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On power-law type relationships and the Ludwigson explanation for the stress-strain behaviour of AISI 316 stainless steel

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The influence of prior cold work on the stress strain behavior of a type 316 stainless steel is investigated at 300 K. The Ludwigson relation and the often cited explanation for the Ludwigson type stress strain behavior found in stainless steels to a changeover from planar slip to cross slip is reexamined. It appears that the Ludwigson type behavior is more a consequence of a structure sensitive "hardness state" of the material. This "hardness state" is expressed in terms of an equivalent plastic strain ε_0. The Swift equation that essentially incorporate such a correction term for strain and describes well, the stress strain curve and the work hardening in a type 316 stainless steel.

1. Introduction

The tension test is a very important mechanical test for structural materials. The significant phenomenon in the development of stress-strain curve is strain hardening, which essentially arises from the increase in dislocation density with strain. The microstructural evolution with deformation which determines the flow stress σ and the rate of work hardening at a given plastic strain ε, is determined by the interaction of dislocations with solutes, secondary phases, other dislocations and various other defects. If the tensile plastic deformation is sufficiently large, an empirical equation

$$\sigma = K\varepsilon^n \tag{1}$$

proposed by Hollomon [1], is frequently used in applications where an explicit expression for true stress (σ) as a function of true plastic strain (ε) is needed. K and n are fitting constants usually termed as strain hardening coefficient and the strain hardening exponent respectively. K and n are easily determined from a double logarithmic plot of the experimental true stress-true strain data. Many deviations from the Holloman relation have been reported. Double n behaviour has been previously reported in iron and mild steel [2], zirconium alloys [3], copper [4] and type 316 stainless steel [5] and requires the Equation 1 be replaced by

$$\sigma = K_1\varepsilon^{n1}, \ldots \varepsilon \leq \varepsilon_1$$
$$\sigma = K_2\varepsilon^{n2}, \ldots \varepsilon \geq \varepsilon_1$$

where ε_1 is the strain at which the two straight lines in the double logarithmic plot intersect. Even a three stage behaviour was observed by Kashyap and Tangri [6] in type 316L stainless steel. Other empirical relationships, which are modifications of Equation 1, for example:

Ludwigson equation [7]

$$\sigma = K\varepsilon^n + \exp(K_1 + n_1\varepsilon) \tag{2}$$

Ludwik equation [8]

$$\sigma = \sigma_0 + K\varepsilon^n \tag{3}$$

and Swift equation [9]

$$\sigma = K(\varepsilon + \varepsilon_0)^n \tag{4}$$

have also been used to describe the stress strain data.

These relationships are purely empirical in nature and are not based on physical arguments involving dislocation theory. The relative efficacies of various constitutive equations have been compared earlier [10–12]. Efforts are made time and again to understand the physical significance of these empirical constants by correlating with the microstructure [13], dislocation mean free path [14], precipitates [15, 16], grain size [16, 17], alloying elements [18–21], ageing treatments [22], test temperature and strain rate [12, 23], radiation effect [24], yield strength [25], strength ratio [26, 27], fracture toughness [28] etc. vis-à-vis the strain

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hardening. The anomalous variation of flow parameters/strain hardening with temperature and strain rate is considered as a manifestation of dynamic strain ageing (DSA) [23, 29–31].

The understanding of the evolution of microstructure with strain hardening is relevant to many practical problems related to the processing and use of materials. For instance, work hardening rate has an influence on formability, ductility and toughness. The exponent n plays a crucial role in sheet metal forming. The larger the n value, the more the material can elongate before necking and the material can be stretched farther before necking starts. In cup drawing high n values may reduce wrinkling; a high n value results in higher strain hardening in the cup wall, so the material does not fracture easily when the blank holder force is increased. A good machineable material should have low fracture toughness and low strain hardening index n, while the reverse is true for the tool material. It is well known that the tendency to elastic spring-back increases with increase in strength coefficient and with decrease in the elastic stiffness. The value of spring-back angle decreases with increasing value of strain hardening exponent.

