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Prospective middle school mathematics teachers' reflective thinking skills: descriptions of their students' thinking and interpretations of their teaching

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Abstract In this study, we examined prospective middle school mathematics teachers' reflective thinking skills to understand how they learned from their own teaching practice when engaging in a modified lesson study experience. Our goal was to identify variations among prospective teachers' descriptions of students' thinking and frequency of their interpretations about how teaching affected their students' learning. Thirty-three participants responded to open-ended questionnaires or interviews that elicited reflections on their own teaching practice. Prospective teachers used two forms of nuance when describing their students' thinking: (1) identifying students' specific mathematical understandings rather than general claims and (2) differentiating between individual students' thinking rather than characterizing students as a collective group. Participants who described their students' thinking with nuance were more likely to interpret their teaching by posing multiple hypotheses with regard to how their instruction affected their students' learning. Implications for supporting continued growth in reflective thinking skills are discussed in relation to these results.

Keywords Prospective teachers · Pre-service teachers · Reflective thinking · Reflection · Middle grades · Analysis of teaching skills · Analyzing teaching · Lesson study · Mathematics methods course

Introduction

Supporting the development of prospective teachers' reflective thinking skills is an important goal for mathematics teacher educators, since reflective thinking is considered to be central to the improvement of mathematics teaching (Artzt 1999; van Es and Sherin 2008). Reflective thinking is a practice through which teachers continuously learn from their own teaching practice and gradually improve it over time. Teachers who practice

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reflective thinking use inquiry as a tool to engage critically with key questions and issues in practice (Jaworski 2006), such as questions about whether and how students learned from teaching. Reflective thinking is a systematic means for achieving a broader understanding of teaching situations and improving the quality of teaching, which is a practice used by action researchers (Krainer 2006).

We examined prospective teachers' enactment of the following subset of skills involved in reflective thinking: (a) identifying nuanced differences in students' mathematical understandings when describing their students' thinking and (b) interpreting their teaching by posing hypotheses regarding how their actions as a teacher contributed to students' learning. We defined "nuance" as distinguishing between students and describing their mathematical thinking with specificity. The selection of this subset of skills was proposed by Rodgers's (2002) reflective cycle for teaching and a model for mathematics teacher education that aims to help prospective teachers learn how to teach from studying teaching (Hiebert et al. 2007; Hiebert et al. 2003). These skills have been found to be challenging for prospective teachers to develop (e.g., Mewborn 2000; Star and Strickland 2008), yet they support learning from one's own teaching practice. Examining how prospective teachers reflect on their teaching through looking closely at how they describe and interpret their own teaching can result in useful pedagogical learner knowledge (Grimmett and MacKinnon 1992) for teacher educators.

Our research is grounded in the belief that mathematics teacher educators should foster reflective thinking among prospective teachers and impart skills and dispositions that support continuing to learn from their own teaching practice once they enter the profession (Hiebert et al. 2007). We wonder, along with Clark (1988), "How can we help our students to *prepare themselves* to think and act in ways that will eventually become good teaching?" (p. 11, italics in original). One possible answer to this question is to support the development of reflection-on-action, a deliberate process of looking back at problematic events and actions, analyzing them, and making decisions (Schön 1987), particularly because, according to Rodgers (2002), reflection-on-action is practice for reflection-in-action.

Reflective thinking skills: describing and interpreting

The subset of skills for reflective thinking that we examined in this study (describing students' thinking and interpreting their teaching) aligns with two of the four phases in a reflective cycle for teaching put forth by Rodgers (2002). The four phases are presence, description, analysis, and experimentation. When teachers implement these four phases, teachers slow down their practice and become more attentive to students' learning. Teachers become more present by learning to attend and notice students' thinking in relation to subject matter. Challenges and opportunities to develop presence in prospective and practicing teachers have been examined previously (e.g., Star and Strickland 2008; Stockero 2008; van Es and Sherin 2002, 2008), with results indicating the importance of sustained opportunities to engage in reflection. The second phase entails describing diverse elements of a classroom experience. In the third phase, analysis, teachers generate a number of conjectures, or interpretations, to explain what they described. These four phases are generally nonlinear, particularly as teachers go back and forth between description and analysis, but experimentation occurs last and doubles as the next experience, when teachers try their new ideas for supporting students' learning. In this study, we focused on the phases of description and analysis.

Describing students' thinking: naming with specificity

In the description phase of the reflective cycle, teachers *name* and *differentiate* complex elements in an experience (Rodgers 2002) with specificity (van Es and Sherin 2002, 2008) as they empirically observe whether their students learn and what they learn (Hiebert et al. 2007). One goal of learning to describe is learning to *name* specific details about students' mathematical thinking. When teachers name students' mathematical thinking with specificity, they move beyond describing whether students had correct or incorrect answers (e.g., Crespo 2000) or beyond a "get it or don't" conception of students' thinking (Otero 2006, p. 247). Specific descriptions of what students understand allow for instructional inventions that are targeted to address students' specific struggles or build upon their particular strengths. We believe that stating specifically *what* students did and did not learn about mathematics is part of describing students' thinking with nuance.

Describing students' thinking: differentiating between students

Another goal of learning to describe involves learning to *differentiate* between students and to see students as individuals rather than characterizing students as a collective group. If teachers see beyond the collective "they" to the individuals in their classrooms, they can more readily connect with every learner; alternatively, viewing students as a collective group allows teachers to depersonalize students and, as a result, releases them from having to relate to students as individual learners (Rodgers 2002). Differentiating between students is a social justice concern in which teachers strive to reach all learners (Davis 2006). Descriptions that differentiate between students can support teachers in designing instructional interventions that are targeted to specific students. We believe that differentiating between *which* students did and did not learn is another part of describing students' thinking with nuance.

