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Goal Directed Programming for Determining Process Efficiency Using Data Envelopment Analysis

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Abstract There are numerous measurement methods for process performance control. One of which, widely used in quality control programs, is process capability (C_p) index. The advantage of this index is due to the high amount of information extracted from it. Since C_p is independent from a particular measurement unit it can be used to compare several quite different processes. While the relative efficiency of the process performance based on the C_p for a period is satisfying, the process may lose efficiency in the next period for variety of reasons and could not keep up with the standard limits. The objective of this paper is to develop a new approach for measuring the relative efficiency of peer decision making units (DMUs) based upon process capability indices. A case study demonstrates the applicability of the proposed approach.

Keywords Data envelopment analysis · Goal programming · Multi-criteria decision analysis · Process capability (C_p)

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1 Introduction

Appropriate measurement of capability of process can be important issue in quality management and can increase the efficiency of the whole system. Also, it brings competitive advantage for a company. Although, a number of studies have been focused on the performance evaluation of manufacturing processes based upon indices of capability of process [23], but this sort of studies are rare in Iran. Thus, this study wishes to fill this void.

Measurement of process capability (C_p) has been proposed in the context of statistical process control. Process capability is the long-term performance level of the process after it has been brought under statistical control. This means process capability is the range over which the natural variation of the process happens as determined by the system of common causes. Process capability is also the ability of the combination of people, machine, methods, material, and evaluations to generate a product that will constantly meet the design requirements or customer expectation. Process capability is a scientific and a systematic method that uses control charts to identify and eliminate unnatural causes of variation until a state of statistical control is reached.

There are a couple of reasons that the capability of process should be recognized. Some of them can be expressed as follows [2, 17]:

- Process capability evaluation allows us to summarize process capability in terms of meaningful percentages and metrics.
- To forecast the extent to which the process will be able to hold tolerance or customer requirements, based on the law of probability, we can compute how often the process will meet the specifications or the expectations of the customers.
- Process capability helps decision maker to choose the most appropriate process.
- By knowing the capability of processes, decision maker can better determine performance requirements of new machines, parts, and processes.

The proposed discussion by Sullivan [30, 31] was the start of a movement from C_p to C_{pk}. Kane [16, 17] addressed new issues in process capability with focus on the application and statistical specification of multi-criteria process capability. Also, he discussed various applications of C_p and C_{pk} in Japan and US industries. These two indices are used in many cases such as tube-bending process [6], machining [21], heat treatment [10], electronics manufacturing [34], medicine manufacturing [20], and also a number of researchers discussed the estimations of process capability indices for evaluating process quality based on one single sample [3, 17, 22, 24, 25].

Charnes et al. [5] proposed data envelopment analysis (DEA) as an evaluation method for measuring relative efficiency of a set of homogeneous decision making units (DMUs) with multiple inputs and multiple outputs. If the efficiency score equals to one, the DMU under evaluation is efficient. Otherwise, it is inefficient. Since then this method has been studied by a number of researchers in many fields. For example, see [7, 9, 11]. In particular, DEA has been used in railways assessment [15, 19], supplier selection [12–14, 32], airlines evaluation [27], and airports assessment [18, 33]. Table 1 describes some of advantage and disadvantage of the DEA method.

Table 1 Advantages and disadvantages of DEA method

Advantages	Disadvantages
No need to explicitly specify a mathematical form for the production function.	Results are sensitive to the selection of inputs and outputs.
Proven to be useful in uncovering relationships that remain hidden for other methodologies.	You cannot test for the best specification.
Capable of handling multiple inputs and outputs.	The number of efficient DMUs on the frontier tends to increase with the number of inputs and output variables.
Capable of being used with any input-output measurement unit.	When there is no relationship between explanatory factors (within inputs and/or within outputs), DEA views each DMU as unique and fully efficient and efficient scores are very close to 1, which results in a loss of discriminatory power of the method.
The sources of inefficiency can be analyzed and quantified for every evaluated unit.	

Goal programming (GP) is a technique that determines a goal for each of the objective functions. According to the priority of the multiple objectives, deviations from the goals are minimized. This technique was first presented by Charnes et al. [4]. In this regard, Stewart [28] and Cooper [8] have discussed the relations between DEA and multiple criteria decision analysis (MCDA). The difference between goal programming and DEA is that goal programming considers future planning while the DEA evaluates past performance. Cooper [8] addressed various applications of DEA and goal programming, and also proved that structure of additive model of DEA is the same as goal programming. Stewart [29] extended the standard DEA model and took into account long-term goals of top management. He developed a new benchmarking approach in DEA context for future planning. There is no guarantee for current efficient DMUs to remain efficient in future. Azadi et al. [1] developed a goal directed benchmarking theory proposed by Stewart [29] for benchmarking and selecting suppliers in the presence of fuzzy data.

