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What is actually discussed in problem-solving courses for prospective teachers?

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Abstract The aim of this study is to characterize the discourse of two problem-solving courses for prospective teachers. The data, consisting of audio recordings and field notes, were examined from a dialogical approach combined with the theory of contextualization. I show not only the substantial differences between the two classroom discourses but also elaborate on plausible reasons for the divergency found. The findings then serve as a basis for a discussion of how to develop a problem-solving course within the mathematics teacher program.

Keywords Contextualization · Dialogical approach · Discourse · Problem solving · Prospective teachers · Student teachers

Introduction

Problem solving is a particularly complex and interesting concept in mathematics education. That is, problem solving is not only seen as one important strand of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001) but also as a productive way to develop other mathematical competencies (Lester & Lambdin, 2004). Further, due to the complexity of the term problem solving (cf., e.g., Carlson & Bloom, 2005; Schoenfeld, 1992; Stanic & Kilpatrick, 1989) one can assume that the teachers' beliefs and interpretations of problem solving have a strong impact on the activities of classrooms. Therefore, the preparation program for teachers should discuss and, if necessary, change the prospective teachers' views of mathematics, including the role of problem solving (Cooney, Shealy, & Arvold, 1998; Crawford, 1996; Thompson, 1992). Considering that "more teachers and better mathematics teaching are needed if mathematical proficiency is indeed to become a widely held competence" (Adler, Ball, Krainer, Lin, & Novotna, 2005, p. 360) together with the

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fact that in Sweden ‘‘the lack of research on mathematics teacher education is astonishing’’ (Bergsten et al., 2004, p. 22), a detailed analysis of problem-solving courses in Swedish teacher programs is of interest, not only for Swedish educators, but for anyone engaged in mathematics teacher education.

In examining the problem-solving courses, I chose to focus on the classroom discourse since one may argue that the content of a course is determined by what is happening in the classroom rather than what is, for instance, written in the formal documents about the course. Therefore, the aim of this study is to characterize the classroom discourses of two problem solving courses for prospective teachers that were, on the surface, very similar. The analysis shows that the discourses of both courses could be characterized in terms of three broad categories: subject-oriented, didactically oriented, and problem-solving-oriented discourses. However, the analysis also shows that the distribution of the discourses in those categories substantially differs between the two courses. In relation to the aim of characterizing the discourses I elaborate on plausible reasons for these divergences. This elaboration, together with the characteristics found, then serves as a basis for two discussions. The first concerns the importance of not only talking about mathematics in teacher programs in quantitative terms (how much mathematics should prospective teachers study) but also in qualitative terms (what to include in those mathematics courses). The second is how the findings of the study may be of interest for educators and researchers aiming at developing problem solving courses for prospective teachers. That is, I discuss a number of practical aspects one should consider when designing or teaching a problem solving course for prospective teachers.

The presentation below is divided into four sections. First, I introduce the data and how it was chosen. Second, I discuss the approach used for analyzing the discourses. Third, I present the analysis of three transcripts and highlight the most important results of the study. Fourth, in the concluding discussions I discuss several practical issues in relation to the characters of the discourses.

The data

In my search for courses directed towards problem solving I contacted each teacher training program in Sweden by mail asking the question: I wonder if your university offers any mathematics courses directed towards problem solving for prospective teachers? Their replies indicated that six out of the 28 training institutions had a specific course directed towards problem solving.[1] In two of the cases, the teacher educators did not want me to study their course, and in a third case the course could not be studied for practical reasons. Consequently, I was able to collect data from three of those courses.

During the collection of data, I found out that two of the three courses had much in common regarding the aim of the course. That is, two of the courses were focused on teaching about problem solving while the third course used tasks as a means of teaching mathematics at university level. Since I was more interested in what prospective teachers learn about problem solving, rather than how they learn university mathematics through problem solving, the two courses about problem solving were

[1] Even though I have not examined this quantitative relation deeply, this observation indicates a need for developing problem-solving courses in Swedish teacher education.

chosen for closer examination. Of course, this is not to say that the third course would not be interesting to study; in fact it would be very interesting to see whether/how mathematics is learned through problem solving (cf. Stein, Boaler, & Silver, 2003).

As mentioned above, two of the courses were about problem solving. On a general level, the instructional ideas of the two courses were very similar. That is, both courses had basically two types of teaching; teacher educators' lectures and prospective teachers' task presentations at the chalkboard. The prospective teachers worked on tasks in groups before presenting them at the chalkboard. These tasks were mathematically fairly simple, designed either for primary or secondary students. The idea in both courses was that the solutions to the tasks presented at the chalkboard should serve as a basis for classroom discussion about, e.g., different ways of solving the task.

The classroom discussions of the two courses were recorded on audiotape and I also took field notes. It could be argued that video recording could have been useful but audiotape recordings were used in order to minimize the disturbance of the classroom setting. In addition, one of the teacher educators asked me not to use video cameras. I was present during the data collection and acted as a nonparticipating observer. I observed and collected data in the beginning, in the middle, and at the end of the courses. In total, I collected approximately 10 hours of audio recordings from each course.

