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Solving a Bicriteria Problem of Optimal Service Centers Location

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Abstract The problem of service centers location is formulated as a bicriteria optimization problem of finding a dominating set in graph. We investigate the properties of this problem and propose the methods for its solving. The results of computational experiment for instances with random data are presented.

Keywords Integer programming · Multiple-criterion optimization · Graphs · Service center · Location problem · L-class enumeration algorithm · Trade offs

1 Introduction

A problem of optimal location of service centers (for example, location of telecenters or telephone exchanges) can be formulated as a problem of finding the dominating set with minimum weight in a graph. In [16] we considered the practical problem of location of telecenters in the case of the Greek islands in the Aegean sea. The telecenters can essentially influence on regions development. We suggested the discrete optimization problem with one criterion and presented the results of computational experiments.

In this paper, we suggest and study a mathematical model of the problem of service centers location as a bicriteria dominating set problem in which a total cost of centers location is minimized and a level of satisfaction of requirements by the service centers is maximized. We propose an integer linear model of this problem and the methods for its solving. The results of computational experiments are presented for instances with random data. The paper is an expansion of article [18].

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The dominating set problem is a special case of the set covering problem (SCP) [6].

The dominating set problem (DSP) can be formulated as follows. Consider a graph $G = (V, E)$, where $V = \{v_1, ..., v_n\}$ is the set of vertices and $E \subseteq \{(v_i, v_j) : i, j = 1, ..., n; i \neq j\}$ is the set of edges. A set $S \subseteq V$ is called the dominating set if for any $v_i \notin S$ there is a vertex $v_j \in S$ such that $(v_i, v_j) \in E$. The DSP consists in finding the dominating set with the minimal cardinality. If for each v_j a positive cost c_j is assigned then the weighted DSP is to find a dominating set of minimum total cost.

Sometimes it is more convenient to consider the weighted DSP as the following integer linear programming (ILP) problem:

$$\text{minimize} \quad cx$$

subject to

$$Ax \geq e,$$

$$x \in \{0, 1\}^n,$$

where $x_j = 1$ if $v_j \in S$ and $x_j = 0$ otherwise, $j = 1, ..., n$. Here A is the incidence matrix of vertices of the graph G in which we put $a_{ii} = 1, i = 1, ..., n$, c is the n-vector of costs, e is the n-vector of 1s. The integer linear program of the SCP has the same form but A is an arbitrary Boolean $(m \times n)$-matrix.

Different exact algorithms have been proposed for solving the SCP using branch and bound approach [1, 4, 19], cutting planes [2], L-class enumeration [22] and other techniques. However, the SCP is an NP-hard problem and application of exact algorithms to the large-scale instances is often time-consuming. A number of heuristic algorithms are developed for approximate solving the large-scale problems within relatively short running time. The solutions of good quality may be obtained using Lagrangian heuristics [5], neural networks [14] or metaheuristics such as genetic algorithm (GA) [3, 11, 15, 21]. Most of the successful versions of exact algorithms combine the exact techniques with heuristic methods (see e.g. [1, 4, 12, 13]). A great survey of the literature in multicriteria optimization is presented in [7].

Let us formulate the problem of optimal location of service centers. Let $P_1, ..., P_n$ be the set of demand points that constitute the total region demand for service centers. The service centers can be located in these points. Denote by c_j the location cost and by u_j the coefficient of the efficiency for the center at point P_j, $j = 1, ..., n$. Let d_{ij} be the distance between points P_i and P_j, $i, j = 1, ..., n$, $i \neq j$ (for example, Euclidean). A service center can satisfy the demand of a point if the distance between this center and the point does not exceed some given value d. The problem consists in finding the centers location for which the total cost of the centers opening is minimized and the total efficiency of the centers location is maximized provided that all demands are satisfied.