Interpreting effects of instruction

In the analysis phase of the reflective cycle, teachers generate a number of different interpretations in the form of "conjectures" (Ball and Lampert 1999) or hypotheses to explain how teaching supported students' learning (Rodgers 2002; Hiebert et al. 2007). Through analysis, teachers integrate and generate local knowledge about teaching and learning that is grounded in their own experience, and this knowledge is equally important as general knowledge, which is aligned with principles of action research (Krainer 2006). This is similar to Davis's (2006) conceptualization of teacher learning as becoming more skilled at integrating knowledge of learners and learning, subject matter, assessment, and instruction; she characterized unproductive reflection as descriptive lists without knowledge integration and productive reflections as connections among knowledge. Hypothesizing about the effects of teaching on students' learning may be a challenge for prospective teachers, as they have limited experiential bases for developing conjectures (Smith 1996) and for internalizing their own authorities to generate, reason about, and test hypotheses to examine children's mathematical thinking (Mewborn 2000).

Another reason why analysis is challenging is due to a potential temptation to evaluate rather than interpret teaching. Van Es and Sherin (2002) distinguish between evaluating and interpreting teaching. When they evaluate teaching, teachers judge what was good, bad, or could have been done differently, and teachers' evaluations are often based on their beliefs rather than evidence of students' thinking. When interpreting their teaching, teachers make inferences that are connected to evidence of students' thinking and pose hypotheses regarding how an instructional strategy influenced students' learning. Evaluating and interpreting both entail a higher degree of inference than describing, but we believe that interpreting is more likely to support learning from instruction than evaluating, since evaluations do not draw upon evidence.

The ability to engage in reflective thinking may be part of a developmental trajectory for teachers. Fuller's (1969) concerns model of teacher development depicts stages in which beginning teachers' concerns are first focused on themselves and then shift to teaching before they are able to consider concerns about their students. Feiman-Nemser and Buchmann (1986) posit that, without guidance, it is challenging for novice teachers to simultaneously think about their students while thinking about oneself as a teacher and the subject matter. Recent challenges to Fuller's concerns model of teacher development (e.g., Conway and Clark 2003; Haritos 2004) indicate that teachers' progression through stages of concerns may be nonlinear; it may be possible for some novice teachers to simultaneously attend to concerns about students while attending to concerns about teaching and themselves under particular conditions.

Lesson study: an opportunity to learn from practice

Lesson study, a well-established practice in Japan, is one option for engaging teachers in an opportunity to learn by closely examining local teaching practices. As described by Fernandez (2002), during lesson study, teachers work together on analyzing the effectiveness of a lesson or set of lessons by identifying a goal for their lesson study activity, planning a detailed lesson as a group, jointly observing the taught lesson and taking detailed notes, debriefing to share their observations, and finally revising the lesson plan. At times, the revised lesson is taught by another teacher with an additional round of observations and debriefing. According to Fernandez (2005), lesson study can be an educative experience for teachers because it is practice-centered, focuses on lessons as the unit of analysis, reflecting a "natural unit of teaching that teachers think about on a daily basis" (p. 283), and provides a concrete opportunity for teachers to experiment with ideas rather than simply discuss them.

However, Fernandez (2005) also cautions that what teachers learn from lesson study depends upon the way they engage in the process, and she suggests that limitations in teachers' knowledge or skills might be an obstacle to their learning from process. For instance, when a group of fifth and sixth grade US teachers engaged in a lesson study of a mathematics lesson, they faced challenges with adopting a researcher perspective that would help them learn from the lesson study process (Fernandez et al. 2003). They faced difficulties with making specific observations and with using specific evidence for assessing the effectiveness of the lesson. Prior research indicated that lesson study helped prospective teachers enhance their subject matter knowledge for teaching and link their knowledge of theory and practice through analyzing teaching (Fernandez 2005). There is a need to understand more about which reflective skills prospective teachers can demonstrate during the lesson study process.

Methods

The following questions structured our investigation about prospective teachers' reflective thinking about their own teaching during a modified lesson study experience: (a) How did prospective teachers describe their students' thinking? Did they describe students' thinking with nuance by differentiating between individual students or naming specific mathematical understandings? (b) Did they interpret their teaching by posing hypotheses regarding how their teaching actions contributed to student learning? (c) Did skills for describing students' thinking appear to correspond or support the skill of interpreting teaching? If so, how?

Data were collected during a methods course for prospective middle school mathematics teachers in a teacher education program at a university in the Mid-Atlantic region of the USA. Thirty-three prospective teachers (30 female, 3 male) participated in this study. Two sections of the methods course were offered during the semester of data collection, with 17 prospective teachers in each section. One of the sections of the course was taught by the first author. The second author did not participate in the instructional intervention, but she has experience teaching prospective mathematics teachers. The instructor of the second section was an adjunct instructor who co-planned with the first author. All course materials and interventions were developed in collaboration or shared between both instructors.

Context of mathematics teacher education experiences

Prior to their experience in this methods course for prospective middle school mathematics teachers, the participants had already taken five mathematics education courses as a part of their teacher education program. Four of the courses were mathematics content courses for elementary and middle school teachers: (a) number, place value and operations, (b) concepts and operations of rational numbers and proportional reasoning, (c) geometry, and (d) algebra. The fifth course was the first mathematics curriculum and methods course for grades K-8. Throughout their mathematics education courses, prospective teachers are asked to analyze students' thinking about mathematics, and they are provided with opportunities to learn to develop teaching strategies to teach mathematics for conceptual understanding in balance with procedural fluency.

Prior to this course, prospective teachers previously taught two mathematics lessons in their field placement and reflected on their teaching as a part of their first mathematics methods course. The teaching episode they reflected upon in this study was not the first time these prospective teachers had taught mathematics in classrooms. In most cases, however, this was the first time these prospective teachers had taught mathematics in middle school classrooms. These prospective teachers had been asked to reflect on their teaching in light of students' learning in this and their previous mathematics methods course.

Instructors engaged the participants in a modified lesson study (Lewis 2002) experience in which prospective teachers taught from the same lesson plan. One modification to the typical lesson study experience was that this lesson plan was initially drafted by others; this lesson was written the previous semester by a group of middle school mathematics prospective teachers while they were student teaching. Prior to teaching, these prospective teachers examined the lesson plan, considering the alignment between the activities in the plan and the lesson's learning goal. The instructors taught the lesson to prospective teachers so that they experienced it as students, and then asked prospective teachers to suggest initial revisions to the lesson plan. Prospective teachers shared the lesson plan with their cooperating teachers in the field placement and some made adjustments to the lesson plan based on their cooperating teachers' feedback.