Analytical approach

Problem solving

There is a huge amount of literature about mathematical problem solving (see, e.g., Carlson & Bloom, 2005; Lester, 1994) and in this study I take a broad perspective by distinguishing between three views of problem solving including: the different *roles* of problem solving in mathematics teaching, the *processes* of solving a problem, and the *competencies* needed for solving problems (see also Ryve, 2006a). Before going into the different perspectives on mathematical problem solving I will specify the use of the concept of mathematical problem in this study.

I follow Schoenfeld's (1993) definition of what a mathematical problem is. Schoenfeld states:

> For any student, a mathematical problem is a task (a) in which the student is interested and engaged and for which he wishes to obtain the resolution, and (b) for which the student does not have a readily accessible mathematical means by which to achieve that resolution. (p. 71)

Hence, the concept of problem is relative in two ways. First, following Schoenfeld's definition, a task could be a problem for one person and at the same time not for another since one of them has a ready method for solving it and not the other. Further, following the theory of contextualization presented below, a problem denotes an interpreted task. Hence, different agents may interpret the same task differently.

Wyndhamn, Riesbeck, and Schoultz (2000) have studied the ways in which the concepts of problem and problem solving are used in Swedish curricula. They identified three different roles of problem solving in relation to mathematics teaching and learning. First, students should study mathematics for developing skills in solving problems. Second, there should be teaching about problem solving. Third, students should learn mathematics through problem solving. This framework helped me in selecting the data for this study (see Sect. The data) as well as for examining how problem solving is viewed in relation to mathematics within the courses.

In accounting for the processes of problem solving the classical framework of Pòlya (1945) is used. This four-step process could be summarized as follow: understanding the problem, devising a plan, carrying out the plan, and reflecting upon your solution. This framework is used in two ways; to examine as to whether the problem solving process is discussed explicitly and to structure the analysis of the students' presentations at the chalkboard.

Schoenfeld (1992) extends Pòlya's (1945) work of problem solving by directing us toward five competencies that are needed for becoming a successful problem solver. That is, five interrelated categories including resources, heuristics, control, beliefs, and practices are introduced as cornerstones for teaching and analyzing mathematical problem solving. The term resource is used to characterize the mathematical tools the problem solver could use when approaching a task, including facts, procedures, and skills. Building on the work of Pòlya, Schoenfeld (1992) denotes several heuristics (problem-solving strategies) including, e.g., working backwards, searching for analogies, and decomposing and recombining. Mason and Davis (1991) also recognize specializing as an important heuristic and relate it to the process of generalizing. Further, on the same line as Bjuland (2002), questioning and visualization are included as heuristics.

In comparing successful problem solvers with less successful ones, Schoenfeld (1985) finds that the degree of control has a major impact of the problem solvers' success. Control is, in this case, closely related to the terms of metacognition and self-regulation, which have been elaborated extensively in the research literature (e.g., De Corte, Verschaffel, & Eynde, 2000; Schoenfeld, 1987). Typical control activities "include making plans, selecting goals and subgoals, monitoring and assessing solutions as they evolve, and revising or abandoning plans when the assessments indicate that such actions should be taken" (Schoenfeld, 1985, p. 27). Control decisions have an impact on the solution at a global level where typical considerations are: Why should one plan be implemented and not the other? Should I stop and try another plan or not? How much time should I spend on different parts of the problem solving process?

One may think that resources, heuristics and control should cover the problem solving activities. Nevertheless, research shows (e.g., Schoenfeld, 1992) that formal and relevant knowledge is simply ignored in real-world situations. This is the reason why Schoenfeld (1985, 1992) introduced a fourth category called beliefs. The beliefs of the students, related to mathematics and mathematical learning, are often divided into three categories: beliefs about the self in connection to the learning of mathematics and problem solving, beliefs about contexts, and beliefs about mathematics as a discipline (cf. De Corte et al., 2000). The fifth category is called practice and refers to students' enculturation into the mathematics practice. More specifically, students need to be guided into productive dispositions, habits, and approaches to think about mathematics. These processes could be seen as a kind of socialization into doing mathematics.

A dialogical approach

I tested several analytical approaches for studying the complexity of the two classroom discourses, such as the approaches of Cobb, Stephan, McClain, and Gravemeijer (2001) and Sfard and Kieran (2001),[2] before I decided to use a dialogical approach (Linell, 1998) combined with the theory of contextualization. Before I go into details about the dialogical approach, I want to note in passing that the dialogical approach has previously been successfully used in mathematics education for studying prospective teachers solving tasks in groups (Bjuland, 2002, 2004) and for studying classroom discourses (Cestari, 1997).

The dialogical approach is an analytical approach that stresses interactional as well as both situational and sociocultural features when examining authentic spoken interaction. The approach rests on three fundamental dialogical principles: sequentiality, joint construction, and act-activity interdependence. In short, the principle of sequentiality stresses that each piece of discourse[3] must be understood in relation to its position in a sequence. Moreover, Linell (1998) accentuates that discourses are always jointly constructed. This implies that analysts should focus on how, rather than if, discourses are co-constructed. Finally, the principle of act-activity interdependence denotes that a piece of discourse gets parts of its meaning in relation to the communicative activity in which it is produced and at the same time contributes to the realization of that communicative activity. These three principles are operationalized into a number of analytical constructs, which I found very useful for structuring and interpreting the classroom discourses of this study. Below, I discuss those constructs.