Using the distances d_{ij} and the value d we obtain the graph $G = (V, E)$ with the vertices set

$$V = \{P_1, ..., P_n\}$$

and the edges set

$$E \subseteq \{(P_i, P_j) : i, j = 1, ..., n; \; i \neq j\}.$$

The edge (P_i, P_j) belongs to E iff $d_{ij} \leq d$. Denote by A the incidence matrix of vertices of the graph G. Put $a_{ii} = 1$, $i = 1, ..., n$. Now the considered problem can be formulated as bicriteria problem of finding dominating set in the graph G.

Introduce Boolean variables

$$x_j = \begin{cases} 1, \text{ if a center locates in point } P_j, \\ 0, \text{ otherwise ,} \end{cases}$$

where $j = 1, ..., n$.

The efficiency function of our problem is given by

$$f_u(x) = \sum_{j=1}^{n} u_j x_j.$$

The maximization of this function we replace by the minimization $(-f_u(x))$. Now the ILP model of the considered problem can be written as follows:

$$\text{minimize} \quad f_1(x) = \sum_{j=1}^{n} c_j x_j, \tag{1}$$

$$\text{minimize} \quad f_2(x) = -\sum_{j=1}^{n} u_j x_j, \tag{2}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \geq 1, \quad i = 1, ..., n, \tag{3}$$

$$0 \leq x_j \leq 1, \quad j = 1, ..., n, \tag{4}$$

$$x_j \in Z, \qquad j = 1, ..., n. \tag{5}$$

In what follows, it is assumed that $c_j > 0$, $u_j > 0$, c_j, u_j are integer, $j = 1, ..., n$.

Note that problem 1, 3–5 is an integer programming model of the weighted DSP.

The paper is organized as follows: in Section 2, analysis of the model is carried out. In Section 3, we give the necessary information on the L-class enumeration algorithm for one criterion SCP. In Section 4, some approaches to solving the problem are suggested. In Section 5, results of computational experiment for instances with random data are presented. Finally, some conclusions and perspectives are considered.

2 Analysis of the Model

Consider the general multicriteria discrete optimization problem

$$\text{minimize} \quad F(x) = (f_1(x), ..., f_m(x))$$

subject to

$$x \in X,$$

where X is some finite set and $m \geq 2$.

A point $\bar{x} \in X$ is called Pareto-optimal solution if there is no $x \in X$ $(x \neq \bar{x})$ for which

(1) $f_k(\bar{x}) \geq f_k(x)$ for all $k = 1, ..., m$,
(2) $f_k(\bar{x}) > f_k(x)$ for at least one $k \in \{1, ..., m\}$ hold.

Let $\tilde{X}$ denote the set of the Pareto-optimal solutions (POS) and let X^0 be a full set of alternatives (FSA) (see, for example, [10]). The FSA is defined as a set X^0 $(X^0 \subseteq \tilde{X})$ with the minimal cardinality and $F(X^0) = F(\tilde{X})$, where $F(X') = \{F(x) \mid x \in X'\}$, $X' \subseteq X$. It is clear that

$$X^0 \subseteq \tilde{X} \subseteq X.$$

Many multicriteria discrete optimization problems have the following property [10]: for any X there exists some set of coefficients of the objective vector-function $F(x)$ such that the following equalities hold

$$X^0 = \tilde{X} = X.$$

The problem 1–5 has the indicate property. In particular, consider the given problem for which the cost functions coefficients are $c_j = u_j = 2^{j-1}$, $j = 1, ..., n$. It is easy to see that in this case any feasible solution belongs to the FSA. Therefore for arbitrary $c_j > 0$, $j = 1, ..., n$ the maximal cardinality of the FSA is equal to $2^n - 1$ and if $c_j = 1$, $j = 1, ..., n$, then it is equal to n.

