J Math Teacher Educ (2008) 11:333–341
DOI 10.1007/s10857-008-9082-z

Expanding the instructional triangle: conceptualizing mathematics teacher development

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Published online: 3 June 2008

Abstract As mathematics educators think about teaching that promotes students' opportunities to learn, attention must be given to the conceptualization of the professional development of teachers and those who teach teachers. In this article, we generalize and expand the instructional triangle to consider different interactions in a variety of teacher development contexts. We have done so by addressing issues of language for models of teachers' professional development at different levels and by providing examples of situations in which these models can be applied. Through the expansion of our understanding and use of the instructional triangle we can further develop the concept of mathematics teacher development.

Keywords Instructional triangle · Mathematics education · Professional development · Teacher development

In this article, we begin to develop a model for thinking about the improvement of mathematics teachers' education and professional development. We do so by expanding a model used for K-12 mathematics instruction to consider interactions in a variety of professional development contexts. We use this model as a thinking tool to consider interactions in the instruction of mathematics teachers as well as of those who teach teachers. We address issues of language in conceptualizing teachers' professional development at different levels and provide examples of situations in which the proposed models can be applied.

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The instructional triangle

In discussing interventions to improve instructional capacity in education, Cohen and Ball (1999) defined instruction as “the interactions among teachers and students around educational materials” (p. 5). They characterized teachers as knowledgeable professionals who shape instruction by how they interpret and respond to students and materials. Students, they explained, bring resources and experiences to these interactions, influencing what teachers can do. Materials represent what students are engaged in, as presented in texts, videos, computer-based media, etc. in the form of tasks, problems, and questions posed. These materials influence instruction by the way they constrain or enable learning opportunities. In this model, although the three elements are important, what defines instruction is not these elements in isolation. Rather, instruction is the interaction among the elements. It is a dynamic concept.

Further developing their vision of instruction, Cohen and Ball (2001) highlighted the importance of context for instruction as they considered that instruction takes place in environments that “offer potential constraints, opportunities, and resources” (p. 75). They redefined instruction as the interactions between teachers, students, content, and environments over time, and proposed that the active element of instruction comes from the way students and teachers “use” each other, the tasks in which they engage, and the environment in which they operate. Within this definition of instruction, teachers’ practices represent “the way they frame and use academic tasks, acquaint themselves with what students know and can do, enact the instructional discourse, and mediate the environment” (p. 75).

Hiebert et al. (2005) built on these ideas to consider instruction as a complex activity in which different pieces need to come together. They noted that instruction takes its shape from the knowledge teachers and students bring to the situation, the tasks on which they work, discourse structures, assessment practices, the physical materials available, and so on. “It is the interaction among these elements, the *system*, rather than the individual elements acting alone, that defines the learning conditions for students” (Hiebert et al., 2005, p. 113)

Similarly, focusing on interactions, Jaworski (1994) created a triad that highlighted three elements of teaching: management of student learning, sensitivity to students, and engagement in challenging mathematics. These elements of teaching can be associated with the relation between the student and the mathematics, the teacher and the student, and the teacher and the mathematics, respectively, in the context of a mathematics classroom. In her work with teachers, Jaworski focused her attention on the activities in which teachers engaged as part of their teaching.

The vision of instruction as involving teachers, students, and content within environments has been represented in mathematics education by an instructional triangle model (Δ model) in which teachers, students, and mathematics are at the vertices and the sides represent the interactions among them—see Fig. 1 (Kilpatrick, Swafford, & Findell, 2001, p. 314). In this case, mathematics instruction can be understood as the contextualized interactions among the teacher, the students, and the mathematics. The value of the Δ model for mathematics educators is that it expands the idea of instruction, taking the focus off the teacher as an independent actor in the classroom, and placing it on the interactions between teachers, students, and mathematics, as well as on the contexts in which these interactions occur. This model also highlights the importance of students’ interactions among themselves and with mathematics for thinking about instruction in the discipline.