Methodological constructs

I use three methodological constructs for denoting pieces of discourse of different size. First, a turn is basically a period of time when one speaker holds the floor. A turn is both reflexively related to the local (sequentiality) and global discourse (act–activity interdependence). That is, the analyst needs not only to relate explicitly the turn to other surrounding turns but also to the context in which the turn is produced (see further below). A number of turns constitute a topical episode. Topical episodes are used to identify sections of discourse that deal with a specific content. However, content should not be seen as something static, defining what the discourse is about, but rather something created through the interaction. In this study I introduce the construct of a *task presentation episode* for denoting a piece of discourse that covers

[2] Although Cobb et al. (2001) stress that their interpretative framework "is a conceptual tool we use to understand what is going on in the classrooms" (p. 122) I found it hard to apply it to my data. Their methodological approach is certainly productive in conducting design experiments but the theoretical constructs within it did not fully help me in characterizing the discourses. However, the close relation between the psychological and social perspective in Cobb et al. (2001) can be compared to the relation between contextualizations and communicative projects in this study. The framework of Sfard and Kieran (2001) seems to be more appropriate for conducting analysis of small-group work.

[3] The term discourse is defined as "a stretch of concrete, situated and connected verbal, esp. spoken, actions" (Linell, 1998, p. 6). This fairly substantive definition of discourse could be contrasted to scholars within the Foucaultian tradition using it to capture more implicit ways of speaking. The construct of communicative activities (Linell, 1998) is here used for capturing such implicit ways of speaking (cf. Gee, 1997).

the prospective teachers' presentation of solutions to one task. Typically, a number of topical episodes constitute a task presentation episode and I will use this distinction to clarify the characteristics of the discourse in (Results of the study). To conclude, I use three constructs to denote different pieces of discourse. Notice, that so far I have only introduced constructs used for structuring and describing the data. Below I discuss constructs and theories used for interpreting the data.

Theoretical constructs

The theoretical construct of a *communicative project* is used for ascribing meaning to pieces of discourse (Linell, 1998). In other words, communicative projects are units of meaning rather than units of expression. The construct of communicative project serves to capture the planning, the process, and the product of collaboratively carried out efforts to solve communicative tasks. Several communicative projects could be ascribed to the same piece of discourse since the projects could vary in character (e.g., social, mathematical) as well as size (e.g., local to global). Further, communicative projects are in this study both ascribed to topical episodes and task presentation episodes. It should be noted that the theoretical construct of communicative project offers the possibility of ascribing meanings to discourses, but the construct does not in itself provide interpretations. Therefore, communicative projects are connected to the theoretical constructs of communicative activity and contextualization, which are discussed below, as well as to the problem solving frameworks presented above.

In reading the following paragraph it may be helpful to think of, for instance, a school lesson, a court trial, or a doctor–patient interaction. *Communicative activities*[4] should here be seen as ways of interacting where certain historical and cultural norms, routines, and interactional patterns have been established. These routinized ways of interacting have been established, according to Luckmann (1992), to solve recurrently occurring communicative problems. The establishment of such communicative activities typically follows the pattern of being first "interactionally developed, then historically sedimented, often institutionally congealed, and finally interactionally reconstructed in situ"(Linell, 1998, p. 239). Within a communicative activity certain topical episodes are co-constructed by means of special verbal (and nonverbal) artefacts. Moreover, the participants of such communicative activities are typically taking different interactional and social roles (Linell, 1998). For instance, consider the phenomenon that, in a classroom, it is not uncommon that the one who knows the answer is the one who asks the questions. To conclude, the theoretical construct of communicative activity is used in this study to enable me to take into account less-explicit features of the mathematical discourses.

Linell (1998) accentuates the homogeneity of communicative activities but also notices that interpretations of tasks and utterances could vary among participants depending on how they (implicitly) define the communicative activity. This implies that different participants within a communicative activity may construct "contrasting and competing versions of the 'same' events in the world" (Linell, 1998,

[4] The concept of communicative activity shares common features with concepts such as orders of discourse (Fairclough, 1992), meta-discursive rules (Sfard, 2000), and communities of practice (Wenger, 1998). It is, however, out of the scope of this paper to go into details about the relation between those concepts.

p. 256). In this study a specific theoretical framework is used for accounting for such different ways of interpreting the setting, namely the theory of *contextualizations* (e.g., Halldén, 1999).

The theoretical framework of contextualization stems from studies (e.g., Halldén, 1982; Wistedt, 1987) showing that students' interpretations of a task could vary, both in relation to the teacher's and to other students' interpretations. The act of interpreting a task is referred to here in terms of contextualization. A contextualized task is called a problem (Halldén, 1988). Hence, students confronted with the same task could be working with different problems. These different ways of contextualizing the task are related to how the students interpret the conceptual, situational, and the cultural aspects embedded in the particular setting (Halldén, 1999). One may ask the question: why not simply use the concept of interpretation instead of contextualization? In short, the theory of contextualization helps the researcher to pay attention to contextual features of the agents' interpretations. Further, the theory of contextualization is closely related to analytical principles of intentional analysis, which is discussed below where I summarize the analytical approach taken in this study.