In [8, 10] the solvability of the multicriteria discrete optimization (MDO) problem was investigated by the method of the linear convolution of the criteria. This approach consists in the replacement of the objective vector-function by the artificial one-dimensional cost function

$$f^{\lambda}(x) = \sum_{k=1}^{m} \lambda_k f_k(x),$$

where $\lambda_k \geq 0$, $k = 1, ..., m$ are fixed coefficients and $\sum_{k=1}^{m} \lambda_k = 1$. The MDO problem is called solvable by the method of the linear convolution of the criteria if for any individual problem every point from the POS (or from the FSA) can be obtained as an optimal solution of the problem with function $f^{\lambda}(x)$ for some values λ_k.

Problem 1–5 is not solvable by the method of the linear convolution of the criteria. Indeed, consider the problem with the parametric values $c_j = 2^{j-1}$, $u_j = 3^{j-1}$, $j = 1, ..., n$, $n \geq 3$ by restrictions Eqs. 4, 5 and

$$\sum_{j=1}^{n} x_j \geq 1. \tag{6}$$

It should be noted that this problem is the bicriteria DSP on the complete graph. It is easy to see that any Boolean point x $(x \neq 0)$ is a Pareto-optimal solution to this problem. But point $x' = (1, 0, ..., 0, 1)$ can not be obtained by solving the following problem

$$\text{minimize} \quad f^{\lambda}(x) = \sum_{j=1}^{n} q_j x_j,$$

subject to Eqs. 4–6 for any $\lambda \in [0, 1]$, where $q_j = \lambda c_j - (1 - \lambda) u_j$.

Indeed, the point x' is an optimal solution to this problem, if the coefficients of the function f^{λ} satisfy the following conditions

$$q_1 = 2\lambda - 1 \leq 0,$$

$$q_n = \lambda \left(2^{n-1} + 3^{n-1}\right) - 3^{n-1} \leq 0,$$

$$q_j = \lambda \left(2^{j-1} + 3^{j-1}\right) - 3^{j-1} \geq 0, \;\; j = 2, ..., n-1.$$

Thus, we conclude that $\lambda \leq \frac{1}{2}$ and $\lambda \geq \frac{3}{5}$. So the needed value λ does not exist.

3 The *L*-class Enumeration Algorithm to Solving the SCP

The L-class enumeration algorithm to solving the ILP problem is based on usage of a special partition of space R^n which is called the L-partition [17].

Denote by Z^n the set of all integer points of space R^n. Each integer point $z \in Z^n$ forms a separate class of the L-partition. Points $x, y \notin Z^n$ $(x \succ y)$ belong to the same fractional L-partition class of space R^n if there is no point $z \in Z^n$ such that $x \succeq z \succeq y$. Here and below $\succ$, $\succeq$ are the symbols of the lexicographical order.

We denote by X/L the quotient set induced by L-partition for a set $X \subset R^n$. The elements of X/L are called L-classes of the set X. There exist $(n+1)$ types of L-classes in R^n. For example, there are three types of them in R^2: integer points, vertical intervals and vertical stripes. On Fig. 1 L-classes V_1 and V_5 are vertical stripes, V_2 and V_4 are vertical intervals and V_3 is integer point.

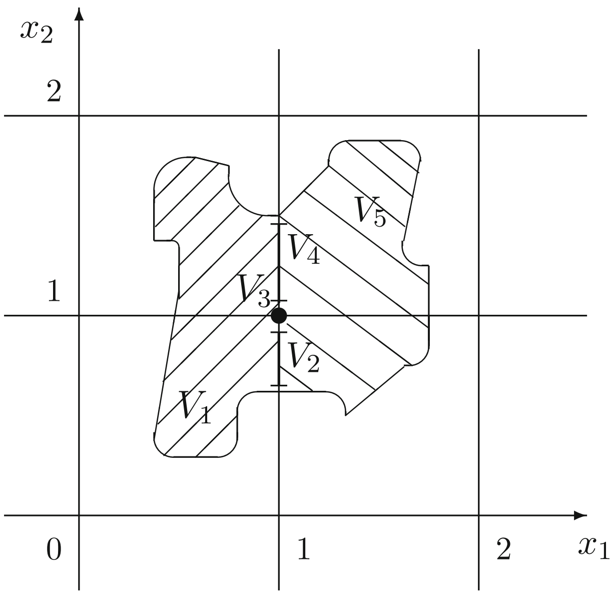

Fig. 1 The L-partition for a set X from R^2

This L-partition has some important properties, for example,

(1) if X is bounded, then X/L is finite;
(2) on X/L a linear order may be introduced: for any nonempty $V', V'' \in X/L$ we define $V' \succ V''$ if $x' \succ x''$ for any $x' \in V'$ and $x'' \in V''$.

