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Alignment, cohesion, and change: Examining mathematics teachers' belief structures and their influence on instructional practices

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Abstract This collective case study reports on an investigation into the relationship between mathematics teachers' beliefs and their classroom practices, namely, how they organized their classroom activities, interacted with their students, and assessed their students' learning. Additionally, the study examined the pervasiveness of their beliefs in the face of efforts to incorporate reform-oriented classroom materials and instructional strategies. The participants were five high school teachers of ninth-grade algebra at different stages in their teaching career. The qualitative analysis of the data revealed that in general beliefs were very influential on the teachers' daily pedagogical decisions and that their beliefs about the nature of mathematics served as a primary source of their beliefs about pedagogy and student learning. Findings from the analysis concur with previous studies in this area that reveal a clear relationship between these constructs. In addition, the results provide useful insights for the mathematics education community as it shows the diversity among the inservice teachers' beliefs (presented as hypothesized belief models), the role and influence of beliefs about the nature of mathematics on the belief structure and how the teachers designed their instructional practices to reflect these beliefs. The article concludes with a discussion of implications of teacher education.

Keywords Mathematics teachers' beliefs · Teacher beliefs · Belief systems · Teacher practice · Belief models

Over the last few decades, more emphasis has been placed on the role teachers play in the learning process. Teachers organize and shape the learning context and therefore have enormous influence on what is being taught and learned. With this recognition, the mathematics education community began to invest more time and resources into teacher research. Specifically, mathematics education researchers, educational psychologists, and

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those involved in teacher education have become increasingly aware of the influence of teachers' beliefs on their pedagogical decisions and classroom practices (Cobb et al. 1991; Nespor 1987; Pajares 1992; Philipp 2007; Philipp et al. 2007; Raymond 1997; Thompson 1992; Torff 2005; Wilson and Cooney 2002).

In this regard, it is believed that for there to be improvement in mathematics achievement, classroom practices must reflect reform recommendations. This would require a change in the instructional practices of many mathematics teachers; a change that can only be actualized if we come to a better understanding of not only the types of beliefs these teachers have but also how these beliefs are related to each other and practice. This article reports on a study that investigated the relationship among Algebra 1[1] teachers' beliefs and their classroom practices, namely, how they organized their classroom activities, interacted with their students, and assessed their students' learning. Additionally, in this study, I examined the pervasiveness of their beliefs in the face of efforts to incorporate more reform-oriented strategies.

Findings from the analysis concur with previous studies in this area that have identified a clear relationship among these variables. This study contributes to the body of literature by illuminating the clustered organization of these teachers' beliefs into an interdependent belief network. This network is presented as hypothesized models reflecting the derivative nature of the teachers' sets of mathematical beliefs. The researcher sought to understand the teachers' beliefs from their own descriptions and experiences to identify dimensions of the phenomenon not covered by pre-existing theory (Ezzy 2002). Specific theoretical perspectives that informed this approach are described below.

Review of the relevant literature

Beliefs

Despite the popularity of the study of beliefs and associated constructs (including, knowledge, dispositions, and values) there has been no universal definition that scholars who study it have agreed upon. Therefore, reflecting the varying orientations of these scholars, I define beliefs as embodied conscious and unconscious ideas and thoughts about oneself, the world, and one's position in it, developed through membership in various social groups; these ideas are considered by the individual to be true (see Pajares 1992; Thompson 1992; Green 1971 for descriptions of these perspectives). Beliefs are personal, stable, and often reside at a level beyond the individual's immediate control or knowledge. They are considered to be very influential in determining how individuals frame problems and structure tasks and are thought to be strong predictors of human behavior (Rimm-Kaufman and Sawyer 2004; Thompson 1992; Torff and Warburton 2005). In this regard, it is thought that how a teacher conceptualizes mathematics has direct impact on her teaching and so if there is to be any change in his/her instructional practices, beliefs must first be addressed. This is no easy feat as beliefs develop over years of schooling and experiences

[1] Algebra 1 is a course focused on the study of elementary algebra concepts and skills (including, but not limited to relations and functions, functions as rates of change, generalization of patterns, and using symbolic algebra to represent and explain mathematical relationships) typically taken by ninth-grade students in the US. For a full description, see *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics 2000).

Fig. 1 Green's quasi-logical structure of beliefs

in various communities, and so tend to remain intact despite educational attainment or teaching experience (Torff and Warburton 2005).

Organizational structure of beliefs

Beliefs tend to be highly resistant to change. This resolute nature of beliefs can be partly attributed to their organization in multidimensional systems (Green 1971). Green's metaphor of belief systems provides a useful framework for understanding relationships between different beliefs and between beliefs and behavior. The first dimension of the belief system describes how beliefs are organized in a similar way to that of premises and conclusions. This organization is considered quasi-logical because it is not based on the content of the belief but *how* these beliefs are held (see Fig. 1).

The second dimension, the psychological strength of a belief, is also related to *how* beliefs are held and not the content of the belief. Beliefs held with great psychological strength are considered *core* beliefs and the others are called *peripheral* beliefs. These two dimensions are considered separate and can vary independently of one another. Due to these two mutually exclusive characteristics of belief systems, individuals can hold two incompatible, inconsistent beliefs without internal conflict, granted they are never required to examine them concurrently.

The third dimension to this system is the way beliefs are clustered. This grouping process provides protection and support for their incompatibility and inconsistencies. Due to the "protective shield" that the individual provides these clusters, it is possible to hold conflicting core beliefs. This segregation of beliefs is often upheld by another belief. For example, a teacher believes that "schools should be an environment where students are provided with all opportunities to excel" and she also holds the belief that "students who are not in the gifted classes should not be recommended for advanced math courses." The teacher holds these two seemingly incompatible beliefs and for her there is no apparent contradiction since they are held at bay by another belief—"ability is fixed".

Teachers' mathematics beliefs

Each individual holds a range of beliefs that influences his/her perceptions of the experiences they have with others and the world in general. These beliefs have been investigated across different areas of educational research and form a broad literature base that includes studies on personal epistemology (Fives and Beuhl 2008; Hofer and Pintrich 2002; Muis 2004), epistemological world views (Perry 1970; Schraw and Olafson 2002), theories of intelligence (Dweck and Leggett 1988), self-efficacy beliefs (Bandura 1997), and domain-specific beliefs (Raymond 1997; Thompson 1992).