Short summary of the analytical approach

The analysis of the data could best be described as a cyclic process between transcripts and field notes, audio recordings, and theoretical and methodological constructs. I use Fig. 1 to summarize the most essential components of this analytical process.

The constructs of turns, topical episodes, and task presentation episodes were used for structuring the analysis of the mathematical discourses, thus giving us *accounts of* the mathematical discourses. The reflexively related concepts of contextualization, communicative projects, and communicative activities were used to ascribe meaning to the mathematical discourse, thus giving us *accounts for* the mathematical discourses. In the process of going from descriptions to interpretations, or from accounts of to accounts for (Mason, 2002), I follow the principles of *Intentional analysis* (see, e.g., Ryve, 2006b; Wistedt & Brattström, 2005; Scheja, 2002). Intentional analysis could be described as an explicit way of approaching the participants' turns, topical episodes, and task presentation episodes by setting them

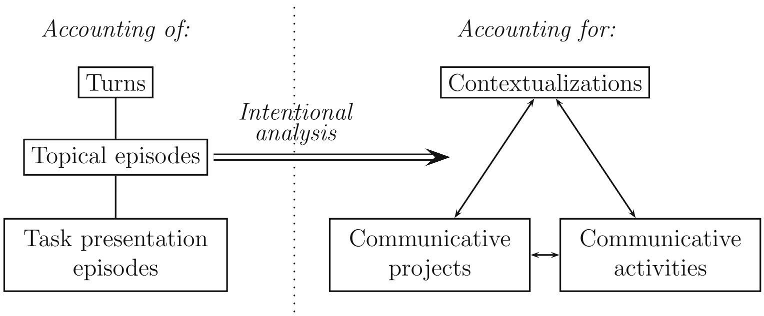

Fig. 1 Structuring and interpreting mathematical discourses

in a wider perspective by means of the theoretical constructs in order to ascribe meaning to them, thus turning them into communicative projects.[5]

It may be helpful for the reader to be reminded of the fact that the theory of contextualizations could be used on different kinds of tasks, such as the task of presenting a mathematical task at the chalkboard or the task of arranging a problem solving course for prospective teachers. In a similar manner, communicative projects could be ascribed to different units of discourse (e.g., topical episodes or task presentation episodes).

Analysis and results

Transcripts and analysis

I have selected three transcripts to illustrate the findings of this study. The two first transcripts are from course 1, while the third transcript is taken from course 2. As described above, the classroom lessons in both course 1 and course 2 were typically centered on the prospective teachers' presentations of their solutions to tasks given to them by the teacher educator. The tasks given to the prospective teachers were normally taken from the textbooks of primary school or secondary school students, so just calculating a mathematical answer to the task did not usually cause any difficulties.

In introducing the transcript I first present the task that the participants are discussing:

> Three friends are carrying a 19 m long flagpole in a block where each and every street is 7 m wide and perpendicular to each other. Could they turn around a street corner if the flagpole is carried in a horizontal position?

Here we enter the discussion when the prospective teacher (PT) at the chalkboard is about to calculate the hypotenuse of a right-angled, isosceles triangle (7 m). The prospective teacher has come to the conclusion that the length of the hypothenuse is $\sqrt{98}$ and asks somebody in the audience (A) for help when the teacher educator (TE1) interrupts.

[43] PT—... square root of 98, I need help with that.
[44] TE1—Yes but it is an even (A—it becomes 9.899) ne, ne, ne no decimals.
[45] PT—How much was it?
[46] TE1—No, look at the number under the square root sign and say something about it.
[47] PT—It is 98 (TE1—Yes) yes it is smaller than 10.
[48] TE1—eeh say something sensible about it.
[49] PT—It is close to 10.
[50] TE1—98 is much bigger than 10.

[5] There are a few points I want to comment on here. First, in the actual process of ascribing meaning (intention) to, e.g., turns I hypothesize an intention connected to this turn. I then test this intention by relating it to, e.g., other turns and the communicative activity. Second, it is important to stress that intentions should not be seen as something referring to what is in the mind of the agents but rather an entity ascribed to, for instance, turns. Sfard and Kieran (2001) put it in this way ''intention is not meant to be in any way prior to the utterance ... it comes into being in this act''(p. 48).

[51] PT—Yes, yes 98 is bigger than 10 but it is close to 100.
[52] TE1—Yes and not just close to 100, say something precise about it.
[53] PT—It is an even number.
[54] TE1—Yes! Divide it by two then, what do you get?
[55] PT—Yes okay then I get 49.
[56] TE1—And 49.
[57] PT—Yes the square root of 49 is 7.
[58] TE1—Exactly, thus it is equal to.
[59] PT—7 times the square root of 2.
[60] TE1 – Yes then it is exact, 7 square root of 2, okay.