Let us consider the idea of the L-class enumeration (LCE) method for ILP problem [17]. Let M be the polyhedron of the related continuous problem of the ILP problem. The main step of the basic algorithm is to pass from one L-class of the polyhedron M to another one according to the lexicographically increasing order. The record value of the objective function is also taken into account. The algorithm generates a sequence S of points $x^{(t)}$ from the relaxation set M with the following properties:

(1) $x^{(t)} \prec x^{(t+1)}$, $t = 1, 2, ...,$ where $x^{(1)}$ is the lexicographically minimal point of M, i.e. $x^{(1)} = \text{lexmin } M$;
(2) all $x^{(t)}$ belong to different L-classes.

Note that searching each new current point $x^{(t)}$ the LCE algorithm goes through some part of the polyhedron M where the objective function values are less or equal to $\rho - 1$. Here ρ is the current objective function record which changes if the point $x^{(t)}$ is integer. In the view of the linear order on the set of L-classes the L-class enumeration may be illustrated with the help of the following "tape" (see Fig. 2). Each box of the tape corresponds to a separate L-class, the white boxes represent fractional L-classes and the black boxes represent integer points.

Further we describe the L-class enumeration algorithm [17] for solving the SCP in the following formulation:

$$\text{minimize} \quad f(x) = (c, x)$$

subject to

$$x \in M,$$

$$x \in \{0, 1\}^n,$$

where M is a polyhedron that is defined by conditions 3 and 4. Let $\zeta^0 \in \{0, 1\}^n$ be an approximate solution obtained by some method.

3.1 Below we Provide the Scheme of LCE for the SCP

Step 0. Find $x' = \text{lexmin } M$. Set $r := \min\{(c, x'), (c, \zeta^0)\}$ and $p := \max\{j : x'_j = 1, j = 1, ..., n - 1\}$.

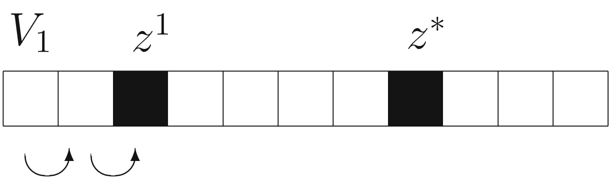

Fig. 2 Solving ILP problem using the L-class enumeration method

Step 1. Find $\varphi := \max\{j : x'_j = 0,\ j = 1, ..., p-1\}$.
If such φ does not exist, go to step 4.

Step 2. Set $x'' := x'$. Solve the linear subproblem:

$$find\ \ x' = \text{lexmin}\{x \in M : (c, x) \le r - 1, x_1 = x''_1, ..., x_{\varphi-1} = x''_{\varphi-1}, x_\varphi = 1\}. \tag{7}$$

2.1 If the subproblem 7 has no solutions, and $\varphi = 1$, go to step 4.
2.2 If the subproblem 7 has no solutions, and $\varphi > 1$, then set $p := \varphi$ and go to step 1.
2.3 If $x' \in Z^n$, then set $r := (c, x')$ and $p := \max\{j : x'_j = 1,\ j \le n-1\}$, go to step 1.

Step 3. Find $\varphi = \min\{j : x'_j \ne \lfloor x'_j \rfloor,\ j = 1, ..., n\}$, and go to step 2.

Step 4. The enumeration is finished: the best obtained integer solution is optimal.