Early research on beliefs, namely, the study of Perry (1970) and Belenky et al. (1986) had two major findings. First, there were gender differences in how epistemological beliefs were organized, and second, beliefs were influenced by educational experiences. These studies, including the early study of Schommer (1990) regarded beliefs as domain general, in that, beliefs about the nature of knowledge were similar irrespective of the particular discipline. More recently, researchers in this area (cf. Beuhl et al. 2002; Hofer and Pintrich 2002) have come to think differently, and posit that how knowledge is conceptualized in discrete areas of study, the content that is taught and how it is taught, have significant influence on an individual's beliefs about knowledge in that discipline. Domain specificity is also observed in other types of personal beliefs, such as self-efficacy beliefs, which refer to an individual's belief about his/her ability relative to a specific task (Bandura 1997).

In this regard, I distinguish between teachers' thoughts about the nature of knowledge, in general, and how individuals come to know (referred to as personal epistemology) and their beliefs about what constitutes mathematical knowledge and how individuals garner this knowledge (hereafter referred to as domain-specific and mathematics-related beliefs). The latter is the focus of this study.

Researchers tend to classify teachers' mathematics beliefs into beliefs about the nature of mathematics, beliefs about mathematics teaching, and beliefs about student learning (Cooney 2003; Cooney et al. 1998; Ernest 1988; Thompson 1992). These beliefs reflect how teachers conceptualize their roles in the classroom, their choice of classroom activities, and the instructional strategies they use. Beliefs are considered central to the way teachers conceptualize and actualize their role in the mathematics classroom, and, therefore, they are integral to any efforts to improving student learning.

Researchers (Cooney 2003; Ernest 1988; Lerman 1983), who study the beliefs teachers hold about mathematics, suggest that they range from viewing mathematics as a static, procedure-driven body of facts and formulas, to a dynamic domain of knowledge based on sense-making and pattern-seeking. Labels have often been assigned to these perspectives; in particular, Ernest (1988) identified and distinguished between three views about the nature of mathematics. They include the instrumentalist view, the Platonist view, and the problem-solving view. Dionne's (1984) three basic perspectives, the traditional, formalist, and constructivist perspectives, can be aligned to Kuhs and Ball's (1986) classification of "dominant views of how mathematics should be taught" (p. 2). Three of the four classifications, learner-focused, content-focused with emphasis on understanding, and content-focused with emphasis on performance, describe models of teaching reflective of the mathematical beliefs outlined by Ernest (1988) and Dionne (1984). The fourth, classroom-focused, is not centered on content or learning, but the efficient organization of classroom activities and procedures, and so is not reflective of any particular belief about the nature of mathematics.

Specifically, the constructivist/problem-solving view conceptualizes mathematics as a dynamic and continually expanding field of human creation and invention encompassing a process of inquiry and coming to know. It supports a learner-focused model of teaching that prioritizes individual sense-making and supports the establishment of a learner-focused environment (Cobb and Steffe 1983). The traditional/instrumentalist view of mathematics holds that mathematics consists of a collection of facts, procedures, and skill sets to be used in the process of achieving an external end, often the solution to a problem. This view supports a style of teaching that focuses on the teacher explaining concepts, with students following rules, and procedures rather than constructing knowledge (Lindblom-Ylanne et al. 2006). Between these two perspectives lies the formalist/platonist view that conceptualizes mathematics as a static, but unified body of knowledge that is there to be discovered, not created. Teaching aligned with this view differs from the former in that it has a dual focus on both the content and student understanding. Although, the primary focus is the content, attention is also given to understanding the facts and procedures underlying the content.

Goals of the study

I used these insights to investigate in-service teachers' descriptions of their mathematics-related beliefs, the degree of alignment of these professed beliefs with their daily instructional practice, and how these beliefs served to facilitate or impede teachers' incorporation of reform-oriented resources and practices.

Research methods

Methodology

This study was a part of a larger project focusing on the effects of mathematical argumentation and writing on the mathematical understanding and achievement of Algebra 1 students. The study can be described as a collective case study, a joint study in which a number of cases are examined to investigate a particular phenomenon (Stake 2000). At the beginning of the study, I was unaware of whether the cases would be similar or different but believed it would provide insight into the relationship between teachers' domain-specific beliefs and the successful implementation of reform. The cases here serve primarily to "…facilitate our understanding of something else" (p. 437). In this case, to come to a better understanding of how teachers' beliefs support or impede the implementation of reform-oriented practices. A qualitative approach was taken for data collection and analysis.[2]

Schools

The two schools in this study were located in a suburban county in the southeastern United States. They were traditional high schools covering the grades 9 through 12 curricula and each had a minority population of over 50%.

Participants

All teachers of ninth-grade algebra (Algebra 1) at both schools were contacted and asked to be participants in this study. Ten of the 14 teachers, who taught Algebra 1, agreed to be participants, but due to scheduling conflicts only five were able to participate. The teachers engaged in an hour-long orientation to the project and received on-going professional development (PD) to (a) aid in the incorporation of writing and discourse tasks into their classroom activities and (b) to develop techniques to facilitate student engagement in these activities. The initial professional development involved a

[2] As a mathematics teacher educator and researcher, I am aware that the same subjectivities that lead me toward this type of research may also misconstrue or distort what I see in the data. In order to monitor and control my subjectivities throughout this process, I was constantly aware of the personal, teacher, and research lenses through which I examined and drew interpretations from the data, and I incorporated procedures to ensure that the interpretations mirrored the participants intended meanings (e.g., member checks).

discussion of the goals of the project, details of the implementation, the research basis underlying the project goals, and the teacher's role in facilitating the activities. The teachers were provided with materials that described techniques for supporting students' discourse in small group discussions and ways to provide meaningful feedback for the writing tasks. Following this meeting, the researcher communicated regularly with the teachers, both face-to-face (at least twice per week) and by email. These conversations were focused on the teachers being reflective about their practice with regard to their role in the classroom, the kinds of tasks, and activities that stimulated students' critical thinking and reasoning and whether or not there was evidence that these changes were helping students develop deeper understandings of the content. The aim was to help teachers develop their skills of facilitation and also to help them be more cognizant of the effectiveness of the activities. From the information garnered in these conversations, the researcher worked individually with the teachers as they had varied concerns and difficulties with the implementation. The following are brief descriptions of these five teachers:

Mr. Brown:[3] Mr. Brown was in his first year of formal teaching. At the time of the interview, he had passed the content examinations required to teach in the state and currently had a provisional teaching certificate. Mr. Brown planned on enrolling in a masters degree program in the following year and obtaining full certification, while completing the program.

Ms. Jones: A veteran teacher of 30 years, Ms. Jones had taught all the mathematics courses offered at high school with the exception of statistics. Following high school, she completed a bachelor's degree in mathematics education, followed by a masters degree, and then a specialist degree in mathematics education.

Ms. Reid: For Ms. Reid, teaching was a second career. She left her job as a management information systems (MIS) specialist 3 years prior to the study to become a teacher. Her preparation for teaching included successfully completing the content examinations required to teach in the state as well as an alternative teacher preparation program.

