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Collaborative teacher inquiry as a tool for building theory on the development and use of rich mathematical tasks

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Abstract This article describes the collaborative inquiry activity of a group of high school mathematics teachers interested in increasing student engagement and problem solving in the classroom. Specific findings related to the nature of the teacher interactions and subsequent impacts on practice are discussed. The findings focus on (a) the nature of the interactions within the teacher group and their impact on instructional decision-making and theory-building, (b) the role of student work in their collaborative inquiry, and (c) the role and impact of the inquiry group's facilitation processes. A description of the professional development provides context for the results. The teachers in this study were able to build individual and collective theories on learners and instruction. The limitations of their inquiry context and their own conversational norms allowed for deeper reflections on classroom practice than on learners and learning. Specific connections between the inquiry work and classroom practice are found.

Keywords Collaborative inquiry · Mathematical tasks · Secondary teachers · Teacher development · Teacher research · Theory building

Introduction

Mathematical tasks can be seen as a classroom tool for eliciting student understanding and discussion (Stein et al. 2008). However, mathematical tasks can also be a professional development (PD) tool framing teachers' interactions around student thinking and challenging their beliefs and assumptions about learners, learning, and teaching (Little 2004; Watson and Sullivan 2008). We present our analysis of the year-long collaborative inquiry activity of a group of secondary mathematics teachers interested in increasing student engagement and problem solving through the use of carefully chosen and implemented mathematical tasks. A description of the associated PD provides context for the results.

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We provide evidence that directly links the context and processes of teacher collaborative inquiry to instructional theory building, teacher interactions, and classroom practice. Specifically, we illustrate that the collaborative inquiry led to teachers constructing specific, usable theories related to scaffolding students' engagement with mathematical tasks. These theories primarily involved sequencing instruction and supporting students' mathematical thinking and problem solving. While consensus on these theories was never reached inside the teacher group, the public awareness and debate of these competing theories through collaborative inquiry promoted teacher reflection and furthered theory-building activity. Limited facilitation of group activity and habits of teacher interaction often hindered the grounding of these instructional theories in specific examples of student thinking.

We examine the nature of the group's interactions as these reveal their emergent and competing theories about the use of rich mathematical tasks in their classrooms. We also discuss the extensive effort these teachers devoted to negotiating the structure of the mathematical tasks in order to make decisions about classroom implementation and supports for student learning. We address the following questions: How do personal and group theories of learning and instruction develop from collaboratively planning and subsequently analyzing mathematical task implementation and student work? What kinds of teacher interactions are important in the construction of these theories? How does theory building impact classroom practice?

Theoretical framework

The value of embedded, ongoing, collaborative PD has been asserted by many (Hawley and Valli 1999; Little 2003; Putnam and Borko 2000). Collaborative teacher inquiry can provide a structure and process that holds promise for supporting teacher learning and improvement in classroom practices (Little et al. 2003; Kazemi and Franke 2004; McLaughlin and Talbert 2006; Slavit et al. 2009). Our examination of mathematics teachers' interactions are framed by their engagement in a collaborative inquiry cycle that focuses on the examination of students' mathematical work.

Inquiry cycle

In supported collaborative teacher inquiry (Slavit et al. 2009), teacher groups begin by wondering (Wells 1999) about questions emergent in their context that serve as guides through an inquiry cycle. Figure 1 illustrates a general path we have observed numerous teacher groups take through an inquiry cycle, and the framework used in support of collaborative teacher inquiry in the PD context described below. Early in the cycle, it is important for teacher groups to build a collaborative stance toward the inquiry and further their individual and collective understandings of the issues at hand. The large arrows in Fig. 1 suggest that an inquiry cycle involves specific, interdependent activities that include a focusing and planning period, the implementation of a common teaching action or other collaborative activity, and a period of assessment or dissemination. However, assessment can occur in all phases of an inquiry cycle, and the timing, nature, and use of this assessment can often define the nature and purpose of the inquiry (Sowder 2007). Further, as the thin arrows in Fig. 1 illustrate, the path through an inquiry cycle might not follow this trajectory, and can often involve "doubling back" periods of

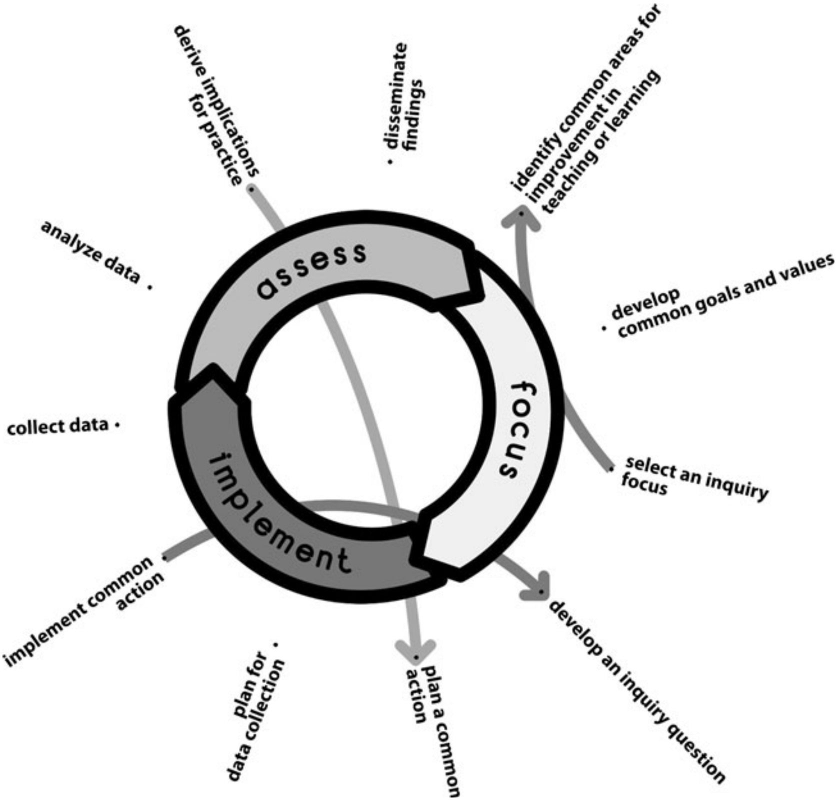

Fig. 1 Common inquiry cycle of a collaborative teacher inquiry group

readjustment. When a teacher group repeatedly doubles back to implementation, a number of mini-inquiry cycles can occur that, collectively, constitute an overall inquiry process. Mini-inquiries are useful to teachers examining targeted issues of practice who desire multiple data points over time.