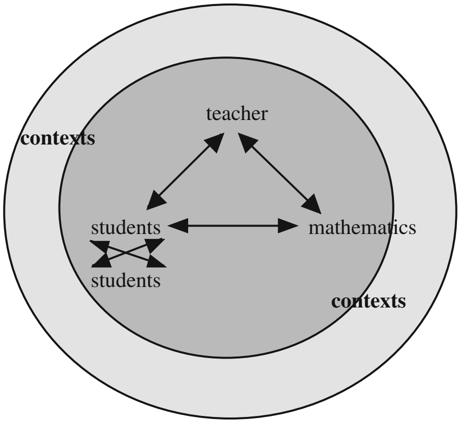

Fig. 1 The instructional triangle

We use the Δ model as a thinking tool to consider different interactions in a variety of professional development contexts for mathematics teachers. We define professional development as any opportunity for teachers to meaningfully interact with content, teaching, or learning. We are using the word "teachers" in a broad sense to include pre-service or practicing teachers at grades Preschool to 16, as well as those who teach other professionals in formal and informal settings. Our goal is to expand our understanding and use of the Δ model, and we do so in two ways. First, we generalize the model to fit other levels of instruction beyond Preschool to grade 12 classrooms. Second, we expand our thinking about the triangle by embedding triangles within the generic instructional triangle (creating teacher development triangle models and teacher education development triangle models) and by adding an additional vertex (creating a tetrahedron model).

The instructional triangle and the instruction of teachers

Ball and Bass (personal communication, February 2004) suggested that an expansion of the Δ model could be used to represent interactions among teacher educators and pre-service teachers in the instruction of teachers. From this idea of expanding the instructional triangle, we began to question how the Δ model could be useful for different levels of instruction. That is, how would the Δ model look beyond the P-12 mathematics classroom for the professional development of mathematics teachers. We first developed the use of a Δ model to represent instruction at the teacher education level. Then we developed the use of a Δ model to represent the instruction of those who educate teachers.

Issues of language immediately surfaced. First, because the student in a teacher education situation was a teacher, using the words "teachers" and "students" in the expansion of the instructional triangle proved to be cumbersome. An explanation was always needed about who the "teachers" and the "students" were. Furthermore, when we continued to expand the instructional triangle to talk about interactions in which teacher educators were the students, we had no language to talk about that. The second language issue that emerged in our expansion of the instructional triangle was the difference that exists between pre-service and in-service teacher education. For example, we often refer to the former as teacher preparation and the later as staff development. Also, many university

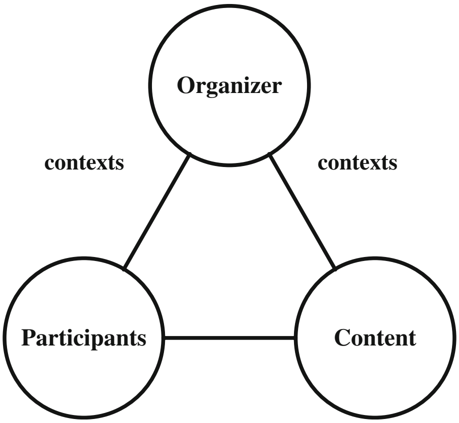

Fig. 2 The generic instructional triangle

professors who teach future teachers, such as university faculty in mathematics departments, do not consider themselves teacher educators. More inclusive terms were needed to talk about the instruction of teachers, and new terms were needed to talk about the education of those who instruct teachers.

From our point of view, pre-service and in-service teacher education are instances of the same phenomenon: teachers' professional need to continually learn and grow. This phenomenon also relates to those who teach teachers. As teachers themselves, teacher educators also need to continually learn and grow. In our expansion of the instructional triangle, pre-service and in-service teacher education are not treated differently, and we use language that represents this idea. We also use language to initiate and support discussion about the preparation and development of those who teach teachers. Thus, as we generalize and expand the Δ model, we pay particular attention to the language we adopt to talk about instruction at various levels.