Mr. Henry: Mr. Henry was in his third year of teaching, but had only taught Algebra 1 and pre-algebra at the high school level. He passed the required content examinations to teach high school, but had not yet completed the requirements for certification. Mr. Henry enjoyed teaching but intended to return to school within the next few years to pursue a doctoral degree.

Mr. Simpson: After earning a bachelor's degree in accounting and working in the field for a few years Mr. Simpson left to teach high school mathematics. Soon after, he went back to university to pursue a Masters degree in mathematics education. He has taught at the same high school for 18 years.

Data collection and analysis

All five teachers engaged in a 45-minute semi-structured interview conducted by the researcher prior to the start of the intervention. The interview questions focused on the teachers' views about mathematics as a discipline (e.g., If you were to think of four words you thought were closely related to mathematics, what would they be?), mathematics pedagogy (e.g., How would you describe your role in the classroom?), and student learning (e.g., How do you think students learn mathematics best?). All transcribed interviews were

[3] The names used in this article are pseudonyms.

sent to the participants so they could verify that the written text captured their intended meaning. All teachers approved the transcripts. Prior to the semi-structured interview, two formal observations were conducted of each teacher. Following the interview, weekly observations (two per week over 10 weeks) were done of each class, and detailed field notes were taken. These observations were followed by informal discussions with the teachers to elicit their thoughts related to specific actions observed and pedagogical decisions made.[4] Notes were taken of the discussions to record the teachers' statements and also to ensure that the researcher's observations of classroom events could be mapped to the teacher's thinking and intentions. Copies of the teachers' lesson plans were collected along with samples of student work. The data on the teachers' math-related beliefs was triangulated through the use of multiple data sources (interview transcripts, field notes, interview notes etc.).

Thematic analysis was employed for analysis of the data. Specifically, using Strauss and Corbin's (1990) open coding technique, the participants' narratives from the transcribed interviews were examined for statements relevant to their beliefs about mathematics. From the open coding, I observed certain patterns among the codes from which categories were developed. The development of categories and refining of the categories was an ongoing iterative process that was repeatedly re-evaluated to ensure they reflected the participant's descriptions of their experiences. Each transcript was read multiple times to verify that for the codes and categories developed the "empirical reality fit the emerging theoretical framework" (Charmaz 2000, p. 514). These themes will be described illustrating how the teachers conceptualized and talked about their mathematics-related beliefs. The field notes from the classroom observations and lesson plans were analyzed and placed in categories that described the teachers' practices in three areas: (a) organizing the classroom environment, (b) role in teacher–student and student–student discourse and interactions, and (c) types and use of assessments. Within these categories, the coding scheme developed from the interviews was applied. Descriptions of particular classroom behaviors and practices that were reflective of these beliefs are also discussed.

Results

The teachers' narratives regarding their mathematics-related beliefs were examined and both commonalities and differences were observed in their descriptions. The themes described below represent the contrasting ways the teachers described these beliefs.

Computation versus a way of thinking

In discussing how they viewed mathematics, several of the teachers described the subject in terms of formulas, procedures, and calculations. When asked to describe mathematics using four words, Mr. Henry stated, "addition, subject, school… and multiplication…." He further elaborated that in thinking about a subject that was most like mathematics, "…I would say my research classes, because we use a lot of statistics…and science because we use a lot of calculations." For him, mathematics constituted a subject that students did in school involving computations and calculations. As such, he viewed its relatedness to other

[4] These informal discussions after the interviews were sometimes combined with the reflective conversations mentioned earlier. These conversations were scheduled primarily to accommodate the teachers' work schedules.

subjects with respect to the number of calculations involved. Mr. Brown and Ms. Reid responded similarly. Specifically, Mr. Brown stated that when he thinks about mathematics, the first thoughts that come to mind are "… addition, subtraction, multiplication, division…" He explained that those thoughts came readily as those are the absolute basic operations, "… that is as basic as all math breaks down to, at least in my mind…." Ms. Reid's response reflected similar notions as she thought the subject most closely related to mathematics was chemistry since "…most of that was dealing with numbers and equations and calculating."

These views of mathematics were translated into their classroom practices in two ways—the kinds of classroom activities they designed and how they interacted with their students. During the period of observation, the classes of these teachers did not engage in any group discussions or organized collaborative activity (outside of those designed for the implementation). Also, during instruction, the teachers lectured and the teacher–student interaction followed an initiate–respond–evaluate (IRE) pattern. Despite efforts to help the teachers engage students in discourse and to facilitate these activities effectively (PD described earlier), the teachers' questioning tended to default to the IRE pattern focused on eliciting final answers (either numeric or algebraic) and providing primarily summative evaluations (either correct or incorrect).

Ms. Reid, Mr. Brown, and Mr. Henry all had quite traditional conceptualizations of mathematics. Viewing mathematics in these rigid and simple ways shaped what the teachers expected students to learn and how they defined mathematical competence. For them, mathematical expertise included being competent with basic arithmetic operations, applying procedures appropriately, and performing accurate calculations; therefore, mathematical expertise was defined by how well the students mastered these skills. Stating correct answers and detailing procedures were regarded as evidence that students had, indeed, mastered these skills and, so, these practices dominated both the classroom communication and the modes of assessment used to evaluate students' work.

In contrast, Mr. Simpson and Ms. Jones considered the thought processes and mental actions of the individual as fundamental aspects of mathematics. When asked to describe mathematics, Ms. Jones responded "it is problem solving, but it is also about teaching students how to think and that is often difficult…" Mr. Simpson also spoke about mathematics being a thinking process, focusing on how an individual approaches and navigates their way through problem situations, rather than about calculations and correct answers. However, although their perspectives seemed to be aligned, how these views were manifested was different. Mr. Simpson designed activities and taught in a way to elicit this kind of thinking regardless of the type of student and the content. He thought about mathematical knowledge as more of a process than a product, as knowledge embodied in one's approach to problem situations both within and outside the classroom. For Mr. Simpson, mathematical expertise was akin to the actions mathematicians engage in when they do mathematics. He stated,

> … I get back to the idea of thinking, they [mathematicians] look at a problem and they think about how to solve the problem before they do anything… a mathematician doesn't rush into a problem… they see a problem and they kind of look above it and say I can probably go this way, and this is probably the best route to solve this problem instead of saying well I know this formula let's try this or let's try that just because that is the easy way to do it or that is what comes to my mind first.

Mr. Simpson's classroom practices reflected this belief. Through continuous engagement in discourse he allowed his students to take charge of their learning, encouraging them to not simply settle for the final answer but to examine their own thinking and the thinking of their peers. Prioritizing these practices in his classroom was indicative of his belief that critical thinking and reasoning were fundamental features of the epistemology of the discipline.