In the present study we examine the stress strain behaviour of prior cold worked Type 316 austenitic stainless steel and the deviation from Hollomon relation. In particular we examine the Ludwigson equation and show that the explanation originally offered by Ludwigson [7] and also invoked by others [17–23] that the deviation from Holloman equation is a consequence of planar slip at low strains which changes over to cross slip at large strains, may not be valid. The results favour the Swift equation and the suggestion that with a correction factor ε_0 the stress strain curve follow Equation 4. The dependence of the fitting constants as a function of prior cold work (PCW) vis-à-vis the yield strength of the material is also examined.

2. Experimental

Type 316 stainless steel having chemical composition (wt.%) C: 0.06, Cr: 16.9, Ni: 11.95, Mo: 2.2, Mn: 1.60, S: 0.002, P: 0.036, N: 0.082, obtained from Alloy Steel plant Durgapur as 12.4 mm thick plate in the hot rolled, mill annealed and pickled condition was used in this study. No laboratory re-solution annealing was given to the material. Tensile specimens with 25 mm gauge length and 4 mm gauge diameter were machined from the plate with the tensile axis in the rolling direction to have consistent initial microstructure and texture. Specimens in the mill-annealed condition was tested at room temperature in uniaxial tension at a nominal strain rate of 3.2×10^{-3} s^{-1} in an Instron 1195 universal testing machine. Another set of specimens was prior deformed to 7.5, 16.2 or 24.7% true plastic strain each in uniaxial tension in the same machine, unloaded and then retested immediately at the same strain rate. Results from the prior deformed specimens along with the results from the mill-annealed (zero PCW) specimen will be referred to as "multiple specimen data". True stress, true plastic strain and work hardening rate data were computed for mill-annealed and prior deformed tests after correcting for the change in gauge length and cross sectional area. True stress-strain data for the three PCW cases were also obtained from the true stress-strain data of the specimen in the mill-annealed condition by appropriately shifting the origin. These sets of data along with the data for mill-annealed specimen (zero PCW) will be referred to as "single specimen data".

3. Results and discussion

The true stress-true plastic strain curve segments from single specimen data and multiple specimen data are shown in Fig. 1. It is observed that they almost match within the normal specimen-to-specimen variation. Fig. 2 shows the typical double logarithmic plot of true stress — true plastic strain data for various PCW levels from multiple specimens. The single specimen data also followed the same trend. It is observed that the type 316 steel at room temperature does not obey a power relation suggested by Hollomon.

Ludwigson [7] suggested that the observed deviation from the Holloman relation in stainless steels and other low stacking fault energy materials is due to the change over of planar slip at low strains to cross slip and subsequent cell formation at high strain region. He had introduced a correction term that depends on plastic strain, to account for the stress deviation from Hollomon relation at low strains as the second term in Equation 2. Above a transition strain ε_L, the correction term becomes essentially zero and the Hollomon equation is obeyed. The various parameters derived from the plots in Fig. 2 as per Ludwigson relation

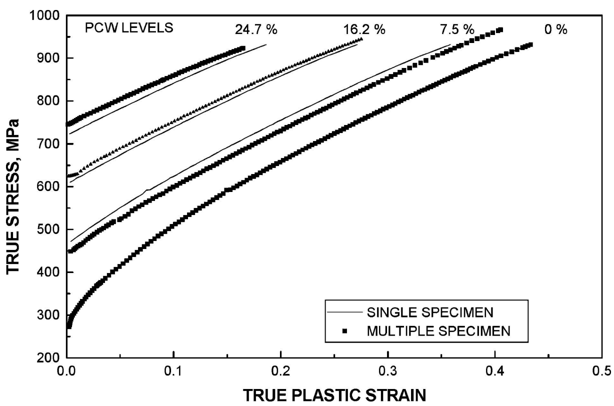

Figure 1 True stress—true plastic strain curve segments for the type 316 stainless steel with different prior cold work levels.