This study combines the DEA and GP to measure process capability indices. Also, this paper develops a new approach for measuring the relative efficiency of peer DMUs based upon process capability indices. Then, a new GP model is provided. The main contributions of this study are as follows:

- For the first time, the long-term goals are determined based on performance capability index.
- A new equation to determine goals for producing the efficiency frontier is developed.

- In Stewart's [29] work, the input goals are larger than input benchmarks and output goals are smaller than the output benchmarks, while in this work the goals are determined conservatively.
- For the first time, this paper proposes a new type of DEA-GP method to assess the process capability of company based upon C_p and C_{pk} indices.

This paper proceeds as follows. In Section 2, the process capability index is introduced. The proposed model is presented in Section 3. Numerical example is given in Section 4. Policy implications are presented in Section 5. Section 6 concludes the paper.

2 Process Capability Index

The C_p index was first introduced by Sullivan [30] in Japan. Advantage of this index is high volume of information it offers and since it is independent from a particular measurement unit, it can be used in comparison with several quite different processes. As an example, consider C_p of 0.9 corresponding to copper layer thicknesses in a plating process and that of 1.3 in resistance (in Ohm) of electrical components. Although the units considered in each process are expressed in inches and Ohms, it is quite obvious that process of manufacturing electrical parts is more capable than plating process. The C_p index is calculated using following formula [26].

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

where USL and LSL are respectively the upper and lower specification limits, and σ is the standard deviation. It should be noted that USL and LSL is determined based on standards. The C_p index can be separately discussed as follows [17].

- If $C_p < 1$, the process would not be able to hold the acceptable limits (Fig. 1).
- If $C_p = 1$, the process would almost be able to hold the acceptable limits (Fig. 2).
- If $C_p > 1$, the process would be able to hold the acceptable limits (Fig. 3). In such case, if C_p >1.33, where distance between specification limits is 8σ, it is usually considered very favorable and most companies seek it as a goal.

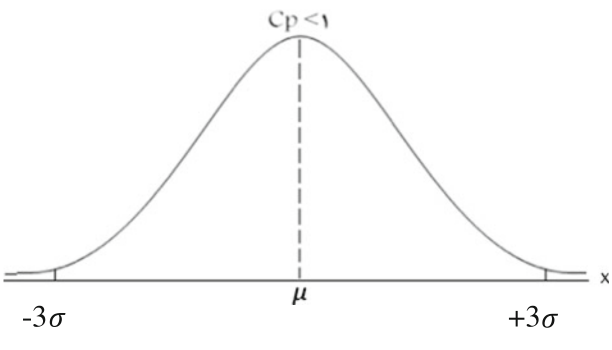

Fig. 1 Distribution of parts for $C_p < 1$

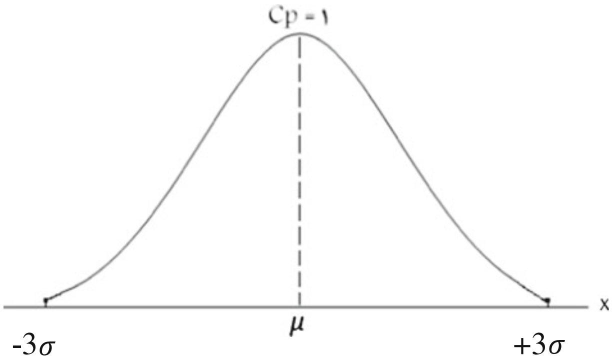

Fig. 2 Distribution of parts for a capable process ($C_p = 1$)

In calculation of C_p index, process situation is not taken into account and only capability of process in meeting the acceptable specifications is considered. Figure 4 shows two processes with $C_p = 1$; one process is set in the middle of specification limits, and the other is close to the upper limit of specification. Figure 4a implies a good quality product and Fig. 4b implies a low quality product.

There is another index, denoted by C_{pk}, which shows process situation.

$$C_{pk} = (1 - k)C_p \tag{2}$$

where k in C_{pk} associates with factor k and is calculated by the following formula [26].

$$k = \frac{\left[\frac{usl+lsl}{2} - \mu\right]}{\frac{usl-lsl}{2}} \tag{3}$$

where μ indicates the mean of the process. It can be shown that k always takes an amount between 0 and 1. The factors which affect capability of a process are mean range ($\overline{R}$) and standard deviation (σ). It is clear that as the mean range and standard deviation are decreased, the capability of the process is increased.