Teacher interactions and student work

Analyzing student work on mathematical tasks is a common aspect of collaborative teacher inquiry cycles, and has been shown to be a promising way of situating critical dialog in practice (Kazemi and Franke 2004; Krebs 2005; Little et al. 2003). Researchers in the Quasar project (Stein et al. 2000; Silver et al. 2005) have discussed “complex,” “worthwhile,” and “intellectually-challenging tasks” as a vehicle for student development. They maintain, however, that these types of problem-solving tasks may not result in students’ achievement of the stated learning goal, particularly when unsupported by appropriate classroom instruction. While teacher interactions around student work in a study by Chamberlin (2005) were not always focused on student thinking, the interactions did impact instructional practice. However, Kazemi and Franke (2004) provide specific evidence on the benefits of focused discussion on student thinking that include a more solid grounding for expectations about student learning and subsequent instructional decision-making. In addition, these latter two studies found that a facilitator was helpful in moving the teacher interactions to critical levels.

Jaworski (2006) uses the term *critical*, in the context of teachers' collaborative inquiry, as "trying to develop, improve, or enhance the status quo" or "avoid[ing] the perpetuation of undesirable states" (p. 191). While some researchers (Kazemi and Franke 2004; Lamb et al. 2009) found that teachers are able to engage critically about practice, teachers' discussions of student work do not always lead to meaningful interactions that challenge the status quo or transform practice (Little 2004). However, when teachers come together as members of a "community of practice" (Lave and Wenger 1991) and take on an inquiry stance as they pose critical questions about their practices emerging from a common puzzle or shared "felt need" (Dewey 1933), they may begin to challenge their individually and jointly-held assumptions as they generate more nuanced or even very different understandings about teaching and learning (Cochran-Smith and Lytle 2009). Such systematic investigations can lead to changes in practice, but more importantly to the building of local and global theories based on teachers' interactions about student thinking (Jaworski 2008).

The professional development context

The focus of our research lies in better understanding collaborative instructional planning relative to the scaffolding of mathematical problem-solving tasks, group analysis of students' mathematical problem solving, classroom practice, and the relationships between them. The following description of the PD context enriches and contextualizes our analysis while providing background on the study's two lead participants, Camron and Bryce (pseudonyms are used throughout).

Overview of PRiSSM

The Partnership for Reform in Secondary Science and Mathematics (PRiSSM) was a 3-year PD project involving 175 mathematics and science teachers in six school districts. The primary goals of the PD were to (a) build leadership and teacher research capacity, (b) establish professional learning communities (PLCs), (c) support the design, enactment, and dissemination of teacher-designed inquiry, focused on improving student learning, and (d) nurture administrative support and plan for the sustainability of the project beyond the funding period. A 12-person steering committee composed of university researchers and district mathematics and science specialists developed and oversaw the project (Nelson et al. 2008).

Each year of PRiSSM began with a summer academy focused on negotiating beliefs and perspectives about high-quality teaching and learning, building community within the teacher groups, developing collaborative inquiry perspectives and skills, and supporting lead teachers' abilities to organize and facilitate collaborative inquiry processes with others. Camron, one of two lead participants in this study, participated in two summer academies and other ongoing PRiSSM events, such as mid-year academies for additional support and end-of-year showcases to disseminate results to district teachers and administrators.

In the first year, lead teachers met in cross-disciplinary (mathematics and science), cross-grade groups approximately once per month as a professional learning community facilitated by a member of the PRiSSM steering committee. The teacher groups worked their way through collaborative inquiry cycles at various speeds and with varied degrees of success. Most groups focused on student learning processes, such as written communication and problem solving. In the second and third years of PRiSSM, lead teachers shifted more into the facilitation role as they formed PLCs within their schools composed of colleagues new to the project. At this stage, Bryce, the other lead participant in this study, began to engage in collaborative inquiry with Camron and other mathematics teachers at their school.

The Madrid PLC

Collaborative inquiry structure

Camron and Bryce were the co-facilitators of the Madrid PLC, which consisted of all eight teachers of the *Integrated 2* (Rubinstein et al. 1995) mathematics course. Camron had over 10 years experience teaching and was the leader of the mathematics department at Madrid, a high school located in the northwestern United States. During the target year, Camron taught half-time while also serving as math coach at Madrid. The latter duty, which focused on individualized support for beginning teachers, overlapped with her PLC facilitation role, as one of her coaching duties was to provide fellow mathematics teachers with tasks that could supplement the problem-solving aspects of the curriculum. Bryce was also an experienced teacher and recognized leader in the department.

While Camron and Bryce co-facilitated all of the PLC meetings, Camron took the lead in establishing the agenda and directing meeting flow as well as providing specific resources, including the mathematical tasks which framed most of the group activity. Ginny, a member of the PRiSSM steering committee and the assigned facilitator, was a regional mathematics specialist and expert on collaborative inquiry. Ginny's schedule became quite busy and she attended Madrid PLC meetings approximately once every 6 weeks, far fewer than she had planned. In an interview late in the school year, Ginny felt she held outsider status and doubted her effectiveness with the group:

> I'm not feeling successful as a facilitator these days. I think that I, or we, still have lots to learn about facilitating PLCs, especially if you aren't a part of the school district community.

All eight instructors of the *Integrated 2* mathematics course were required PLC participants. The inclusive nature of the inquiry group was indicative of the emergent emphasis on collaborative inquiry at Madrid. Partly as a result of PRiSSM, the principal chose to support the entire staff in this form of PD, including the delegation of weekly, 30-min meeting times as well as two additional 90-min sessions for this purpose.