To facilitate representation and to focus the discussion, we begin our expansion of the instructional triangle by generalizing it. A simplified model of the generic Δ model is in Fig. 2. This generalization from teacher to organizer, from students to participants, and from mathematics to content allows the generic instructional triangle to represent interactions in a variety of contexts. In our approach and use of this generalized Δ model, positions are fixed to indicate that different vertices stand for different interacting elements of instruction. In the analysis that follows, the organizer of the instructional activity is always placed at the top vertex of the triangle, the participants are placed to the left, and the content is to the right. This specification clarifies discussion because as the Δ model is expanded to teacher education, for example, the word "teacher" is placed in the position of the participant, contrasting with the initial model where the teacher is the organizer.

Teacher development triangle models

We call the first level of expansion of the instructional triangle the teacher development instructional triangle. At this level, the organizers are teacher developers (anyone interested in the development of mathematics teachers, such as university faculty in mathematics departments, mathematics teacher educators, or district staff developers) and the participants are teachers (at the pre-service or in-service levels). Thus, in our analysis,

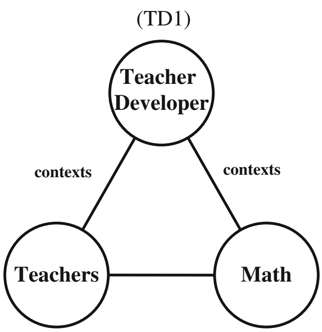

Fig. 3 Two teacher development triangles

the teacher development triangle represents the contextualized interactions between teacher developers, teachers, and content.

In mathematics education, the content upon which teacher developers and teachers interact can vary, and several models can be considered. One possible model (TD1) has mathematics as the content of teacher development. Another model (TD2) has the P-12 classroom instructional triangle as the content (Fig. 3). As with the original instructional triangle, these new instructional triangles represent interactions that are situated in contexts. Thus, in the TD2 Δ model, in addition to the classroom context indicated in the original triangle, there is also the context of the professional development experience.

Mumme and Seago (2002), and Ball (2004) have suggested models similar to TD2. In this model, the use of the original instructional triangle as the content for teacher development underscores the importance of the context of mathematics teaching and of classroom mathematical interactions to the development of teachers. In this case, particular aspects of the original Δ model can be highlighted as the specific content for teacher development. For example, the content of a teacher development initiative could be the interactions between children and mathematics or between teachers and mathematics. A model in which the topic is the interactions between teachers and students, not explicitly related to mathematics, is also possible and often found; however, because our interest is in mathematics-specific models, we do not consider such a model in this article.

A couple of situations can exemplify the teacher development triangle and the different models we propose. When pre-service teachers take an algebra content course at the university, a mathematician is the teacher developer (organizer), undergraduate students taking the course are the teachers (participants), and algebra is the mathematics (content). In a different context, when an in-service teacher participates in a professional development activity on calculators in the classroom organized by the local school district, a staff-development professional from the district is the organizer, school teachers are the participants, and children's misconceptions about calculators is the content.

Having mathematics as the content for the development of teachers, as in model TD1, was an approach intensively used for in-service teacher training during the new math era, a movement in the 1960s to address concerns in the mathematical and technical preparation of teachers (Herrera & Owens, 2001). At that time, due to concerns about teachers' lack of knowledge about new developments in mathematics as a discipline, the discussion about mathematics instruction centered on what mathematics to teach in schools and teachers had to increase their knowledge of mathematics to understand the new concepts that were being added to the school mathematics curriculum.

While the TD1 Δ model for professional development is still common, it now competes with professional development models in which aspects of the original instructional triangle are the content (TD2). As mathematics educators gave more attention to how one learns mathematics, teacher developers expanded their attention from teachers' knowledge of mathematics to include teachers' knowledge of mathematics classrooms and of mathematics teaching and learning. Teachers, it is said, need to increase their pedagogical content knowledge (Shulman, 1987) and their knowledge of mathematics cannot be isolated from their knowledge of the classroom. Research projects such as Cognitively Guided Instruction (e.g., Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Fennema, Peterson, Chiang & Loef, 1989), for example, have focused their teacher development program on the interactions between children and mathematics, whereas Simon and colleagues (e.g., Simon & Blume, 1994) have involved pre-service teachers in reflecting about their own interactions with mathematics and their construction of mathematical ideas.