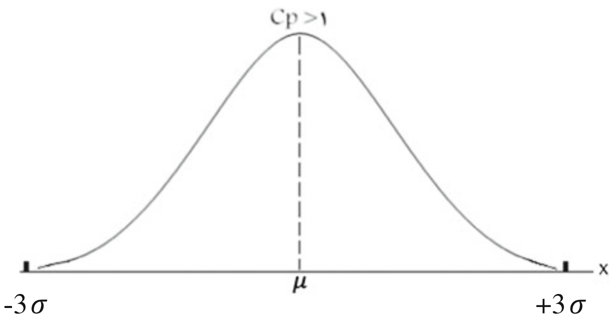

Fig. 3 Optimal product function at minimum costs

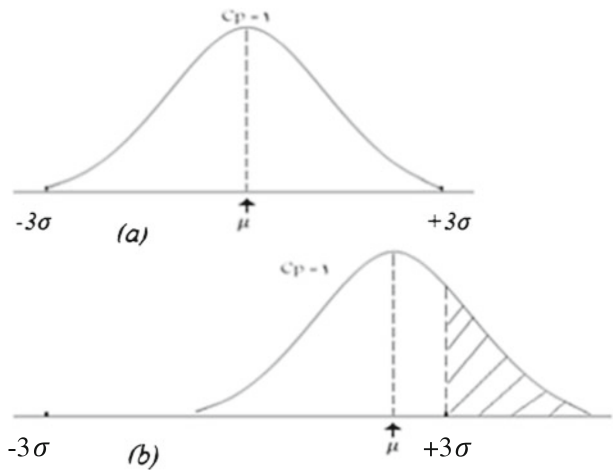

Fig. 4 Distribution of two processes with C_p equal to 1 and different means

2.1 Mean Range ($\overline{R}$)

The range is the size of the smallest interval which contains all the data and provides an indication of statistical dispersion. To find the mean range, we first need to find the lowest and highest values in the data set. The range is found by subtracting the lowest value from the highest value. The ranges are determined mathematically as follows:

$$\overline{R_i} = \max(R_{ij}) \text{ and } \underline{R_i} = \min(R_{ij}) \rightarrow R_i = \overline{R_i} - \underline{R_i}$$

Then

$$\overline{R} = \frac{\sum_{i=1}^{n} R_i}{n}$$

where R_i indicates the value of data. Also, $\underline{R_i}$ and $\overline{R_i}$ indicate lower and upper values, respectively. The $\overline{R}$ denotes mean range.

2.2 Standard Deviation

In statistics and probability theory, standard deviation shows how much variation or dispersion exists from the average (mean), or expected value. A low standard deviation indicates that the data points tend to be very close to the mean; high standard deviation indicates that the data points are spread out over a large range of values. The standard deviation can be calculated by following formula:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2}$$

where $\{x_1, \ldots, x_N\}$ are the observed values of the sample items and $\overline{X}$ is the mean value of these observations, while the denominator N stands for the size of the sample.

3 Proposed Model

In this section, we combine DEA and GP to include long term goals of senior management. This is due to the fact that benchmarking for inefficient DMUs is more than a purely monitoring process, and consists of a component of future planning.

Consider DMU$_k$ (K ∈{1,. . ., n}) which its benchmark is a linear combination of DMUs.

$$x^*_{ik} = \sum\nolimits_{j=1}^{n} \lambda_j x_{ij} \tag{4}$$

$$y^*_{rk} = \sum\nolimits_{j=1}^{n} \lambda_j y_{rj} \tag{5}$$

where x_{ij} denotes the amount of input i consumed by DMU$_j$ and y_{rj} denotes the amount of output r produced by DMU$_j$ and N is the number of DMUs. The x^*_{ik} and y^*_{rk} are the linear combinations of the inputs and outputs which these values could be as input and output benchmarks for the DMU$_k$. The set of inputs and outputs that are defined for all combinations of λ_j are assumed to make the production possibility set (PPS). To determine the weights of λ_j we may employ any of the two conventional DEA methods with the input and output oriented forms as follows.

3.1 Input-Oriented Model

In this model the benchmark is obtained in a way that the most improvement is achieved in the input for the DMU$_k$ with the same outputs. Model (6) is input-oriented CCR (Charnes-Cooper-Rhodes) model proposed by Charnes et al. [5].[1]

$$\begin{aligned} &Minimize \quad E \\ &\qquad Subject\ to: \\ &\qquad\qquad \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} \le E x_{ik}, \qquad (i = 1 \ldots m) \\ &\qquad\qquad \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \le y_{rk}, \qquad (r = 1 \ldots s) \\ &\qquad\qquad \lambda_j \ge 0, \qquad (j = 1, \ldots, n) \end{aligned} \tag{6}$$

where E denotes efficiency measure which is between 0 and 1.

