be discussed later in this article.

As we continued our lesson study over 2 months, there was a gradual shift in how the teachers facilitated consensus building in their video-taped lessons. The video-taped lessons showed that the teachers began to make more effort to help their students explain their ideas clearly, compare and contrast different strategies that they presented, and build consensus on the strategies without the teachers interjecting their judgments into the class discussions. By the end of six lesson study sessions, all the teachers' videos reflected some extent of improvement in having students think about different problem-solving approaches carefully and build consensus *among* themselves as the students explained their rationales to their peers. We also observed that children increasingly spoke about their ideas spontaneously without being prompted by the teachers. The following are some of the examples.

Videotaped classroom interaction #2

The teacher gave the worksheet with a figure of parallelogram, and asked the *hatsumon*, "Use what you know about the areas of rectangles and triangles to find the area of this parallelogram. Explain your solution." When she introduced the lesson to the group, she clearly stated the goal of the lesson (understanding how the area of parallelogram could be calculated using their knowledge about the areas of rectangles and triangles) and several possible responses from the students that she anticipated.

In the video, the students first worked on solving the problem in groups. Then the teacher asked a student from one group to present their strategy using the document camera. Then the student (Gilbert) presented in front of the whole class that they divided the parallelogram to three parts, the triangle on the left, the rectangle in the middle, and the triangle on the right (Fig. 2). Then they first calculated the area of the rectangle by multiplying 4 and 10 and obtained 40 in^2, and then calculated the area of each of the two triangles by multiplying 3 and 4, and obtained 24 in^2 as the sum of the areas of the two triangles. Finally, they added 40 and 24, and obtained 64 in^2 as the answer.

Obviously, this answer is wrong because they failed to divide 12 by 2 as the area of each triangle (or considering 3×4 as the sum of the areas of the two triangles). The teacher asked the following question right after the presentation.

Teacher	So… I see that Eric and Donovan have a lot of questions for you. I'm worried 'cause the people in the audience have different answers… What do you think, Jacob?
Jacob	You didn't divide by two.
Gilbert	…
Teacher	What do you think about that, Gilbert? Did you hear what he just said?

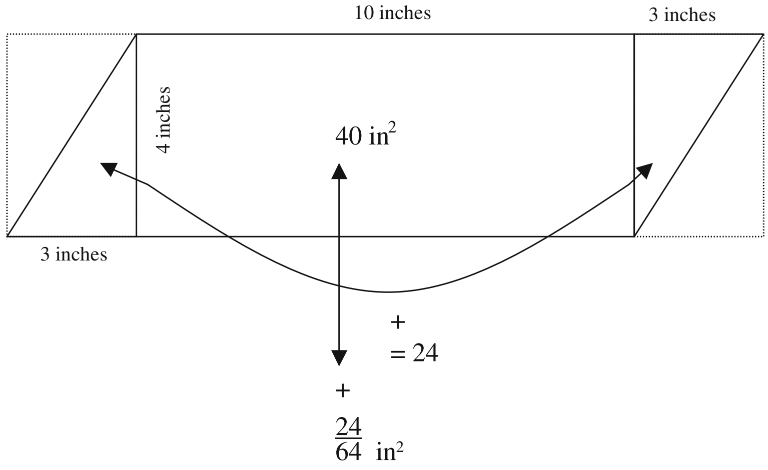


Fig. 2 Gilbert's problem solving

Gilbert	Yeah … What?
Jacob	Where did you get 12 from?
Gilbert	By multiplying 3 and 4. We didn't divide by 2 because… right here in this rectangle, instead of multiplying the halfway, what we know is… we noticed that the height of this rectangle is 4 inches, so if this height is 4 inches, and the width is 3 inches, what we did is we just multiplied and then we didn't have to divide by 2.
Teacher	OK, Jose and Gilbert, I want you to notice what Jacob has done—It's very similar to yours. Jacob and Bao, can you explain how you did it?

Then Jacob presented the details of the strategy in which he and Bao first calculated the area of the rectangle ($4 \times 10 = 40$ in^2), calculated the area of each of the two triangles ($3 \times 4 \div 2 = 6$ in^2), added the areas of the rectangle and the area of the triangles, and obtained 52 in^2. Then the teacher asked.