The learning goal for the lesson was as follows: “Students will begin to develop an understanding of the division of fractions algorithm, or why the invert and multiply algorithm makes sense, for the cases when the divisor is a unit fraction.” The learning goal focused on developing meaning for a mathematical procedure, not for the development of fluency with this procedure. In addition, the learning goal focused on instances when the divisor was a unit fraction. For example, given the problem of 2 $\div$ 1/3, 1/3 is a divisor that is a unit fraction. The learning goal was not written for students to develop an understanding of the division of fractions algorithm for all cases.

The activities in the lesson involved using various representations to explore division of fractions using repeated subtraction. A warm-up activity was designed to activate prior knowledge; students explored the concept of division and part–whole relationships in fractions. The next activity involved guided exploration as a whole class in which prospective teachers walked middle school students through situations such as “How many halves of a paper plate fit into three paper plates?” This particular situation illustrated that three divided by one-half could equal six, because six copies of one-half fit into three. The prospective teachers used visual aids of paper plates in the front of the room during this guided discussion. Then the prospective teachers set up an activity for the middle school students in which the students used pattern blocks to explore division of fractions in small groups. For example, if a yellow hexagon was 1, and 6 smaller, green triangles fit into 1 hexagon, this would show 1 divided by 1/6. Students were asked to explore various relationships such as these, with various whole number dividends and unit fraction divisors, and to write down their observations in a chart. The prospective teachers then conducted a large group discussion about the relationships discussed by the students in their small groups.

At the end of the lesson, students completed a short quiz, or post-lesson assessment, in which they were asked to divide a whole number by a unit fraction for the first three items and then to divide a whole number by a non-unit fraction on the last item. The purpose of the last item was to determine whether they could transfer their skills for drawing a diagram to illustrate division of fractions when the divisor was a unit fraction to a more challenging mathematics problem.

Instructors of the methods course asked the prospective teachers to attend to opportunities in the lesson plan that allowed them to collect data to assess students’ thinking. Prospective teachers were assigned to write a first attempt at a reflection on their teaching. In this reflection, they were asked to make claims in which they described students’ thinking during the lesson and support those claims with evidence. Also, they were asked to interpret the pivotal lesson elements or actions they took as a teacher (if not explicitly written in the lesson plan) that supported students’ learning, as well as suggest revisions to the lesson plan that were aligned with their claims and evidence about students’ thinking. Instructors promoted the ideas that student learning was defined by whether individual students’ thinking indicated that they have developed an understanding of the mathematical learning goal for that lesson and that effective teaching practices were those that helped the most students achieve the lesson’s learning goal.

Prospective teachers received feedback from their instructors on their first attempt at their reflections in relation to whether they differentiated between individual students and whether they provided specific descriptions of their students’ mathematical understanding. After prospective teachers submitted their first reflections, instructors conducted a class discussion in which they revised the lesson plan together as a class. During this discussion, prospective teachers were encouraged to reference evidence about students’ thinking when proposing revisions to the lesson plan.

Data collection

The data analyzed for this study were prospective teachers' second attempts to reflect on a lesson taught in their field placement classrooms. After receiving feedback on their first reflections and after participating in a class discussion in which they revised the lesson plan, prospective teachers were given prompts for a second attempt at a reflection. Data were collected within 3 weeks of when the prospective teachers taught this lesson.

The prospective teachers' second attempt at reflections were in response to open-ended prompts. Examples of prompts were questions such as the following: "Do you think the students learned what you hoped they would learn from this lesson? How do you know? If you could have changed anything about your experiences teaching this lesson in order to improve students' learning, what would you have changed?" (The full set of questions is contained in the Appendix.) The items specifically prompted prospective teachers to attend to their students, the lesson, and their implementation of the lesson because we wanted to characterize *how* they attended to their students rather than *whether* they attended to their students.

The prompts were pilot tested through interviews conducted with seven prospective teachers. Initially, we intended for those data to be collected through interviews, but since the interview responses were relatively short, we administered the prompts in the form of a written questionnaire to the rest of the 26 participants. Prompts were not adjusted as a result of the pilot interviews. To activate participants' thinking about the lesson prior to answering the prompts, prospective teachers were asked to review their reflections and any notes from their class discussion about revising the lesson plan. We analyzed data from the interviews and the questionnaires in this study, since all participants responded to the same prompts.

Data analysis

Data were analyzed through a constant comparative process (Glaser and Strauss 1967) with the goal of identifying prospective teachers' reflective thinking skills. Every participant's open-ended responses were independently coded by the two authors, and consensus was reached for any disagreements. We developed codes to examine whether prospective teachers attended to their students, elements of the lesson, or themselves as teachers when reflecting on their teaching, as well as whether they hypothesized how teaching influenced students' learning.

We looked closely at prospective teachers' descriptions of students' mathematical thinking (specific or general), and whether or not prospective teachers differentiated between individual students or described their students as a collective group. Each author made independent judgments of participants' overall descriptions of students' thinking and resolved disagreements.

In order to maximize reliability, the independent analyses of each author were compared and all disagreements were resolved. We challenged each others' interpretations of the data, as one of us was closer to the context than the other (the second author was not one of the instructors). Authors did not include member checking in the analysis; by the time analyses were complete, participants were student teaching in various school districts and no longer at the university campus, which would have presented a logistical challenge for seeking their points-of-view.

In our final phase of analysis, we examined whether prospective teachers' descriptions of their students' thinking related to whether they posed multiple hypotheses to explain how teaching influenced students' learning. Using *t*-tests, we compared the mean number of hypotheses posed by different groups of prospective teachers, and we grouped them with respect to variations in descriptions of students' thinking. The purpose of these statistical analyses was to determine strengths of relationships that we recognized qualitatively.

