J Math Model Algor (2015) 14:67–89
DOI 10.1007/s10852-014-9258-8

Mathematical Modeling and Computational Algorithm to Solve Multi-Echelon Multi-Constraint Inventory Problem with Errors in Quality Inspection

S. Priyan · R. Uthayakumar

Received: 14 November 2013 / Accepted: 16 April 2014 / Published online: 18 May 2014

Abstract In this research an integrated production-distribution inventory model is developed for a single-vendor single-buyer supply chain system with the consideration of quality inspection errors at the buyer's end, the buyer's warehouse has limited capacity and there is an upper bound on the purchase of products. Mathematical modeling is employed in this study for optimizing the replenishment lot-size and total number of deliveries from the vendor to the buyer in one production run with the objective of minimizing integrated expected total cost of the system while satisfying the constraints. We show that the model of this problem is a constrained non-linear programme and propose a simple Lagrangian multiplier algorithmic technique to solve it. The computational effort and time are small for the proposed algorithm and it is simple to implement. A numerical example is given to demonstrate the application and the performance of the proposed methodology. In addition, sensitivity analysis has been carried out to illustrate the behaviors of the proposed model and some managerial insights are also included.

Keywords Mathematical modeling · Lagrangian multiplier · Quality inspection error

1 Introduction

To achieve business objective, managers have to procure and make best use of resources like money, machines, materials and men. Many of these resources and functions which are under the disposal of managers are inter-related. To attain the common objectives of the organization efficiently, different activities and efforts must be planned and carried on in an orderly manner. Coordination among different business entities such as buyer, vendor,

S. Priyan (✉) · R. Uthayakumar
Department of Mathematics, Gandhigram Rural Institute-Deemed University, Gandhigram-624 302, TamilNadu, India
e-mail: jaisilpriyan@gmail.com

R. Uthayakumar
e-mail: uthayagri@gmail.com

producer, etc. are an important way to gain today's competitive advantages. It involves synchronization of different efforts or actions of the various units of an organization to provide the requisite amount, timing, quality and sequence of efforts so that the planned objectives may be achieved with minimum conflict. In the retailing industry, WalMart and Proctor and Gamble received substantial collaboration benefits by implementing collaborative planning, and replenishment, a business model that intends to help supply chain members to collaborate in both tactical and strategic levels. Therefore, the integrated inventory research has received more attention in the literature. For instance, Goyal and Nebebe [9], Pandey et al. [17], Arora et al. [1], Yan et al. [28], Glock [6] and Uthayakumar and Priyan [24] addressed integrated inventory model under various assumptions.

The traditional Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) inventory models assumed that all products are perfect. It is common to all industries that a certain percent of produced/ordered products are non-conforming (imperfect) quality. Among other researchers, Salameh and Jaber [20] addressed an inventory model which accounted for imperfect quality products using the EPQ/EOQ formulae. Goyal and Cardenas-Barron [7] extended Salameh and Jaber's [20] model and proposed a practical approach to determine EPQ for products with imperfect quality. In contrast to the above models, Goyal et al. [8] examined the model of Goyal and Cardenas-Barron [7], considering vendor-buyer integration policy. Recently, Sana [21] developed a production inventory model of imperfect quality products in a three-layer supply chain. Further, some research (see, for instance, Huang [12], Huang [13], Wang [26] and Sarkar et al. [22]) has been done in multi-stage lot sizing decisions for imperfect production processes with imperfect production quality.

Inspection helps to control the quality of products by helping to fix the sources of defects immediately after they are detected, and it is useful for any factory/or companies that wants to improve productivity, reduce defect rates, and reduce re-work and waste. In the literature, there are several types of inspection plans. Due to the disastrous consequences from accepting a defective component a common practice in industry is to institute repeat (multiple) inspections. Repeat inspection means each characteristic is inspected more than once. The reason for repeat inspections is that inspection is never perfect. There is always the possibility of false rejection (type I error) and false acceptance (type II error). In case of critical components the cost of inspection and the cost of false rejection are much less in order of magnitude than the cost of false acceptance. Because in case of accepting a defective component and mounting it on the system (such as aircraft avionics, the parts of a gas ignition, space shuttles and nuclear reactors) will result in system failure which may cause the loss of the system and human lives. Therefore, many developed countries have always maintenance check of electronic equipment on a Navy aircraft as safety is the highest priority (For instance, see Wee and Widyadana [27]). Hence, the production process is interrupted regularly for maintenance so as to avoid major failures and supply disruptions.

Quality inspection error is an important aspect that demands due consideration in the inventory and supply chain management related literature. Initially, Bennett et al. [4] investigated the effects of inspection errors on a cost-based single sampling plan. Duffuaa [5] addressed the impact of inspection errors on the performance measures of a complete repeat inspection plan. Hsu and Hsu [11] developed two EPQ models with imperfect production processes, inspection errors, planned backorders, and sales returns. Further, some researchers developed EOQ/EPQ inventory model with imperfect quality and inspection errors (see Khan et al. [15], Hsu and Hsu [10] and Khan et al. [14]). In contrast to the existing models, Ben-Daya and Rahim [2] incorporated inspection errors in integrated production, quality, and maintenance decisions in two-stage, as well as in multi-stage, imperfect production-inventory environments. Recently, Khan et al. [14] developed a mathematical model to determine an optimal vendor-buyer inventory policy. They take into quality inspection errors at the buyer's end and learning in production at vendor's end with an objective to minimize the joint annual cost incurred by the supply chain.

Most of the classical inventory models are generally developed for a single constraint. Nevertheless, in real life, this kind of inventory model rarely occurs. Instead of single constraint, many companies, enterprises or buyers deal with several constraints such as available floor/shelf space, capital investment and average number of inventory, etc. Single stage classical inventory models under single resource constraint are presented in well-known text books of this subject. Ben-Daya and Raouf [3] discussed a multi-item inventory model with stochastic demand subject to the restrictions on available space and budget. Recently, Taleizadeh et al. [23] addressed a multi-buyer multi-vendor supply chain problem is considered in which there are several products, each buyer has limited capacity to purchase products, and each vendor has warehouse limitation to store products. Later, some researchers (Pasandideh et al. [18], Pasandideh et al. [19] and Yan et al. [29]) investigated a multi-echelon multi-constraint inventory model under various assumptions.

The aforementioned multi-echelon multi-constraint inventory models showed that the model of the problem is a constrained non-linear program and proposed a genetic algorithm to solve it. The major limitations of the genetic algorithms are the amount of computer memory and the computation time required for large problems. As a result, Uthayakumar and Priyan [25] proposed a Lagrangian multiplier algorithmic technique to solve a multi-echelon multi-product multi-constraint inventory problem under healthcare environment. In this study, we develop a single-vendor single-buyer supply chain system with the consideration of quality inspection errors at the buyer's end, the buyer's warehouse has limited capacity and there is an upper bound on the purchase of products. We develop a mathematical model and recommend the Lagrangian multiplier approach similar to Uthayakumar and Priyan [25] to find the optimal solution of the proposed model. The remainder of this paper is organized as follows: In Section 2, a list of the notations and the description formulation of the model are given. Section 3 provides the solution procedure of the proposed model. The numerical and sensitivity analysis are given in Section 4. Finally, the conclusion of the study is summarized in Section 5.

2 Notations and Model Formulation

In this study, an equal lot-size inventory policy is adopted for a two-stage vendor-buyer supply chain similar to Huang [12]. The vendor follows an EPQ policy to manufacture a single product. The coordination mechanism is such that the vendor receives the buyer's demand and produces the single product at a finite production rate P; the vendor replenishes the order in a number of equal-sized shipments. It is assumed that the vendor's production processes are imperfect and may produce defective products. Thus, once the buyer receives the lot-size Q, a 100 % screening process is conducted. The screening process and demand proceed simultaneously. The screening process is also imperfect in that an inspector may incorrectly classify a non-defective product as defective (Type I error), or a defective product as non-defective (Type II error). The goal of this study is to determine values of the lot-size Q and total number of deliveries n in a production run such that the integrated expected total cost of the supply chain is minimized and the constraints are satisfied.