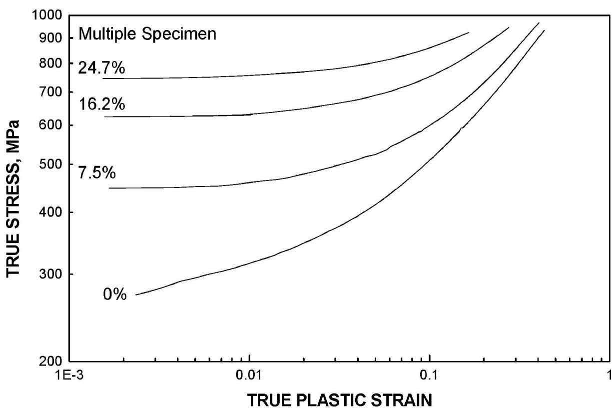

Figure 2 Double logarithmic plot of true stress-true strain data as a function of prior cold work.

(Equation 2) is shown in Table I. Following Ludwigson, the transition strain ε_L was evaluated by setting the value of the ratio r defined as $r = \exp(K_1+n_1\varepsilon)/K\varepsilon^n$ to a small value as near as zero; ε_L and the corresponding transition stress σ_L were determined for two different r-values namely 0.001 and 0.002 and are included in Table I. Truncating the data at small strain ranges affect the fitting parameters. The fitting parameters in Table I are obtained as the best fit parameters and their variation with r is also shown in Table I. ε_L and σ_L values would approach "true values" as r tends to zero; but for any value $r < 0.001$, ε_L and σ_L values to be obtained will be larger than for $r = 0.001$ and therefore does not alter our arguments against the Ludwigson explanation that follow.

It is interesting to note that the strength coefficient (K) and strain-hardening index (n) vary with the amount of PCW or pre strain. It is also noted that the transition strain ε_L, which is about 25% in the mill-annealed material still persist and is as large as 10–11% even after the material was given a prior cold work of 24.7%. If ε_L denotes the change-over to cross slip from planar slip at about 25% strain in the as received material, then either from the material tested after giving a prior cold work of 24.7% in the case of multiple specimen testing or on re-analysing the single specimen data of the mill-annealed material after appropriate shifting of the origin for the strain axis, for 24.7% pre-strain condition, we should have obtained the entire stress strain data fitting to the Hollomon relation. It is observed that in either case this does not happen and the transient region persists to an additional strain as large as 10 to 11%.

Our argument that ε_L does not correspond to change over from planar slip to cross slip is supported by a number of electron microscopic studies [5, 6, 32]. Electron microscopic studies [5, 6] in a type 316 steel by Kashyap and Tangri showed dislocation tangles at low strains of 2–5% and well developed cell structure at large strains (~20%) at 294 K. Detailed TEM studies on the origin of tensile flow stress in 316L stainless steel by Feaugas [32] showed planar slip and single slip in most grains at plastic strain level less than 1.5%. This region was termed as Stage I work-hardening. In Stage II work-hardening which extends from 1.5 to 14% strain, cross slip activation and multiple slip were observed which promoted the formation of heterogeneous dislocation structures. In Stage III which was observed form 14 to 33% strain, the increase in dislocation density was balanced by a dynamic recovery process. In all the three TEM studies, the occurrence of cross slip was seen at strains 1.5–2% which is far below the ε_L values observed for the transition. Hence the suggestion that ε_L denotes the change over from planar slip to cross slip is debatable.

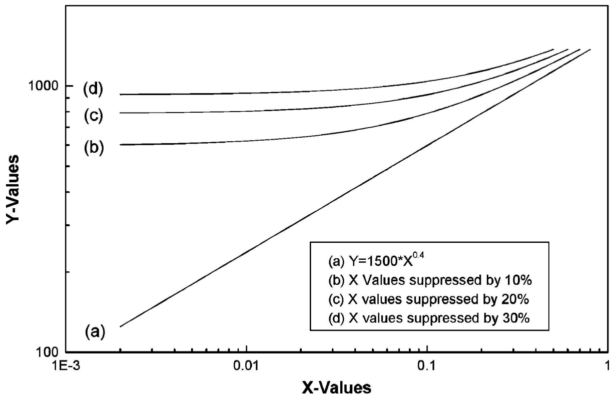

Figure 3 A typical power relation and its transformation with a suppression of x-axis values.