Collaborative inquiry process

Teachers in the Madrid PLC felt their textbook "lacked a great deal of rich tasks" to support their problem-solving goals, and the group turned to Camron for supplements. Madrid's school district produced "power standards" (Reeves 2002) in mathematics that targeted problem solving, increasing the group's felt need for the rich tasks. Group consensus was that the tasks should (a) be "open-ended" with multiple possible strategies, (b) allow students to "muddle" or "struggle," (c) elicit conversation and thinking so that they are "appropriate for group work," and (d) have "important and relevant" content. Hence, the previously-discussed perspectives of the Quasar group regarding mathematical tasks

Table 1 Mini-inquiry cycle of the Madrid PLC that occurred over three 30-min weekly sessions

Week A	Week B		Week C
Previous cycle concludes Cycle begins Presentation of new task, worked by all teachers	Presentation and negotiation of lesson plan related to the new task, including task format and instructional supports	Task given	Debrief student response to rich tasks Cycle ends Next cycle begins

(challenging, worthwhile, and connected to classroom discussion) were also present in the Madrid PLC. Camron selected the tasks for the group, stating, "We believed students needed to be involved in good mathematical tasks in order to have meaningful math talk."

Although the group had a variety of foci and iterations of a research question during the year, the primary focus of the group is captured by the following: "How can the use of rich tasks and group work increase student engagement in the classroom?" It quickly became evident that the group's focus on engagement was inextricably linked to student understanding and problem solving. The bulk of the collaborative inquiry occurred in 30-min weekly meetings during 3-week mini-inquiry cycles (Table 1). Week A was split into two parts and began with the conclusion of the previous cycle by debriefing the use of the prior rich task. A new mini-inquiry cycle began midway through Week A with the introduction of a new task, which the teachers would work as a group. During Week B, Camron would present and lead a discussion of a lesson plan related to the task, and the teachers would brainstorm potential questions and means of supporting their students' work. The teachers would then implement the task in their classrooms, usually in student group-work format. The cycle would conclude during the initial part of Week C with a discussion of students' responses to the task. At times, teachers would bring student work to guide the discussion. Originally, each mini-inquiry cycle lasted 2 weeks, but after 5 weeks of implementation the group realized the impracticality of this timeframe and extended the length by 1 week.

Approximately halfway through the school year, two special 90-min sessions were devoted to in-depth explorations of student work. The teachers sought to develop a better understanding of students' mathematical thinking as well as a collective understanding of how the use of group work and rich tasks might impact student engagement and problem solving in the classroom. Camron stated, "[In these meetings] we first focused on scoring procedures, but as the year went on, and with help from a facilitator, we focused more on characteristics of good student work, how to help students 'muddle,' and group work." Ginny attended and helped plan the structure of both of these meetings.

Methods

A case study approach (Merriam 1998) was used to interpret and convey the individual and collective actions of the Madrid PLC during the third year of the PRiSSM project. Relationships between the authors and the participants were established across the 3 years of PRiSSM which provided for a rich, contextualized perspective prior to data collection in the target year. This grounded theory approach (Strauss and Corbin 1998) utilized prior activity in the interpretation of actions during the target year, and continued to build an empirical basis for the support of conclusions throughout the entire study. Further, we report here on 1 year of one case of a 5-year research initiative developing multiple cases and cross-case analyses (Nelson et al. 2005). In addition to the specific analyses described below, the data have been discussed on multiple occasions among members of this research team.

Participants and setting

Madrid is a medium-sized high school in a suburban/urban setting in a northwestern U.S. state. Madrid has twice the African-American population, half the Hispanic population, and two-thirds the free/reduced meal population of the state's student averages. Scores on state achievement tests are on par with state averages, including mathematics. The eight members of the Madrid PLC were the primary participants in the study. Camron and Bryce were paid research participants, receiving a modest stipend and other resources; the remaining members of the PLC were voluntary research participants (but required PLC participants).

Data sources

Data for this case study are taken from the third and final year of PRiSSM; this marked the third year of involvement for Camron, and the second year of work by the Madrid PLC. Primary data sources consist of the weekly collaborative inquiry meetings of the Madrid PLC, as well as two 90-min sessions. Over 90% of the meetings were audiotaped and transcribed; the first author attended approximately every fourth meeting as a passive observer (Spradley 1980), primarily focused on descriptive documentation of group activity. Interviews were conducted with Camron and Bryce near the beginning and end of the school year that focused on group goals, actions, dynamics, and various aspects of the inquiry cycle. Numerous e-mail exchanges and informal conversations with all of the above participants provided additional insights into group activity and were used prominently in the data analysis.

Data analysis

An "accounts of practice" methodology was employed for data analysis, adapted from Simon and Tzur (1999). In this approach, the researcher explicitly recognizes and honors both the participant and researcher perspectives in the analyses of phenomena, and attempts to synthesize perspectives into a teacher's account of practice through the eyes of the researcher. This study expands the applicability of this approach into a collaborative inquiry setting among a group of teachers exploring the use of mathematical tasks to develop student understanding and elicit classroom discourse.

Using this framework, transcripts of the PLC meetings over the entire academic year were reviewed multiple times. As themes emerged, these were cross-referenced and consolidated internally and across interpretations generated by other members of the research team. Interview transcripts, e-mail conversations, and field notes from classroom observations and PLC meetings were then analyzed to add depth to the identified themes; no new themes were generated at this stage. Eventually, overarching themes related to instructional scaffolding and the analysis of student work were identified and matched to specific exemplars from the data corpus and presented to the primary participants for an iterative member checking process to ensure trustworthiness (Denzin 1978). These themes correspond to the main findings in this article.

Results

Analyses of the dialog inside the Madrid PLC over the course of the year reveal two important aspects of how teacher interactions surfaced and furthered competing instructional theories related to scaffolding rich mathematical tasks. First, we consider the teachers' collective decision-making about the relationships between the structure of the mathematical tasks, ways in which the tasks could be introduced to students, and how students' learning could be scaffolded. We then discuss the group's analyses of students' mathematical work. In doing so, we provide insight into the specific ways the teacher group's interactions gave rise to theories that informed their classroom practice. We particularly consider the ways in which they collaboratively made or did not make meaning of their students' mathematical thinking in the negotiation of instructional goals and activities.