2.1 Notations

The following notations would be used throughout the model:

n	Total number of deliveries from the vendor to the buyer in one production run, an integer, a decision variable
Q	Lot-size of a shipment from the vendor to the buyer, a decision variable
A	Buyer's ordering cost per order
h_b	Buyer's holding cost per unit time
h_v	Vendor's holding cost per unit time
S	Vendor's setup cost per setup
s_c	Buyer's unit screening cost ($)
D	Demand for the vendor (units/year)
P	Vendor's production rate (units/year), $P > D$
C_v	Vendor's production cost per unit time ($/year)
$C_a(Q)$	False acceptance cost, which is proportional to the lot size Q($/delivery)
θ	False acceptance cost rate (per monetary unit invested in inventory) per unit time
$C_r(Q)$	False rejections cost, which is proportional to the lot size Q($/delivery)
θ_0	False rejections cost rate (per monetary unit invested in inventory) per unit time
p	Buyer's purchasing price per unit item ($/unit)
F	Buyer's fixed transportation cost per delivery ($/delivery)
v	Buyer's unit variable cost for order handling and receiving ($/unit)
α	Percentage of defective products supplied by the vendor
α_c	The percentage of defective products observed by the buyer through screening $\alpha_c = (1-\alpha)m_1 + \alpha(1-m_2)$
m_1	Probability of a Type I error (classifying a non-defective product as defective)
m_2	Probability of a Type II error (classifying a defective product as non-defective)
$f(\alpha)$	Probability density function of α
$f(m_1)$	Probability density function of m_1
$f(m_2)$	Probability density function of m_2
x	Buyers screening rate (units/year)
T	Time between successive shipments (years)
T_p	Vendor's production time in a cycle
T_d	Vendor's non-production time in a cycle
B	Buyer's maximum available budget to purchase products
f_c	Space occupied per product
W	Buyer's total available storage space

2.2 Inventory Pattern and Base Model

Once the buyer orders a lot size of Q units, the vendor produces the items in a lot size of nQ units in each production cycle of length nQ/D with a constant production rate P units per unit time, and the buyer will receive the supply in n lots each of size Q units. The vendor ships in its first lot as soon as it has Q units. Figures 1 and 2 illustrate the inventory patterns at the buyer and vendor, respectively. Figure 2 leads to the the first lot size of Q units is ready for shipment after time Q/P just after the start of the production. During the production period, $T_p = nQ/P$, the vendor's inventory is building up at a constant production rate P which is higher than demand rate D (i.e. $P > D$), and simultaneously supplies a lot of size

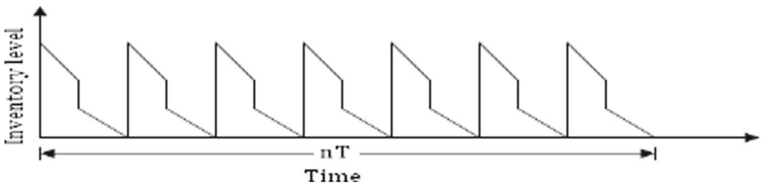

Fig. 1 Buyer's inventory in vendor's one cycle

Q units to the buyer on expected every Q/D units of time. Subsequently, during the non-production period, T_d, the vendor continues his shipments to the buyer on expected every Q/D units of time until the inventory level falls to zero. Figure 1 demonstrates the behavior of the buyer's inventory, which is the same as in Salameh and Jaber [20] and Khan et al. [14]. The buyer starts screening at the beginning of the cycle and discards or salvages the defective lot at the end of this screening process.

Figure 3 demonstrates the same manners of inventory in another appearance. That is, the triangle XOY and the rectangle OCYZ are the total inventory at the vendor's end whereas the shaded rectangles are the total inventory supplied to the buyer (see Salameh and Jaber [20] and Khan et al. [14]). Thus, the total inventory or stock level with the vendor will be determined by using Fig. 3. Alternatively, the buyer's stock level will be calculated by using Fig. 1.

Similar to the method of Goyal et al. [8] the total inventory for the vendor in a cycle is the sum of areas of the triangle XYO and the rectangle OYCZ in Fig. 3, that is,

$$\text{Area}_{XOY} = \frac{1}{2}\left(\frac{nQ}{P}\right)(nQ) = \frac{n^2Q^2}{2P}$$

$$\text{Area}_{OCYZ} = nQ\left[(n-1)\left(\frac{Q}{D} - \frac{Q}{P}\right)\right] = \frac{nQ^2(n-1)(P-D)}{PD}$$

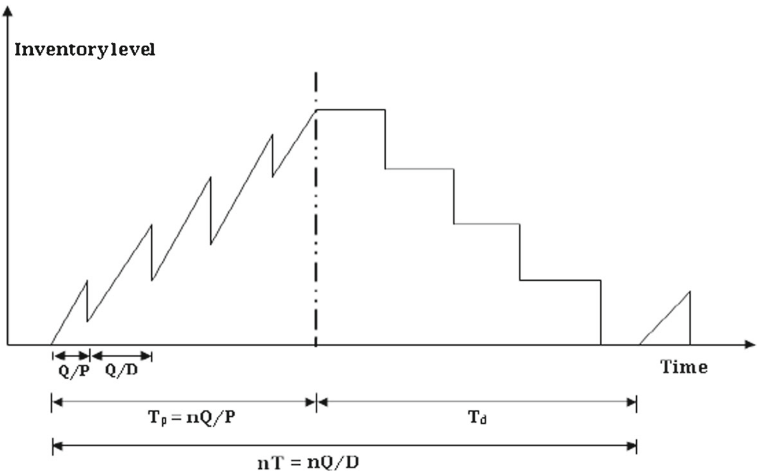

Fig. 2 Vendor's inventory level in a one cycle with time

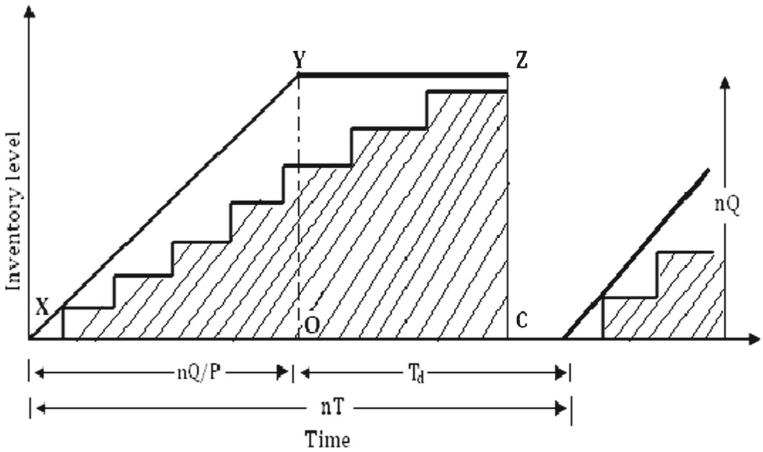

Fig. 3 Accumulation and supply of vendor's inventory in a cycle

The total inventory moved to the buyer in a cycle by the vendor is $\frac{n(n-1)Q^2}{2D}$. Therefore, the vendor's total inventory in a cycle is

$$I_v(Q,n) = \frac{n^2Q^2}{2P} + \frac{nQ^2(n-1)(P-D)}{PD} - \frac{n(n-1)Q^2}{2D}$$
$$= \frac{nQ^2}{2D}\left\{(n-1)-(n-2)\frac{D}{P}\right\}$$

Hence, the vendor's total cost in a cycle is the sum of the setup, holding and production costs

$$TC_v(Q,n) = S + \frac{h_v nQ^2}{2D}\left\{(n-1)-(n-2)\frac{D}{P}\right\} + \frac{nQC_v}{P} \tag{1}$$