In order to prove this point further, a power function ($Y = 1500.X^{0.40}$) has been generated for X values ranging from 0.002 to 0.8. The data are plotted in a double logarithmic graph as shown in line (a) Fig. 3. In Fig. 3 curves b–d correspond to appropriate shifting of the origin for the X axis to 0.10, 0.20 and 0.30 as in single specimen data described above which in mechanical testing parlance would correspond to giving 10, 20 and 30% PCW.

It is observed that these curves b–d are similar to the curves shown in the log true stress — log true plastic strain plots (Fig. 2) corresponding to a prior deformed material and will yield different values for the fitting constants. Hence one could argue that the apparent transient behaviour on double logarithmic plot is generic to materials, which would have hardened due to some kind of prior deformation. Curves b–d in Fig. 3 can be brought back to the curve (a) by correcting the

TABLE I Strain-hardening parameters derived from Ludwigson relationship for prior cold worked stainless steel at room temperature

	K (MPa)	n	K_1	n_1	$r = 0.001$ ε_L	$r = 0.001$ σ_L (MPa)	$r = 0.002$ ε_L	$r = 0.002$ σ_L (MPa)
(a) From multiple specimen								
Prior cold work (%)								
0	1356 ± 1.73	0.453 ± 0.0008	5.20 ± 0.006	−19.3 ± 0.19	0.2499	724	0.2260	697
7.5	1343 ± 2.82	0.375 ± 0.0008	5.65 ± 0.011	−29.0 ± 0.57	0.2172	752	0.1969	727
16.2	1219 ± 2.91	0.209 ± 0.0011	5.48 ± 0.018	−40.0 ± 1.12	0.1328	792	0.1184	774
24.7	1173 ± 2.67	0.132 ± 0.0015	5.35 ± 0.019	−50.0 ± 2.06	0.1021	862	0.0910	850
(b) From single specimen								
Prior strain (%)								
0	1356 ± 1.73	0.453 ± 0.0008	5.20 ± 0.006	−19.3 ± 0.19	0.2499	724	0.2260	697
7.5	1354 ± 2.06	0.366 ± 0.0012	5.60 ± 0.014	−20.1 ± 0.42	0.2163	858	0.2340	820
16.2	1271 ± 2.85	0.244 ± 0.0019	5.55 ± 0.017	−29.0 ± 0.12	0.1967	855	0.1729	827
24.7	1182 ± 1.92	0.147 ± 0.0013	5.35 ± 0.02	−49.0 ± 0.15	0.1117	855	0.0989	839

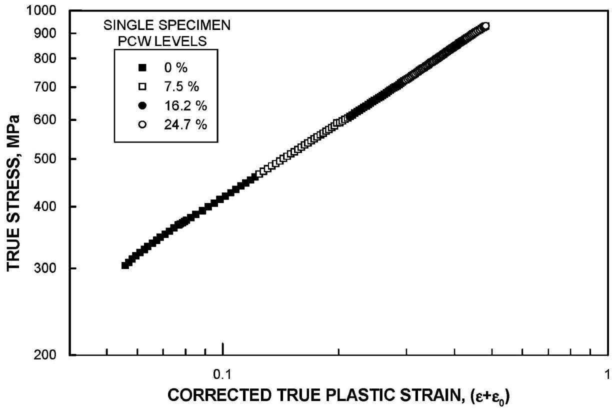

Figure 4 Double logarithmic plot of true stress and corrected true plastic strain for single specimen data.