Planning instruction and scaffolding student learning

Three instructional theoretical perspectives were repeatedly debated over the course of the year, all of which related to scaffolding issues embedded in the construction and delivery of the rich tasks. These involved (a) the degree to which related skills and concepts should be taught prior to the implementation of a task, (b) the degree to which a task should mesh with the current instructional topic or be used as review or foreshadowing of different content (termed "connected" or "disconnected" by the Madrid PLC), and (c) the degree to which a task should be "broken down" into specific steps or presented as a "large problem" (termed "prescribed" or "unprescribed"). Our results show that various individual and collective theories around these issues were continuously being built by the participants throughout the year. While there is strong evidence that the inquiry context supported the development of instructional theories, the data also suggest that this context limited the group's ability to make use of student thinking in their theory development. This result is important to professional developers who seek to understand linkages between teachers' use of student work and subsequent instructional change.

To illustrate the manner in which this occurred, we examine a 3-week mini-inquiry cycle that occurred in the latter half of the school year. As the PLC inquiry focus was already decided, our analysis centers on the *Implementation* and *Assessment* inquiry stages (Fig. 1), with special attention to the theory-building aspects of the *Derive Implications for Practice* sub-stage. Analyses of the entire year's conversations suggest that these teacher interactions are representative of some important features of the collaborative inquiry conducted by the Madrid PLC, including collaborative instructional decision-making, the analysis and use of student work, and the role of facilitation. In general, the teachers honored the conversational norms they intentionally generated at the beginning of the year, including turn taking, active listening, and arriving on time (Table 2).

In Week A of the target mini-inquiry cycle, Camron began with an introduction of the rich task. After the group spent a few minutes working silently on the problem, Camron initiated a discussion of the teachers' solutions:

Table 2 Rich mathematical task used by the Madrid PLC in a 2-week mini-inquiry cycle

Come Fly With Me

Every year over 2,000 men and women apply to the U.S. Air Force to become airplane pilots. Applicants are eligible only if they satisfy three conditions: they must have 20-20 vision with glasses or contact lenses; they must have no allergies requiring medication; and they must not get altitude sickness when flying

This year exactly 1,400 people applied, and of those:

570 did not have *20-20 vision*

798 had *allergies*

65 had *altitude sickness*

120 did not have *20-20 vision* and had *allergies*

32 did not have *20-20 vision* and had *altitude sickness*

45 had *allergies* and *altitude sickness*

25 had all three disqualifiers

To receive credit, you must answer the following five questions correctly:

1. How many applicants actually qualified?
2. What percent of the applicants actually qualified?
3. How many applicants only had allergies?
4. How many applicants had exactly two disqualifiers?
5. What is the probability a person who is chosen at random has exactly one disqualifier?

Explain clearly how you solved each of the problems above. You may use words, pictures and/or numbers.

Camron: How did you get 95 here?

Bryce: I said 25 had all 3, and it said 120 did not have 20/20 and had allergies. We already counted 25.

Camron: 120 did not have 20/20? Oh, this is *not* 20/20 and *had* altitude sickness [pointing to a piece of Bryce's diagram], I get it. So it's a pretty straightforward problem. And I could find a more complicated problem, or we could just expose them to this and see how they think of it. As soon as they come up with the Venn diagram strategy it's pretty easy.

Kris: You want to expose them to some smaller kinds of –

Camron: *Integrated 2* last year had exposure to Venn diagrams, I think. If they recognize it, they will know how to do it …

Bryce: Did I do it wrong? This is *allergies* and *20/20*. It says 120 did not have 20/20 and had allergies –

Camron: No, it's *not* 20/20. It's *not*. I read it exactly that way, so I was thinking it would have to be outside the circle … So how do we feel about that? What's our gut? I'm thinking about as we get them ready for the [state achievement test], while we're on logical reasoning, do we want to do a Venn diagram? Do we want to do a more difficult task? Do we want to give them a different puzzle that's just logical reasoning? What do you guys think?

This conversation illustrates several important features of the group's interaction during the early stages of the mini-inquiry cycle. First, it shows the manner in which Camron controlled the flow of the discussion in terms of both time and topic. It also illustrates how, through Camron's facilitation, teachers' problem-solving strategies were blended with discussions of instructional perspectives and potential implementation strategies, with

Table 3 2 × 2 matrix drawn by Camron to illustrate an instructional tension inside the Madrid PLC

	Connected to current content	Disconnected from current content
Prescribed (task broken down into sub-steps)		
Unprescribed (task presented as a single problem)	Camron is HERE	

additional but consistent attention to state learning requirements. Finally, this interaction shows the importance of group members' content knowledge and problem-solving ability when discussing possible student solutions.

The degree to which students should "struggle" was a common catalyst for discussions of the three theoretical perspectives related to scaffolding mentioned previously. In an interview in November, Camron discussed two of these perspectives in specific terms:

> We've debated how much the problems should be prescribed or unprescribed, and whether or not they should be connected to what the book is doing … I don't think other teachers will use the task if they don't see a connection to the content they are teaching.

Camron's vision of a prescribed task was one that "asks the students to fill in blanks" and is "broken down to where it isn't a problem anymore." During this interview, Camron drew a 2 × 2 grid to illustrate the possible theoretical perspectives the teachers could take on this issue (Table 3). While Camron held an "unprescribed" and "connected" perspective, she was aware that the group did not have a commonly held vision. However, efforts to negotiate these aspects of instruction were never explicitly planned by Ginny or Camron; no protocols or targeted activities were utilized to prompt or support these discussions, and the teachers negotiated these issues in unstructured, unplanned moments. This suggests that, in certain contexts, teachers can have little trouble developing instructional theory, but may need specific support to ground this theory in others' perspectives or a formal analysis of student thinking.