In [43] PT says 'square root of 98, I need help with that'. At least some, I guess all, prospective teachers in the audience interpret turn [43] as a request for numerical value. However, while one of the prospective teachers in the audience gives the numerical value, TE1 initiates the communicative project of transforming $\sqrt{98}$ into $7\sqrt{2}$ by turn [46] 'look at the number under the root and say something about it'. We can at least conclude two things in relation to turn [46]. First, the turn must to a high degree be interpreted in relation to the communicative activity in which it is produced and, second, PT seems to have vague ideas about TE1's contextualization of doing something with $\sqrt{98}$. The second statement could be supported by the fact that TE1 needs to perform some communicative strategies, such as specifying the expression 'say something about it' by turn [48] 'say something sensible about it' and by turn [52] 'say something precise about it', in order to continue the communicative project. In [54] TE1 seems delighted about PT's contribution in [53] "It is an even number". Notice though that in [44] TE1 was about to introduce the fact that 98 is an even number. The discourse connected to turns [53]–[60] proceeds without complications and PT and TE1 continue the communicative project of turning $\sqrt{98}$ into $7\sqrt{2}$.

The transcript is typical for course 1 for three reasons. First, the topical episode is characterized as subject oriented. That is, subject-oriented topical episodes are focused on mathematics ideas rather than on, e.g., how mathematics should be taught to school students or discussions of mathematical problem solving. Naturally, each and every task presentation episode includes topical episodes that are subject oriented. However, in course 1, typically all topical episodes of a task presentation episode are subject oriented and, therefore, subject-oriented communicative projects are ascribed to these task presentation episodes. Second, it shows the asymmetrical co-construction of topical episodes. Expressed differently, the contextualization of TE1 to a large extent determines the communicative projects. However, in course 1, this is just typical for the discussions followed by the presentations. In presenting the task at the chalkboard, the prospective teachers have a significant latitude for different contextualizations. That is, they have a relative freedom of deciding how to present the solutions at the chalkboard (see also Sect. Results of the study). Third, the topical episode analyzed above deals with fairly 'easy' mathematical ideas. That is, most subject-oriented communicative projects concern primary and secondary school mathematics, rather than university mathematics.

The second transcript, analyzed below, is taken from the beginning of course 1. In this particular case, one group of three prospective teachers (PT1, PT2, PT3) has presented two ways of solving the task:

> In a charity meeting money was collected for welfare purposes. It was decided that each and every person should give as much money as the number of participants attending the meeting. In that way 1600 Skr (Swedish crowns) was collected. How many participants attended the meeting?

Here we enter the discussion when the teacher educator (TE1) initiates the communicative project of discussing, with the prospective teachers at the chalkboard and the audience (A1, A2, A3...), alternative ways of solving the task.

[22] TE1—By the way, have you got any alternative solution for this?
[23] A1—I just think that it became a little bit clearer when I used x for the number of participants and y for the amount of money they should pay and after a while I realized that x and y must be the same.
[24] TE1—Two unknowns instead of one makes it even more sophisticated.
[25] A2—I thought that one could draw a graph also.
[26] TE1—Yes.
[27] PT1—I was also thinking in that direction.
[28] TE1—Yes, mhm, then you are working with x squared.
[29] A3—Write a table with the number of people and money then x times x.
[30] TE1—By the way, when is x introduced?
[31] PT1—Yes and graphs, they did not exist.
[32] TE1—No, but x, when does one start to use one unknown?
[33] A4—Secondary school.
[34] TE1—In secondary school they work with pretty simple problems and the question is if x squared comes in to play.
[35] A3—That is the reason why you should use this table.
[36] TE1—Yes, yes that's right.
[37] A3—It will be easier for the children to understand.
[38] TE1—Mhm.
[39] PT1—I think it is easier to take a chance, perhaps it is not so good but it feels like it is, first one does not know which two numbers one will have and then it will be terribly complicated to name them x and y, I think that one should just take a chance.

By [22] TE1 initiates the overall communicative project of discussing alternative ways of solving the task. One of the prospective teachers in the audience (A1) presents his way of solving the task [23]; by introducing two unknowns, x and y. It is here worth noting that A1 says 'it became a little bit clearer when I used...'. In doing so, A1 contextualizes the task as a problem that could be solved by using any mathematically appropriate method. Put differently, A1 does not adapt his way of solving the task to secondary school students since two unknowns are far too complicated for secondary school students in Sweden. This way of contextualizing the task may seem natural if we consider that the course takes place at a mathematics department, which reasonably influences the communicative activity. In [24] TE1 continues the communicative project introduced by A1 by recognizing that two unknowns make the solution more sophisticated. However, TE1 does not stress that this way of solving the task is too complicated. A2 introduces the communicative project of discussing her way of solving the task, namely by drawing a graph. In [27] PT1 comments on this by saying 'I was also thinking in that direction'. The communicative project of discussing graphs for solving the task is continued in [31] where PT1 questions whether graphs are introduced for secondary-school students in Sweden. Hence, PT1 has been thinking about the possibilities of using a graph but rejects the idea since he does not believe that secondary school students are familiar with them. It seems reasonable, therefore, that PT1 contextualizes the task as a problem that should be solved in an understandable way for secondary school students. Further, in [39] PT1 explicitly criticizes the suggestion of introducing two unknowns for solving the task, which supports the contextualization of PT1. In relation to this, we can see that A3 seems to have the same contextualization as PT1 of the task (see turn [37]).