3.2 Output-Oriented Model

In this model the benchmark is obtained in a way that the most improvement is achieved in the output for the DMU$_k$ with the same inputs. Model (7) is

[1]This paper assumes that DMUs have constant returns to scale. Therefore, CCR model is used. Assuming variable returns to scale needs to develop new models based upon BCC model.

output-oriented CCR (Charnes-Cooper-Rhodes) model proposed by Charnes et al. [5].

$$\begin{aligned} &Maximize \quad F \\ &\quad Subject\ to: \\ &\qquad \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ik} \qquad , (i = 1 \ldots m) \\ &\qquad \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \le F y_{rk} \quad , (r = 1 \ldots s) \\ &\qquad \lambda_j \ge 0 \quad (j = 1, \ldots, n) \end{aligned} \tag{7}$$

where F denotes efficiency measure which is equal or bigger than 1.

At this juncture, the goals should be determined for finding out the benchmarks. The goals could be determined according to the conditions of the DMUs and the following expressions.

$$g_{ik} < x^*_{ik}, \qquad (i = 1, \ldots, m) \tag{8}$$

$$h_{rk} > y^*_{rk}, \qquad (r = 1, \ldots, s) \tag{9}$$

where g_{ik} and h_{rk} are the ith input goal and the rth output goal for DMU$_k$, respectively. According to the Eq. (8), the ith input goal should be smaller than the linear combination set of inputs and According to the Eq. (9), the rth output goal should be higher than linear combination set of outputs. Also it should be noted that the goals should obtain realistic values. In other words, they should not be much far from the efficient frontier.

The input-oriented GP model for finding appropriate weights, with respect to pre-determined goals is as follows.

$$\begin{aligned} &\min imize \quad && \theta + \varepsilon \left[\sum_{i=1}^{m} \delta_i^k + \sum_{r=1}^{s} \delta_r^k \right] \\ &subject\ to && \sum_{j=1}^{n} x_{ij}\lambda_j - \delta_i^k \ge g_{ik}\theta \quad , \quad (i = 1, \ldots, m) \\ & && \sum_{j=1}^{n} y_{rj}\lambda_j - \delta_r^k \ge y^*_{rk} \quad , \quad (r = 1, \ldots, s) \\ & && \sum_{j=1}^{n} x_{ij}\lambda_j \le x^*_{ik} \quad , \quad (i = 1, \ldots, m) \\ & && \lambda_j \ge 0 \quad , \quad (i = 1, \ldots, n) \end{aligned} \tag{10}$$

where δ_i^k and δ_r^k are respectively the input and output deviational variables which are unconstrained in sign. The g_{io} and h_{ro} are defined as input and output goals of DMU$_k$. In the first constraint of Model (10), the weights of the inputs are determined in a way that the left handside becomes greater than or equal to the input goal. In the second constraint, the weight of the outputs are determined in a way that the left handside becomes greater than or equal to the output benchmark of DMU_k obtained from the expression (5). In the third constraint, the weight of the inputs are determined in a way that the left hand side becomes less than or equal to the value of the input benchmark of DMU_k obtained from the expression (4).

The output-oriented GP model for finding appropriate weights, with respect to pre-determined goals is as follows.

$$
\begin{array}{lll}
\max imize & \theta + \varepsilon \left[\sum\limits_{i=1}^{m} s_i^k + \sum\limits_{r=1}^{s} s_r^k \right] & \\
subject\ to & \sum\limits_{j=1}^{n} x_{ij}\lambda_j - \delta_i^k \leq x_{ik}^* & ,\quad (i = 1, \ldots, m) \\
& \sum\limits_{j=1}^{n} y_{rj}\lambda_j - \delta_r^k \leq h_{rk} & ,\quad (r = 1, \ldots, s) \\
& \sum\limits_{j=1}^{n} y_{ij}\lambda_j \geq y_{ik}^*\theta & ,\quad (i = 1, \ldots, m) \\
& \lambda_j \geq 0 & ,\quad (j = 1, \ldots, n)
\end{array}
\tag{11}
$$

where s_i^k and s_r^k are respectively the input and output deviational variables which are unconstrained in sign. In the first constraint of Model (11), the weights of the inputs are determined in a way that the left hand side becomes less than or equal to the input benchmark of DMU_k obtained from expression (4). In the second constraint, the weight of the outputs are determined in a way that the left hand side becomes less than or equal to the output goal. In the third constraint, the weight of the outputs are determined in a way that the left hand side becomes greater than or equal to the value of the output benchmark of DMU_k obtained from expression (5).