After introducing and working through a possible alternative task provided by Kris, the group tried to address the issue of how to scaffold their students' problem-solving attempts. The teachers' comments in this discussion made explicit reference to the two tensions in Table 3, but no efforts were made to critically examine either issue at this time. Putting aside Kris' alternative task, Camron began Week B with the following remarks that supported the use of the original task in its unprescribed form:

> Camron: This is "Come Fly with Me." I guess my take on this was that people wanted to introduce [Venn diagrams] ahead of time. I found some Venn stuff here that we could use to introduce, I'm open to whatever, but my thinking is I'd like to see if they could discover [a solution] on their own …
> Kris: That's kind of what we talked about the last time we were together, not necessarily direct them to say this is a Venn problem.
> Camron: Right, or do a Venn as an introductory set problem. Then you're kind of telling them it's a Venn.
> Kris: But if you did it a week before.
> Camron: You're going to do this ["Come Fly with Me"] this week. But you could do it Monday –
> Kris: And give this on Friday.

> Bryce: I think I'm going to give mine cold turkey, because my kids are used to, "Here's a problem, what did I learn last week?"

This discussion involves the first and third scaffolding issues mentioned previously, as attention to prior mathematical content and the prescribed nature of the task are both present in this interaction. While this discussion produced momentum, primarily generated by Camron, for the use of the task without explicit instructional references to the use of Venn diagrams, Table 2 shows that the format of the task was somewhat prescribed, with Question 5 being preceded by four sub-tasks that facilitate its solution. Hence, through a critical analysis of both the task and instructional scaffolding issues related to the introduction and use of Venn diagrams, the group was moving toward a "prescribed" and "disconnected" approach, with the issue of preteaching related mathematical skills and concepts yet unresolved. After shifting to a discussion of student strategies on the previous rich task, the group returned to the preteaching dilemma:

> Kris: Do you think it would be nice to have some sort of organizational chart, or some kind of organizational pictorial? Don't say the word "Venn," how could you –
> Camron: They do say it in the problem, I left it on there. I don't know how you guys feel about this, "You must explain how you solved the problem and/or provide a diagram to explain your reasoning." So that kind of is a hint in itself. I could even ask that, "Why do you think they say the word diagram? What would be a diagram you could write?" I don't know.
> Lucy: I think I'd like to wait and see what they come up with.
> Bryce: I want to give them some playtime.
> Camron: I think this is a nice problem to do. It's very possible they see a Venn on the [state achievement test]. There was one last year, but I tried to make up a rubric for it and I failed … What exactly do we hope to see?
> Lucy: A Venn.
> Camron: They don't have to do a Venn, but is the Venn itself so powerful we don't expect written work? Or do we want to see a written explanation in –
> Lucy: I guess it would be what is in the diagram, right? You're asking for a diagram.

Kris, by introducing the notion of an organizational pictorial, resurfaced the scaffolding issue of preteaching Venn diagram skills and concepts. The teachers seemed to agree to minimize this kind of scaffolding, but evidence presented later shows that some teachers who remained silent during this interaction held an alternate theoretical perspective and provided significant scaffolding to their students. The role of the state achievement test in the instructional decision-making process also continues to be present in this interaction.

Following the exchange between Camron and Lucy on the degree to "expect written work," the teachers turned their focus squarely on assessment. George begins with a focus on evaluation, but the group shifts to considerations of scaffolding and interpreting student thinking:

> George: I guess Camron's question is how do you break this down into 4, 3, 2, 1 [rubric scoring levels]. There's not –
> Kris: It says and/or, but if we really are wanting that written communication, at the very least I think you want them to have a sentence that says, "There are 739 what." Say something to me, label some of your work.
> Bryce: You can break that down into two sentences. You must explain how you solved this problem –
> Camron: So what would we look for?

George: We may include a diagram to help–
Camron: In your explanation?
George: In your explanation.
Camron: So what would that explanation be? I used a Venn diagram –
George: That's what you're going to get. What do we want, versus what do we think kids are going to give us?
Camron: So talk to me about what we want, what do we want to see kids do?
Lucy: We want them to say, "This is the portion of the Venn diagram I used to illustrate this." I don't know. Is that what we're looking for?

The lack of a common theory regarding scaffolding introduced different assumptions about how much information to provide to students, use of mathematical representations, and perceptions of students' conceptual understanding and problem-solving ability. These unresolved differences led to questions such as, "What are we looking for?" and a somewhat mixed assessment perspective that combined numeric scoring with understanding student thinking. Although consensus was never achieved, the teachers' willingness to engage in critical discussion around this topic allowed the questions to be raised and made public at this time.

After Camron provided the group with the answers to the task, more discussion of possible student strategies and modes of instructional implementation occurred. As time was running out, Camron was forced to conclude the meeting without a common resolution for the implementation of the rich task, nor a group decision on whether or not to pre-teach Venn diagram skills and concepts.

Therefore, the teachers were left to enter the *Data Collection* stage of the mini-inquiry without a shared vision for how to enact the common intervention. The meeting during Week C, which Bryce, George, and Stan did not attend, was dedicated to discussions of student responses to the task. Camron, who taught a double-period section composed of students with low scores on standardized mathematics tests, began by sharing her experiences.

Camron: I said think about a strategy. There's one particular strategy that might be better in this particular problem, and it's difficult to know, but I'm not going to tell you what that strategy is … Then, when they got into groups, one person had figured it out that it should be a Venn, out of the eight groups. Then two or three others got the idea, "I think we should use a Venn. What about a Venn diagram?" … And once they could see the Venn, most of them knew how to manipulate it and use it correctly. Once we got the word "Venn" out, they made good progress. Would you say that's how your's was?
Tasha: No, they were confused, then I [used the word "Venn"]. They knew how to construct their Venn diagram, but they were like, "No, that's way too confusing for us."
Camron: The Venn?
Tasha: Yeah.
Camron: So they haven't had practice as freshman in the honors? So they haven't had practice probably using that.
Tasha: This is my stellar, advanced –
Camron: Right.
Tasha: This student here is my 110%, and he was like –
Camron: What were they trying then?
Tasha: They were just taking a look and trying to do that subtraction in their head … and then, when some went to do the Venn, they got themselves lost. So today, we ran out of time.