X values by the amount X_0 which corresponds to the exact shift given to the X axis. To be more explicit, curves b–d would coincide with line (a) if we replot them as 1500 $(X + X_0)^{0.40}$ {with $X_0 = 0.10$, 0.20 and 0.30 for b–d respectively}, a relation equivalent to the Swift equation (4). This leads us to suggest that the four plots in Fig. 2 would yield straight lines if the strain values were corrected by a ε_0 term as in the Swift equation (4). For the data shown in Fig. 2, and for the similar single specimen data, the constants K, n and ε_0 in equation (4) were calculated by a non-linear curve fit by iteration method. These values are shown in Table II. It is observed that ε_0 for the mill-annealed material (zero PCW) from the single specimen data is 0.0488 and the ε_0 values for the three PCW conditions are higher than 0.0488 by the amount of pre-strain given. [$\varepsilon_0(0.075) = 0.0488 + 0.075$] and so on. Therefore it appears that ε_0 is a measure of the initial "hardness state" of the specimen. We also find in Fig. 4 that with the corrected true plastic strain $(\varepsilon + \varepsilon_0)$ the double logarithmic plot yields a single straight line for the single specimen data with $K = 1366$ and $n = 0.523$.

Any comments on the "specimen to specimen" variation in ε_0, n, K etc. at this stage can only be speculative. Localized variation in a plate in microstructure or "hardness state" due to processing variables (stretching treatment after mill-annealing and water quenching) is a possibility. But it is to be mentioned that even in a well annealed type 316 stainless steel after laboratory solution annealing treatment and after different heat treatments on laboratory solution annealed material to obtain different grain sizes with fully austenitic microstructure, we have reported earlier [17] that the transition strain increased from 0.097 to 0.145 with increase in grain size from 0.025 to 0.650 mm. Like all other investigators, at that time we also interpreted the results in the variation of transition strain as due to cross over from planar slip to cross slip. Also Man *et al*. [4] have observed the Ludwigson type behaviour in copper in fully recrystallized samples as well as cold worked samples annealed to different levels of recovery (hardness state). Therefore the original Ludwigson explanation for the transition strain ε_L may not necessarily be a consequence of planar slip changing over to cross slip. Whether there is any correlation between the well established multistage work hardening in fcc polycrystals [32, 33] as revealed in a logarithmic plot between the rate of work hardening and plastic strain and the Ludwigson deviation from the Hollomon equation, needs further study.

According to Equation 4, when $\varepsilon = 0$, an yield stress can be derived as $K(\varepsilon_0)^n$. The derived and the calculated 0.2% yield stress values are also shown in Table II and they agree well.

Another interesting observation is that when we look at the true stress-true plastic strain plots in Fig. 1, it is seen that the 7.5% PCW multiple specimen is weaker than the single specimen while the 16.2 and 24.7% PCW multiple specimens appear to be stronger than the single specimen contrary to the conclusion based on the correction terms ε_0 for the respective specimens. The answer to this paradox or two contradictory observations is the point made by several researchers [33–35] that the rate of work hardening θ is a more appropriate parameter that represents the "hardness/strength/microstructural state" of a specimen than stress or strain. The variation of work hardening rate with flow stress for multiple specimen and single specimen testing showed that work hardening rates of PCW material superimpose on the curve corresponding to that of the as received material. A one-to-one matching is observed in both the cases. A typical example is shown in Fig. 5. A similar super position is obtained for the θ–ε data from single specimen as well as multi specimen testing when essentially an offset of the strain axis is done with the correction term ε_0 as shown in Fig. 6. This clearly indicates that ε_0 is a true measure of the 'hardness state' of a material. Further the fact that θ–σ

TABLE II Empirical constants following the constitutive equation $\sigma = K(\varepsilon+\varepsilon_0)^n$ from a single and multiple specimen tensile testing