Then, the buyer's total cost in a vendor's cycle is the sum of ordering, holding, screening and the shipment costs

$$TC_b(Q,n) = A + nh_b\left\{\frac{Q(1-\alpha)T}{2} + \frac{\alpha Q^2}{x}\right\} + nQs_c + n(F+vQ) \tag{2}$$

Accordingly, the annual total cost in a cycle for the vendor-buyer integrated system is given by sum of Eqs. (1) and (2), mathematically

$$TC(Q,n) = TC_v(Q,n) + TC_b(Q,n)$$
$$TC(Q,n) = S + \frac{h_v nQ^2}{2D}\left\{(n-1)-(n-2)\frac{D}{P}\right\} + \frac{nQC_v}{P} + A$$
$$+nh_b\left\{\frac{Q(1-\alpha)T}{2} + \frac{\alpha Q^2}{x}\right\} + nQs_c + n(F+vQ)$$

Since α is a random variable with probability density function $f(\alpha)$, the integrated expected total cost of the supply chain per cycle, after some algebraic simplification, is

$$TC(Q,n) = S + A + nF + \frac{nQ^2}{2D}\left[h_v\left\{(n-1)-(n-2)\frac{D}{P}\right\} + \frac{2h_b DE[\alpha]}{x}\right] + \frac{nQC_v}{P} + \frac{nh_b Q(1-E[\alpha])E[T]}{2} + nQ(s_c+v)$$

In addition, as we have $E[T] = \frac{(1-E[\alpha])Q}{D}$, using $nE(T)$ as the total cycle time, the integrated expected total cost, by using the renewal theory as in Maddah and Jaber [16], would be

$$E\left[TC(Q,n)\right] = \frac{D}{(1-E[\alpha])Q}\left\{\frac{(S+A)}{n} + F\right\} + \frac{D}{(1-E[\alpha])}\left(s_c + v + \frac{C_v}{P}\right) + \frac{Q}{2}\left[\frac{h_v}{(1-E[\alpha])}\left\{(n-1)-(n-2)\frac{D}{P}\right\} + h_b\left\{(1-E[\alpha]) + \frac{2DE[\alpha]}{x(1-E[\alpha])}\right\}\right] \tag{3}$$

2.3 Inspection Errors

The performance of an inspection plan is greatly influenced by inspection errors. An inspector can commit two types of errors. Type I error is the probability of classifying a non-defective item as defective and type II error is the probability of classifying a defective item as non-defective. These errors may have an adverse effect on the ability of an inspection plan to ensure product quality. However, the screening process in most of the supply chain literature is assumed to be error-free, for instance Goyal et al. [8] and Huang [12]. In this section, it is assumed that the inspectors at the buyer's end commit errors while screening the vendor's product. That is, they will categorize some non-defective units as defectives, i.e. $(1-\alpha)m_1$, while some defective units as non-defectives, i.e. αm_2. In other words, they will characteristic a percentage of defective to the vendor which is dissimilar from the actual one. Thus, the fraction of defective units as professed by the inspectors would be

$$\alpha_c = (1-\alpha)m_1 + \alpha(1-m_2)$$

and

$$E[\alpha_c] = (1-E[\alpha])E[m_1] + E[\alpha](1-E[m_2])$$

So the time interval between successive shipments would now be

$$E[T] = \frac{(1-E[\alpha_c])Q}{D} = \frac{\{(1-E[\alpha])E[m_1] + E[\alpha](1-E[m_2])\}\,Q}{D}$$

The errors in screening will cause the buyer's costs of misclassifications. That is, the cost of false rejection of non-defective items (Type 1 error) and the cost of false acceptance of defective items (Type II error). In this study, we assume that the cost of false acceptance C_a and false rejections C_r are function of lot-size Q. That is, $C_a(Q) = \theta Q$ where $0 \le \theta \le 1$ and $C_r(Q) = \theta_0 Q$ where $0 \le \theta_0 \le 1$. In case of critical components, such as aircraft and healthcare industries where safety is the highest priority, the cost of false acceptance is much more than that of false rejection. To accommodate this fact, the value of θ is chosen to be much higher than that of θ_0 ($\theta \gg \theta_0$) in our numerical example. Thus accounting for these costs, the expected total cost given in Eq. (3), would now be written as

$$\begin{aligned} ETC(Q,n) = {} & \frac{D}{(1-E[\alpha_c])Q}\left\{\frac{(S+A)}{n}+F\right\}+\frac{D}{(1-E[\alpha_c])}\left(s_c+v+\frac{C_v}{P}\right) \\ & +\frac{Q}{2}\left[\frac{h_v}{(1-E[\alpha_c])}\left\{(n-1)-(n-2)\frac{D}{P}\right\}\right. \\ & \left. +h_b\left\{(1-E[\alpha_c])+\frac{2DE[\alpha_c]}{x(1-E[\alpha_c])}\right\}\right] \\ & +\frac{\theta Q E[\alpha]E[m_2]D}{(1-E[\alpha_c])}+\frac{\theta_0 Q(1-E[\alpha])E[m_1]D}{(1-E[\alpha_c])} \end{aligned} \tag{4}$$

As mentioned earlier, our goal is to determine values of the lot-size Q and total number of deliveries n in a production run such that the integrated expected total cost of the supply chain (as expressed in Eq. (4)) is minimized and the constraints are satisfied. The constraints are:

(i) The capacity of the storage to store all products is limited, mathematically, $f_c Q \leq W$
(ii) The purchasing cost for all products is also limited, mathematically, $pQ \leq B$

Now our problem is to find the optimal of Q and n in a production cycle that minimize the integrated expected total cost (as expressed in Eq. (4)) and satisfy the constraints on storage and budget, that is the mathematical formulation of the problem becomes

$$\begin{aligned} Min\ ETC(Q,n) = {} & \frac{D}{(1-E[\alpha_c])Q}\left\{\frac{(S+A)}{n}+F\right\}+\frac{D}{(1-E[\alpha_c])}\left(s_c+v+\frac{C_v}{P}\right) \\ & +\frac{Q}{2}\left[\frac{h_v}{(1-E[\alpha_c])}\left\{(n-1)-(n-2)\frac{D}{P}\right\}\right. \\ & \left. +h_b\left\{(1-E[\alpha_c])+\frac{2DE[\alpha_c]}{x(1-E[\alpha_c])}\right\}\right] \\ & +\frac{\theta Q E[\alpha]E[m_2]D}{(1-E[\alpha_c])}+\frac{\theta_0 Q(1-E[\alpha])E[m_1]D}{(1-E[\alpha_c])} \\ \text{Subject to} \quad & f_c Q \leq W \\ & pQ \leq B. \end{aligned} \tag{5}$$

3 Solution Procedure

The problem (as expressed in Eq. (5)) is a nonlinear programming model, and this problem is hard to solve by a exact method. Therefore, we present a simple Lagrangian multiplier technique similar to Uthayakumar and Priyan [25] to solve the given problem. In this section, the following cases discuss the detailed solution approach of this nonlinear problem.

Case 1 In this case, we temporarily ignore the constraints $f_c Q \leq W$ and $pQ \leq B$ then determine the optimal solutions of Q and n which minimizes the integrated expected total cost $ETC(Q,n)$.