Teacher	What do you think about that, Gilbert?
Gilbert	I have a question… They are same as ours, but they put like 6 in the middle (*of the triangle*). My question is why they put 6 right there. And they did get a different answer.
Teacher	They got a different answer. Donna, what do you want to say?
Gilbert	They added 40 plus 6—and then they put 52 instead of 46.
Teacher	They added $40 + 6 + 6$.
Gilbert	Oh… But why do you put 6 inches when the half point is 5 inches?
Student A	That's what I am confused about. Why did you put 6 inches in the middle?
Teacher	Alright… go ahead, Jacob. So you are answering why 6 inches instead of 12.
Jacob	I just multiplied 3 and 4… and I divided it by 2.
Teacher	Gilbert, did you hear that? I think that was your question…
Gilbert	Why did they put 6 in this?
Jacob	I just multiplied 3 and 4, and I got 12, and I divided it by 2.
Gilbert	But how did you get the 2?
Jacob	'Cause I wanna get it in half.
Gilbert	Why did you divide?
Jacob	Cause …you need to divide it by 2.
Teacher	What did we find out when we talked about the area of a triangle yesterday? Wasn't it the half of the area of the rectangle? Isn't half as same as dividing by 2?
Students	Yes.

All the students agreed with the reason why Gilbert's answer was different. After that discussion, the teacher asked another student to present her different strategy. The student cut and pasted the triangle on the right to the left to form a rectangle whose area was equivalent to the area of the parallelogram (Fig. 3). The teacher then asked everyone to consider the easiest formula for obtaining the area of a parallelogram ("What is the easiest way to calculate the area that everyone can agree on?").

In this episode, the teacher facilitated the discussion without giving immediate feedback to Gilbert's wrong answer. Rather, the teacher asked other students to consider the presented strategy. The teacher's role was a facilitator to "polish" the students' understanding via the whole class discussion, using the wrong answer as the opportunity for consensus building. During the discussion, there were spontaneous interactions between the students on the problem solving. This makes a stark contrast with the previous episode, where the

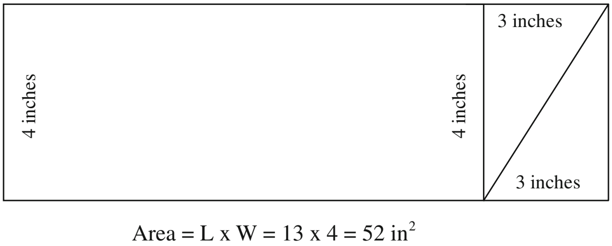


Fig. 3 Another student's problem solving

teacher simply judged and explained the presented strategy and asked the students if they understood her point. The teacher's written reflection indicates

> My students struggled with the process at the beginning because they were so used to learning math through direct instruction and guided practice. They were also so used to only watching students present their work that the teacher chose because they were right. Once students got used to the idea that the students sharing may not be sharing the right answer, or the best way, they became empowered to question and clarify. This process also allowed me a chance to really see what my students could do using what they already know to solve problems they had never taught before. As a teacher, I feel that my students learn so much more when I step back and let them discover concepts on their own.

This reflection indicates the teacher's effort not to give immediate feedback to the students, but instead to help students share ideas and build integrated understanding of the concept in the whole class discussion. By "stepping back" (and holding back from playing the expert in the room), the teacher learned that the students can learn more by using their prior knowledge and become empowered to construct their understanding in inquiry-based lessons.

Videotaped classroom interaction #3

The teacher showed the following table (Fig. 4) to her 5th grade students and gave the students the *hatsumon*, "On which day did Jill walk the greatest distance? On which day did she walk the least distance?" with the guiding questions of "(1) How can we solve the problem by making a model? (2) Which model gives the best representation of the fractions?" Before this lesson, the students had learned the concept of fraction as a quantity, but had not learned how to compare proper fractions with different denominators yet.

The students first began solving the problems in small groups, and then the teacher asked two of the groups to present their strategies in front of the whole class. The first group's strategy was to compare the fractions using four number lines that indicated the

Day	Mon	Tue	Wed	Thu
Miles Walked	$2\frac{1}{2}$	$3\frac{1}{8}$	$2\frac{3}{4}$	$2\frac{3}{8}$

Fig. 4 Table used for the *hastumon*

locations of each fraction on each of the lines arranged vertically, but not aligned precisely (Fig. 5). Right after this presentation, the teacher had the second group present their solution using only one number line that indicated the locations of the four fractions on a single line (Fig. 6). After the group presentations were over, the teacher initiated the following whole class discussion.