The linear subproblems on step 2 may be solved, for example, using the lexicographical dual simplex method. Often in the case of SCP it turns out that these subproblems have no feasible solutions. To save the computation time the linear subproblems are examined by the following group testing heuristic [22], which allows to test more than one L-class at a time. This heuristic is applied on step 2 in the case if an L-class where $x_\varphi = 1$, and $x_j = x''_j$, for all $j = 1, \ldots, \varphi - 1$ is not found. Then we solve the following subproblem:

$$\text{find } x' = \text{lexmin}\left\{x \in M : (c, x) \le r - 1, x_1 = x''_1, ..., x_{j_0-1} = x''_{j_0-1}, \sum_{j=j_0}^{j_0+n_0-1} x_j \ge 1\right\}, \tag{8}$$

where j_0 and $n_0 > 0$ are such that $j_0 = \min\{j < \varphi : x''_j = x''_{j+1} = ... = x''_{j+n_0-1} = 0$, and no $k \in [j + n_0, \varphi - 1]$ exists that $x''_k = 0\}$.

In the case if in the set $\{x \in M : (c, x) \le r - 1\}$ there is no such L-class $V \succeq x''$ that for all $x \in V$ we have $x_j = x''{}_j$, $j = 1, ..., j_0 - 1$, then problem 8 has no feasible solutions. Otherwise the next L-class would be found, and the process would continue from it.

Before solving problem 8 we first check the existence of admissible solutions to it. Let us consider the following supplementary linear subproblem:

$$\text{minimize} \sum_{j=1}^{n} c_j x_j \tag{9}$$

subject to

$$x \in M, \sum_{j=j_0}^{j_0+n_0-1} x_j \ge 1, \tag{10}$$

$$x_j = x''_j,\ j = 1, ..., j_0 - 1. \tag{11}$$

If the optimal goal function value for Eqs. 9–11 exceeds $r-1$, then the problem 8 has no solutions. In order to obtain a lower bound for the optimum of Eqs. 9–11 one can use an approximate solution to the dual problem. This solution may be obtained for example by the knapsack greedy algorithm.

The LCE method always finds the optimum of SCP and finishes enumeration after visiting not more than $|M/L|$ L-classes.

The hybrid algorithm for the SCP is based on the described above L-class enumeration algorithm and uses the GA and a Lagrangean heuristic to obtain an initial approximate solution. The linear subproblems in L-class enumeration are tested by means of Lagrangean relaxation. The results of computational experiments showed that the given method of hybridization seems to be promising.

4 The Approaches to Solving the Bicriteria Problem of Optimal Service Centers Location

Since obtaining all elements from the set of POS and (or) the FSA may be difficult, therefore the special subsets of the POS, in particular, the lexicographical set of the alternatives (LSA) are used.

For a vector-function $F(x) = (f_1(x), ..., f_m(x))$ a Pareto-optimal solution x^* is called the lexicographical minimum (LM) of the problem if

$$F(x^*) = \text{lexmin } \{F(x) \mid x \in X\}.$$

Let $X_{\min}$ be the set of all lexicographical minima for all different orders of components of the objective vector-function. The LSA is a set X_{lex} ($X_{\text{lex}} \subseteq X_{\min}$) with minimal cardinality and $F(X_{\text{lex}}) = F(X_{\min})$ (see, for example, [9, 20]).

If the vector-function of problem 1–5 is of the form $F(x) = (f_2(x), f_1(x))$ then the unique LM exists and it is (1,1,...,1). In practice, the order $F(x) = (f_1(x), f_2(x))$ is more useful. We consider some methods of obtaining the corresponding LM.

Let the MDO problem have a finite set of feasible solutions. In [9, 20] it was showed that in this case any point from a LSA may be found by the algorithm of linear convolution of the criteria. Using [20] the coefficients of linear convolution function for considered problem 1–5 can be defined in the following way:

$$\lambda_1 = \frac{\alpha+1}{\alpha+2}, \ \lambda_2 = \frac{1}{\alpha+2},$$

where $\alpha = \sum_{j=1}^{n} u_j - [\tilde{u}_0] + 1$. Here $\tilde{u}_0$ is the optimal value of the objective function of the linear program:

$$\text{minimize} \sum_{j=1}^{n} u_j x_j$$

subject to Eqs. 3 and 4.