Both Camron and Tasha provided minimal guidance to their students in choosing the use of a Venn diagram as a potential strategy, but Tasha was much less pleased in her perception of student progress than was Camron. Camron's students, who had a longer amount of time to work, were able to construct and interpret a Venn diagram in ways in which Tasha's students could not. Tasha's decision to "throw the word [Venn] out" suggests that she provided a bit more scaffolding than Camron, who allowed her students more time to decide on this diagrammatic approach and never gave specific direction to use a Venn diagram. Camron and Tasha both gave descriptions of student thinking, but neither used specific pieces of student work nor talked about any one specific strategy in detail. Kris then offered her insights:

Kris: I ran out of time … There was so much information given. They're like, "Wait a minute," so I was like, "How do you think you can break that down?"
Tasha: So many of them automatically went to, "There's too many. There's too many. There's more disqualified than the 1400." Ninety-five percent automatically jumped to that. It's like, "No, you're looking at it wrong." "Oh, so some of these are probably overlapped." Maybe that's good thinking.
Kris: That's about as far as we got. Then I just hated letting them go out the door with it, but I did.
Camron: Did they get to the Venn idea?
Kris: No, but they got to the idea, "Hey, wait a minute, these groups do overlap."

The teachers' use of student strategies in these examples is illustrative of a general pattern found in the data, examined in more detail later, that shows the role of student work in the teachers' theory-building activity. Camron, Tasha, and Kris all provided detailed generalizations of their students' thinking without referencing any particular student work sample, including the students' conclusion that there were "too many" because they failed to see the "overlap" in the groups. Camron then used a facilitator move to draw Dusty and Lucy, the remaining two teachers, into the conversation:

Camron: Lucy, you probably didn't have time to do it yet?
Lucy: No, we're testing today.
Camron: [To Dusty] Did you have time to do it?
Dusty: I'd gone over this [Venn diagrams].
Camron: You did! So then they knew? It was a breeze?
Dusty: No, a lot, not a lot, several students subtracted wrong. Several couldn't find a percent.
Camron: But their difficulty wasn't in the organization structure.
Dusty: No, I did a warm-up that was Venn, so –
Camron: Cheater [laughter].

Making one's beliefs and theoretical perspectives public is necessary for critical dialog to evolve. Dusty's silence on the scaffolding issue 1 week earlier illustrates the need for facilitators to garner broad participation, perhaps through protocols or other tools, when negotiating practice and co-developing instructional theory. Otherwise, false consensus can be built.

Overall, however, the interactions from this 3-week inquiry cycle illustrate the manner in which the teachers were able to challenge the status quo of their classroom practices (Jaworski 2006), and Camron's playful chastising of Dusty underscores the degree to which the teachers felt safe to question each other's instructional theories and decisions. The Madrid PLC was consistent in its ability to utilize a rich task as a basis for critical dialog about various aspects of classroom practice, often grounded in generalized descriptions of student thinking. While a shared vision of practice was not fully developed and the teachers still held a variety of beliefs, multiple theories were being negotiated and a shared sense of each other's practices was emerging. Without the structures afforded by collaborative inquiry, such conversations could not have occurred. However, as evidenced in the next section, the teachers were limited in their ability to hold critical dialog about student thinking or the specifics of the student work they brought to the meetings.

In an interview during this inquiry cycle, Camron reflected on her colleagues' various theories on scaffolding, particularly regarding preteaching skills and concepts:

> Camron: I think we're still not in agreement with how that [scaffolding issue] should look.
> DS: Is that a big issue inside the PLC?
> Camron: I think it has come out as a big issue in our PLC. I think we can agree to disagree and keep moving forward, and hopefully people's thinking will solidify, or we'll kind of come into agreement. I do think it's an important topic and I don't think it's necessarily one right answer, but I think that conversation, always putting it out there … is important.

The comments of Camron show that the PLC tended to address teaching and learning issues in the context of examinations of classroom practice and/or student work. Neither Ginny nor Camron designed explicit protocols or structured activities that might deconstruct these theoretical issues in a more intentional way, and Ginny's infrequent occurrences at the meetings minimized her impact on the nature of the teacher interactions. Hence, conversations that entered the level of critical dialog did so mostly as a result of the teachers' own collective ability to make their beliefs and theoretical perspectives public, use classroom data to frame issues, and take the risk to challenge each other's theories. These comments also illustrate the degree to which Camron was cognizant of the multiple perspectives held inside the Madrid PLC and her focus on continual dialog, not necessarily resolution, on key theoretical issues.

Teachers' analyses of student work

Analysis of the year-long teacher interactions related to students' mathematical work on the rich tasks suggests three broad themes. These involved the teachers' use of (a) vague terms and general descriptions of student work and thinking, (b) student work as an immediate springboard to discussions of instructional approach and theory, and (c) student work as a springboard to discussions of student understanding and thinking. While the latter interactions led to the most targeted discussions of student responses, they occurred least frequently.

As the manner in which the 3-week mini-inquiry cycles were structured, the teachers devoted approximately three-fourths of the time to negotiating "what we want our students to get from this task" and subsequent implementation strategies, while the other one-fourth (only about 15 min of every mini-inquiry cycle) was spent analyzing and discussing student work. While supporting the group's need to plan instruction, this time allotment limited the data-based theory-building opportunities of the group. Overall, the teachers used vague language when speaking about students' solutions (Krebs 2005) and tended not to challenge a given teacher's analysis of their own student's strategy. A detailed analysis of the degree and manner in which the teachers' examined student work follows.

General descriptions of student thinking

Blanket phrases to describe student thinking and problem-solving strategies were found frequently throughout the teachers' interactions. For example, the statement "they really struggled with that" can be found numerous times in the PLC dialog. These comments added little depth to their collective understanding of student thinking or understanding, or on what aspects of the task were particularly difficult.