3.3 Computational Framework

Figure 5 describes a computional flow of the proposed approach. The proposed approach starts with primal and dual of the CCR model. The objective of the proposed models (6) and (7) is finding the appropriate weights for inputs and outputs of DMU under evaluation. Then for generating production possibility set based upon chosen appropriate weights, the Eqs. (4) and (5) are applied. Next, the inputs and outputs goals can be determined by using expressions (8) and (9). After specifing goals, for finding appropriate inputs and outputs weights that consider determined goals, the models (10) and (11) are run. Finally, the new efficienct frontier regarding per-determined goals is produced by using Eqs. (4) and (5). The proposed approach has several properties that are expressed as follows:

- Decision maker preferences which we express in terms of goals can be imposed on the DMUs so that they may be beyond the current PPS.
- Such preferences (goals) may be expressed in terms of inputs and/or outputs.
- Benchmarking is not restricted for only inefficient DMUs.

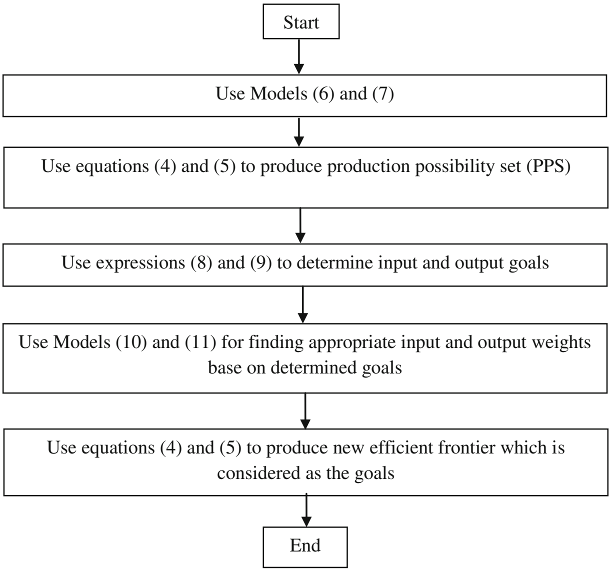

Fig. 5 Summary of discussions

4 Case Study

4.1 Data Set

One of the main contributions of this paper is to develop a GP model for determining long term goals. This study combines DEA and GP method for evaluating the performance capability of can caps manufacturing company based on C_p and C_{pk} indices in Iran during 2011. This is the first study which discusses the relationship between DEA-GP and process capability (C_p and C_{pk}) indices. We select mean range ($\overline{R}$) and mean standard deviation ($\overline{\sigma}$) as inputs and C_p and C_{pk} as outputs. Table 2 depicts the dataset of external diameter of can caps manufactured in "Easy T Can" company. This company is in Hashtgerd Industrial Town, Iran. The dataset in the Table 2 are results of random sampling. The $\overline{X}$ and R are mean and range of dataset from 5 iterations, respectively.

This company is the first decision making unit (DMU_1) in our case study. For the sake of brevity, the dataset of other DMUs are not considered. The standard outside diameter with tolerance of 0.1 is 84 mm.

4.2 Results

Notice that the values of C_{pk}, C_p, $\overline{R}$, and $\overline{\sigma}$ are 1.11, 1.16, 0.0234, and 0.014 according to Eqs. (1), (2) and (3), respectively. These values are listed in Table 3 as the

Table 2 External diameter of can caps related to DMU_1

Sample	Iterations						
	1	2	3	4	5	$\overline{\mathbf{X}}$	R
1	84.030	84.002	84.019	83.992	84.008	84.010	0.038
2	83.995	83.992	84.001	84.011	84.004	84.001	0.019
3	83.988	84.024	84.021	84.005	84.002	84.008	0.036
4	84.002	83.996	83.993	84.015	84.009	84.009	0.022
5	83.992	84.007	84.015	83.989	84.014	84.003	0.026
6	84.009	83.994	83.997	83.985	83.993	83.996	0.024
7	83.995	84.006	83.994	84.000	84.005	84.000	0.012
8	83.985	84.003	83.993	84.015	83.998	83.997	0.030
9	84.008	83.995	84.009	84.005	84.004	84.004	0.014
10	83.998	84.000	83.990	84.007	83.995	83.998	0.017
11	83.994	83.998	83.994	83.995	83.990	83.994	0.008
12	84.004	84.000	84.007	84.000	83.996	84.001	0.011
13	83.983	84.002	83.998	83.997	84.012	83.998	0.029
14	84.006	83.967	83.994	84.000	83.984	83.990	0.039
15	84.012	84.014	83.998	83.999	84.007	84.006	0.016
16	84	83.984	84.005	83.998	83.996	83.997	0.021
17	83.994	84.012	83.986	84.005	84.007	84.001	0.026
18	84.006	84.010	4.0188	84.003	84.000	84.007	0.018
19	83.984	84.002	84.003	84.005	83.997	83.998	0.021
20	84	84.010	84.013	84.020	84.003	84.009	0.020
21	83.988	84.001	84.009	84.005	83.996	84.000	0.033
22	84.004	83.999	83.990	84.006	84.009	84.002	0.019
23	84.010	83.989	83.990	84.009	84.014	84.002	0.025
24	84.015	84.008	83.993	84.000	84.010	84.005	0.022
25	83.982	83.984	83.995	84.017	84.013	84.006	0.035
26	84.012	84.015	84.030	83.986	84.000	84.009	0.044
27	83.995	84.010	83.990	84.015	84.001	84.002	0.025
28	83.987	83.999	83.985	84.000	83.990	83.992	0.015
29	84.008	84.010	84.003	83.991	4.0068	84.004	0.019
30	84.003	84.000	84.001	83.986	83.997	83.997	0.017
						$\overline{\overline{\mathbf{X}}} = 74.0018$	$\overline{\mathbf{R}} = 0.0234$