Now we discuss the solution procedure of this case. Initially, for fixed Q, $ETC(Q,n)$ can be proved to be a convex function of n, which indicate that there must be an optimal $n = n^*$ to meet the following equation:

$$\begin{cases} ETC(Q,n^*) \geq ETC(Q,n^*+1) \\ ETC(Q,n^*) \geq ETC(Q,n^*-1) \end{cases}$$

Property 1. *For fixed Q, $ETC(Q,n)$ is convex in n.*

Proof Taking the first and second partial derivatives of $ETC(Q,n)$ with respect to n, we have

$$\frac{\partial ETC(Q,n)}{\partial n} = -\frac{D(S+A)}{(1-E[\alpha_c])Qn^2} + \frac{Qh_v}{2(1-E[\alpha_c])} - \frac{Qh_v D}{2P(1-E[\alpha_c])}$$

and

$$\frac{\partial^2 ETC(Q,n)}{\partial n^2} = \frac{2D(S+A)}{(1-E[\alpha_c])Qn^3} > 0.$$

Therefore, for fixed Q, $ETC(Q,n)$ is convex in n.
This completes the proof of Property 1. □

Now, for fixed n, we take the first partial derivative of $ETC(Q,n)$ with respect to Q, and obtain

$$\begin{aligned} \frac{\partial ETC(Q,n)}{\partial Q} &= -\frac{D}{(1-E[\alpha_c])Q^2}\left\{\frac{(S+A)}{n}+F\right\} \\ &\quad +\frac{\theta E[\alpha]E[m_2]D}{(1-E[\alpha_c])} + \frac{\theta_0(1-E[\alpha])E[m_1]D}{(1-E[\alpha_c])} \\ &\quad +\frac{1}{2}\left[\frac{h_v}{(1-E[\alpha_c])}\left\{(n-1)-(n-2)\frac{D}{P}\right\}\right. \\ &\quad \left. +h_b\left\{(1-E[\alpha_c]) + \frac{2DE[\alpha_c]}{x(1-E[\alpha_c])}\right\}\right] \end{aligned} \tag{6}$$

Hence, for fixed n, $ETC(Q,n)$ is convex in Q since

$$\frac{\partial^2 ETC(Q,n)}{\partial Q^2} = \frac{2D}{(1-E[\alpha_c])Q^3}\left\{\frac{(S+A)}{n}+F\right\} > 0$$

On the other hand, for fixed n, we obtain optimal Q by setting Eq. (6) to zero as

$$Q = \left\{\frac{2D\left(\frac{(S+A)}{n}+F\right)}{\theta E[\alpha]E[m_2]D + \theta_0(1-E[\alpha])E[m_1]D + G(n)}\right\}^{1/2} \tag{7}$$

where $G(n) = h_v\left((n-1)-(n-2)\frac{D}{P}\right) + h_b\left((1-E[\alpha_c])^2 + \frac{2DE[\alpha_c]}{x}\right)$

Thus, for fixed n, when all constraints are ignored, Eq. (7) gives optimal value of Q such that the integrated expected total cost is minimum. Furthermore, based on the convexity behavior of objective function with respect to the decision variables the following algorithm is developed to find the optimal values for Q and n.

Algorithm 1

Step 1. Set $n = 1$.
Step 2. Determine Q from Eq. (7).
Step 3. Compute the corresponding $ETC(Q, n)$ by putting Q in Eq. (4).
Step 4. Set $n = n + 1$, repeat the step 2 and 3 to get $ETC(Q, n)$.
Step 5. If $ETC(Q, n) \leq ETC(Q, n - 1)$, then go to step 4, otherwise go to step 6.
Step 6. Set $(Q^*, n^*) = (Q, n - 1)$, then the set (Q^*, n^*) is the optimal solution for multi-echelon multi-constraint inventory system when both constraints are ignored.

Case 2 In this case, we consider the buyer's floor space and ignore budget constraint. In order to solve this kind of problem we optimize the following function adding a Lagrange multiplier β:

$$\begin{aligned} Min\ ETC(Q, \beta, n) = & \frac{D}{(1 - E[\alpha_c])Q}\left\{\frac{(S + A)}{n} + F\right\} + \frac{D}{(1 - E[\alpha_c])}\left(s_c + v + \frac{C_v}{P}\right) \\ & + \frac{Q}{2}\left[\frac{h_v}{(1 - E[\alpha_c])}\left\{(n - 1) - (n - 2)\frac{D}{P}\right\}\right. \\ & \left. + h_b\left\{(1 - E[\alpha_c]) + \frac{2DE[\alpha_c]}{x(1 - E[\alpha_c])}\right\}\right] \\ & + \frac{\theta Q E[\alpha]E[m_2]D}{(1 - E[\alpha_c])} + \frac{\theta_0 Q(1 - E[\alpha])E[m_1]D}{(1 - E[\alpha_c])} + \beta(f_c Q - W) \end{aligned} \tag{8}$$

Now we discuss the solution procedure of this case. Initially, for fixed n, the optimal Q can be determined by solving $m + 1$ equations in $m + 1$ unknown variables given by $\frac{\partial ETC(Q,\beta,n)}{\partial Q} = 0$ and $\frac{\partial ETC(Q,\beta,n)}{\partial \beta} = 0$. Without loss of generality, we present the final results

$$Q = \left\{\frac{2D\left(\frac{(S+A)}{n} + F\right)}{\theta E[\alpha]E[m_2]D + \theta_0(1 - E[\alpha])E[m_1]D + G(n) + \beta f_c}\right\}^{1/2} \tag{9}$$

where the β value can be determined by solving the following equation:

$$f_c\left[\sqrt{\frac{2D\left(\frac{(S+A)}{n} + F\right)}{\theta E[\alpha]E[m_2]D + \theta_0(1 - E[\alpha])E[m_1]D + G(n) + \beta f_c}}\right] - W = 0 \tag{10}$$

Further, as in the first case, we can prove that $ETC(Q, \beta, n)$ is a convex function of n. Now, based on the convexity behavior of objective function with respect to the decision variables the following algorithm is developed to find the optimal values for Q, n and β.

Algorithm 2

Step 1. Set $n = 1$.
Step 2. Calculate β value solving Eq. (10).
Step 3. Compute Q from Eq. (9) by using the value of β.
Step 4. Compute the corresponding $ETC(Q, \beta, n)$ by putting Q and β in Eq. (8).
Step 5. Set $n = n + 1$, repeat the step 2 to 4 to get $ETC(Q, \beta, n)$.
Step 6. If $ETC(Q, \beta, n) \leq ETC(Q, \beta, n - 1)$, then go to step 5, otherwise go to step 7.
Step 7. Set $(Q^*, \beta^*, n^*) = (Q, \beta, n - 1)$, then the set (Q^*, β^*, n^*) is the optimal solution for multi-echelon multi-constraint inventory system when budget constraint is ignored.

Case 3 In this case, we consider the buyer's budget and ignore floor space constraint. In order to solve this kind of problem we optimize the following function adding a Lagrange multiplier γ:

$$\begin{aligned} Min\ ETC(Q,\gamma,n) = {} & \frac{D}{(1-E[\alpha_c])Q}\left\{\frac{(S+A)}{n}+F\right\}+\frac{D}{(1-E[\alpha_c])}\left(s_c+v+\frac{C_v}{P}\right) \\ & +\frac{Q}{2}\left[\frac{h_v}{(1-E[\alpha_c])}\left\{(n-1)-(n-2)\frac{D}{P}\right\}\right. \\ & \left.+h_b\left\{(1-E[\alpha_c])+\frac{2DE[\alpha_c]}{x(1-E[\alpha_c])}\right\}\right] \\ & +\frac{\theta QE[\alpha]E[m_2]D}{(1-E[\alpha_c])}+\frac{\theta_0 Q(1-E[\alpha])E[m_1]D}{(1-E[\alpha_c])}+\gamma(pQ-B) \end{aligned} \tag{11}$$

Now we discuss the solution procedure of this case. Initially, for fixed n, the optimal Q can be determined by solving $m+1$ equations in $m+1$ unknown variables given by $\frac{\partial ETC(Q,\gamma,n)}{\partial Q} = 0$ and $\frac{\partial ETC(Q,\gamma,n)}{\partial \gamma} = 0$. Without loss of generality, we present the final results

$$Q = \left\{\frac{2D\left(\frac{(S+A)}{n}+F\right)}{\theta E[\alpha]E[m_2]D+\theta_0(1-E[\alpha])E[m_1]D+G(n)+\gamma p}\right\}^{1/2} \tag{12}$$

where the γ value can be determined by solving the following equation:

$$p\left[\sqrt{\frac{2D\left(\frac{(S+A)}{n}+F\right)}{\theta E[\alpha]E[m_2]D+\theta_0(1-E[\alpha])E[m_1]D+G(n)+\gamma p}}\right]-B=0 \tag{13}$$

In addition, as in the first case, we can prove that $ETC(Q, \gamma, n)$ is a convex function of n. Based on the convexity behavior of objective function with respect to the decision variables the following algorithm is developed to find the optimal values for Q, n and γ.