Results

We examined how prospective teachers described students' thinking and whether they interpreted their teaching by posing hypotheses to explain how their teaching supported students' learning. Below, we present prospective teachers' descriptions of their students' thinking, including whether prospective teachers used nuance to describe their students' thinking by naming their specific mathematical understandings and differentiating between individual students. Then we present data that indicate whether and how prospective teachers interpreted their teaching through the use of hypotheses or evaluated their teaching by posing judgments disconnected from references to evidence of students' thinking. Finally, we share results about the correspondence between prospective teachers' reflective thinking skills of describing students' thinking and interpreting their teaching.

Describing students' thinking

We prompted prospective teachers to describe their students' thinking. There were variations in their descriptions of their students' thinking in terms of whether they used nuance (named their students' specific mathematical understandings or differentiated between individual students). Below, we provide qualitative data in the form of quotations from prospective teachers' questionnaires to illustrate the variation in descriptions of students' thinking as well as quantitative data to demonstrate the frequency with which prospective teachers used nuance to describe students' thinking.

Naming specific mathematical understandings

One of the ways prospective teachers' descriptions of students' thinking varied was whether or not they named their students' specific mathematical understandings. Thirteen out of 33 prospective teachers (39.3%) *specifically* described their students' mathematical thinking, and the remaining 20 out of 33 (60.6%) *generally* described their students' thinking. General descriptions of students' mathematical thinking have references to whether or not students had correct or incorrect answers, usually on a short assessment given at the end of the lesson, and/or a general statement about whether students understood (or not) without describing what the students did or did not understand. Specific descriptions of students' mathematical understandings have details about the mathematics that students did or did not appear to understand in relation to the learning goal of the lesson.

When prospective teachers provided general descriptions of students' mathematical thinking, these descriptions did not illuminate what their students did or did not understand in relation to the lesson's learning goal. An example of a general description was when a prospective teacher said, "I do not think the students learned what I hoped they would from this lesson." In this case, the prospective teacher states that her students did not learn, but she does not indicate what they do not understand in relation to the lesson's learning goal. In another example of a general description, a prospective teacher said, "Based on how they did on the post-assessment, the students understood division of fractions well." This is a case of generally describing students' thinking in terms of achieving correct answers. An additional example was when a prospective teacher said, "The students did not fully understand the major concepts and struggled with the assessment." In this example, the prospective teacher describes the students' thinking both in terms of not understanding the learning goal and not achieving correct answers.

Specific descriptions of students' mathematical thinking involved details about students' thinking in relation to the lesson's learning goal. A prospective teacher said, "I think students were able to understand the section of the lesson where they had to find how many parts were in a certain number of wholes." "How many parts" referred to the unit fractions, or the manipulative used to represent the unit fractions. Additional specific descriptions indicated that the students were able to make sense out of the representations in relation to the symbols in the number sentence for the division problem, but then the students struggled to connect the representations to the algorithm for division of fractions. Some prospective teachers noticed the limits of their students' learning, such as one who said, "The only fractions they were able to work with were unit fractions… When they have to divide a whole by a fraction such as 2/3, for example, the students were unsure how to do this." The learning goal focused on developing an understanding for division of fractions when the divisor was a unit fraction, but there was one additional transfer question on the assessment at the end of the lesson to see if the students would be able to use a diagram strategy for a divisor that was not a unit fraction. Another prospective teacher specifically described her students' thinking in a manner that combined the ideas in the previous two examples when she said,

> I know that the students did in fact learn to divide whole numbers by unit fractions. They did great on the final assessment and were able to explain the relationship between a unit fraction and a whole… They did not seem to learn how to divide by fractions whose numerator was a number other than one.

These specific descriptions provide potential for designing instruction that builds upon or connects to students' mathematical thinking in contrast to general descriptions of students' thinking, which offer few suggestions to teachers for improving or changing a lesson. In the case above, the teacher could focus on interventions that give students the experience of seeing that division of fraction involves consideration of the number of copies of the divisor that fit into one (thus, the reciprocal of the divisor). This would build upon the students' understanding of division of fractions when the divisor is a unit fraction and move toward situations when the divisor is not a unit fraction.

Differentiating between individual students

Prospective teachers' descriptions of their students' thinking varied such that some differentiated between individual students' thinking and others characterized their students as a collective group. Ten out of 33 (30.3%) prospective teachers *differentiated* between individual students' thinking and 23 out of 33 (69.7%) prospective teachers described their *students as a collective* group. When prospective teachers described their students as a collective group, descriptions included the pronoun "they" or references to "the students," as if all students in their classroom thought similarly. (The examples presented in the previous section, prospective teachers' descriptions of general and specific mathematical understandings, also provide evidence of describing students as a collective group.)

When prospective teachers differentiated between individual students' thinking, descriptions distinguished between students in the class in terms of subgroups or numbers of students. For example, a prospective teacher said, "I would say about half of the students learned what I taught them, which was a bigger success rate than I expected." In this case, the prospective teacher estimated a subgroup of students who did learn from the lesson (implying that the other students did not learn), which indicates some differentiation between the students. Another prospective teacher said, "…many of the students did not completely understand everything I taught them… only about eight students in the entire class completed the assessments successfully." In this case, after making a claim that many students did not understand what was taught, the prospective teacher differentiated between the students by indicating the number of students who had correct answers.

Frequency in variation of prospective teachers' descriptions of students' thinking is presented in Table 1 below.

The 10 prospective teachers who differentiated between individual students also described students' mathematical understanding generally. The 13 prospective teachers who specifically described students' mathematical thinking also described their students as a collective group. No prospective teachers in this sample differentiated between individual students *and* specifically described students' mathematical understandings. Twenty-three prospective teachers in this sample used either one form of nuance or the other when describing their students' thinking.

Interpreting teaching

Prospective teachers moved beyond description and interpreted their teaching. Most participants posed at least one hypothesis to explain how their teaching actions supported students' learning. Some prospective teachers posed multiple hypotheses. Prospective teachers also went beyond description through posing evaluations, or judgments about what was effective or ineffective about their teaching without referencing a connection to students' thinking.