I want to stress two features in connection to the transcript above. First, PT1 seems to adjust the suggested solution for a fictitious secondary-school student. Communicative projects that in some way are related to the teaching of mathematics, rather than mathematics as such (cf. subject-oriented communicative projects), are denoted *didactically oriented communicative projects* (see also Sect. Results of the study). Second, the overall communicative project of discussing different solutions to the same problem is an example of what I chose to denote a *problem-solving-oriented communicative project*.

Before going into more-comprehensive discussions of the results connected to the whole data material, I introduce one more transcript. The transcript below follows after the presentation of solutions to the task:

> David and Jenny take classes in the same school but have to travel different distances to school. Both of them cycle to the same bus stop. When David has reached the bus stop he has cycled $1/3$ of the way to school. However, when Jenny reaches the bus stop she has cycled $2/5$ of the way to school. Who travels the greatest distance to school?

At the chalkboard there are several solutions to this task produced by the prospective teachers, and the teacher educator (TE2) initiates the communicative project of characterizing the solutions by directing a question to the audience (A21, A22, A23...) .

[8] TE2—Which kinds of representations and expressions have they used?
[9] A21—Figures.
[10] TE2—Figures, there exists a graphic solution since he has drawn the distances.
[11] A22—Algebraic.
[12] TE2—An algebraic solution, why?
[13] A22—He has got x and y.
...
[47] TE2—The two solutions at the chalkboard, the first one he has himself called graphic, what would you call the other one?
[48] A23—Algebraic.
[49] TE2—Yes, he has been using the picture as a support and the interesting thing here is that there is a relation between the graphical solution and the algebraic solution, one has not completely solved the problem by means of the picture, but one has understood the problem by means of the picture and then one has worked with the algebraic solution.

TE2 introduces [8] the communicative project of characterizing the mathematical solutions at the chalkboard. In [9], one of the prospective teachers in the audience gives a suggestion, and in [10] TE2 adjusts that answer to fit into the framework of different kinds of solutions. That is, earlier in the course, TE2 introduced a framework of different kinds of mathematical solutions including; concrete solution, algebraic solution, graphic solution, and logical solution. The kind of solution suggested by A22 fits into the framework and TE2 does not need to transform the answer. Instead, TE2 asks for a clarification of why the particular solution could be categorized as algebraic [12]. In [47], TE2 comments upon two other solutions for solving the same task and encourage the prospective teachers in the audience to specify the type of mathematical solution. In [48], an answer is given and in [49] TE2 takes to opportunity to comment on how two different ways of representing the task were used for solving it.

The third transcript was introduced here to give an example of how TE2 and the prospective teachers co-construct communicative projects aimed at differentiating between kinds of mathematical solutions. Such communicative projects are, as described above, called problem solving oriented communicative projects. In general, my analysis of the data shows that such communicative projects are very common in course 2 and introduced both by TE2 and by the prospective teachers. In addition to the framework of different types of mathematical solutions, other types of frameworks were introduced and used in course 2, e.g., specifying steps in the problem solving process, defining a mathematical problem (in relation to mathematical exercise), different ways of teaching mathematical problem solving. Towards the end of the paper, I will discuss the importance of using such explicit conceptual frameworks when teaching mathematical problem solving in teacher education.

Results of the study

By means of the transcripts and the analysis above I will now summarize the most important results of this study. Through the examination of all the data material, three broad types of communicative projects emerged, namely subject-oriented communicative projects, didactically oriented communicative projects, and problem-solving-oriented communicative projects. The occurrence of these types of communicative projects is not surprising; it is the quantitative distribution of communicative projects that are of particular interest. However, before going into discussion of the distribution, I need to characterize the different communicative projects.

Subject-oriented communicative projects deal with the mathematical aspects of the tasks. Put differently, they do not deal with mathematics in relation to the secondary school students' learning of mathematics or in relation to problem solving activities but rather with the mathematical objects and procedures needed for producing an answer to the task. Both the first transcript and A1's turn [23] in the second transcript could serve as examples for this category. In the latter case, A1 does not adjust his way of solving the task in relation to a secondary student. Further, in both courses, two types of subject-oriented communicative projects could be found. One deals with fairly easy mathematics, that is, closely associated with the tasks presented at the chalkboard (cf. the first transcript). The other deals with

Table 1 Distributions of communicative projects

	Subject oriented	Didactically oriented	Problem solving oriented
Course 1	92% (23 of 25)	4% (1 of 25)	4% (1 of 25)
Course 2	0% (0 of 18)	33% (6 of 18)	67% (12 of 18)

university mathematics, for instance, proof of induction or continued fraction, which is typically introduced by the teacher educators. As indicated above, the former kind of subject-oriented communicative projects, focused on primary- and secondary-school mathematics, is much more common.

Didactically oriented communicative projects, on the other hand, deal with mathematics in relation to primary school or secondary school students' learning of mathematics. In the data, there are examples of when the prospective teachers are very explicit about that relation, illustrated here by the turn 'this could be very hard for a high-school student to solve therefore it is important to...' followed by a careful explanation of how to perform the calculations. Studying the second transcript above, we can see that PT1, partly implicitly, relates his way of solving the task to secondary school students, resulting in didactically oriented communicative projects. In both courses, the prospective teachers were given assignments of constructing mathematical tasks for primary and secondary students. However, from my analysis, it seems that these assignments only became an explicit topic of the classroom discourse in course 2. Further, communicative projects aimed at discussing when a certain mathematical idea is introduced in school are also categorized as didactically oriented (cf. turn [30] in the second transcript).