Teacher	Is there a model that all the group chose to represent the problem? … Chris, what do you want to say?
Chris	These are … table?
Teacher	That's not a representation for the fractions.
Chris	Oh, I mean, no, number line.
Teacher	OK, number line.
Chris	And so all they did…. (*silence*)
Teacher	And all did exactly the same number line?
Chris	One group did like four different number lines, and labeled and they compared each other. Another group did like one number line, and put arrows.
Teacher	Juan, that's something?
Juan	So we can each use a number line like different ways.
Teacher	And if we talk about efficiency, what would be the efficient way to solve this problem?
Frank	By looking how much close, do the same…and use…
Anne	And we have to cut the pieces of…and see which one is…
Teacher	I guess my question is on a test, do you want to draw four number lines?
All the children	No!
John	We have to draw one number line and then…and …
Teacher	So this (one number line) would be the efficient way to solve the problem? This group, with one number line? They included all on the same number line.
All the children	Yes!

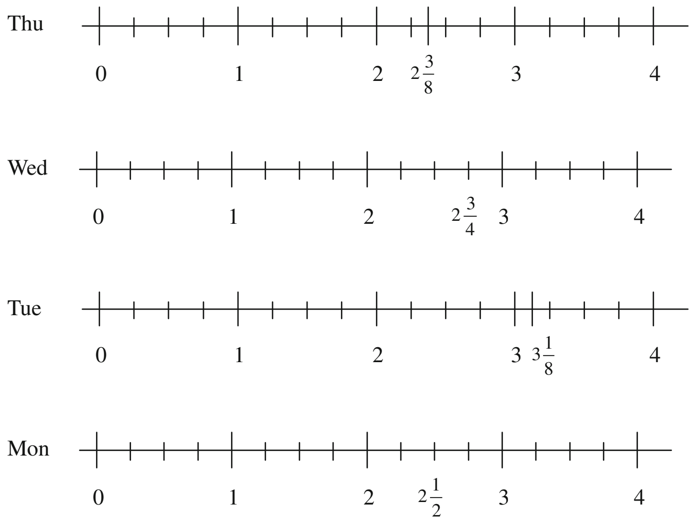


Fig. 5 Mathematical model used by a group

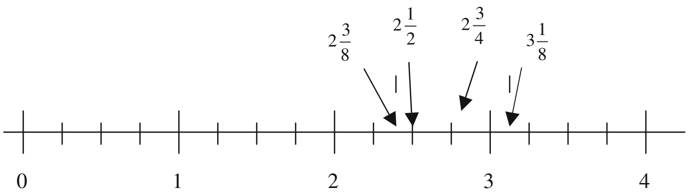


Fig. 6 Mathematical model used by another group

This episode presents the teacher's attempt to incorporate *neriage* in her lesson, i.e., helping the students think about and co-determine the most efficient representation to compare the fractions. Unlike the first episode (#1), the teacher reserved her judgment on the strategy and instead focused on drawing out ideas from multiple students (e.g., "Chris, what do you want to say?" "Juan, that's something?"). After the presentations, the teacher focused on giving the students the opportunity to think about and build consensus on the best strategy, i.e., drawing one number line is much more efficient for comparing multiple fractions than drawing four number lines. Although the mathematical content is not very deep and the discussion could have gone deeper (e.g., discussing why it is the case, how to plot fractions precisely for comparing them, how to align the grids of number lines, etc.), the teacher attempted to facilitate the discussions by reserving her judgments, and attempted to help the students think about different strategies and make their own judgment on the efficiency of the presented models. Again, this makes a stark contrast with the first episode, where the teacher simply explained the presented strategy and asked the students if they understood the point. The teacher's written reflections after presenting this video reflect this point:

> When it came time to build consensus, I gradually released control of the discussion to the students with each lesson we did. They naturally chose the order of the presentations by comparing their work with the presentation and then wanting to add or say something different they would jump up and share their work. Their questions, comments, and clarifying became more natural and it helped them build understanding.

The earlier episodes and the teachers' reflections indicate that the teachers shifted their role toward the facilitator of the discussion, and the students became more "natural" in comparing and contrasting different strategies during the consensus building discussions. In short, the lesson study discussions served as the opportunity for the teachers' self-transformation.

By the end of the six lesson study meetings, all the other teachers' reflections contained similar points. In fact, a couple of teachers indicated how overcoming the old habit of dominating the students' discussions required courage and focusing on students' reasoning process. All the teachers indicated how difficult this shift in their role was, but reported that it resulted in rewarding outcomes, such as the enrichment of students' understanding, higher levels of students' participation and motivation in the learning activities in their classrooms. The videotaped lessons that all of the six teachers presented in the final session also reflected these points.