inputs and outputs of DMU_1. Similarly, for other 29 companies (DMUs) the same process is repeated. The values of σ and $\overline{R}$ are considered as inputs and the values of C_p and C_{pk} are considered as outputs. Obviously, as the values of $\overline{R}$ and σ are decreased the associated DMU gets more capability. The results indicate that the

Table 3 The dataset obtained from 30 DMUs

DMU	Inputs		Outputs	
	$\overline{R}$	$\overline{\sigma}$	C_p	C_{pk}
1	0.0234	0.014	1.16	1.11
2	0.0450	0.019	0.87	0.71
3	0.0410	0.017	0.95	0.94
4	0.0550	0.023	0.70	0.56
5	0.0470	0.020	0.82	0.74
6	0.0500	0.021	0.79	0.39
7	0.0480	0.022	0.75	0.30
8	0.0350	0.015	1.1	0.94
9	0.0390	0.016	0.99	0.70
10	0.0430	0.018	0.90	0.54
11	0.0490	0.021	0.79	0.63
12	0.0580	0.024	0.67	0.53
13	0.0351	0.015	1.1	0.99
14	0.0399	0.017	0.98	0.86
15	0.0420	0.018	0.95	0.70
16	0.0400	0.017	0.97	0.89
17	0.0510	0.021	0.76	0.76
18	0.0480	0.020	0.81	0.32
19	0.0600	0.025	0.66	0.51
20	0.0540	0.023	0.72	0.70
21	0.0420	0.018	0.95	0.57
22	0.0430	0.019	0.87	0.57
23	0.0630	0.027	0.61	0.36
24	0.0600	0.025	0.66	0.26
25	0.0500	0.021	0.78	0.46
26	0.0330	0.014	1.17	1.14
27	0.0550	0.023	0.70	0.26
28	0.0460	0.019	0.85	0.34
29	0.0660	0.028	0.58	0.58
30	0.0410	0.017	0.95	0.57
Average			0.85	0.63

average C_p and C_{pk} for all DMUs are 0.85 and 0.63, respectively. It means all the DMUs have a weak performance capability.[2]

[2]If $C_p < 1$, the process would not be able to hold the acceptable limits (see Section 2).

Now, for instance, consider the DMU_{10} depicted in Table 4. The input and output benchmarks are determined by Models (6) and (7). The goals of DMU_{10} should be according to Eqs. (8) and (9). The Models (10) and (11) are run for finding appropriate input and output weights with respect to determined goals. To produce new efficiency frontier for DMU_{10}, the Eqs. (4) and (5) are used. Therefore, the calculated values ($\overline{R}$, $\hat{\sigma}$, C_p, and C_{pk}) of goal-based benchmarks in Table 4, indicate new efficiency frontier of DMU_{10}. In other words, if DMU_{10} reduces the values of $\overline{R}$ and $\hat{\sigma}$ from 0.043 to 0.029 and 0.018 to 0.012 and also increases C_p and C_{pk} from 0.9 to 1.27 and 0.054 to 1, respectively, it could be concluded that the DMU_{10} would remain efficient in the next period.

4.3 Correlation Between Indices

The purpose of this subsection is to describe relationship between input and output variables. Table 5 provides correlation coefficients between each pair of measures. For instance, the $\widehat{\sigma}$ is statistically correlated with $\overline{\mathbf{R}}$ which the correlation coefficient between two measures is 0.764. This implies that any increase or decrease in the amount of $\overline{\mathbf{R}}$ will cause the increase or decrease in the amount of $\hat{\sigma}$. On the other hand, negative correlations represent the inverse relationship between variables. For instance, the inverse correlation between C_p and $\hat{\sigma}$ indicates that any decrease in the amount of $\hat{\sigma}$ causes the amount of process capability index to be increased which finally leads to the improvement of the production process efficiency.

5 Policy Implications

One of the main objectives of this study is to evaluate the two measures of C_p and C_{pk} of a sample. Since we divided the capability performance into the C_p and C_{pk}, one can easily recognize how an activity can be improved. Given that the average of C_p and C_{pk} of 30 samples are 0.85 and 0.63 respectively, the Fig. 6 is divided into four subsections. The dots in Fig. 6 are samples obtained from the Table 3.