The TD1 and TD2 models are often used simultaneously at the pre-service teacher education level when college students take mathematics content and mathematics methods courses. Although there are many variations in pre-service teacher education programs, mathematics content courses are typically organized in a TD1 format, whereas methods courses are TD2.

The instructional triangle and the instruction of those who teach teachers

The expansion of the instructional triangle to talk about teacher development brings to mind the question of whether we can continue to expand the model and use the Δ model to talk and think about instruction for those who teach teachers. The Center for Proficiency in Teaching Mathematics (CPTM) has worked on raising the awareness that teachers of teachers, as with any teacher, need to continually work on their professional growth as educators. From this perspective, the education of teacher developers needs to be conceptualized (Cochran-Smith, 2003; Stein, Smith, & Silver, 1999). One way to think about this concept is to further expand the instructional triangle to this level of instruction, that is, to generate a *teacher-educator development* triangle. In this case, we have teacher-educator developers as organizers and teacher developers as participants, interacting around content. Questions about (a) who the teacher-educator developers are and (b) what the content of these interactions is need to be answered as the concept of educating teacher developers is construed.

Teacher-educator development triangle models

One can begin addressing the abovementioned questions by thinking about possible content for this instructional triangle. At this third level, the expanded Δ model can have three different formats. One model considers mathematics as the content (TDE1). The other two models (TDE2 and TDE3) have as content the mathematical development of teachers, which can be either of the two teacher-development triangles previously proposed (Fig. 4). In the case of TDE3 Δ model, there are three levels of context to be considered in the instruction: the classroom context, the teacher development context, and the context of the teacher-educator development.

An example of a TDE activity is a mathematician (organizer/teacher-education developer) working with district curriculum specialists (participant/teacher developers) on their

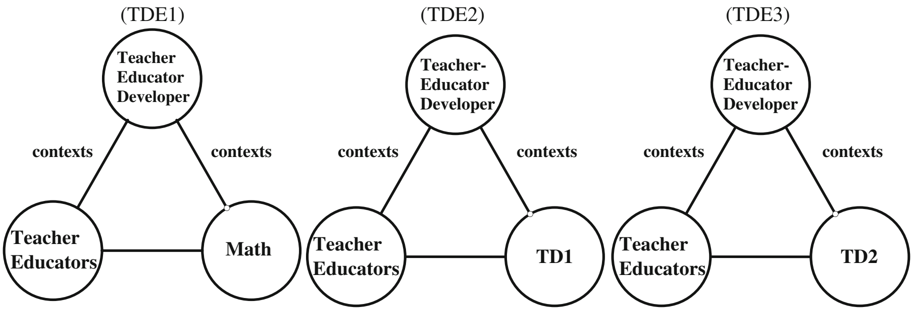

Fig. 4 Three teacher-educator development triangles

knowledge of geometry (content/mathematics). A second example of teacher-educator development is a mathematics educator (organizer/teacher-educator developer) working with teacher leaders (participant/teacher developers) on rethinking mathematics teacher development initiatives within their schools (content/teacher development triangle).

Examples of teacher-educator development can be found in the summer institutes organized by CPTM. In summer 2003, teacher developers had the opportunity to observe and interact with in-service middle school teachers as they participated in a geometry course. In 2004, teacher developers observed a mathematics content class on fractions for pre-service elementary teachers and had the opportunity to discuss and influence what happened in the class. In both cases, the organizers of the institutes (faculty and graduate students at the University of Georgia and the University of Michigan) were the teacher-educator developers. The participants were teacher developers. They constituted a diverse group encompassing university faculty members from mathematics and mathematics education departments, two-year college faculty members, and district and school personnel.

While the content of the Summer Institute 2003 was the teacher development triangle with mathematics as the content, the content of the Summer Institute 2004 was the teacher development triangle in which the K-12 instructional triangle was the content. Using the Δ models, the 2003 Summer Institute was a TDE2 model, and the 2004 Summer Institute was a TDE3 model. Being able to use the Δ model to differentiate these two examples of teacher-educator development activities and to talk about what each one accomplished was one of our initial reasons for pursuing the idea of expanding the Δ model to teacher-educator development settings.