Another approach to finding the lexicographical minimum is the sequential optimization. In this case the MDO problem is replaced by a sequence of one-criterion problems. We use the hybrid algorithm for the SCP [12] for finding the lexicographical minimum of problem 1–5 by means of the sequential optimization approach.

Finding the LM of problem 1–5 consists of two steps

Step 1. Solve SCP Eqs. 1, 3–5 by the hybrid algorithm from [12]. Denote by x^* the optimal solution to this problem. Put $c_0^* = f_1(x^*)$, $u_0^* = f_2(x^*)$.

Step 2. Solve the following problem

$$\text{maximize } f_2(x) = \sum_{j=1}^{n} u_j x_j \tag{12}$$

subject to Eqs. 3–5 and the additional condition

$$\sum_{j=1}^{n} c_j x_j = c_0^*. \tag{13}$$

This problem can be solved by some known method for general integer programming problem (branch and bound algorithms, cutting plane algorithms, LCE algorithm). On the other hand, problem 3–5, 12 and 13 can be decomposed into an optimization problem and a problem of solving a system of linear constraints.

Note that the integer points of plane Eq. 13 can be found by solving problem 1, 3–5 by a modification of the LCE algorithm, in case $\rho = c_0^* + 1$ and ρ does not change during the solving process. Now consider the following decomposition method to solving problem 3–5, 12 and 13.

Starting from the point x^* the LCE algorithm solves SCP Eqs. 1, 3–5 with $\rho = c_0^* + 1$. Let x' be a current integer point obtained by the algorithm. If x' satisfies the inequality

$$\sum_{j=1}^{n} u_j x_j \geq u_0^* + 1,$$

then $u_0^* := f_2(x')$ and the move towards the next element of the L-partition of the relaxation polyhedron of SCP Eqs. 1, 3–5 is made.

Another decomposition method is connected with using knapsack problem 4, 5, 12 and 13.

Other known approach to finding the solutions set of the MDO problem (trade-off method) consists in the following. We pass from bicriteria problem 1–5 to the problem $\mathcal{P}$:

$$\text{maximize } f_2(x) = \sum_{j=1}^{n} u_j x_j \tag{14}$$

subject to Eqs. 3–5,

$$c_0^* \leq \sum_{j=1}^{n} c_j x_j \leq c_0^* + \delta, \tag{15}$$

where δ is the integer value of the trade-off, and $[c_0^*, c_0^* + \delta]$ is the interval of the cost which can be used for opening the service centers.

By changing the values of parameter δ we can obtain a set of the feasible solutions and some Pareto-optimal solutions to problem 1–5 among them. Note that if

$c_j = 1, \quad j = 1, ..., n$ then the optimal solution to problem $\mathcal{P}$ is a Pareto-optimal solution to Eqs. 1–5 and it satisfies the condition

$$\sum_{j=1}^{n} c_j x_j = c_0^* + \delta.$$

If all c_j are arbitrary, then finding a Pareto-optimal solution requires in addition to solve problem 1, 3–5 and 15 with the constraint

$$\sum_{j=1}^{n} u_j x_j \geq u_0^*.$$

Here u_0^* is equal to the optimal value of the objective function of the problem $\mathcal{P}$.

It is easy to see that the decomposition methods can be also used for the problem $\mathcal{P}$. Here, the initial value of ρ is equal to $c_0^* + \delta + 1$ and the enumeration of L-classes is started with the lexicographically minimal point of the relaxation polyhedron of problem $\mathcal{P}$. Note that the algorithm gives all Pareto-optimal solutions that satisfy condition Eq. 15.

The similar approach can be also used when starting from function 2.