Ms. Jones on the other hand, believed that this form of analytical thought was more applicable depending on the context. Elaborating on why she often found teaching mathematics difficult, she stated,

> …it is not difficult when you are teaching algebra because you are given an algorithm to follow but when you are teaching geometry, when you are teaching students how to prove things it is about teaching students how to think about things in a logical, systematic manner and that is very difficult.

Ms. Jones presents an interesting case as she designed her activities and instructed differently depending on the particular subject she was teaching. Her algebra and geometry classes were organized differently; algebra classes were typically teacher-centered (although at times she did require her students to justify particular mathematical moves), but geometry classes, although they often began with lectures, these lectures were brief, and were followed by small group work. Students generally worked independently and then were expected to report on their solutions once completed. A distinct difference in the teaching practices for both classes was the type of questions she asked her students. Although Ms. Jones did ask for explanations in both classes, she tended to ask more probing questions to the geometry students and persisted until the students produced valid justifications for their responses. In cases, where they were unable to, they were provided with guidance or directed toward further investigation.

Demonstration versus guidance

Mr. Henry found it increasingly difficult to develop teaching strategies that met the diverse learning needs of his students. However, there was one class he favored, because during that block he got to *teach*. He described what it was like to teach this class, “[I] show them how to do things and have them sit and watch and pay attention and then practice…, and ask questions if they don’t understand.” Mr. Henry’s description of his role in the classroom reflected how he believed students acquire mathematical knowledge—not through active construction but through passive receipt. Given his view of mathematics as a pre-existing body of facts and procedures with expertise being acquisition of these facts, it follows, then, that good teaching would constitute providing students with this information, and ensuring that they commit it memory.

Similarly, Mr. Brown was influenced by his own experiences in middle- and high-school. He remembered these classes being very structured, similar to the way he currently organized his classes. In these math classes, they would “work and cover a particular lesson” followed by “practice and homework” and then the teacher would “ensure you mastered it” before moving on. Although this was the general format of the classes, he recalled that in high school they had a lot of open discussions about math questions and there were times his class would spend significant amounts of time on a particular problem, yet they would remain engaged. He thought this teaching style was possible because these classes were advanced and the students “were more willing to learn” and “engage in the discussion; [they were] more eager”.

Mr. Brown's decision to organize his class in a teacher-centered manner was reflective of two beliefs; low-achieving students are not interested or willing to learn so direct instruction is most effective, and students learn mathematics best through demonstration and practice. These beliefs were evident in both his descriptions of his classes and in his daily practices. He described his usual format, "we [he and the students] get some notes, work on some examples and then have them practice and then review their practice". This style of showing students what to do and then having them follow was a common teaching technique among Mr. Brown, Ms. Henry and Ms. Reid. These classroom practices aligned well with what the teachers believed was important about mathematics, emphasizing the correct use of procedures and accurate computations. This subset of teachers was very committed to this particular teaching approach and throughout the period of observation they did not deviate much from it.

Both Mr. Simpson and Ms. Jones spoke about teaching differently, equating teaching with the notion of "guiding". Mr. Simpson focused on giving his students enough freedom to explore and come to understandings on their own. Ms. Jones elaborated on what she meant by guiding her students,

> Well, I have to be the one to provide, I have to be the one to guide them through, guide them through the material and to answer their questions… to be knowledgeable enough about the material to answer their questions. Not only tell them that they are wrong but tell them why they are wrong and to help them find their mistakes and correct them.

This difference in meaning ascribed to the notion of guiding was seen in his efforts to engage students and in the fidelity with which he facilitated the argumentation activities. Mr. Simpson was comfortable using questioning to get his students to think—he believed that meaningful mathematical activity involved students making sense of mathematical concepts and constructing their own ideas. Through questioning and withholding summative evaluations he could force students to rely on their own cognitive abilities to reason about mathematical problems. Therefore, his role was to plan and orchestrate lessons that supported these thought processes. In this regard, the writing and argumentation activities provided opportunities for these beliefs to be enacted.

Ms. Jones used a similar metaphor when describing teaching. In her view, teaching was like coaching and as a coach her job was to prepare her students in a disciplined and rigorous way to perform at their best level. This included "being knowledgeable" about the subject so she could organize the students' activities, "guide them through the material", and "help them find their mistakes and correct them". Similar to a coach, she knew that there would be obstacles and times of defeat and in those moments her responsibility was to "inspire, encourage and motivate my students to do."

Similarly, Mr. Simpson believed that teaching went beyond just delivering the content. He wanted his students to have options, and so it was necessary for him to provide them with a broad knowledge base and thinking skills so that they were sufficiently equipped to make the life choices that best suited them. He believed that doing mathematics involved engaging in powerful analytical thinking about problems and so teaching was about helping students to hone these skills. This included helping them to think about problems that arose, within the mathematics classroom or otherwise, in a systematic and analytical way, choosing the best and most efficient path to a solution. For him, mathematics was a way of thinking so he impressed on his students that we engage in "mathematics every single day".

Practice versus understanding

As teachers, creating a learning environment, where student understanding and achievement is maximized, is paramount. One key feature that underlies how these environments are organized and maintained are the beliefs teachers hold about how students learn mathematics best. Several of the teachers emphasized the importance of practice in the development of expertise in mathematics. They considered practice an integral part of the learning process and so their students routinely had daily classwork and homework assignments. In Ms. Reid's classroom, order, organization, and structure were fundamental features of her instructional design. Ms. Reid believed that mathematics is hierarchical and so students had to master foundational concepts before they could move on to more advanced concepts. Therefore, for her, good teachers were those who had "organization and structure".

Although, within mathematics education, we believe it essential that students construct particular understandings before they move on to more advanced concepts, for Ms. Reid, "organization and structure" manifested itself in a profusion of classroom routines. Difficulties arose when students were to assemble in groups to discuss their ideas and responses to mathematical tasks. Ms. Reid was not fully convinced that engaging in argumentative discussions around these tasks were valuable and thought it would be more useful for students to study on the tasks individually. Ms. Reid viewed mathematics as an absolute and established body of facts that students come to know through practice, as opposed to mathematical knowledge being socially constructed, developed through the negotiation of meanings with others through discourse (see Bishop 1988 and Wilder 1981 for discussions of this perspective). It was therefore difficult for her to connect how sharing one's own ideas and listening to the ideas of others were important for constructing mathematical understandings.

Similar to Ms. Reid, Mr. Henry thought that practice was an important part of becoming "good" at mathematics. He too saw practice as integral to the learning process and seemingly placed practice above the process of developing understanding. Mr. Henry stated,

> …practice, the way they learn it is probably less important than their practicing it [mathematics]. I can show them, they can come and show me, they can do it together, they can read it, they can write it. I think that if they don't go home and practice it they don't learn it.