	K (MPa)	n	ε_0	$K(\varepsilon_0)^n$ (MPa)	0.2% YS (MPa)
		(a) Single Specimen			
Pre-strain (%)					
0	1354 ± 2.8	0.515 ± 0.003	0.0488 ± 0.0098	286	274
7.5	1358 ± 1.4	0.517 ± 0.0009	0.1235 ± 0.00016	461	470
16.2	1372 ± 1.0	0.526 ± 0.0006	0.21 ± 0.0002	604	610
24.7	1378 ± 1.1	0.533 ± 0.0008	0.296 ± 0.00025	720	723
		(b) Multiple specimens			
PCW (%)					
0	1354 ± 2.8	0.515 ± 0.003	0.0488 ± 0.0098	286	274
7.5	1378 ± 0.8	0.633 ± 0.0032	0.1671 ± 0.0016	444	442
16.2	1368 ± 1.0	0.524 ± 0.0056	0.2198 ± 0.0036	618	628
24.7	1346 ± 1.6	0.409 ± 0.006	0.2337 ± 0.004	743	743

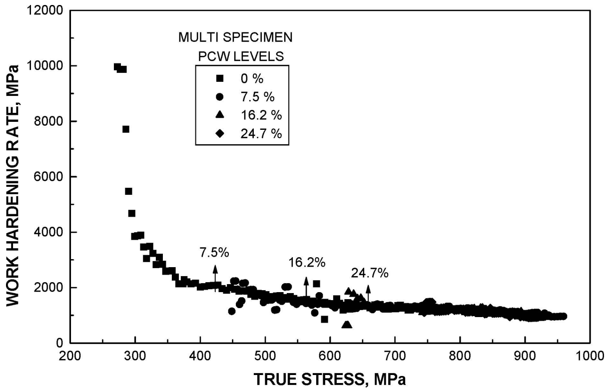

Figure 5 Variation of work hardening rate with true stress for multiple specimen data.

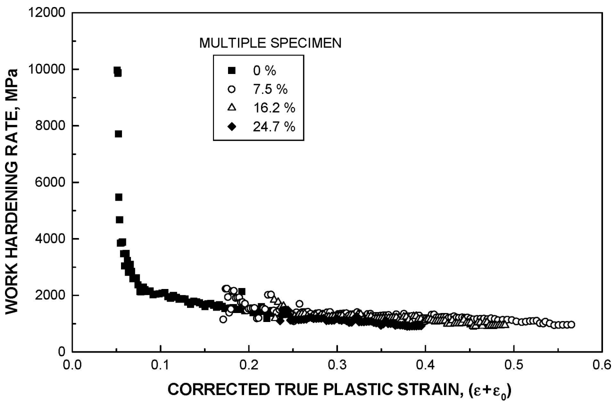

Figure 6 Variation of work hardening rate with corrected true plastic strain for multiple specimen data.

plots do not need an offset where as θ–ε plots do, is also not surprising since in the "mechanical equation of state" [33] σ is a state variable but ε is not.

4. Conclusion

Though the stress strain behavior of type 316 stainless steel at room temperature can be adequately described by Ludwigson type relation, our studies show that the deviation from Hollomon relation need not necessarily arise from a transition from planar to cross slip as suggested by Ludwigson. It appears that the Ludwigson type behaviour is more a consequence of some "hardness state" of the material. This "hardness state" is expressed in terms of an equivalent plastic strain ε_0. The Swift equation essentially incorporates such a correction term for strain and describes well, the stress strain curve and the work hardening in a type 316 stainless steel.