Hypotheses explaining how teaching supported students' learning

Twenty-eight out of 33 (84.8%) prospective teachers interpreted their teaching by posing at least one hypothesis to explain how their teaching supported or did not support students' learning (defined as students' thinking aligned with the lesson's learning goal). An example of a hypothesis posed by a prospective teacher is as follows:

> If the manipulatives were used more when filling out the chart, then the students would have had less trouble answering questions… The students would have been given an extra visual to help them answer the questions. Also, the manipulatives could have been used to push the students to answer the questions, since they were often quiet and looked confused when each question was asked.

Table 1 Variation in prospective teachers' descriptions of students' thinking

	General mathematical understandings	Specific mathematical understandings
Described students as collective group	10	13
Differentiated between individual students	10	0

This prospective teacher hypothesized that her students struggled to learn due to her implementation of the manipulatives during the lesson. Explicit connections to students' thinking were made when the prospective teacher mentioned that students struggled to answer the questions during the lesson. The prospective teacher conjectured that the manipulatives could have been integrated more effectively into the lesson to support students' engagement with the questions. Six prospective teachers hypothesized that the manipulatives used in the lesson supported (or had the potential to more effectively support) students' learning.

Prospective teachers' hypotheses addressed why students did and did not achieve the lesson's learning goal. A prospective teacher posed a hypothesis explaining why students did learn when she said, "I do feel that the questions I asked and my guidance did aid the students in developing much of their knowledge about division of fractions and reciprocals." In this case, the prospective teacher explicitly connected to students' thinking by stating that students developed knowledge during the lesson. Four prospective teachers posed hypotheses regarding how their questioning strategies helped students learn. Another prospective teacher posed a hypotheses explaining why students did not learn when she said, "The students were not able to transfer the knowledge from the exploration to the post-test. I think this is mainly because they were not given an opportunity to practice with dividing fractions." In this case, the prospective teacher explicitly connected to students' thinking by noting that the students were unsuccessful on the lesson's assessment, or post-test. Four prospective teachers posed hypotheses about how the lack of opportunities for students to practice during the lesson inhibited their learning. These examples of hypotheses are not an exhaustive list of the hypotheses posed by prospective teachers in this sample, but they provide illustrations of the nature of hypotheses posed.

Evaluations of teaching

All prospective teachers in this sample made at least one evaluation by posing judgments about their teaching that were not explicitly connected to students' thinking. For example, nine prospective teachers gave a response similar to this one: "I think that this lesson was somewhat ineffective because it had so many parts to it that did not transition well into one another." In addition, six prospective teachers evaluated the design of the post-assessment negatively, such as the prospective teacher who said, "…the design of the post-assessment was very poor. The entire lesson was based on conceptual understanding, but the last four problems the students were to complete were based on procedural understanding." Also, six prospective teachers gave a response similar to the following response: "I ended up showing them how division of fractions is the same as multiplying by the reciprocal. I was simply giving the students a formula to use, which seemed like the easy way out as a teacher." These three examples are not an exhaustive list of the prospective teachers' evaluations, but they provide a sample of the nature of evaluations that prospective teachers posed.

Evaluations such as those presented above appear to be based upon principles that prospective teachers held about effective or ineffective teaching (Jacobs and Morita 2002; Jacobs et al. 1997). Their evaluation related to poor transitions was based on the principle that activities in a lesson should flow, connect, and build upon each other. The prospective teachers who evaluated the post-assessment negatively based their evaluation on a principle that lesson activities should align, and, in particular, a lesson's assessment should align with a lesson's learning goal. Those who evaluated their teaching negatively for taking "the easy way out as a teacher" appeared to be wrestling with their role as a teacher and how much they should explicitly reveal to students in balance with allowing their students to discover connections. Evaluations of teaching that do not explicitly take students' thinking into account are limited in their potential for improving students' learning. It was possible that prospective teachers implicitly had students' thinking in mind when evaluating their teaching; however, they did not explicitly make this connection in their evaluations.

Correspondence between describing and interpreting

Describing and interpreting are two skills in the reflective cycle described by Rodgers (2002). Interpreting occurs during the analysis phase of the reflective cycle. Description and analysis are said to occur in a nonlinear fashion. For instance, teachers may go back and revise their descriptions after engaging in analysis. Given the potential for back and forth interplay between description and analysis, we wondered whether the skills of describing and interpreting corresponded. To investigate this potential correspondence, we examined the degree of nuance in prospective teachers' descriptions of students' thinking in relation to the number of hypotheses they posed that explained how teaching influenced students' learning. More nuanced descriptions of students' thinking involved naming specific mathematical understandings *or* differentiating between students, and less nuanced descriptions provided general descriptions of students' mathematical understandings *and* described their students as a collective group. (The frequency of forms of nuance that prospective teachers used when describing students' thinking was reported previously in Table 1.)

The prospective teachers who described their students' thinking with more nuance were more likely to pose more hypotheses to explain how their teaching influenced students' learning. Table 2 presents the mean number of hypotheses posed per participant for prospective teachers in a series of contrast groups.

As Table 2 indicates, neither describing students' specific mathematical understandings nor differentiating between individual students, when considered by themselves, corresponded with a significant difference in the number of hypotheses posed (although in each case, prospective teachers using more nuanced descriptions posed more hypotheses).

Table 2 Number of hypotheses posed by contrasting groups of prospective teachers

Descriptions of students' thinking	*N*	Mean # of hypotheses (per participant)	SD	*t*(31)	*p*
Specificity in descriptions					
General mathematical understandings	20	1.50	1.10		
Specific mathematical understandings	13	2.15	1.46	−1.46	0.153
Differentiation in descriptions					
Described students as collective group	23	1.61	1.37		
Differentiated between individual students	10	2.10	0.99	−1.02	0.317
Degree of nuance in descriptions					
Less nuance (general mathematical understanding and collective group)	10	0.90	0.88		
More nuance (specific mathematical understanding or individual students)	23	2.13*	1.25	−2.81	0.009

* $p \leq 0.05$

Significant results were only obtained when combining the two groups who described students' thinking with either form of nuance in contrast with those who did not describe students' thinking with any form of nuance. The effect size *r* for this comparison was 0.45, classified as large effect. The mean number of hypotheses was less than one among the group of prospective teachers who described their students' thinking with less nuance and the mean number of hypotheses was more than two among the group of prospective teachers who described their students' thinking with more nuance. This suggests that describing students' thinking with nuance (of any kind) corresponds with posing *multiple* hypotheses to explain how teaching influenced students' learning.