Algorithm 3

Step 1. Set $n = 1$.
Step 2. Calculate γ value solving Eq. (13).
Step 3. Compute Q from Eq. (12) by using the value of γ.
Step 4. Compute the corresponding $ETC(Q, \gamma, n)$ by putting Q and γ in Eq. (11).
Step 5. Set $n = n + 1$, repeat the step 2 to 4 to get $ETC(Q, \gamma, n)$.
Step 6. If $ETC(Q, \gamma, n) \leq ETC(Q, \gamma, n-1)$, then go to step 5, otherwise go to step 7.
Step 7. Set $(Q^*, \gamma^*, n^*) = (Q, \gamma, n-1)$, then the set (Q^*, γ^*, n^*) is the optimal solution for multi-and echelon multi-constraint inventory system when space constraint is ignored.

Case 4 In this case, we consider both constraints such as budget and floor space constraint. To solve this kind of problem we optimize the following function adding a Lagrange multipliers β and γ:

$$\begin{aligned} Min\ ETC(Q,\beta,\gamma,n) = {} & \frac{D}{(1-E[\alpha_c])Q}\left\{\frac{(S+A)}{n}+F\right\} + \frac{D}{(1-E[\alpha_c])}\left(s_c+v+\frac{C_v}{P}\right) \\ & + \frac{Q}{2}\left[\frac{h_v}{(1-E[\alpha_c])}\left\{(n-1)-(n-2)\frac{D}{P}\right\}\right. \\ & \left. + h_b\left\{(1-E[\alpha_c]) + \frac{2DE[\alpha_c]}{x(1-E[\alpha_c])}\right\}\right] \\ & + \frac{\theta Q E[\alpha]E[m_2]D}{(1-E[\alpha_c])} + \frac{\theta_0 Q(1-E[\alpha])E[m_1]D}{(1-E[\alpha_c])} \\ & + \beta(f_c Q - W) + \gamma(pQ - B) \end{aligned} \tag{14}$$

Now we discuss the solution procedure of this case. Initially, for fixed n, the optimal Q can be determined by solving $m+2$ equations in $m+2$ unknown variables given by $\frac{\partial ETC(Q,\beta,\gamma,n)}{\partial Q} = 0$, $\frac{\partial ETC(Q,\beta,\gamma,n)}{\partial \beta} = 0$ and $\frac{\partial ETC(Q,\beta,\gamma,n)}{\partial \gamma} = 0$. Without loss of generality, we present the final results

$$Q = \left\{\frac{2D\left(\frac{(S+A)}{n}+F\right)}{\theta E[\alpha]E[m_2]D + \theta_0(1-E[\alpha])E[m_1]D + G(n) + \beta f_c + \gamma p}\right\}^{1/2} \tag{15}$$

where the β and γ values can be determined by solving the following equations simultaneously:

$$f_c\left[\sqrt{\frac{2D\left(\frac{(S+A)}{n}+F\right)}{\theta E[\alpha]E[m_2]D + \theta_0(1-E[\alpha])E[m_1]D + G(n) + \beta f_c + \gamma p}}\right] - W = 0 \tag{16}$$

and

$$p\left[\sqrt{\frac{2D\left(\frac{(S+A)}{n}+F\right)}{\theta E[\alpha]E[m_2]D + \theta_0(1-E[\alpha])E[m_1]D + G(n) + \beta f_c + \gamma p}}\right] - B = 0 \tag{17}$$

In addition, as in the first case, we can prove that $ETC(Q, \beta, \gamma, n)$ is a convex function of n. Based on the convexity behavior of objective function with respect to the decision variables the following algorithm is developed to find the optimal values for Q, β, γ and n.

Algorithm 4

Step 1. Set $n = 1$.
Step 2. Calculate β and γ values solving Eqs. (16) and (17).
Step 3. Compute Q from Eq. (15) by using the values of β and γ.
Step 4. Compute the corresponding $ETC(Q, \beta, \gamma, n)$ by putting Q, β and γ in Eq. (14).
Step 5. Set $n = n + 1$, repeat the step 2 to 4 to get $ETC(Q, \beta, \gamma, n)$.
Step 6. If $ETC(Q, \beta, \gamma, n) \leq ETC(Q, \beta, \gamma, n - 1)$, then go to step 5, otherwise go to step 7.
Step 7. Set $(Q^*, \beta^*, \gamma^*, n^*) = (Q, \beta, \gamma, n - 1)$, then the set $(Q^*, \beta^*, \gamma^*, n^*)$ is the optimal solution for multi-echelon multi-constraint inventory system when both constraints are considered.

3.1 Main Computational Procedure

When there are two constraints imposed simultaneously in multi-echelon inventory system, based on the above four cases, the main computational procedure to solve the problem is as follows:

Step 1. Ignoring both constraints and determine the optimal values using algorithm 1. If Q satisfy both constraints, then the obtained values of Q and n are optimal solutions such that the integrated expected total cost is minimum and go to step 5.
Step 2. Else solve the optimization problem subject to floor space constraint and ignore budget constraint. That is, determine the optimal values using algorithm 2. If Q satisfy budget constraint, then the obtained values of Q, β and n are optimal solutions such that the integrated expected total cost is minimum and go to step 5.
Step 3. Else solve the optimization problem subject to budget constraint and ignore floor space constraint. That is, determine the optimal values using algorithm 3. If Q satisfy floor space constraint, then the obtained values of Q, γ and n are optimal solutions such that the integrated expected total cost is minimum and go to step 5.
Step 4. If none of the three steps aforementioned is applicable, both constraints are active. Then solve the optimization problem subject to both constraints such as floor space and budget. That is, determine the optimal values using algorithm 4 and the optimal solutions Q, β, γ and n has been found such that the integrated expected total cost is minimum and go to step 5.
Step 5. Stop.

4 Numerical Analysis

In this section, we conduct numerical analysis to illustrate the solution procedure. The values of the following parameters which are almost similar to those used in Goyal et al. [8] and

Salameh and Jaber [20]: $D = 1000$ units/year, $P = 3200$ units/year, $C_v = \$100000$ /year, $x = 175200$ units/year, $A = \$25$/order, S = \$400/setup, h_b =\$5/unit/year, h_v =\$4/unit/year, s_c =\$ 0.5/unit, F = \$25/shipment, v = \$2/unit, $\theta = 0.3$ and $\theta_0 = 0.05$, f_c = 0.4 square feet, $p = \$20$/unit, $B = \$2000$ and $W = 40$ square feet.

In addition, if defective percentage and inspection errors (Type I and Type II) follow uniform distribution with

$$f(\alpha) = \begin{cases} \frac{1}{\lambda}, & 0 \le \alpha \le \lambda \\ 0, & \text{otherwise.} \end{cases}$$

$$f(m_1) = \begin{cases} \frac{1}{\eta}, & 0 \le m_1 \le \eta \\ 0, & \text{otherwise.} \end{cases}$$

$$f(m_2) = \begin{cases} \frac{1}{\delta}, & 0 \le m_2 \le \delta \\ 0, & \text{otherwise.} \end{cases}$$

then we have

$$E[\alpha] = \int_0^{\lambda} \alpha f(\alpha) d\alpha = \int_0^{\lambda} \frac{\alpha}{\lambda}\, d\alpha = \frac{\lambda}{2},$$

$$E[m_1] = \int_0^{\eta} m_1 f(m_1) dm_1 = \int_0^{\eta} \frac{m_1}{\eta}\, dm_1 = \frac{\eta}{2},$$

$$E[m_2] = \int_0^{\delta} m_2 f(m_2) dm_2 = \int_0^{\delta} \frac{m_2}{\delta}\, dm_2 = \frac{\delta}{2},$$

Specifically, if $\lambda = \eta = \delta = 0.04$, we have $E(\alpha) = E(m_1) = E(m_2) = 0.02$.