The samples that are in the upper-left quadrant (the DMUs 20, 17, and 5) describe lower C_p and higher C_{pk} index respect to other samples. An appropriate policy implication for these samples is that they need to improve C_p without worsening the C_{pk} index. The samples placed in the lower-right quadrant (the DMUs 22, 10, and 30)

Table 4 Goal-based benchmarks for DMU_{10}

DMU_{10}	$\overline{\mathbf{R}}$	$\widehat{\sigma}$	C_P	C_{pk}
Current performance	0.043	0.018	0.9	0.54
Input-oriented benchmarks	0.039	0.01	–	–
Output-oriented benchmarks	–	–	1	0.81
Goals	0.030	0.008	1.2	1
Goals-based benchmarks	0.029	0.012	1.27	1

Table 5 Correlation coefficients between indices

	$\overline{\mathbf{R}}$	$\widehat{\sigma}$	C_p
$\widehat{\sigma}$	0.764	–	–
C_p	−0.697	−0.977	–
C_{pk}	−0.710	−0.974	0.978

have higher C_p but lower C_{pk}. The proper policy for these samples is that they need to improve C_{pk} without worsening the C_p index. Those samples placed in the lower-left quadrant, have both low C_p and C_{pk}. The proper policy for these samples is that they need to improve C_p and C_{pk}, simultaneously. Finally, the best samples are located in the upper-right quadrant. These samples can be considered as benchmarks for other samples.

6 Concluding Remarks

This study discussed a combined use of DEA and GP to measure performance capability index. In this work, the process performance of various DMUs was studied. In order to remain efficient DMU in the next period, the long-term goals were determined, and then the new GP model based on pre-determined goals was proposed. In this model, a DMU could have higher process performance in the next period by determining appropriate goals. As well, a problem solving algorithm was presented. The results of proposed approach indicated that the DMU_{10} should reduce the values of $\overline{R}$ and $\hat{\sigma}$ from 0.043 to 0.029 and 0.018 to 0.012 and also increases C_p and C_{pk} from 0.9 to 1.27 and 0.054 to 1, respectively, in order to remain efficient in the next

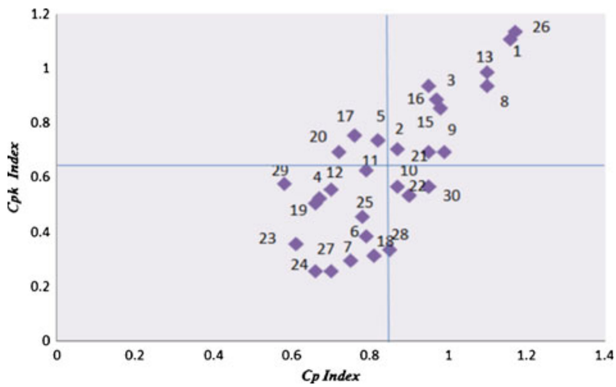

Fig. 6 C_p vs. C_{pk} index

period. To improve the performance of the company, the sources of poor performance should be identified.

Further researches can be done based on the results of this paper. Some of them are as follows:

- Similar research can be repeated in the presence of stochastic data.
- Similar research can be repeated in the presence of dual-role factor.

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Appendix

To give an idea about the calculations, consider the DMU_{10}:

At first, input and output benchmarks should be determined by the Models (6) and (7).

$$\begin{aligned}
&Input\ oriented:\\
&\min E\\
&0.0234^*\lambda_1+0.0450^*\lambda_2+0.0410^*\lambda_3+\ldots+0.0410^*\lambda_{30}\leq 0.0430^*E,\\
&0.0140^*\lambda_1+0.0190^*\lambda_3+0.0170^*\lambda_3+\ldots+0.0170^*\lambda_{30}\leq 0.0180^*E,\\
&1.16^*\lambda_1+0.87^*\lambda_2+0.95^*\lambda_3+\ldots+0.95^*\lambda_{30}\geq 0.90,\\
&1.11^*\lambda_1+0.71^*\lambda_2+0.94^*\lambda_3+\ldots+0.57^*\lambda_{30}\geq 0.54\\
&E: URS
\end{aligned}$$