Discussion and implications

Results from this study indicate that most of these prospective teachers are moving toward reflective thinking, because most of the prospective teachers used one form of nuance when describing students' thinking and posed at least one hypothesis when interpreting their teaching. Approximately 2/3 of the participants described their students' thinking with nuance. We had the following purposes for focusing on nuance: (a) Describing students' thinking with mathematical specificity was deemed to be beneficial because this information could help teachers plan a targeted mathematics lesson to address students' thinking. (b) Differentiating between students could help teachers target their interventions to particular students as well as encouraging teachers to ensure that all of their students learned, not just most of their students or some of their students. Also, over 4/5 of the participants posed at least one hypothesis explaining how their teaching influenced students' learning. We examined prospective teachers' hypotheses because the act of engaging in these sort of interpretations helps them improve their instruction; if teachers focus their reflective thinking on considering which teaching practices support students' learning or which do not, they can adjust their teaching so that more students can learn. We believe these results demonstrate the potential for prospective teachers to use reflective thinking skills in response to prompts. We are curious about opportunities to improve prospective teachers' reflective thinking skills to help them go beyond the emergence of reflective thinking demonstrated by results in this study.

Improve descriptions of students' thinking

Since we expected prospective teachers to engage in reflective thinking by posing hypotheses to explain how their instruction affected their students' thinking, we believe we should do the same. Why did most of these prospective teachers describe their students' thinking with some form of nuance while others did not? What is most curious about this result is that when prospective teachers described their students' thinking with nuance, they used either one form of nuance or the other; they either described students' thinking with specificity or they differentiated between students. Instruction in the course addressed both forms of nuance in terms of assessing students' thinking. We conjecture that prospective teachers did not achieve *both* forms of nuance in this reflection due to this opportunity to reflect being an initial occasion in which they were specifically requested to use both forms of nuance. In their previous mathematics methods course, they were encouraged to use mathematical specificity, but not consistently asked to differentiate between students. Asking students to use both forms of nuance simultaneously when describing students' thinking may result in students emphasizing one or the other during their initial opportunities to do so, potentially due to limitations of attention. We conjecture that if prospective teachers pay attention to the detail of students' thinking, they can only do so for a few students and then draw an inference for the rest of the class. On the other hand, if prospective teachers try to differentiate between individual students, then they may not be able to pay attention to the specific details of each student's mathematical thinking, because the details of each individual's thinking would be too much information.

We conjecture that teachers would become increasingly effective at using both forms of nuance to describe their students' thinking after engaging in multiple reflective cycles. Prior research supports the position that multiple opportunities to engage in reflective cycles promote growth in teachers' reflective thinking skills (e.g., Sherin and Han 2004; van Es and Sherin 2002, 2008). Specifically in relation to our findings, we believe that, over time as teachers are able to recognize students' thinking more fluently and have extended opportunities to listen to the thinking of each student, they may be able to coordinate both forms of nuance. However, given a drop-in lesson with relatively unfamiliar students, it is unclear whether even an expert teacher would be able to attend to both individual students and the details of their mathematical thinking. Also, when teachers use nuanced information about students' thinking to revise their lesson or plan the targeted intervention for a future lesson, they are more likely to see the value of the information, whereas if they did not initially describe their students' thinking with both forms of nuance, they may recognize the need to have more detailed information. These conjectures should be examined through future research.

We also conjecture that prospective teachers' pedagogical content knowledge may explain why some participants were able to more effectively describe their students' thinking with mathematical specificity. Prior research indicates that pedagogical content knowledge influences prospective teachers' reflective thinking; for instance, McDuffie (2004) learned that prospective teachers are more effective at using their pedagogical content knowledge when reflecting *on* action after teaching and less likely to call upon their pedagogical content knowledge when reflecting *in* action during teaching. Furthermore, we believe that strong pedagogical content knowledge would support naming students' understanding with specificity because knowing the details of mathematical concepts is necessary for recognizing them in students' thinking; further research is needed to investigate this assumption.

Movement toward posing multiple hypotheses

The skill of hypothesizing explanations about the effects of teaching on students' learning is emerging among these participants. We again conjecture that prospective teachers' reflective thinking skills would improve after engaging in multiple reflective thinking cycles. Also, we conjecture that if the prospective teachers had extended the reflective cycle to the stage of experimentation, as advocated by Rodgers (2002), the value of posing hypotheses to explain how teaching supported students' learning might have been elevated for prospective teachers. As previously stated, these conjectures should be investigated empirically.

Corresponding skills

These prospective teachers' skills for describing students' thinking corresponded with their skills for interpreting students' thinking by posing hypotheses to explain how teaching influenced student learning. One of our conjectures to explain this result is that prospective teachers whose skills corresponded may have been those who internalized their instructors' expectations. Alternatively, prospective teachers who posed more nuanced descriptions were more focused on specific lesson outcomes, which then led them to wonder how those outcomes occurred. Future research could examine whether these skills co-occur among other prospective or practicing teachers in other contexts.

Supporting continued growth in reflective thinking

The results in this study demonstrate evidence of what emergent reflective thinking skills look like when prospective teachers initially engage in reflective thinking skills in a scaffolded context. Whether and how these prospective teachers use these reflective thinking skills autonomously during their busy lives in their own classrooms, with more constraints and without prompting, is an open question. To improve upon these results, we believe that *sustained* opportunities to engage multiple cycles of reflective thinking over time is essential to support teachers' learning so that they see value in the process as well as receive feedback in their use of reflective thinking skills. This view of the importance of sustained opportunities to reflect over time is supported by the findings of prior research (e.g., Stockero 2008). Also, we wonder whether explicit conversations about social justice concerns associated with differentiating among individuals could encourage prospective teachers to engage in this practice on their own.