He made a distinction here between learning a concept and understanding a concept, in that understanding can only come through practice. In his view, a student can learn a concept but if he or she does not follow up with practice the knowledge will disappear. Mr. Henry added, "I don't care how well they get it at school if they don't go home and practice it, it all just goes away". This view of learning, similar to Ms. Reid's, places the students at the receptive end of the process, passively engaging in practice for the mastery of skills.

Teachers' beliefs about student learning were also manifested in their use and interpretations of the results of assessments. These assessments could be informal like oral questioning or a paper and pencil test. For Mr. Simpson these formal or informal assessments were an opportunity for the students to tell him what they knew; he was looking for evidence that the students were thinking about the concepts correctly and not necessarily focused on the right answer. He explained, "… so it's the process more so than the answer… and as you get better and better we'll worry about the answer, but the process is really important right now." In contrast, Mr. Henry privileged getting the correct answer over understanding. He stated, "because the grades matter the most to me. It's great if you are excited about learning and I'll try to talk that up if I can, but the bottom line is you gotta be right, you gotta get it right." He saw his role in the classroom, as the person there to make sure the students "get it", but for him "getting it", meant getting the correct answer and not necessarily making sense of the concepts. Using assessments in this way were somewhat incompatible with the goals of implementation tasks as these tasks were designed to be used formatively by both the teachers and the students; hence, Mr. Henry had difficulty implementing the tasks as designed.

Discussion

Cohesiveness of teachers' beliefs

Researchers who have theorized about belief systems suggest that beliefs are organized in a form of structural order (Green 1971; Rokeach 1968). Green (1971) proposes a three-dimensional organization (described earlier) that provides a framework from which to discuss the organization of the teachers' beliefs. All of the teachers held fairly strong beliefs about what constituted mathematical knowledge. Although there were differences among the five teachers, all had strong beliefs that stemmed from their own early experiences in schooling, with their own teachers, and with mathematics. Several of the teachers had beliefs about mathematics that are not considered conducive to the type of mathematics teaching and learning supported by the National Council of Teachers of Mathematics (see *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics 2000). Some of these beliefs included mathematics is computation, and the goal of doing mathematics problems is to obtain the correct answer (Ernest 1988; Frank 1988). Namely, Mr. Henry, Mr. Brown, and Ms. Reid, in thinking about mathematics, seemed to focus more on the skills and procedural aspect of the subject and less so on the cognitive processes. In other words there was less emphasis placed on reasoning and problem solving skills and more on practice. These teachers believed that mathematical knowledge was an absolute, established set of concepts that was rigid and infallible, and their classroom practices reflected these beliefs. For them, the salient features of mathematics were the formulas, procedures, rules, and seeming objectivity intrinsic in the subject that set it apart from others. This view of the mathematics shaped how they designed their instructional activities, the tasks they engaged their students in, the quality of interaction they encouraged in the classroom, the types of evaluation methods they employed, and the fidelity with which they incorporated and facilitated the reform-oriented materials and practices.

For example, Mr. Henry saw mathematics as knowledge of concepts, rules, and procedures to solving mathematical problems. From their statements and the observations of their teaching practices, he believed (like Ms. Reid and Mr. Brown) that expertise in the discipline equated to having expert knowledge of these rules and skill sets, including how and when to apply them appropriately. As such, in attempts to help their students develop mathematical expertise, these teachers saw their role as that of possessor and giver of this knowledge, providing students with the mathematical content and ensuring they created opportunities for students to store this knowledge. This process of "storage" often being memorization achieved through repeated practice of these procedures. They conceived of learning as applying the correct procedures in the right context while maintaining

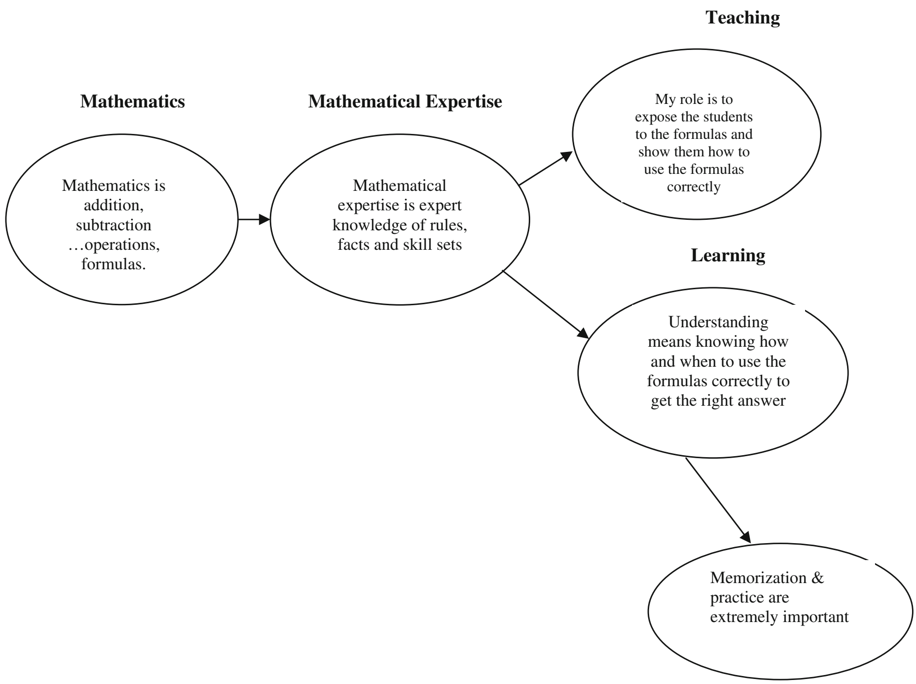

Fig. 2 Hypothesized mathematical belief system of Mr. Henry, Mr. Brown and Ms. Reid

computational accuracy, therefore mathematical understanding came through practicing these procedures. All these beliefs formed a cohesive network or structure that served to support and sustain itself (see Fig. 2). For these teachers, their beliefs about teaching and learning appeared to be derived from their beliefs about the nature of mathematics (Green 1971).

Although Mr. Simpson's beliefs about mathematics differed considerably from the other teachers, they did cluster in similar ways. He viewed mathematics as a thinking and problem solving activity, prioritizing meaning making and finding suitable approaches over merely obtaining a correct answer. His beliefs about teaching and learning were shaped from this mathematical perspective. Holding mathematics as a social construction (Bishop 1988; Wilder 1981), he viewed it not as isolated bits of facts and concepts but as an interconnecting and evolving set of relationships from which individuals construct personal meaning. In light of this, he saw his role as the person responsible for designing activities that focused on constructive sense-making. He did not see this necessarily as an individual process, but acknowledged the need for collaboration both in the learning process and as a source of verification and evaluation. As a guide he was able to help students embark on this learning process and aid in successfully navigating through it. These ideas and the role that he took on in the classroom were a reflection of his mathematical point of view, demonstrating again the cohesiveness and clustering nature of their beliefs (see Fig. 3).