Calculating optimal inventory policies for multi constraint in a multi echelon inventory system requires efficient solution procedures that can handle large scale inventory systems, reduce the associated computational time, and reduce modeling complexity due to the dependency between echelons [25]. We proposed computational algorithms based on a Lagrangian multiplier approach similar to Uthayakumar and Priyan [25] to solve the two-echelon multi-constraint inventory problem. The computational effort and time are small for the proposed algorithm and it is simple to implement. The algorithms were coded in MATLAB. Here we describe the computational testing done to evaluate the performance of the algorithm in the numerical example. The solution procedure of the numerical example through main computational procedure described in Section 3.1 is given in Table 1.

Now, from Table 1, we can recognized that the integrated expected total cost $ETC(Q, n)$ is clearly lower for $n = 6$ than for $n = 5$ and $n = 7$, and the proposed algorithm reports this as an approximate minimum. In other words, from Table 1, we can verify that

$$ETC(Q, n = 5) > ETC(Q, n = 6) < ETC(Q, n = 7)$$

Accordingly, when both constraints are ignored, we can choose the optimal values for the optimal lot size $Q^* = 96$, total number of deliveries $n^* = 6$ and the corresponding minimum integrated expected total cost $ETC(Q^*, n^*) = \$37256$.

Now we consider both floor space and budget constraints. Then

$$f_c Q^* < 40$$
$$p Q^* < 2000$$

Table 1 Illustration of the solution procedure for the numerical example

n	Q	β	γ	$ETC(.)$
1	359	-	-	37939
2	221	-	-	37490
3	164	-	-	37342
4	131	-	-	37282
5	111	-	-	37260
6	96	-	-	37256*
7	85	-	-	37263
8	77	-	-	37278

*Minimum integrated expected cost

The optimal solution for the given example is not affected by the constraints as the optimal lot-size of Q^* satisfy both constraints, so the constraints can be ignored. Suppose that the optimal lot size does not satisfy the floor space constraint and satisfy budget constraint and we apply the same procedure to find the optimal solution using algorithm 2. Similarly, if the optimal lot size does not satisfy the budget constraint and satisfy floor space constraint, then we apply the same procedure to find the optimal solution using algorithm 3. If the optimal lot size does not satisfy either of the constraints, we apply the same procedure to find the optimal solution using algorithm 4. Hence, the optimal solution for vendor-buyer inventory system involving quality inspection errors with multi-constraints is the lot size $Q^* = 96$, total number of deliveries $n^* = 6$ and the corresponding minimum integrated expected total cost $ETC(Q^*, n^*) = \$37256$.

4.1 Sensitivity Analysis

We now study the effects of changes in the system parameters D, P, C_v, S, A and θ on the optimal lot-size Q^*, total number of deliveries n^*, Lagrangian multipliers β^* and γ^*, and the expected total cost $ETC(.)$ of the given example.

4.1.1 Effects of Demand on Optimal Solution

In order to study how various demand D affect the optimal solution of the proposed model. The demand sensitivity analysis is performed by changing the parameter of D by +50 %, +40 % +30 %, +20 %, +10 %, −10 %, −20 %, −30 %, −40 % and −50 %, and keeping the remaining parameters unchanged. The results of the demand analysis are shown in Table 2 and the corresponding curve of the minimum expected total cost is also plotted in Fig. 4.

4.1.2 Effects of Production Rate on Optimal Solution

In order to study how various production rate P affect the optimal solution of the proposed model. The production rate sensitivity analysis is performed by changing the parameter of P by +50 %, +40 % +30%, +20 %, +10 %, −10 %, −20 %, −30 %, −40 % and −50 %, and keeping the remaining parameters unchanged. The results of the production rate analysis are shown in Table 3 and the corresponding curve of the minimum integrated expected total cost is plotted in Fig. 5.

Table 2 Effects of demand on optimal solution of given example

D	Q^*	β^*	γ^*	n^*	$ETC(.)$
+50 %	70	0.34	0.26	12	55083
+40 %	78	0.13	0.16	10	51570
+30 %	81	0.08	0.11	9	48012
+20 %	96	–	–	7	44454
+10 %	79	0.05	0.08	8	40839
−10 %	68	0.28	0.15	8	33577
−20 %	96	–	–	5	30024
−30 %	91	–	–	5	26394
−40 %	99	–	–	4	22751
−50 %	91	–	–	4	19094

4.1.3 Effects of Production Cost on Optimal Solution

In order to study how various production cost C_v affect the optimal solution of the proposed model. The production cost sensitivity analysis is performed by changing the parameter of C_v by +50 %, +40 % +30 %, +20 %, +10 %, −10 %, −20 %, −30 %, −40 % and −50 %, and keeping the remaining parameters unchanged. The results of the production cost analysis are shown in Table 4 and the corresponding curve of the minimum integrated expected total cost is plotted in Fig. 6.

4.1.4 Effects of Setup Cost on Optimal Solution

In order to study how various setup cost S affect the optimal solution of the proposed model. The setup cost sensitivity analysis is performed by changing the parameter of S by +50 %, +40 % +30 %, +20 %, +10 %, −10 %, −20 %, −30 %, −40 % and −50 %, and keeping the remaining parameters unchanged. The results of the setup cost analysis are shown in Table 5 and the corresponding curve of the minimum integrated expected total cost is plotted in Fig. 7 as well.

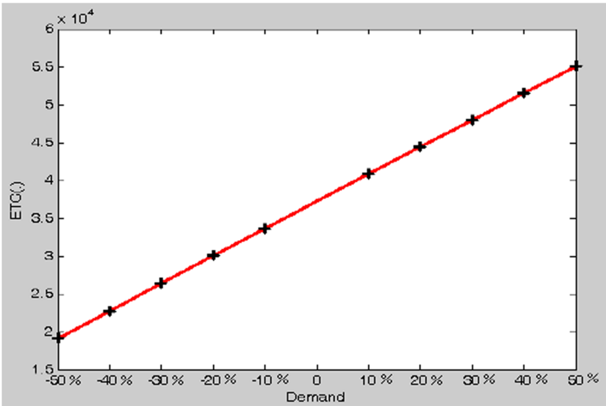

Fig. 4 Curve of minimum $ETC(.)$ for various demand

Table 3 Effects of production rate on optimal solution of given example

P	Q^*	β^*	γ^*	n^*	$ETC(.)$
+50 %	56	0.46	0.38	10	26251
+40 %	63	0.26	0.25	9	27906
+30 %	70	0.20	0.19	8	29725
+20 %	80	0.09	0.11	7	31843
+10 %	95	–	–	6	34321
−10 %	97	–	–	6	40843
−20 %	80	–	–	7	45326
−30 %	91	–	–	7	51083
−40 %	86	–	–	8	58755
−50 %	85	–	–	9	69479

4.1.5 Effects of Ordering Cost on Optimal Solution

In order to study how various ordering cost A affect the optimal solution of the proposed model. The ordering cost sensitivity analysis is performed by changing the parameter of A by +50 %, +40 % +30 %, +20 %, +10 %, −10 %, −20 %, −30 %, −40 % and −50 %, and keeping the remaining parameters unchanged. The results of the ordering cost analysis are shown in Table 6 and the corresponding curve of the minimum integrated expected total cost is plotted in Fig. 8 as well.

4.1.6 Effects of Parameter θ on Optimal Solution

In order to study how various value of θ affect the optimal solution of the proposed model. The parameter θ sensitivity analysis is performed in Table 7. In addition, the minimum integrated expected total cost $ETC(.)$ is plotted for different values of θ in Fig. 9.