Figure 7 indicates the mean hours that the teachers spent for Japanese inquiry lessons in their classes over the six lesson study meetings. As the graph shows, there was a drop at the

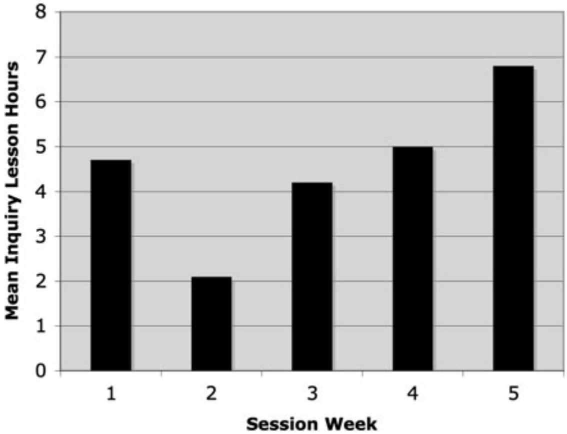


Fig. 7 Mean hours of inquiry lessons

initial stage, but the time for inquiry lessons gradually increased and eventually reached a level higher than the initial level as they experienced more lesson study meetings. Though there were some individual differences across the teachers, this supports the aforementioned analysis, i.e., the teachers generally became more confident in implementing inquiry lessons as they went through the lesson study meetings.

Neriage strategies

The following section summarizes the key points on which the group built consensus during the lesson study discussions, based on the author's notes. Again, although the paths to build consensus to these points were different, the following points not only illustrate what the six US teachers agreed as key points for effectively incorporating consensus building discussions in their classrooms, but also serve as useful guiding posts (i.e., zen and the art of *neriage*) for other educators attempting to achieve similar goals.

1. *Know what you are asking*: The quality of "*hatsumon*" (initial question or problem) that teachers give greatly determines the quality of students' thinking in the consensus building activity. It is important to know the specific mathematical/pedagogical goal for giving the *hatsumon* and the inquiry lessons. Teachers need to be clear about specific points that they want their students to notice, discover, or realize in the lesson (*kizuki*) in relation to the lesson goal and the curriculum map, before initiating the consensus building discussions. Consider how the students could develop deep mathematical concepts through the inquiry lessons and how consensus building discussions could be facilitated to achieve the goal. It helps to write down the educational goal of the activity (e.g., understanding the importance of comparing multiple fractions by using one representation) and its relationship to the curriculum map before the lesson. The use of scenarios or stories could also make a *hatsumon* appealing to the students and improve the quality of the consensus building discussions.
2. *Anticipate students' responses during lesson planning:* In the videotaped discussions, there were many instances where students presented diverse strategies and the teacher did not know how to respond. This could create too much confusion and deviation of

the whole class discussions into unconstructive directions. Before the lesson, teachers should solve the problem themselves and predict the students' likely responses. Plan how to make use of the anticipated responses in actual consensus building discussions. This helps the teacher judge whether to intervene and how to guide the students' problem solving at each point of the discussions. For beginning teachers who are yet to build up a wealth of common mathematical misconceptions and pedagogical content knowledge, this could be a challenging task as was the case in this study. However, actually teaching and reflecting on the lesson in collaboration with other educators in lesson study or the similar forms of professional development could help teachers develop richer pedagogical content knowledge and the capability to anticipate students' correct and incorrect responses, as suggested by researchers (Inoue 2009; Kinach 2002; Stones 1989). In other words, going through recursive cycles of teaching and reflecting on mathematical lessons with other teachers could help even the teachers without much experience in inquiry lessons become capable of anticipating students' responses for effectively facilitating consensus building discussions.

3. *Releasing control to students*: The group agreed on the importance of releasing control to their students so that the students could freely compare and contrast different problem-solving strategies from multiple angles. For instance, when asking questions during consensus building discussions, teachers need to wait for students' responses patiently. Become silent when it is needed and wait for their thinking/answers without readily imparting teacher's judgments. Even when inefficient strategies are presented, do not make immediate judgments on the inadequacy. Rather, have students listen to such strategies carefully and actually "experience" the inefficiency as a whole class. Then facilitate discussions on the presented strategies. Accept students' answers as the students explain them instead of providing instant judgments about whether the answers are right or wrong. Give students plenty of time to discuss inefficient strategies. Without carefully listening to and experiencing multiple problem-solving strategies, each student's experience in mathematics classrooms would be limited to the particular problem-solving strategy that she/he initially chose.
4. *Don't hesitate to provide traffic control:* Sometimes students' explanations of their mathematical reasoning are too quick or inaudible to the whole class. In such cases, repeat or rephrase the students' answers slowly and in a concise, step-by-step manner. Ask students to draw neat pictures using rulers or other mathematical tools, so that they present as accurate representations of the quantity as possible in examining and sharing their ideas. If a student's point does not make sense or is too confusing, ask the student or other students to explain or comment on the presented strategy in a non-confrontational way. Use guiding questions, such as "What is cool about this?", "What's different about this answer?", "Which of these make sense?", "What is your favorite strategy?", "Tell me why you disagree?", etc. depending on the needs of the situation. Before the whole class consensus building discussion, it helps to have the students think/speak with a partner/group first and build consensus with their peers, especially when the students are not used to talking about mathematics. For whole class discussions, carefully choose the order of students' presentations or select them. The sequence of presentations could guide students' thinking toward an integrated and deeper understanding. Sometimes students present totally wrong solutions and their colleagues become really confused by the presented strategies. The students' confusion or confrontation during consensus building discussions is not necessarily bad, since it can serve as an essential catalyst for developing deeper understanding, but it needs to be contained. When the students become really confused, do not hesitate to