Ms. Jones, on the other hand, held a view of mathematics that was manifested in the different teaching roles she adopted across her classes. Ms. Jones's mathematical beliefs were a conglomerate of perspectives. She believed that mathematics was about problem

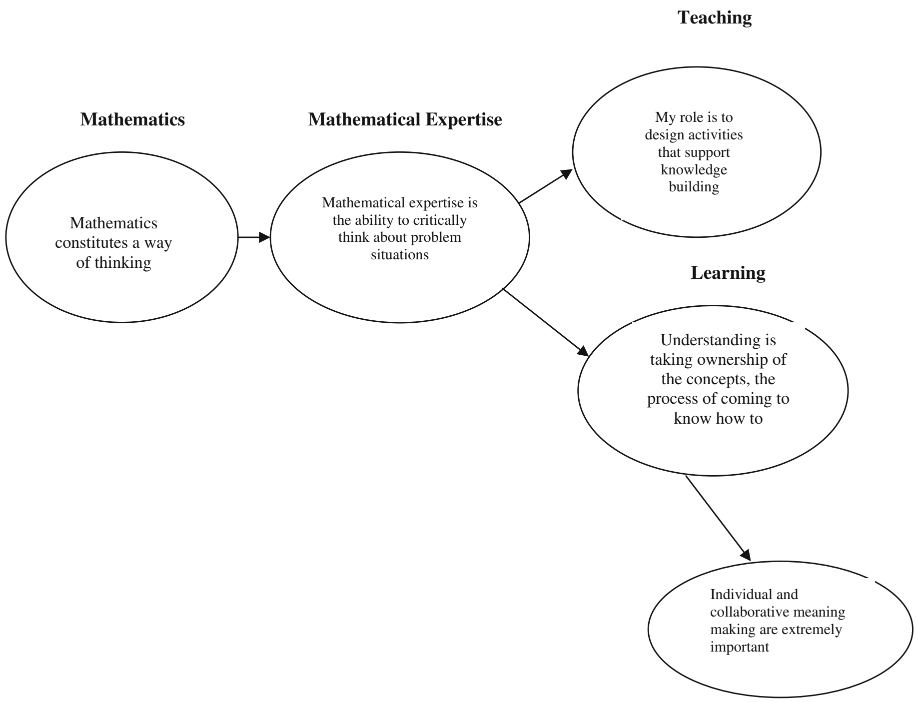

Fig. 3 Hypothesized mathematical belief system of Mr. Simpson

solving and learning to think critically, but she also thought of mathematics as a huge bank of knowledge that was rooted in numbers. These differing beliefs about mathematics were aligned with different branches of mathematics and mentally facilitated her flip-flopping between teaching roles depending on the subject area (geometry or algebra). Similar to the other teachers, her beliefs about teaching and learning were related to her beliefs about mathematics as a discipline. Ms. Jones believed that her role as a teacher was to guide her students toward understanding and to encourage, motivate, and support them. Simultaneously she also believed that it was her responsibility to have the knowledge base so she could show the students how to solve problems, identify student errors, and show the students how to correct them. These somewhat differing views of mathematics pedagogy were connected to specific areas of mathematics and they seemingly did not present any internal conflict. As beliefs are organized according to how the individual sees their connections, two opposing beliefs may be held simultaneously without conflict (Green 1971). This is possible because they are often held apart by another belief; in the case of Ms. Jones this belief was perhaps, mathematics is not a cohesive domain of knowledge (see Fig. 4).

The hypothesized models presented demonstrate how these teachers' beliefs about the nature of mathematics, mathematics teaching, and mathematics learning were organized in a derivative manner where beliefs about teaching and learning appeared to stem from beliefs about the epistemology of mathematics. Although the models of the teachers' belief systems are quite similar in their order and organization, the content of their beliefs reflected a range of perspectives about mathematics. It was the specific content of these beliefs that had the greatest influence on their instructional practice.