Student work as a springboard for discussions of instruction

The focus of teachers' interactions around student work would often shift from student thinking to instructional approach. The following dialog occurred during a mid-year PLC meeting and pertained to a task which involved the calculation of the volume of water in a pipe. The teachers began the discussion focused on students' solution strategies:

> Dusty: Okay, the volume piece. I think almost every group got it. They didn't convert it first, but the answer of 7.2 seconds, 1 of 23 groups in both of my classes, that's all, could figure out the time exactly. Some kids said it has to be less than 20 seconds, and they could tell me why. But as far as coming up with an exact answer, not too many.

Lucy then quickly shifted the discussion towards issues of instruction:

> Lucy: Did you give them any hints as far as dimensional analysis? Have you done that?
> Dusty: I did not. But one group was ready for extra help. I talked to them about it and they remembered doing it last year, kind of.
> Camron: I had a lot of kids use proportion, so it's not a super-complicated dimension problem, I don't think.
> Lucy: No, it isn't. I was wondering, I was thinking that maybe giving them some practice, some little hints beforehand –
> Camron: I have done that in class, at the math review stuff –
> George: I don't know if you're using the *Integrated 1* book, but there's a volume question in there dealing with an aquarium and how many …

George's comments spurred a lengthy discussion about instructional approaches for teaching dimension and volume. While the focus of the dialog began with student solution strategies, it quickly shifted to teaching actions and specific instructional approaches. This piece of dialog also illustrates the manner in which comments about student work sometimes strayed into generalization ("almost every group," "some kids") or lacked real depth in the description and analysis of student work ("I had a lot of kids use proportion").

Even with facilitation from Ginny, the group had difficulty centering their conversations on student thinking, as the following dialog from a PLC meeting later in the year illustrates:

> Lucy: I'm sorry. We are in high school. If you can't find area and perimeter of a square and triangle, there is something wrong.
> Camron: Ginny, what is your advice on this? Do we go on, or do we [go back] and teach [area and perimeter]? It's so frustrating.
> Ginny: Especially since it is a middle school thing. But it also is well documented at the state level that it's an issue, that you don't know the difference between area and perimeter …

Camron: You have 80 feet of crime scene tape, what is the largest crime scene you can tape off? That was a quiz question, except the kids know that's going to be a square because they know to maximize area, you have to have a square.
Ginny: You're assuming they do.
Bryce: My kids didn't know that.
Stan: They do because we've done problems like it, but I think if you're asking what shape will maximize area, I don't know if I'd have two or three kids that would know.
Ginny: And is that really important? If they could problem solve and get to that same answer, do they need to memorize?
Camron: Did you grade it if they didn't give you the area, or did you grade it if they gave you the 20 by 20 [the square of 20 feet by 20 feet]?
[All talking]
Lucy: Here [showing a piece of student work]. They put 20 by 20, they drew the square, some gave me area, but in my mind when we did review I was thinking the same thing, nothing specifically asked for area.

While opportunities arose to explore specific student strategies to the task in the above interaction, the teachers chose to focus on perceived student deficiencies or generalized student strategies. Here, Camron and Lucy continued to focus on the more evaluative aspects of their student-work analysis, despite Ginny's prompting to examine their instructional goals. Even with facilitation, teachers can have difficulty shifting away from evaluation and toward a better understanding of specific student thinking. Ginny recognized this and attempted to refocus the group on specific examples of student work and mathematical thinking:

Ginny: If you guys took that quiz and really broke it apart and looked for where the kids were not being successful, is it in computing the area and perimeter, or is it in those more challenging problems? Then is your question, did we even ask a good question? I think before you make decisions about how much you re-teach, you really have to look at those things.
Lucy: I worked on the exact same one, and only two got it right.
Camron: Which is, I think, that would go on in my class, because when we're instructing they're not always engaged.
Lucy: That's the problem! We just did it on the board, why didn't you write down the answer? That is really pathetic when you can't write down the answer we just got. You don't even have to show me work because we just did it.

Frustrations with a lack of student engagement were frequent in discussions of student work, sometimes becoming the focal point of the teachers' interaction. However, more importantly, the above teacher dialog illustrates the tendency of the teachers to fail to look at student strategies, focusing more on correctness and suppositions about students in general, even when making group decisions about future instructional directions.

Student work as a springboard for discussing student thinking

Most interactions that centered on student thinking occurred in the two 90-min sessions, both of which were facilitated by Ginny. However, as will be seen in the following dialog, the teachers tended to focus more on evaluation than on specific student thinking, even when protocols for looking at student work were used:

Ginny: The more you talk, the more you can come to common agreement about these things, so Bryce won't have to guess about what Kevin is doing. Now, we really want to look at what this work tells you about student thinking, student understanding, and to clarify your learning goals … Maybe we could look at three papers and talk about strategies rather than their score. And I think it would be best if you chose them randomly, don't try to find interesting or representative samples.
Camron: Maybe if we have some focusing questions?
Ginny: At the bottom [of the handout] I think we do.

The above dialog contains several facilitation moves by Ginny to focus teacher interactions away from evaluation and toward student thinking. As the teachers examined their students' written responses with an analysis chart provided by Ginny, distinctions emerged between student justification for "themselves," which were usually quite abbreviated, and justification to "please" or "convince" another, which usually contained expository language and increased detail. Ginny then made another facilitation move toward examining specific student thinking by stating, "Ask yourself what are students getting out of this. That's what this is about. Why would you give them (referring to a numeric score) what you give them?" In a box on the analysis sheet labeled "Evidence," George and Bryce provided specific descriptions of student work and thinking, but the other teachers included value judgments. For example, Lucy wrote "Good communication" and "High" in her evidence boxes. As the discussion continued, the teachers characterized student thinking as "high," "medium," or "low," based on Ginny's initial suggestion. Eventually, examples of specific student strategies were identified, such as the use of a counting strategy and making a list. Camron then asked the group if there were any trends in the analysis:

Lucy: Yeah, we all scored pretty close.
Camron: I mean trends in the student thinking.
Bryce: No.
Kevin: They were all over the place.

Lucy's initial answer reflects a perspective more toward evaluation than understanding student thinking. While Bryce and Kevin were referring to student thinking in their response, they were not able to delve deeply into the specific characteristics they saw, and the group was unable to draw any practical inferences from their analyses.