The first author has begun to investigate whether and how novice teachers, graduates of this teacher education program, use reflective thinking in the context of their own teaching, and results suggest that they are attending to their students' thinking over issues of classroom management and student engagement when prompted to reflect on their teaching (Jansen 2007). We would like teachers to develop skills for using inquiry as a tool to move toward inquiry as a way of being (Jaworski 2006) beyond their teacher education coursework.

Research supporting the development of presence, or "learning to notice" (Star and Strickland 2008; Stockero 2008; van Es and Sherin 2008), suggests that prospective and practicing teachers can make progress toward engaging in reflective thinking through participating in analysis of video cases. More specifically, analysis of video cases helps teachers develop specificity in describing students' thinking (Sherin and Han 2004) and to move toward providing their own interpretation of these events (van Es and Sherin 2002, 2008). Engaging in multiple cycles of analysis for the same video appears to support moving toward specificity in characterizing evidence of students' learning (Santaga et al. 2007). A common finding across projects in which teachers engage in analyzing videos is that video acts as a common text for practicing skills that support the development of "professional vision" (Sherin and Han 2004, p. 179). In the case of this study, we are advocating for professional vision to include noticing both specific details about students' mathematical thinking and differentiating between students' thinking as well as interpretations about instruction that focus on conjectures about how critical teaching moves affected students' thinking. Engaging collectively in the process of analysis around a shared text can allow for prospective teachers to challenge each others' descriptions of students' thinking or hypotheses about the effects of their instruction on their students' learning.

Limitations

One limitation of this study is the design of the lesson itself. In particular, the lack of alignment between the post-lesson assessment and the lesson's learning goal, as observed by prospective teachers, constrained the possibilities for using that data to learn about students' thinking in relation to the learning goal. Although a lesson with some faults provides opportunities to learn through revising the lesson, the fault could have been caught by prospective teachers and their instructor prior to teaching. This difficulty highlights the benefit of conducting multiple rounds of study on a given lesson, as a revised lesson will often not only provide for improved student learning but also better opportunities to collect evidence about that learning.

The relatively small sample size is a limitation to this study, since the small sample size limited the power of the statistical analyses to reveal significant differences for each different type of nuance. We chose to use statistical analyses to assess the relative strength of patterns that we observed qualitatively, but the purpose of this particular study was not necessarily to generalize. Rather, following the intent of qualitative research, we were able to "elucidate the particular, the specific" (Creswell 2007, p. 126), such as identifying the nature of nuance in describing students' thinking and the associated prevalence of hypotheses for interpreting effects of instruction on students' thinking. Future research could be conducted to examine whether skills for describing correspond with skills for interpreting in other contexts, whether either type of nuance (differentiation or specificity) is more strongly correlated with interpretation skills, or whether describing students' thinking with more than one form of nuance is a challenge for other prospective teachers.

It is possible that prospective teachers could have been reluctant to provide an open critique about a lesson provided by their instructors. Given that prospective teachers posed a number of negative evaluations and some prospective teachers described their students' lack of achievement of the lesson's learning goal, it appears that the participants did not feel hesitant to be candid about the lesson. It was our hope that engaging prospective teachers in a form of lesson study would provide an invitation to be candid. The lesson was designed by prospective teachers, a group of student teachers from the previous semester, which we hoped would reduce threat associated with openly critiquing a lesson provided by instructors.

In addition, there was a time delay between teaching the lesson and collecting this data set about prospective teachers' reflective thinking skills. This time delay was directly connected to phases of the instructional intervention. We wanted to analyze their second attempt to engage in reflective thinking after receiving feedback on their first attempt and having a class discussion about revising the lesson based on evidence of students' thinking. It appears that these prospective teachers engaged in reflective thinking to some degree, although it is unclear whether they would have shown stronger reflective thinking skills with less of a time delay.

Conclusions

Although these prospective teachers exhibited some promising evidence of emergence of reflective thinking skills, they would benefit from opportunities to develop these skills further. We have learned that it may be challenging for prospective teachers to enact multiple forms of nuance when describing students' thinking, as these participants enacted only one form of nuance or another (either differentiated between individual students or specifically described students' mathematical understandings). We also learned that using either form of nuance when describing students' thinking corresponded with posing multiple hypotheses, which demonstrated that reflective thinking skills are aligned. However, given that practicing teachers experience challenges with engaging in some of these reflective thinking skills during lesson study (Fernandez et al. 2003), we believe it is promising to find that these prospective teachers could exhibit strong reflective skills so early in their careers.

Our research is motivated by our desire to practice what we teach. Since we encourage prospective teachers to engage in reflective thinking after teaching, including specifically describing students' thinking, differentiating between individual students, and posing hypotheses about the effects of teaching on students' learning, we want to do the same. We studied whether and what our students (prospective teachers) were learning about reflective thinking so we could use this data to consider new interventions for our mathematics education courses. Specific descriptions of our prospective teachers' thinking and differentiating between subgroups of prospective teachers allowed for considering multiple revisions to our instruction to support the growth of our prospective teachers' reflective thinking skills. The skills we believe are useful for prospective teachers to use when reflecting on their practice are useful for us, as mathematics teacher educators, as well.

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Appendix

Questionnaire/interview items

1. Consider your first lesson that you taught in your field placement about division of fractions. Did the lesson go well or not? Explain.
2. What did you hope students would learn from this lesson?
3. More specifically, do you think the students learned what you hoped they would learn? How do you know? Why do you think they did or did not learn this?
4. If you could have changed anything about your experiences teaching this lesson in order to improve students' learning, what would you have changed? Which of these changes would be most beneficial to promote students' learning? Why?
5. How would you evaluate your *teaching* during this lesson?

   (a) Very effective
   (b) Somewhat effective
   (c) Neither effective or ineffective
   (d) Somewhat ineffective
   (e) Very ineffective

Explain your reasons for this choice.

6. For this question, think about the design of the lesson itself rather than your teaching: Overall, to what degree would you say *this lesson* was an effective lesson?