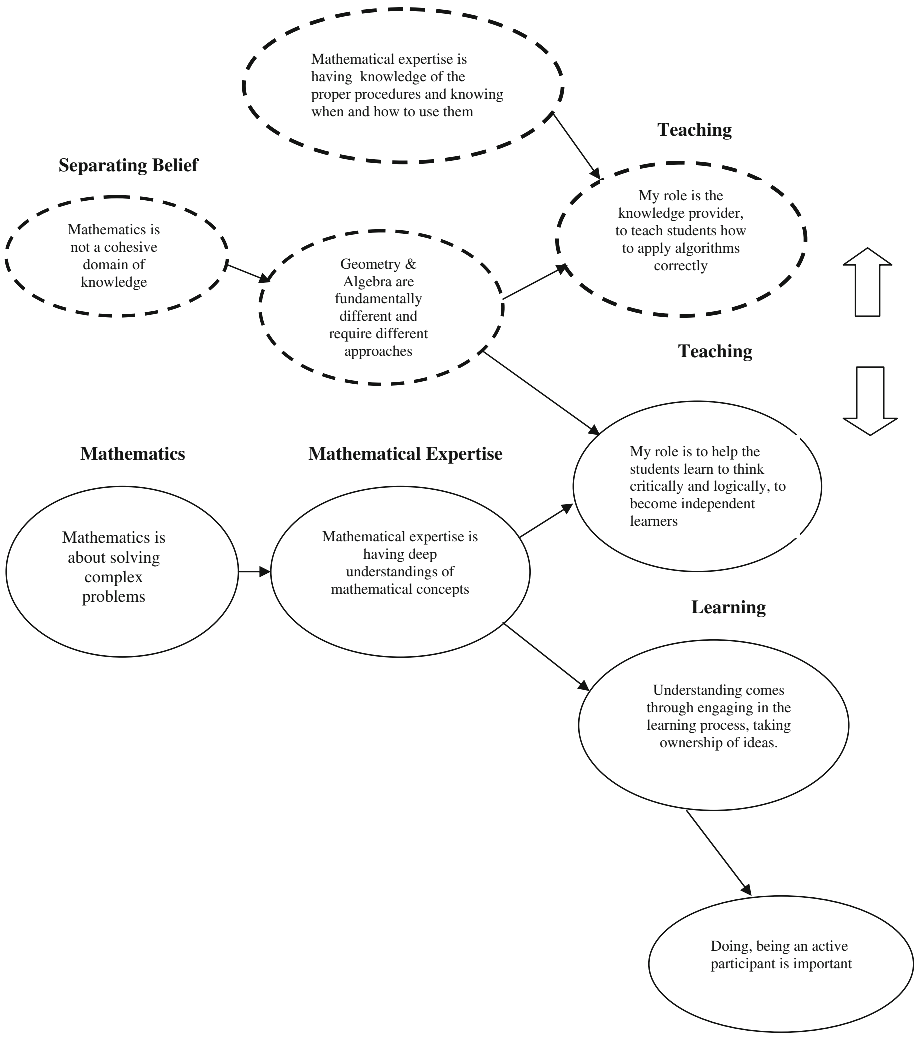

Fig. 4 Hypothesized mathematical belief system of Ms. Jones

Alignment between beliefs and practice

Research has been inconclusive with respect to the degree of alignment between the mathematical beliefs of teachers and their practices (Chapman 2002; Ernest 1989; Raymond 1997; Stipek et al. 2001; Thompson 1984, 1992). Of note is that in past research teachers often espoused reform-oriented beliefs that were not actualized in their instructional practices. In contrast, the five teachers presented here described varying mathematics beliefs and these professed beliefs were in fairly close alignment with their instructional practices. With the exception of Mr. Simpson (and Ms. Jones in some cases), the teachers in this study expressed beliefs that could be considered as anti-reform. Researchers who study beliefs and belief change have warned about the risk of participants reporting what they believe the researchers want to hear (Cochran-Smith 1991) and suggest that studies should not merely rest on verbal accounts but on evidence that these principles have been incorporated into the teachers' own teaching and/or learning (Anderson and Piazza 1996). The teachers in this study were very forthcoming in expressing their views. Having had reasonable success on standardized tests they were very efficacious about their teaching and so were confident in expressing their beliefs about mathematics that had served their students well.

From the analysis, we see that the teachers' mathematics-related beliefs were organized in a primary-derivative order where their beliefs about mathematics teaching and learning were derived from their mental models of mathematics. Mr. Henry, Ms. Reid and Mr. Brown all had early student experiences that shaped their mathematics-related beliefs. They described their role within the classroom as the person with the knowledge, responsible for showing the students how to apply the procedures, perform the calculations accurately, and then provide opportunities for practice. All three classrooms on repeated observation matched this format exactly, with very little deviation. The students were exposed to concepts through demonstration and were expected to sit, listen, and take notes.

While knowledge construction does take place in different kinds of settings, these students were rarely provided with opportunities to make sense of the material independently; the meaning they ascribe to the concepts were the meanings the teachers provided. Emphasis was placed on knowing the procedures and the appropriate formulas and student explanations tended to be limited to when and how to use them. After being introduced to the concept, "understanding" was attained and reinforced through practice of textbook-like problems. This understanding was evaluated by how well the student was able to apply the appropriate skill sets to new problems and on the correctness of the final answer. There was little student–student discourse of a conceptual nature with these conversations consisting mainly of off-task behavior and answer checks. Teacher–student talk was primarily conducted in an IRE (initiate–respond–evaluate) manner where the teacher would initiate the questions, the students would respond, and an evaluation provided. In situations, where the teachers were required to facilitate students' conceptually rich discussions, they still defaulted into the IRE pattern and accepted single answers with no justifications. Mr. Henry, Ms. Reid, and Mr. Brown's actions were aligned with a more traditional (Dionne 1984) or instrumentalist (Ernest 1988) view of mathematics.

Similarly, Mr. Simpson's classroom practices were for the most part quite reflective of his mathematics-related beliefs. Being from a family of mathematics educators, he also had early influences that shaped these beliefs. Standing back and allowing his students to develop their own personal understanding, and providing opportunities for students to tell him what they knew were the major principles that governed how Mr. Simpson organized his classroom. Along with Mr. Simpson, Ms. Jones also emphasized the importance of student ownership over his/her own ideas. As such, they tried to position themselves more as guides for their students, allowing the students to develop their own individual understanding of the content, a feature they considered vital to student learning.

However, while Mr. Simpson's beliefs were repeatedly actualized in his classroom practices, Ms. Jones's beliefs were manifested quite inconsistently in her classes. With her younger ninth-grade students, instruction primarily followed a lecture format, although she did use a variety of questioning techniques that often pushed students to extend their thinking. Therefore, although Ms. Jones did routinely assign straightforward, procedural questions it was clear that getting the right answer and understanding the solution in the context of the problem were equally important. Of significance, however, was the disparity in the design and organization of Ms. Jones's classroom activities in her different classes; this disparity being dependent on the type of students and the subject (described earlier). She made a distinction between teaching geometry where she taught students how to think, as opposed to algebra where you could show them an algorithm and have them follow it.

Ms. Jones held conflicting beliefs about mathematics, student learning and mathematics teaching, which were influenced by the type of students and the particular subject area of mathematics she was teaching. Individuals may hold beliefs that are contradictory since they are not perceived by the individual to be conflicting. These beliefs may remain intact due to another belief (Green 1971). From the results of follow-up questioning with Ms. Jones, there appeared to be dual separating beliefs; one was domain-related (the conceptual nature of algebra and geometry are fundamentally different) and the other student-related (underachieving students learn best through direct instruction).

This phenomenon has been previously documented as researchers have observed that beliefs can differ with regard to specific groups of students (Fuchs et al. 1998; Torff and Warburton 2005). For example, teachers often believe that cognitively demanding tasks should only be assigned to high-achieving students as low-achieving students are unable to adequately address or successfully complete these tasks (Torff 2005). This was a factor in the pedagogical decisions Ms. Jones made about how to teach different subject areas.

Implications for teacher education

For the five teachers in this study, there was greater alignment than misalignment between their mathematics-related beliefs and their instructional practices, indicating that for these teachers beliefs served as a fairly reliable predictor of the type of instruction that took place in the classroom. However, despite the fact that the majority of these teachers did not express beliefs that are reflective of mathematics teaching and learning proposed in *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics 2000), they were all diligent, passionate, hardworking, and dedicated teachers. They were, in a sense, using the resources available to them to ensure their students' success.

The results of this study tend to confirm the notion that teachers' beliefs about mathematics not only greatly influence the way teachers design and execute their lessons but also provide new insights that have implications for teacher education and professional development. It presents teachers' mathematical beliefs as potentially a cohesive structure where the beliefs are connected in a derivative order with pedagogical and student learning beliefs stemming from beliefs about the nature of mathematics. However, although the types of beliefs and the general structure were consistent across teachers the content of the beliefs differed.