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References

1. H. HOLLOMON, *Trans. AIME* **162** (1945) 268.
2. Y. BERGSTROM and B. ARONSON, *Metall.Trans.* **3** (1972) 1951.
3. S. L. MANNAN and P. RODRIGUEZ, *Trans. Indian Inst. Metals.* **26** (1973) 49.
4. J. MAN, M. HOLZMANN and B. VLACH, *Phys. Stat. Sol.* **19** (1967) 543.
5. B. P. KASHYAP, K. MCTAGGART and K. TANGRI, *Phil. Mag. A.* **57** (1988) 97.
6. B. P. KASHYAP and K. TANGRI, *Acta Metall. Mater.* **43** (1995) 3971.
7. D. C. LUDWIGSON, *Metall. Trans.* **2** (1971) 2825.
8. P. LUDIK, "Elemente der Technologischen Mechanik" (Verlag Von Julius, Springer Leipzig, 1909). p. 32.
9. H. J. SWIFT, *J. Mech. Phys. Solids* **1** (1952) 1.
10. N. S. MISHRA, S. MISHRA and V. RAMASWAMY, *Metall. Trans. A.* **20** (1989) 2819.
11. V. SIVAPRASAD, S. VENUGOPAL and S. VENKADESAN, *Metall. Mater. Trans A.* **28** (1997) 171.
12. B. K. CHOUDHARY, E. ISSAC SAMUEL, K. B. HANU SANKARA RAO and S. L. MANNAN, *Mater. Sci. Technol.* **17** (2001) 223.
13. S. LATHA, K. LAHA, K. B. S. RAO and S. L. MANNAN, *Int. J. Pres. Ves Piping* **67** (1996) 155.
14. ZANG FAN, HUANG MINGZHI and SHIDEKE, *Mater. Sci. Engng.* **A122** (1989) 211.
15. J. W. SIMMONS, *Scripta Metall. Mater.* **32** (1995) 265.
16. S. S. WU, S. Y. CHEN and D. GAN, *Mater. Sci. Engng.* **A127** (1990) L1.
17. S. L. MANNAN, K. G. SAMUEL and P. RODRIGUEZ, *Scripta Metall.* **16** (1982) 255.
18. J. W. SIMMONS, *Acta Mater.* **45** (1997) 2467.
19. A. SOUSAN, S. DEGALLAIX and T. MAGNIN, *Mater. Sci. Engng.* **A142** (1991) 169.
20. J. LESAGE, D. CHICOT, O. BARTIER, M. A. ZAMPRONIO and P. E. V. DE MIRANDA, *ibid.* **A282** (2000) 203.
21. S. NAGARJUNA, M. SRINIVAS, K. BALASUBRAMANIAN and D. S. SARMA, *Scripta Mater.* **38** (1998) 1469.
22. P. N. SINGH and VAKIL SINGH, *ibid.* **34** (1996) 1861.
23. E. ISSAC SAMUEL, B. K. CHOUDHARY and K. B HANU SANKARA RAO, *ibid.* **46** (2002) 507.
24. T. S. BYUN, K. FARRELL, E. H. LEE, J. D. HUNN and L. K. MANSUR, *J. Nucl. Mater.* **298** (2001) 269.
25. D. J. LLOYD, *Scripta Mater.* **48** (2003) 341.
26. A.C. BANNISTER, J. RUIZ OCEJO and F. GUTIERREZ-SOLANA, *Engng. Frac. Mech.* **67** (2000) 547.
27. M. B. TOLOCZKO, M.L. HAMILTON and G. E. LUCAS, *J. Nucl. Mater.* **283–287** (2000) 987.
28. T. PARDOEN and J. W. HUTCHINSON, *Acta Mater.* **51** (2003) 133.
29. B. K. CHOUDHARY, S. K. RAY, K. BHANU SANKARA RAO and S. L. MANNAN, *Berg Huttenmann Monatsh.* **137** (1992) 439.
30. K. G. SAMUEL, S. L. MANNAN and V. M. RADHAKRISHNAN, *Int. J. Pres. Ves. Piping.* **52** (1992) 151.
31. P. RODRIGUEZ, *Bull. Mater. Sci.* **6** (1984) 653.
32. X. FEAUGAS, *Acta Mater.* **47** (1999) 3617.
33. U.F. KOCKS and H. MECKING, *Progr. Mater. Sci.* **48** (2003) 171.
34. P. RODRIGUEZ and S. K. RAY, *Bull. Mater. Sci.* **10** (1988) 133.
35. E. W. HART, *Acta Metall.* **18** (1970) 599.

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