Problem-solving communicative projects require meta-shifts in the discourse. For instance, in the first transcript above TE1 initiates [22] the communicative project of discussing different ways of solving the same task, hence a shift from presenting solutions to discussions of different solutions in relation to one another. Other examples of such problem solving shifts found in the data are; to specify the different mathematical ways of solving the task (cf. the third transcript), different possibilities of interpreting a task, and considering the mathematical soundness of an answer.

The three categories presented above cover all task presentation episodes of the data. In the Table 1, the distribution of the communicative projects ascribed to the task presentation episodes in the two courses is illustrated.

As can be seen in the table, there are substantial differences between the courses. For instance, course 1 is totally dominated by subject-oriented communicative projects while no such communicative projects constitute the discourse in course 2. Does this mean that they were not discussing mathematics in course 2? This is not the way the table should be interpreted. Each task presentation episode in course 2 consists partly of subject-oriented topical episodes. However, within those task presentation episodes a considerable number of either didactically or problem-solving topical episodes are to be found. Therefore, the communicative projects related to those task presentation episodes are categorized as either didactically oriented or problem solving oriented. In the concluding discussions I elaborate on plausible reasons for the divergence found between the courses.

The constructs within the dialogical approach do not only help us to examine what the discourse is about but also how it is co-constructed. In the second transcript from course 1, we can see that both the teacher educator and the prospective teachers actively contribute to the discourse. In studying the whole data material of course 1 it could be seen that the co-construction of the discourse becomes increasingly asymmetrical throughout the course (cf. the first transcript). That is, the teacher educator, to a large extent, initiates and directs the communicative projects of the discussions in the middle and at the end of the course. However, concerning the presentation of the tasks at the chalkboard, the prospective teachers have great latitude for different contextualizations. In other words, the prospective teachers in course 1 have relative freedom to choose what to focus on in the presentations while the teacher educator controls the follow-up discussions. It is hard to produce causal explanations for this development in Course 1, but I can produce some reasonable explanations. In course 1 just one group presented their solution(s) (often just one solution) to a particular task, which I think influenced the nature of the participation in the classroom in several ways. First, as the course proceeded, the groups did not put much energy into preparing the tasks of the other groups, leading to difficulties in commenting on the other groups' solutions. Second, since just one way of solving the task is usually presented there were fewer opportunities to compare solutions. Third, in course 1 the prospective teachers are not explicitly introduced to frameworks for characterizing solutions. As I indicated earlier, and will be discuss below, the role of such frameworks seems to be crucial for engaging the prospective teachers in the discourse.

The analysis of course 2 reveals that TE2 initiates and directs most of the communicative projects. However, in course 2 there is no gradual shift towards more asymmetrical co-construction of the discourses but rather the opposite: the prospective teachers become more active in the construction of the discourses. This finding could be explained by using the complement to what was discussed in relation to the development in course 1, namely, all groups work with all tasks, several solutions to the same task are presented making it easier to highlight the strengths and weaknesses of the solutions, and the prospective teachers are introduced to frameworks designed for commenting on mathematical solutions. As regards the engagement of the prospective teachers in the discourse, the latitude for different contextualizations of how to present the task at the chalkboard is, in contrast to course 1, much more restricted in course 2. Expressed differently, the prospective teachers are given clear instructions that the solutions should be adjusted for school students, that they are supposed to present several solutions, and that they should try to characterize their solutions by means of the framework for different types of mathematical solution.

Concluding discussion

The analytical approach taken in this study did not only help me to realize that the collective communicative projects of the two classroom discourses studied differed in several substantial ways but also how the contextualization of the participants was related to those communicative projects. The divergences found could be given plausible explanations by referring to the principle of act-activity, by relating the communicative projects to the participants' contextualization of the communicative activities of the two classrooms. That is, the analysis indicates that the nature of the communicative activities is to a great degree dependent on the teacher educators' contextualization of the task of arranging a problem solving course for prospective teachers. Of course, the teacher educator's contextualization of arranging a course always affects the communicative activity, but the ambiguity and complexity of the concept of problem solving imply that the teacher educators' contextualizations are of particular importance. For instance, in Sweden, at least, mathematics courses for prospective teachers are often developed in association with a textbook. However, when it comes to problem solving for prospective teachers there is no well-established textbook[6] so the teacher educators of this study used instructional material from a variety of sources such as curricula, scientific articles, secondary school textbooks, national tests in mathematics and tasks formulated by themselves. So, if we agree that instructional materials are important, and that the teacher educators had little guidance in choosing the material, one may argue that the teacher educators' contextualizations of problem solving are of particular importance when it comes to the content of the course. Let me elaborate on this further by focusing on the communicative activities of the two courses.