5 Computational Experiment

We solved the practical problem of location of telecenters in the case of the Greek islands in the Aegean sea (24 greatest islands) [16]. The Data was collected from the Greek Organization of Statistics and from the Greek Organization of Telecommunications. We varied the maximal admissible distance d between every pair of potential telecenters from 100 to 250 km. Note that in this problem all costs c_j were equal, i.e. in Eq. 1 we minimized the number of located telecenters. For example, in the case of $d = 150$ the efficiency will be maximal when the number of telecentres is equal to 5. This solution is of practical importance.

Below the results of computational experiments are presented for instances with random data. The approach including the trade-off method and the first type of decomposition for solving the bicriteria dominating set problem was tested on a series of instances with random data. We used the hybrid algorithm for the DSP implemented in Borland Pascal and tested on Pentium IV (3 GHz). The experiments were carried out on series with $n = 100, \ 250, \ 300, \ c_j, u_j \in [1, 100], \ j = 1, ..., n$, the probability of appearance of an edge in graph was equal to 0.1 or to 0.2. The value δ was equal to 5 % of optimal value c_0^*. The number of the obtained Pareto-optimal solutions for a given value δ varied from 3 to 11. The front of Pareto-optimal solutions in the criterial space for four instances with $n = 250$ is presented on Fig. 3.

The average times of solving for series were: 17; 514 and 1139 s. The average increase of the value of objective function 14 was from 12 to 20 %. The solving time grows when the probability of appearance of an edge in graph and (or) the upper bound on c_j decrease. For example, in the case $n = 250$ and the probability equal to 0.2 the average times of solving decreased in 6 time.

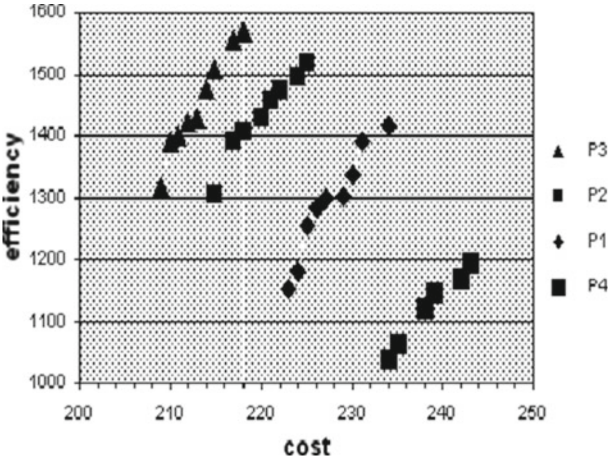

Fig. 3 The front of Pareto-optimal solutions

6 Conclusion

In this paper the problem of optimal location of service centers is considered. The discrete optimization formulation of this problem is suggested as a bicriteria dominating set problem in graph. Some properties of the problem are investigated, in particular, it is shown that the maximal cardinality of the full set of alternatives may be exponential.

We used the integer linear programming model of our problem and the L-partition approach. The trade-off method, the decomposition and LCE algorithm were used for finding a subset of the Pareto-optimal solutions set. The computational experiment showed that the algorithm based on the trade-off method and decomposition of first type is promising.

In [23] we have shown that analogous results are valid for the more general bicriteria set covering problem. In addition, some greedy heuristics for solving this problem were suggested. In future it is prospective to use the other search heuristics and parallelization of the algorithms.

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References

1. Balas, E., Carrera, M.C.: A dynamic subgradient-based branch and bound procedure for set covering. Oper. Res. **44**(6), 875–890 (1996)
2. Balas, E., Ho, A.: Set covering algorithms using cutting planes, heuristics and subgradient optimization: a computational study. Math. Program. Study **12**, 37–60 (1980)
3. Beasley, J.E., Chu P.C.: A genetic algorithm for the set covering problem. Eur. J. Oper. Res. **94**(2), 394–404 (1996)
4. Beasley, J.E., Jörnsten, K.: Enhancing an algorithm for set covering problems. Eur. J. Oper. Res. **58**, 293–300 (1992)
5. Caprara, A., Fischetti, M., Toth, P.: Algorithms for the set covering problem. In: Technical Report OR-98-3, DEIS-Operations Research Group (1998)