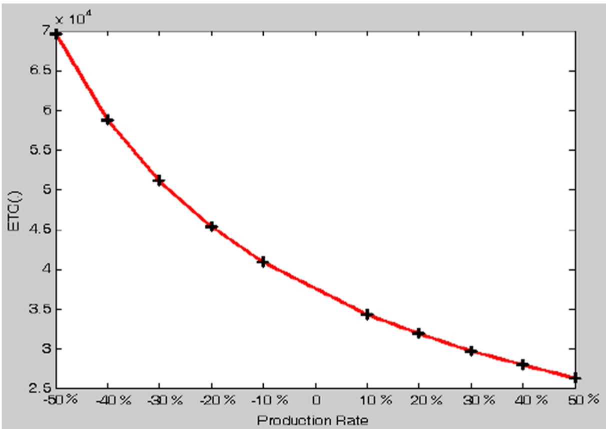

Fig. 5 Curve of minimum $ETC(.)$ for various production rate

Table 4 Effects of production cost on optimal solution of given example

C_v	Q^*	β^*	γ^*	n^*	$ETC(.)$
+50 %	96	-	-	6	53519
+40 %	96	-	-	6	50266
+30 %	96	-	-	6	47014
+20 %	96	-	-	6	43761
+10 %	96	-	-	6	40509
-10 %	96	-	-	6	34004
-20 %	96	-	-	6	30751
-30 %	96	-	-	6	27499
-40 %	96	-	-	6	24246
-50 %	96	-	-	6	20994

4.2 Managerial Insights

In this section, we present some managerial insights of the proposed model based on the numerical results and sensitivity analyses.

(i) Table 2 shows that when the demand D decreases, the total number of deliveries n and expected total cost $ETC(.)$ also decrease. This fact may occurs in real life business, because, if the buyer's demand decrease, then the buyer orders low amount of quantity with the vendor, so that the number of deliveries and costs are automatically decrease.

(ii) If we produce the products at higher production rate, then we can reduce the total cost of the system (see Table 3).

(iii) From Table 4, it is interesting to note that an reduce in the production cost C_v tends to reduce the expected total cost $ETC(.)$ without affecting the lot-size Q and total number of deliveries n. This is expected because Eqs. (7), (9), (12) and

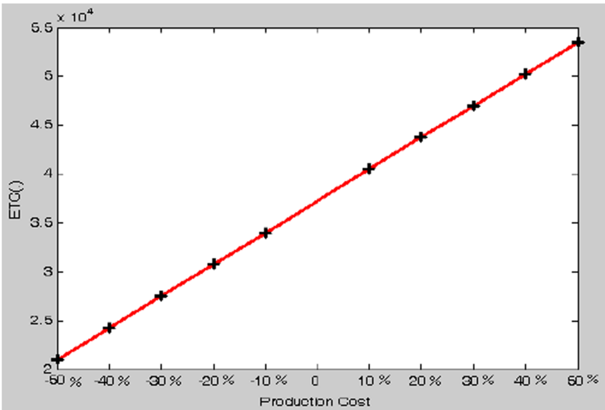

Fig. 6 Curve of minimum $ETC(.)$ for various production cost

Table 5 Effects of setup cost on optimal solution of given example

S	Q^*	β^*	γ^*	n^*	$ETC(.)$
+50 %	99	–	–	7	37594
+40 %	96	–	–	7	37532
+30 %	94	–	–	7	37467
+20 %	65	0.49	0.21	10	37291
+10 %	99	–	–	5	37329
−10 %	64	0.29	0.21	9	37075
−20 %	68	0.13	0.17	8	37025
−30 %	98	–	–	5	37013
−40 %	93	–	–	5	36923
−50 %	57	0.21	0.32	8	36629

(15) show that C_v is an independent of lot-size Q. Further, the optimal solutions for all different values of C_v satisfy both constraints such as floor space and budget.

(iv) Table 5 shows that when the setup cost S decreases, the expected total cost $ETC(.)$ also decrease. This fact is expected because the inventory total cost automatically decrease when the producer control his/her setup cost in practice.

(v) From table 6, it is interesting to note that reduce in the ordering cost A tends to reduce the lot-size Q and expected total cost $ETC(.)$ without affecting the total number of deliveries n.

(vi) Table 7 shows that when the value of θ increases, the expected total cost $ETC(.)$ increase without affecting the lot-size Q and total number of deliveries n. This is expected because Eqs. (7), (9), (12) and (15) show that θ is an independent of lot-size Q. Further, the optimal solutions for all different values of θ satisfy both constraints.

(vii) The proposed model can be used in industries like aircraft, healthcare, automobiles, computers, textiles, footwear, printers, refrigerators, mobile phones, televisions, air

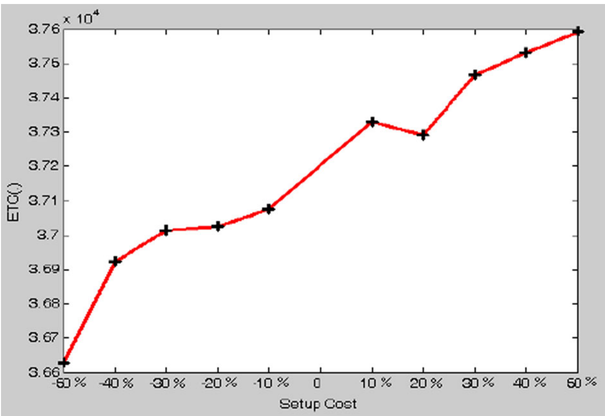

Fig. 7 Curve of minimum $ETC(.)$ for various setup cost

Table 6 Effects of ordering cost on optimal solution of given example

A	Q^*	β^*	γ^*	n^*	$ETC(.)$
+50 %	97	–	–	6	37279
+40 %	97	–	–	6	37274
+30 %	97	–	–	6	37270
+20 %	97	–	–	6	37265
+10 %	96	–	–	6	37261
−10 %	96	–	–	6	37251
−20 %	96	–	–	6	37247
−30 %	96	–	–	6	37242
−40 %	95	–	–	6	37237
−50 %	95	–	–	6	37233

conditioners, washing machines, tyres and bulky products such as printed circuit boards., etc.

5 Conclusion

Inspection is an important step in ensuring product quality especially in aircraft and healthcare industries where safety is the highest priority. Since safety is involved, effective strategies need to be set to improve quality and reliability of aircraft and healthcare inspection/maintenance and for reducing errors. Nevertheless, an important issue lacking in the supply chain literature relates to the integration of such prototypical and ubiquitous human factor as errors in quality inspections. The proposed model addressed a two-level supply chain consisting of a single vendor and a single buyer with the consideration of quality inspection errors at the buyer's end, the buyer's warehouse has limited capacity and there is an upper bound on the purchase of products. Under these conditions, we formulated the

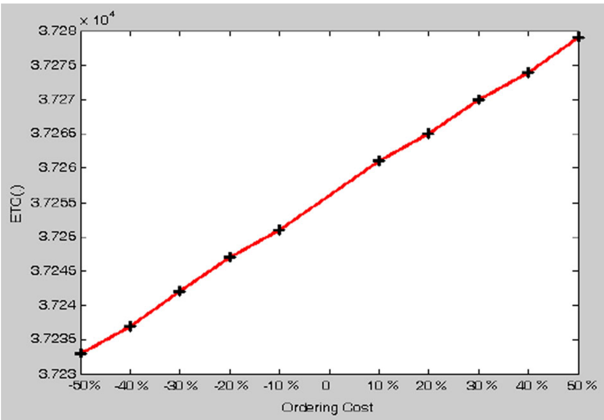

Fig. 8 Curves of minimum $ETC(.)$ for various ordering cost

Table 7 Effects of θ on optimal solution of given example

θ	Q^*	β^*	γ^*	n^*	$ETC(.)$
0.1	96	–	–	6	37248
0.2	96	–	–	6	37252
0.3	96	–	–	6	37256
0.4	96	–	–	6	37260
0.5	96	–	–	6	37264
0.6	96	–	–	6	37268
0.7	96	–	–	6	37272
0.8	96	–	–	6	37276
0.9	96	–	–	6	37280

mathematical problem as a non-linear programming model and recommended a Lagrangian multiplier algorithm similar to Uthayakumar and Priyan [25] to solve it. The proposed algorithm is simple as well as it does not require tedious computational effort and obtains the solution in a very short time. In other words, it does not require any intensive computational effort due to the fact that only evaluates maximum 13 times the total cost function in any instance. All instances were solved using a lap top computer with the following technical characteristics: Intel$^{@}$ Core^{TM} 2 Duo CPU, P8700 @ 2.53 GHz, 3.45 GB of RAM.