Results: $\lambda_8^*=0.3963^*10^{-3}$, $\lambda_{25}^*=0.1247^*10^{-1}$, $\lambda_{30}^*=0.9367$

$$\begin{aligned}
&Output\ oriented:\\
&\max F\\
&0.0234^*\lambda_1+0.0450^*\lambda_2+0.0410^*\lambda_3+\ldots+0.0410^*\lambda_{30}\leq 0.0430,\\
&0.0140^*\lambda_1+0.0190^*\lambda_3+0.0170^*\lambda_3+\ldots+0.0170^*\lambda_{30}\leq 0.0180,\\
&1.16^*\lambda_1+0.87^*\lambda_2+0.95^*\lambda_3+\ldots+0.95^*\lambda_{30}\geq 0.90^*F,\\
&1.11^*\lambda_1+0.71^*\lambda_2+0.94^*\lambda_3+\ldots+0.57^*\lambda_{30}\geq 0.54^*F\\
&F: URS
\end{aligned}$$

Results: $\lambda_8^*=0.3963^*10^{-3}$, $\lambda_{25}^*=0.1870^*10^{-1}$, $\lambda_{30}^*=1.41$

Now, the obtained results from the Models (6) and (7) should be placed in the Models (4) and (5) as follows:

$$\begin{aligned}
&x_1^*=0.3565*10^{-4}*0.0350+0.1247*10^{-1}*0.500+0.9367*0.0410\Rightarrow x_1^*=\overline{R}=0.39\\
&x_2^*=0.3565*10^{-4}*0.015+0.1247*10^{-1}*0.021+0.9367*0.017\Rightarrow x_2^*=\sigma=0.01\\
&y_1^*=0.3963*10^{-3}*1.1+0.1387*10^{-1}*0.78+1.041*0.95\Rightarrow y_1^*=C_p\approx 1\\
&y_2^*=0.3963*10^{-3}*0.94+0.1387*10^{-1}*0.46+1.041*0.57\Rightarrow y_2^*=C_{pk}=0.81
\end{aligned}$$

After determining long term goals by decision maker, the Models (10) and (11) are run:

$$\begin{aligned}
&\min = \mathrm{f} + 0.0001 * (\mathrm{s1} + \mathrm{s2} + \mathrm{s3} + \mathrm{s4});\\
&0.0334 * \lambda 1 + 0.0450 * \lambda 2 + 0.0410 * \lambda 3 + \ldots + 0.041 * \lambda 30 - \mathrm{s1} \geq 0.030,\\
&0.014 * \lambda 1 + 0.019 * \lambda 2 + 0.017 * \lambda 3 + \ldots + 0.017 * \lambda 30 - \mathrm{s2} \geq 0.008,\\
&1.16 * \lambda 1 + 0.87 * \lambda 2 + 0.95 * \lambda 3 + \ldots + 0.57 * \lambda 30 + \mathrm{s4} \geq 0.81,\\
&0.0334 * \lambda 1 + 0.0450 * \lambda 2 + 0.0410 * \lambda 3 + \ldots + .041 * \lambda 30 \leq 0.039 * \mathrm{f},\\
&0.014 * \lambda 1 + 0.019 * \lambda 2 + 0.017 * \lambda 3 + \ldots + 0.017 * \lambda 30 \leq 0.01 * \mathrm{f};\\
&f : URS
\end{aligned}$$

$$\begin{aligned}
&\max = \mathrm{f} + 0.0001 * (\mathrm{s1} + \mathrm{s2} + \mathrm{s3} + \mathrm{s4});\\
&0.0334 * \lambda 1 + 0.0450 * \lambda 2 + 0.0410 * \lambda 3 + \ldots + 0.041 * \lambda 30 - \mathrm{s1} \leq 0.039,\\
&0.014 * \lambda 1 + 0.019 * \lambda 2 + 0.017 * \lambda 3 + \ldots + 0.017 * \lambda 30 - \mathrm{s2} \leq 0.01,\\
&1.16 * \lambda 1 + 0.87 * \lambda 2 + 0.95 * \lambda 3 + \ldots + 0.95 * \lambda 30 + \mathrm{s3} \leq 1.2,\\
&0.11 * \lambda 1 + 0.71 * \lambda 2 + 0.95 * \lambda 3 + \ldots + 0.57 * \lambda 30 + \mathrm{s4} \leq 1,\\
&0.0334 * \lambda 1 + 0.0450 * \lambda 2 + 0.0410 * \lambda 3 + \ldots + 0.041 * \lambda 30 \geq 0.030 * \mathrm{f};\\
&0.014 * \lambda 1 + 0.019 * \lambda 2 + 0.017 * \lambda 3 + \ldots + 0.017 * \lambda 30 \geq 0.008 * \mathrm{f};\\
&f : URS
\end{aligned}$$

Again, the obtained results from the Models (10) and (11) should be placed in the Models (4) and (5) for determining goal-based benchmark as follows:

$$\begin{aligned}
x_1^* &= \overline{R} = 0.29\\
x_2^* &= \sigma = 0.012\\
y_1^* &= C_p = 1.27\\
y_2^* &= C_{pk} = 1
\end{aligned}$$

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