intervene and guide them to a constructive direction. This could capture the students' "learning moment".

5. *Always follow-up*: Consensus building discussions should not end when consensus is built. Generalize the mathematical principles agreed in the consensus building discussions to different cases of problem solving at the end of the lesson and possibly incorporate exercises of similar problems so that children can see the value of their discussions as leading to accurate, efficient problem solving. Also, after each lesson, teachers could develop a new *hatsumon* for a new mathematical inquiry activity for the next lesson. Lesson study with colleagues who are teaching different grade levels can help teachers plan the lesson and consensus building discussions in relation to the curriculum map across multiple grade levels.

Conclusion

There has been an increasing number of lesson study initiatives as a part of professional development of teachers outside the school system of Japan (e.g., Fernandez et al. 2003; Buczynski et al. 2007). This lesson study project particularly focused on how to elicit negotiation and social construction of meaning through consensus building discussions in inquiry-based problem-solving activities. Through reflecting on their lessons and other teachers' lessons in this video-based lesson study, US teachers went through a process of self-transformation and developed many realizations for incorporating consensus building in a way that is congruent to what has been emphasized for students' meaning-making and engagement, such as social negotiation of meaning (Cobb et al. 1991; Voigt 1991), social involvement in mathematical discourses (Turner et al. 1998), and authoritative teaching that support student autonomy and personal interest (Walker 2008).

What should be noted is that what teachers learned in this project was congruent to the key features of successful inquiry-based lessons that researchers theorized, including teachers anticipating and connecting students' responses (Stein et al. 2008), reflective and instructive communication (Brendehur and Frykholm 2000), reflective discourse and collective reflection (Cobb et al. 1997), disciplinary engagement (Engle and Connant 2002), and didactical situation to socially validate mathematical ideas (Brousseau 1997). This study could be seen as an attempt to implement an effective inquiry model in mathematics classrooms using *neriage* as a cross-cultural framework for teachers to effectively orchestrate whole group discussions for deep mathematical learning.

It should be noted that a culture-specific epistemology and set of values underlie the social dynamics and the nature of the mathematical discourse in Japanese classrooms as well as professional development of teachers (Lewis 1995). Because of such cultural factors, some may argue that what works in Japanese schools does not work in US schools. In this project, we learned that the cross-cultural lesson study focusing on *neriage* (or consensus building) in mathematical inquiry lessons could be an effective model for professional development in the US context. The study identified different sets of "tacit knowledge" that the participating teachers need to know at various points in planning and delivering mathematical inquiry lessons. In fact, acquiring the aforementioned set of tacit knowledge changed the six US teachers' view of mathematical inquiry lessons: all of the US teachers initially viewed teaching inquiry activities as risky and time-consuming, but at the end of the lesson study, they indicated that if they considered the aforementioned points when planning and delivering inquiry activities, it can be a "easier and faster" way to teach mathematical algorithms and techniques, building on students' deep understanding of the mathematical concepts and the rationale behind the algorithm established in the consensus building.

This type of transformation could be replicated in different cultural contexts if teachers go through the similar type of lesson study focused on consensus building. Although the effectiveness of lesson study could depend on a variety of factors such as the developmental level of the student population, the socio-cultural norm of the classroom, and teachers' experience and knowledge, this type of lesson study possesses enormous potential for transforming the ways teachers view and teach inquiry lessons. If this study with only six meetings can bring out positive changes in the ways the teachers orchestrate whole group discussions, then a long-term lesson study with a similar goal and structure could generate even more significant changes.

To conclude, this lesson study project highlights the potential for improving the quality of mathematical inquiry lessons using video-based, cross-cultural lesson study in non-Japanese contexts. This article concludes by calling for continued attempts to see how different groups of teachers could plan and deliver mathematical inquiry lessons more effectively by incorporating *neriage* in different cultural contexts.

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