   (a) Very effective
   (b) Somewhat effective

(c) Neither effective or ineffective
(d) Somewhat ineffective
(e) Very ineffective

Explain your reasons for this choice.

References

Artzt, A. F. (1999). A structure to enable preservice teachers of mathematics to reflect on their teaching. *Journal of Mathematics Teacher Education, 2*(2), 143–166.

Ball, D. L., & Lampert, M. (1999). Multiples of evidence, time and perspective. In E. C. Lagemann & L. S. Shulman (Eds.), *Issues in education research: Problems and policies* (pp. 371–398). San Francisco: Jossey-Bass.

Clark, C. M. (1988). Asking the right questions about teacher preparation: Contributions of research on teacher thinking. *Educational Researcher, 17*(2), 5–12.

Conway, P. F., & Clark, C. M. (2003). The journey inward and outward: A re-examination of Fuller's concerns-based model of teacher development. *Teaching and Teacher Education, 19*(5), 465–482.

Crespo, S. (2000). Seeing more than right and wrong answers: Prospective teachers' interpretations of students' mathematical work. *Journal of Mathematics Teacher Education, 3*(2), 155–181.

Creswell, J. (2007). *Qualitative inquiry & research design. Choosing among five traditions* (2nd ed.). Thousand Oaks: Sage Publications.

Davis, E. A. (2006). Characterizing productive reflection among pre-service teachers: Seeing what matters. *Teaching and Teacher Education, 22*(3), 281–301.

Feiman-Nemser, S., & Buchmann, M. (1986). The first year of teacher preparation: Transition to pedagogical thinking? *Journal of Curriculum Studies, 18*(3), 239–256.

Fernandez, C. (2002). Learning from Japanese approaches to professional development: The case of lesson study. *Journal of Teacher Education, 53*(5), 393–405.

Fernandez, C. (2005a). Lesson study: A means for elementary teachers to develop the knowledge of mathematics needed for reform-minded teaching? *Mathematical Thinking and Learning, 7*(4), 265–289.

Fernandez, M. L. (2005b). Learning through microteaching lesson study in teacher preparation. *Action in Teacher Education, 26*(4), 37–47.

Fernandez, C., Cannon, J., & Chokski, S. (2003). A US–Japan lesson study collaboration reveals critical lenses for examining practice. *Teaching and Teacher Education, 19*, 171–185.

Fuller, F. F. (1969). Concerns of teachers: A developmental conceptualization. *American Educational Research Journal, 6*(2), 207–226.

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research.* New York: Aldine.

Grimmett, P. P., & MacKinnon, A. M. (1992). Craft knowledge and the education of teachers. *Review of Research in Education, 18*, 385–456.

Haritos, C. (2004). Understanding teaching through the minds of teacher candidates: A curious blend of realism and idealism. *Teaching and Teacher Education, 20*(6), 637–654.

Hiebert, J., Morris, A. K., & Glass, B. (2003). Learning to learn to teach: An "experiment" model for teaching and teacher preparation in mathematics. *Journal of Mathematics Teacher Education, 66*(3), 201–222.

Hiebert, J., Morris, A. K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education, 58*(1), 47–61.

Jacobs, J. K., & Morita, E. (2002). Japanese and American teachers' evaluations of videotaped mathematics lessons. *Journal for Research in Mathematics Education, 33*(3), 154–175.

Jacobs, J. K., Yoshida, M., Stigler, J. W., & Fernandez, C. (1997). Japanese and American teachers' evaluations of mathematics lessons: A new technique for exploring beliefs. *Journal of Mathematical Behavior, 16*(1), 7–24.

Jansen, A. (2007). Factors that influence novice middle school mathematics teachers' analyses of their instruction and opportunities to learn from their own teaching. In T. Lamberg, & L. R. Wiest (Eds.), *Proceedings of the twenty-ninth annual meeting of the North American chapter of the international group for the psychology of mathematics education* (pp. 98–101). Stateline (Lake Tahoe), NV: University of Nevada, Reno.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education, 9*(2), 187–211.
Krainer, K. (2006). Editorial: Action research and mathematics teacher education. *Journal for Research in Mathematics Teacher Education, 9*(3), 213–219.
Lewis, C. (2002). *Lesson study: A handbook for teacher-led improvement of instruction*. Philadelphia, Pennsylvania, USA: Research for Better Schools.
McDuffie, A. R. (2004). Mathematics teaching as a deliberate practice: An investigation of elementary pre-service teachers' reflective thinking during student teaching. *Journal of Mathematics Teacher Education, 7*(1), 33–61.
Mewborn, D. S. (2000). Learning to teach elementary mathematics: Ecological elements of a field experience. *Journal of Mathematics Teacher Education, 3*(1), 27–46.
Otero, V. K. (2006). Moving beyond the "get it or don't" conception of formative assessment. *Journal of Teacher Education, 57*(3), 247–255.
Rodgers, C. R. (2002). Seeing student learning: Teacher change and the role of reflection. *Harvard Educational Review, 72*(2), 230–253.
Santaga, R., Zannoni, C., & Stigler, J. (2007). The role of lesson analysis in pre-service teacher education: An empirical investigation of teacher learning from a virtual video-based field experience. *Journal of Mathematics Teacher Education, 10*(2), 123–140.
Schön, D. (1987). *Educating the reflective practitioner: Toward a new design for teaching and learning in the professions*. San Francisco: Jossey-Bass.
Sherin, M. G., & Han, S. Y. (2004). Teacher learning in the context of a video club. *Teaching and Teacher Education, 20*(2), 163–183.
Smith, J. P., I. I. I. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education, 27*(4), 387–402.
Star, J. R., & Strickland, S. K. (2008). Learning to observe: Using video to improve preservice mathematics teachers' ability to notice. *Journal of Mathematics Teacher Education, 11*(2), 107–125.
Stockero, S. L. (2008). Using a video-based curriculum to develop a reflective stance in prospective mathematics teachers. *Journal of Mathematics Teacher Education, 11*(5), 373–394.
van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education, 10*(4), 571–596.
van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education, 24*(2), 244–276.