For students to become powerful mathematical thinkers, it is desirable that their teachers possess beliefs that support the development of problem-centered, learner-oriented classroom environments (Cross 2008). Therefore, teachers who do not hold such beliefs should be engaged in programs that aim to transform these beliefs. This is not an easy feat as observed in the current study. By the end of the project, the teachers (with the exception of Mr. Simpson) were only beginning to question the effectiveness of their current practices and reported that although they had learnt alternative methods of designing and orchestrating instruction they were not confident they could adopt these practices holistically given the curricular and institutional constraints. This reflection on the part of the teachers was prompted by the student work. As participants in the larger project students engaged in discussion and writing around conceptually-rich tasks. These tasks were given to students as review activities after completion of the topics. Overall the students performed rather poorly which provided sufficient evidence to the teachers that the students had been applying procedures and formulas without understanding (See Cross 2008 for a full description of the project). With this recognition, Mr. Brown, Ms. Reid, and Mr. Henry increased their attempts to facilitate the students' discussions and provide feedback on their writing. However, for these teachers, reflection on their current practices led to the conclusion that they needed to do a better job of explaining the concepts rather than providing opportunities for students to make sense of the concepts.

It was clear that although the teachers welcomed the new practices they were filtered through the old belief system, resulting in minimal overall change (Yerrick et al. 1997). This demonstrates that providing evidence that contradicts teachers' current beliefs is an important component of the process, but alone, will not lead to any real or sustained change. The cases of Mr. Simpson and Ms. Jones provide interesting examples of this filtering effect. In the case of Mr. Simpson, his instructional techniques fit well with the reform practices and therefore they were enacted quite consistently. On the other hand, only a subset of Ms. Jones's beliefs aligned with the implementation practices and so we saw this filtering effect in her algebra classes in that, although she encouraged her students to justify their responses, it was clear that only algorithmic proficiency and computational accuracy were valued as these were all she evaluated.

Holding that these beliefs form a cohesive unit where teaching and learning beliefs are derived from teacher's conceptions of mathematics, it suggests that if the beliefs about the nature of mathematics change then those derivative beliefs would also begin to be modified. In this regard, the belief change process may have greater success if it were organized around re-conceptualizing teachers' views about mathematics as a discipline. Numerous scholars (Ambrose 2004; Anderson and Piazza 1996; Ashton and Gregoire 2003) have documented the lack of success in belief change having targeted beliefs through mathematics methodology courses, and so targeting prospective teachers' beliefs in mathematics content courses (rather than methodology courses) should result in greater success. Therefore, it is important that pre-service teachers be engaged in the study of mathematics that will foster a disposition toward mathematics as inquiry. Similarly, practicing teachers should undergo continuous professional development that allows them to see the social and constructive aspects of the discipline.

An additional point of consideration is that beliefs tend to be organized in clusters. From the analyses, different models emerged illustrating how the mathematical beliefs of these five teachers *may* be structured. However, these beliefs are only a subset of a host of beliefs teachers take with them into their professional careers (for example, beliefs related to student learning in general, general epistemological beliefs, and teacher efficacy beliefs) that impact their daily classroom decisions and practices. In light of this, any program designed to specifically address teachers' mathematical beliefs is already a deficient model, as there are other clusters of beliefs that will influence their classroom behavior. As such, belief change must be an ongoing process of awareness, confrontation and reflection. In this regard, teacher education programs are able to begin the process of belief change but the school environment and communities of which pre-service teachers become members are extremely important to sustained success of any belief change effort. Teachers must be continuously engaged in experiences that challenge their beliefs and cause them to reflect on them; only then can change be lasting.

Additionally, although beliefs have enormous influence on teaching practices, there were other factors that influenced how these teachers envisioned and enacted their roles within the classroom. All five teachers identified teachers from their own student experiences who were pivotal in their decisions to become teachers and who they modeled in their own teaching practices. Teachers tend to instruct in ways similar to the way they were taught (Cooney 2003; Cooney et al. 1998; Torff and Warburton 2005) so it is important that during their teacher education program they be provided with good instructional models that they can emulate. Understanding the impact prior teaching models have on teachers' current ways of instructing is an important area for future research. The teachers also spoke extensively about the constraints that existed which prevented them from having their ideal classroom. Mr. Simpson was fairly consistent in maintaining the classroom environment he envisioned for his students but admitted that he frequently fell behind in the prescribed curriculum; this is often an issue in more learner-oriented classrooms. Curriculum coverage and time were factors that clearly impacted the fidelity with which the teachers incorporated the new practices. It is clear that for these teachers in addition to their beliefs being a constraint for them adopting more reform-based teaching practices, there were also institutional factors that served as deterrents.

Teaching experience also played a role in the fidelity with which the materials and resources were implemented. Both Mr. Simpson and Ms. Jones had considerable years of teaching experience and incorporated the materials and practices with the greatest fidelity. Their years in the classroom allowed for minimal classroom management issues and useful insights about how to incorporate the materials into their regular classroom activities with ease. Distinct about these teachers was that their beliefs were well-grounded and evidentially held, as they were based on their experiences with students throughout their careers. In stating their beliefs these teachers often provided examples from past experiences to justify their decisions to instruct in particular ways. In this regard, professional teaching experience is an important factor to consider when engaging teachers in professional development geared toward reform.

It is apparent that there is no clear linear relationship between beliefs and practice, and other factors do influence how teachers perceive and enact their roles in the classroom. The findings of this research align well with the work of others that describe these other factors, including internal psychological constructs, such as goals, emotions, teacher identity, and teacher efficacy (Aquire and Speer 2000; Schutz et al. 2006; Williams et al. 2008; Woolfolk-Hoy et al. 2006) and also external factors, such as school and department culture, curriculum mandates, class sizes etc. (Hart 2002; Valderrama-Aguelo et al. 2007). Additional research needs to be done in this area to detail and describe these perceived constraints and the extent to which they directly impact the instructional decisions teachers make.

The results also have implications for belief change efforts within mathematics teacher education. Due to the perceived connection between beliefs and instruction practice and instructional practices and student learning, teacher belief change is considered vital in our efforts toward reducing mathematics underachievement. Three of the five participants were graduates of mathematics teacher education programs but only one had fully embraced reform-oriented principles and practices, demonstrating that there is still much room for improvement in how we educate our pre-service teachers.

Finally, these five participants across only two schools produced three different models of beliefs. The number of different models most likely will increase as the pool of teachers gets larger from a mathematics department to an entire school district. Considering the diversity of other teacher belief models, it begs the question of whether efforts toward belief change that assume that one approach may work for all teachers are realistic. In this regard, developing a better understanding of the role of belief structure in the support and maintenance of beliefs and how the organization of beliefs impacts teacher behavior will serve as an aid in making the process of belief change more successful.

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