Conclusions and implications

This case study focused on the third and final year of the Madrid PLC's participation in a large-scale PD initiative centered on supported collaborative teacher inquiry. Several broad implications can be made from this case. Perhaps most importantly, our results show specific and direct links between teacher inquiry and classroom practice, a much-needed finding given the current prevalence of collaborative teacher inquiry. Second, the case provides specific information on how an inquiry context is inextricably linked to the way teachers interact, form theories, and make use of student work. Specifically, the teacher interactions presented opportunities for teachers to hear others' similar or dissimilar conjectures and assessments of appropriate scaffolding, and then negotiate personal and collective theories around these issues. However, the constraint of 30-min meeting times, state learning requirements, and limited facilitation shaped what occurred. Third, the case illustrates why teachers struggle with meaningfully analyzing student work or incorporating their analysis into their instructional perspective. While this result differed from teacher to teacher, the evidence suggests that facilitation and differing views of instruction and assessment presented the biggest challenges to the group. Finally, while the teachers were required to attend the meetings, the self-directed nature of the work stimulated passion, energy, and buy-in, despite this requirement.

The teachers in this case study exhibited explicit connections between their collaborative inquiry and their instructional practice. The rich tasks and collaboratively-developed lesson plans illustrate this important connection at the level of classroom practice. In addition, the teachers developed specific instructional techniques that they jointly explored and into which they inquired. However, the evidence suggests that the teachers were also able to build and negotiate usable theories, as individuals and as a group, about scaffolding mathematical tasks and eliciting student discourse. Such direct linkages between supported collaborative teacher inquiry and classroom practice are important in determining the broad implications of this mode of PD, and this study provides a detailed examination of both the inquiry context and teacher interactions that determined this result.

The interactions inside the PLC surfaced several tensions related to instructional practice in a variety of areas, including instructional scaffolding; tensions related to assessment and evaluation also emerged in regard to the explicit representation of student thinking. The teachers freely engaged in critical dialog about their teaching, but much less so in regard to their interpretations of their students' thinking. Although the teachers used student work (sometimes) in their discussions, they never challenged each other's interpretations of their own students' work, and rarely shared specific examples of their own students' work with others, activities found to be particularly useful in teacher development (Krebs 2005; Chamberlin 2005). Working with elementary teachers, Kazemi and Franke (2004) found that teachers first attended to student thinking, then shifted to developing possible instructional trajectories. The interaction patterns of the teachers in this study often took the opposite approach not only, in part, because of the structure of their inquiry cycle and the focus on rich tasks, but also because of the manner in which interactive norms channeled the majority of the critical dialog to issues of instruction and away from specific examples of student thinking.

While deeper analyses of student work may have made the teachers' interactions richer, the teachers in the Madrid PLC were successful in building theory about instruction and student learning, particularly around issues of scaffolding. Specifically, the teachers developed theories regarding the degree to which content should be developed prior to or during the working of the task as well as the degree to which a task should be divided into a series of subtasks. While these kinds of interactions were rich in detail and took on a critical nature, the interactions around student work did not usually rise to these levels of detail or criticality. Similar to findings by Bergqvist (2005), each teacher had occasion to generalize her or his students' understandings in vague terms, such as "my top ten," "my low kids," or "they struggle with basic stuff." Facilitators must be wary of generalized teacher comments and ask for specific data-based support for assertions of student thinking. Some of the conversations generated by the collaborative inquiry may have instantiated these beliefs, while others produced challenges to these assumptions. The evidence presented here suggests that the closer the teachers moved to discussions of specific student work, the more these assumptions were laid bare and open to challenge. However, these types of discussions occurred infrequently, partly because of Ginny's infrequent attendance and limited opportunities to push the teachers to critical analyses of student work. Even when Ginny did make facilitator moves in this regard, the teachers' failed to pursue this line of discussion. And although Camron had experiences in PRiSSM with protocols and other analytic techniques for looking at student work, she was the only member of the Madrid PLC with this background. Further, the 30-min meeting times were often an inhibition to prolonged discussion of various instructional issues. Given this context, it seems reasonable that the teacher interactions evolved as they did, and helps explain the continual lack of consensus regarding the various instructional theories present in the group.

The mode of teacher inquiry described in this study has implications on what we can say about teachers' use of data. Like other teacher groups who have looked at mathematical tasks (Chamberlin 2005), a collection of mini-inquiry cycles defined the structure of the collaborative inquiry. However, unlike Chamberlin (2005), the Madrid teachers met during brief 30-min timeframes and received less support from an outside facilitator. The Madrid PLC did not really engage in a collaborative exploration of student data, although they did engage each person's interpretations of her or his own students' work. This approach proved fruitful in developing specific instructional interventions and building usable theories about the use of rich mathematical tasks, but may have been further enhanced by a collaborative look at specific student work samples (Nickerson 2008). Jaworski (2006) describes inquiry as "a tool that promotes critical alignment with modes of practice and corresponding development of practice" that can lead to "inquiry as a way of being" (p. 204). The Madrid PLC was able to critically examine their *teaching actions*, employing this inquiry orientation to better align their instructional ideals with their instructional theories and actions. However, the group's interactions with *student work* were much less critical. This implies that researchers who describe teacher collaboration as "inquiry" and dialog as "critical" may need to be more nuanced and context-specific, as the alignments and interactions in this particular case moved across various degrees of being critical and inquiry-oriented. In Jaworski's terms, the teachers in this case study developed a way of being inquiry-oriented and engaging in critical dialog around their practice, but less so in regard to their students' thinking. The inquiry context, particularly the facilitation, influenced this development.

However, in this article, one case of one group of teachers engaged in supported collaborative inquiry to improve their practice and, ultimately, their students' learning. While aspects of this case contribute to an emerging database, more empirical information, including case narratives, are needed to create strong theories about the nature of supported collaborative inquiry and its impact on teaching and learning. This is particularly true given the broad and rapid manner in which collaborative teacher inquiry is being implemented in our current educational landscape.

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