There are several extension of this work that could constitute future research related to this field. One immediate probable extension could be to discuss the effect of shortage. Another possible extension of this work may be conducted by considering the vendor's provision of a permissible delay in payments in this integrated inventory model. Also, we can consider multi-echelon supply chains such as; single buyer multiple-vendor, multiple-buyer single-vendor and multiple-buyer multiple-vendor systems. Furthermore, some of parameter of the model may be either fuzzy or random variable. In this case, the model has either fuzzy or stochastic nature.

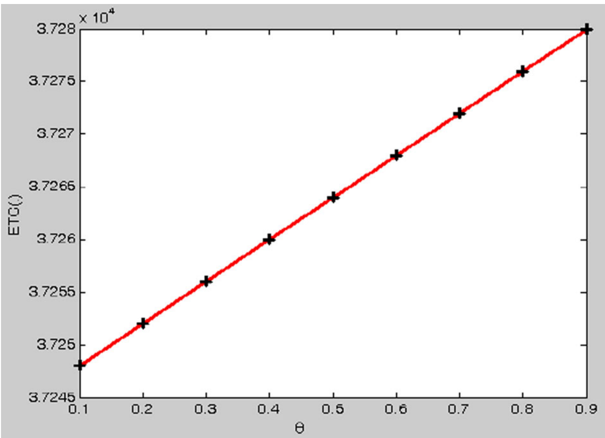

Fig. 9 Curve of minimum $ETC(.)$ for various θ

Acknowledgments The authors would like to acknowledge the Editor of the Journal and anonymous reviewers for their encouragement and constructive comments in revising the paper. This research was fully supported by the University Grants Commission under the scheme of UGC-Basic Science Research.

References

1. Arora, V., Chan, F.T.S., Tiwari, M.K.: An integrated approach for logistic and vendor managed inventory in supply chain. Expert Syst. Appl. **37**, 39–44 (2010)
2. Ben-Daya, M., Rahim, A.: Optimal lot-sizing, quality improvement and inspection errors for multistage production systems. Int. J. Pro. Res. **41**, 65–79 (2003)
3. Ben-Daya, M., Raouf, A.: On the constrained multi-item single period inventory problem. Int. J. Prod. Manag. **13**, 104–112 (1993)
4. Bennett, G.K., Case, K.E., Schmidt, J.W.: Economic effects of inspection error on attribute sampling plans. Nav. Res. Logist. **21**, 431–443 (1974)
5. Duffuaa, S.O.: Impact of inspection errors on the performance measures of a complete repeat inspection plan. Int. J. Prod. Res. **34**, 2035–2049 (1996)
6. Glock, C.H.: The joint economic lot size problem: A review. Int. J. Prod. Econ. **135**, 671–686 (2012)
7. Goyal, S.K., Cardenas-Barron, L.E.: Note on: Economic production quantity model for items with imperfect quality-a practical approach. Int. J. Prod. Econ. **77**, 85–87 (2002)
8. Goyal, S.K., Huang, C.K., Chen, H.K.: A simple integrated production policy of an imperfect item for vendor and buyer. Prod. Plan. Control **14**, 596–602 (2003)
9. Goyal, S.K., Nebebe, F.: Determination of economic production-shipment policy for a single-vendor single-buyer system. Eur. J. Oper. Res. **121**, 175–178 (2000)
10. Hsu, J.T., Hsu, L.F.: A note on An economic order quantity (EOQ) for items with imperfect quality and inspection errors. Int. J. Ind. Eng. Comput. **3**, 695–702 (2012)
11. Hsu, J.T., Hsu, L.F.: Two EPQ models with imperfect production processes, inspection errors, planned backorders, and sales returns. Comput. Ind. Eng. **64**, 389–402 (2013)
12. Huang, C.K.: An integrated vendor-buyer cooperative inventory model for items with imperfect quality. Prod. Plan. Control **13**, 355–361 (2002)
13. Huang, C.K.: An optimal policy for a single-vendor single-buyer integrated production-inventory problem with process unreliability consideration. Int. J. Prod. Econ. **91**, 91–98 (2004)
14. Khan, M., Jaber, M.Y., Abdul-Rahim, A.: An integrated supply chain model with errors in quality inspection and learning in production. Omega **42**, 16–24 (2014)
15. Khan, M., Jaber, M.Y., Bonney, M.: An economic order quantity (EOQ) for items with imperfect quality and inspection errors. Int. J. Prod. Econ. **133**, 113–118 (2011)
16. Maddah, B., Jaber, M.Y.: Economic order quantity for items with imperfect quality: Revisited. Int. J. Prod. Econ. **112**, 808–815 (2008)
17. Pandey, A., Masin, M., Prabhu, V.: Adaptive logistic controller for integrated design of distributed supply chains. J. Manuf. Syst. **26**, 108–115 (2007)
18. Pasandideh, S.H.R., Niaki, S.T.A., Yeganeh, J.A.: A parameter-tuned genetic algorithm for multi-product economic production quantity model with space constraint, discrete delivery orders and shortages. Adv. Eng. Softw. **41**, 306–314 (2010)
19. Pasandideh, S.H.R., Niaki, S.T.A., Nia, A.R.: A genetic algorithm for vendor managed inventory control system of multi-product multi-constraint economic order quantity model. Expert Syst. Appl. **38**, 2708–2716 (2011)
20. Salameh, M.K., Jaber, M.Y.: Economic production quantity model for items with imperfect quality. Int. J. Prod. Econ. **64**, 59–64 (2000)
21. Sana, S.S.: A production-inventory model of imperfect quality products in a three-layer supply chain. Decis. Support Syst. **50**, 539–547 (2011)
22. Sarkar, B., Sana, S.S., Chaudhuri, K.: An imperfect production process for time varying demand with inflation and time value of money - An EMQ model. Expert Syst. Appl. **38**, 13543–13548 (2011)
23. Taleizadeh, A.A., Niaki, S.T.A., Barzinpour, F.: Multiple-buyer multiple-vendor multi-product multi-constraint supply chain problem with stochastic demand and variable lead-time: A harmony search algorithm. Appl. Math. Comput. **217**, 9234–9253 (2011)
24. Uthayakumar, R., Priyan, S.: Permissible delay in payments in the two-echelon inventory system with controllable setup cost and lead time under service level constraint. Int. J. Inf. Manag. Sci. **24**, 193–211 (2013)

25. Uthayakumar, R., Priyan, S.: Pharmaceutical supply chain and inventory management strategies: Optimization for a pharmaceutical company and a hospital. Oper. Res. Health Care **2**, 52–64 (2013)
26. Wang, C.H.: Integrated production and product inspection policy for a deteriorating production system. Int. J. Prod. Econ. **95**, 123–134 (2005)
27. Wee, H.M., Widyadana, G.A.: A production model for deteriorating items with stochastic preventive maintenance time and rework process with FIFO rule. Omega **41**, 941–954 (2013)
28. Yan, C., Banerjee, A., Yang, L.: An integrated production-distribution model for a deteriorating inventory item. Int. J. Prod. Econ. **133**, 228–232 (2011)
29. Yan, C., Liao, Y., Banerjee, A.: Multi-product lot scheduling with backordering and shelf-life constraints. Omega **41**, 510–516 (2013)