Despite these examples, what the different triangles exemplify at the teacher-educator development level needs more discussion and further clarification. Initiatives in educating teacher developers are in their early stages of design, and much work is still to be done. Furthermore, because we do not know much about who teacher-developer educators are and how they interact with teacher developers at this level of professional development, the separation between teacher-education developers and teacher developers is still tentative.

Teacher-educator developers teach teacher developers. They are not teaching P-12 students or P-12 teachers, and one can argue that they are ultimately teachers of teachers—like teacher developers. In this case, perhaps there are no differences between teacher-education developers and teacher developers. However, one can argue that this difference is not currently explicit because there is no established body of knowledge that separates those who work on teacher-educator development from those who work on teacher development. Thus, currently, we use what is known about teacher development to conceive teacher-educator development. The development of a specific body of knowledge might come to distinguish these two professionals. We believe this is an empirical question yet to be explored.

Despite these perspectives, as the work on the education of teacher developers begins, teacher-educator developers and teacher developers are coming to instructional situations with similar types of knowledge and experiences, probably bringing a more collaborative relation to the interactions between organizers and participants in this level of the Δ model. Accompanying the development of these interactions might allow us to better understand the development of instructional relations in which organizers and participants are not at different hierarchical positions.

The role of mathematics as content in teacher-developer education is also not clear. Mathematics is more explicit in model TDE1. As we further expand the Δ model by embedding one triangle within another (models TDE2 and TDE3), mathematics fades away into the triangles. Thus, the role of mathematics in these triangles needs to be considered. While on one hand, we know that considering mathematics as isolated from classroom interactions might not be appropriate for all teacher development or teacher-educator development initiatives, having mathematics become less and less evident is also not a desirable image for conceptualizing teacher development in our field.

Further expansions of the instructional triangle

As we continue to think about ways to expand the instructional triangle, we can consider other elements that impact the professional development of teachers. An important element in professional development is the role of teacher's practice (whether their prior, intended, or actual classroom experiences). By adding another vertex to the generic instructional triangle, we create a tetrahedron model. Each triangular side of the tetrahedron represents a subset of interacting elements from the set of vertices -organizer, participant, content, and practice. The value of this model for mathematics educators is that it expands the idea of professional development, taking the focus off the professional development experience and placing it on development as an individual construction situated in practice. This model also highlights the importance of the organizer modeling appropriate pedagogy and the kinds of mathematics encountered by the participant.

The tetrahedron model elicits new questions for those planning instruction in mathematics at all levels. For example, one can ask "how do teachers make sense of new content from, for and in their practice?" Also, "what expectations do teachers bring to professional development activities and what tensions arise when these expectations are not met?" And finally, "what constitutes a professional development opportunity in mathematics that is consonant with making instructional practice and its development central to that learning?" These are important questions that are already beginning to be examined. Models, such as the tetrahedron model, provide a metaphor for beginning and facilitating this dialogue.

Concluding remarks

Our goal for this article was to conceptualize different levels of mathematics teacher development through the expansion of our understanding and use of the instructional triangle. We have done so by considering the generic instructional triangle and the importance of talking about organizers and participants when we refer to teachers' professional development at all levels. We have proposed the consideration of the teacher development and the teacher-educator development triangles, expanding the original Δ model to consider teachers' professional development at different levels. We showed examples of situations in which the proposed Δ models can be applied. In addition, we offered a tetrahedron model as another venue for expanding the Δ model, adding, for example, teacher's practice as another important element of professional development.

As mathematics educators further think about models for conceptualizing the education and professional development of teachers and teacher developers, we believe attention should be given to issues of language. Clarifying what we mean will help us develop the concept of professional development, thinking about it in creative yet clear ways. An effort toward developing this language and making it explicit was initiated here, and we want to encourage others to join us in this dialogue.

Acknowledgments The authors wish to acknowledge Jeremy Kilpatrick, University of Georgia, for comments on earlier drafts of this manuscript. The development of this article came in part from work and discussions within the National Science Foundation supported Center for Proficiency in Teaching Mathematics. The ideas expressed are those of the authors.

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