6. Christofides, N.: Graph theory. An algorithmic approach. Management Science, Imperial College, Academic press Inc. London Ltd., 2nd printing (1977)
7. Ehrgott, M., Gandibleux, X.: Multiple Criteria Optimization: State of the Art Annotated Bibliographic Surveys. International Series in Operations Research & Management Science, vol. 52. Springer, Heidelberg (2002)
8. Emelichev V.A., Kravtsov, M.K.: On the nonsolvability of the discrete optimization vectors problem on the subsets systems in the class of the algorithms of linear convolution of the criteria. Doclady Mathematics **334**(1), 9–11, (1994) (in Russian)
9. Emelichev, V.A., Kravtsov, M.K., Yanushkevich, O.A.: Lexicographical optima of multicriteria discrete optimization problem. Matematicheskie Zametki. **58**(3), 365–371 (1995) (in Russian)
10. Emelichev, V.A., Perepelitsa, V.A.: The complexity of discrete multicriteria problems. Discrete Math. Appl. **4**(2), 89–117 (1994)
11. Eremeev, A.V.: A genetic algorithm with a non-binary representation for the set covering problem. In: Proceedings of OR'98, pp. 175–181. Springer-Verlag, New York (1999)
12. Eremeev, A.V., Kolokolov, A.A., Zaozerskaya, L.A.: A hybrid algorithm for set covering. In: Proceedings of International Workshop on Discrete Optimization Metods Design, pp. 123–129. Minsk (2000)
13. Eremeev, A.V., Zaozerskaya, L.A., Kolokolov, A.A.: A set covering problem: complexity, algoritms, experimental research. Discretnyi Analiz i Issledovanie Operatsii. **7**(2), 22–46 (2000) (in Russian)
14. Grossman, T., Wool, A.: Computational experience with approximation algorithms for the set covering problem. Eur. J. Oper. Res. **101**(1), 81–92 (1997)
15. Jaszkiewicz, A.: A comparative study of multiple-objective metaheuristics on the bi-objective set covering problem and the pareto memetic algorithm. Ann. Oper. Res. **131**(1–4), 135–158 (2004)
16. Kitrinou, E., Kolokolov, A.A., Zaozerskaya, L.A.: The location choice for telecenters in remote areas. The case of the aegean islands. In: Proceedings of Discrete Optimization Methods in Production and Logistics (DOM-2004), pp. 61–65. Omsk, Nasledie Dialog-Sibir (2004)
17. Kolokolov, A.A.: Regular partitions and cuts in integer programming. Discrete Analysis and Operations Research, pp. 59–79. Kluver Academic Publisher, Netherland (1996)
18. Kolokolov, A.A., Zaozerskaya, L.A.: A bicriteria problem of optimal service centers location. In: Proceedings of 12th IFAC International Symposium. vol. III, pp. 429–434. St. Etienne, France (2006)
19. Mannino, C., Sassano, A.: An exact algorithm for the maximum stable set problem. Comput. Optim. Appl. **3**, 243–258 (1994)
20. Podinovski V.V., Gavrilov, V.M.: Optimization by Consequently Applied Criteria. Sovetskoe Radio Moscow (1975) (in Russian)
21. Solar, M., Parada, V., Urrutia, R.: A parallel genetic algorithm to solve the set-covering problem. Comput. Oper. Res. **29**, 1221–1235 (2002)
22. Zaozerskaya, L.A. On L-class enumeration algorithm for set covering problem. In: Proc. of 11-th Baikal Intern. School-Seminar "Optimization Methods and Their Applications". pp. 139–142 (1998) (in Russian)
23. Zaozerskaya, L.A., Kolokolov, A.A.: Study and solution of bicriteria set covering problem. Problemy Informatiki. **2**, 11–18 (2009) (in Russian)