By analyzing the whole data material, it seems reasonable to summarize the contextualization of TE1 as 'let the prospective teachers present the problems at the chalkboard and see what comes up'. I draw this conclusion from the fact that no explicit instructions of how to present the tasks were introduced and also that no frameworks were used to analyze the solutions. This contextualization lead to instructions that gave the prospective teachers mandate to discuss different interpretations of the task or not, choose to present one or several solutions to the same task, discuss the character of the solution or not, adapt the presentation to fictitious school students or not. So it could be argued that there is a substantial latitude for the prospective teachers to contextualize the enterprise of presenting the solutions at the chalkboard. This seems to be the case in course 1, as this relative freedom leads to more purely subject-oriented presentations. After all, it is more complicated to solve and present several solutions to the same task than it is to present one, so why bother about presenting several solutions if you pass the course anyway?

In contrast, TE2 seems to contextualize the task of arranging a problem solving course as 'to denote and contrast different solutions to the same task'. This interpretation seems reasonable for several reasons. First, several solutions to each task presented at the chalkboard are produced, either by letting all five groups present their solutions or by requesting that each group should present several solutions to the same task. This creates opportunities to contrast different solutions. Second, TE2 introduces and frequently uses problem solving frameworks in commenting about the solutions at the chalkboard. One could therefore argue that the communicative activity established by the measures taken by TE2, more or less, direct the prospective teachers into didactically and problem solving oriented discourses. Expressed differently, the instructions and the conceptual frameworks strongly encourage the prospective teacher to initiate meta-shifts in the discourse, thus initiating didactically oriented and problem-solving communicative projects (see also below).

The findings of this study may be of interest for a number of reasons. First, in Sweden at least, there are numerous discussions, in quantitative terms, of how many

[6] I refer here to textbooks specially designed for teaching Swedish prospective teachers about problem solving. The data of this study were collected in 2004, and in 2005 an interesting candidate for such a book was published.

credit points of mathematics prospective teachers should have. With regard to the findings of the distribution of types of discourse, as well as the nature of the prospective teachers' participation, one may argue that it is absolutely necessary to complement the quantitative discussions by qualitative discussions focusing on what to include in the courses. Second, the results may be interesting for educators arranging, or planning to arrange, problem-solving courses for prospective teachers. Below, I will discuss some issues to consider when analyzing and developing such a didactical problem solving course.

On a fairly general level, the teacher educator must consider if the emphasis of the course should be on mathematical problem solving or didactical problem solving. That is, is the main focus of the course to teach prospective teachers to solve mathematical problems or is the main focus on how to teach mathematical problem solving? The focus of the two courses examined in this study is on didactical problem solving and this may explain why the mathematical problem solving frameworks of 3.1 were of limited help in capturing important characteristics of the data. Below I present a discussion that may help both researchers and educators in analyzing didactical problem solving courses.

First I must clarify why I do not regard the courses of this study as mathematical problem solving courses. The reason is simple: the mathematical tasks discussed in connection with the courses are typically not mathematical problems for the prospective teachers. Instead, the questions of how to present the tasks and how to highlight important educational issues are genuine problems for the prospective teachers. This leads us to the first issue to consider. What we want the prospective teachers to learn should be embedded within the tasks (cf. Lester & Lambdin, 2004). However, as shown in relation to course 1, the tasks chosen create opportunities for several kinds of discussions and the teacher educators need to consider ways of how to introduce the prospective teachers into productive discussion. This could be accomplished by ‘‘initiating shifts in the discourse such that what was previously done in action can become an explicit topic of conversation’’(Cobb, Boufi, McClain, & Whitenack, 1997, p. 269). The study shows that the explicit introduction of conceptual frameworks in course 2 is one way to facilitate such shifts. One may argue that, without such explicit frameworks, the educators' questions may instead ‘‘degenerate into a social guessing game in which students try to infer what the teacher wants them to say and do’’(Cobb et al., 1997, p. 269). This is actually what happened on a number of occasions in course 1 (see the first transcript). Therefore, one should consider carefully the aims of the course and design tasks and conceptual frameworks that facilitate the shifts needed for fulfilling these aims.

Another important issue to discuss is the role of problem solving in the classroom. I suggest that the work of Wyndhamn et al. (2000) could serve as a conceptual framework for introducing discussions of how to use mathematical problems in teaching. For instance, should mathematical problems be used to entertain the best pupils in the class when they have finished the compulsory exercises or should problem solving be an integrated part of the mathematical teaching for all students?

Even though the frameworks of Pòlya (1945) and Schoenfeld (1985, 1992) were of limited help in accounting for what was happening in the problem-solving courses, these conceptual frameworks may help the educator and the prospective teachers to focus their discussions of the problem-solving process and competencies needed for being a successful problem solver.

Finally, if we view learning as becoming a participant in a specific discourse (Kieran, Forman, & Sfard, 2001) it is also important that the prospective teachers become active in the co-construction of that discourse. Here, once again, I claim, by means of the empirical findings of this study, that explicit frameworks facilitate the engagement of prospective teachers in the discourse. From a theoretical perspective this claim also makes sense. That is, if language is seen as a tool for thinking rather than not only a tool for expressing ones thoughts (cf., e.g., Sfard, 2001), one may argue that language, in the form of conceptual frameworks, helps students to think about, and engage in, mathematical classroom discourses.

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