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Electrocardiogram Signal Compression Using Beta Wavelets

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Abstract In this paper, a wavelet based methodology is presented for compression of electrocardiogram (ECG) signal. The methodology employs new wavelet filters whose coefficients are derived with beta function and its derivatives. A comparative study of performance of different existing wavelet filters and the Beta wavelet filters is made in terms of compression ratio (*CR*), percent root mean square difference (*PRD*), mean square error (*MSE*) and signal-to-noise ratio (*SNR*). When compared, the Beta wavelet filters give better compression ratio and also yields good fidelity parameters as compared to other wavelet filters. The simulation result included in this paper shows the clearly increased efficacy and performance in the field of biomedical signal processing.

Keywords Beta wavelet · ECG compression · Huffman encoding

1 Introduction

An electrocardiogram (ECG) is the graphical representation of electrical impulses due to ionic activity in the cardiac muscles of human heart. It is an important physiological signal which is exploited to diagnose heart diseases because every arrhythmia in ECG signals can be relevant to a heart disease [1]. ECG signals are recorded from patients for both monitoring and diagnostic purposes. Therefore, the storage of computerized ECG has become necessary. However, the storage has

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limitation which has made ECG signal compression as an important issue of research in the biomedical signal processing. In addition, the transmission speed of real-time ECG signal is also enhanced and economical due to ECG signal compression.

An ECG signal contains steep slopes QRS complexes and smoother P and T waves. It is recorded by applying electrodes to various locations on the body surface and connecting them to a recording apparatus. There are certain amounts of sample points in ECG signal which are redundant and replaceable. ECG data compression is achieved by elimination of such redundant data sample points. During the past few decades, many schemes for ECG signal compression have been proposed. Most of them are lossy compression techniques in which the reconstructed signal is not exact replica of the original input signal. Generally, these techniques are classified in categories: direct techniques transform and parameter extraction [2–6]. In the direct scheme, several methods such as the amplitude zone time epoch coding (AZTEC), and coordinate reduction time encoding system (CORTES), the turning-point (TP) data reduction algorithm, the scan-along polygonal approximation (SAPA) and differential pulse code modulation (DPCM) were developed. In these techniques, the compression is achieved by eliminating redundancy between different ECG samples in the time domain. They involve simple signal processing and yield minimum distortion with good compression. A detailed review on these techniques is presented in [7–12] and the references there in.

In the last two decades, a substantial progress has been made in the field of data compression. So far, several efficient ECG compression techniques have been reported in literature such as Linear Predictive Coding (LPC), Waveform coding and Subband coding. In these techniques, more sophisticated signal processing techniques are employed. Linear predictive coding is robust tool widely used for analyzing speech and ECG signal in various aspects such as spectral estimation, adaptive filtering and data compression. Several efficient methods [13–16] have been accounted in literature based on linear prediction. While in subband decomposition, spectral information is divided into a set of signals that can be encoded by using a variety of techniques. Based on subband decomposition, various techniques [17–19] have been devised for the ECG signal compression.

In the past, a marked researches have made in the many transformation methods such as Discrete Cosine Transform (DCT), Fast Fourier Transform (FFT) and Discrete Wavelet Transform (DWT) which are extensively used in data compression. Here, compression is achieved by transforming original signal into another domain to compact much of the signal energy into a small number of transformed coefficients. In this way, many small valued transform coefficients can be discarded in the hope of achieving better compression. Various techniques [20–25] have been developed based on FFT and DCT. The discrete wavelet transform has been emerged as a powerful tool for analyzing and extracting information from non-stationary signal such as speech signal and ECG signal due to the time varying nature of these signals. Non-stationary signals are characterized by numerous abrupt changes, transitory drifts, and trends. Wavelet has localization feature along with its time-frequency resolution properties which makes it suitable for analyzing non-stationary signals such as speech and electrocardiogram (ECG) signals [26]. Recently, several other methods [27–32] have been developed based on wavelet for compressing ECG signal. The authors in [33–37] have developed a new orthogonal mother wavelet based on Beta function as well as its derivatives for image compression.

In above context, therefore, this paper presents a wavelet based methodology for ECG signal compression using beta wavelets. The paper is organized as follows. A brief introduction has been provided in this section on the existing compression techniques of ECG signal. Section 2 discusses overview of discrete wavelet transform (DWT), multiresolution analysis and beta wavelet. In Section 3, a compression methodology based beta wavelet is presented. Finally, a comparison of results obtained with beta wavelet and other wavelet filters is carried out in Section 4, followed by concluding remarks in Section 5.

2 Discrete Wavelet Transform and Multiresolution Analysis

The Wavelet Transform has emerged as a powerful mathematical tool in many areas of science and engineering, more so in the field of data compression. The concept of wavelets was first introduced by Grossman and Morlet in 1984 to analyze signal structures of very different scales, in the framework of seismic signals. The basic principal of wavelet transform is that it decomposes the given signal in too many functions by using property of translation and dilation of a single prototype function, called a mother wavelet ($\psi(t)$), defined as

$$\psi_{ab}(t) = |a|^{-1/2}\,\psi\left(\frac{t-b}{a}\right), \qquad a,\ b\ \in \mathbf{R},\ a \neq 0. \tag{1}$$

When the parameters a and b are restricted to discrete values as $a = 2^{-m}$, $b = n2^{-m}$, then, a new family of discrete wavelets are derived as:

$$\psi_{mn}(t) = 2^{m/2}\psi\left(2^m t - n\right),\ m,\ n\ \in\ Z, \tag{2}$$

where, the function ψ, the mother wavelet, satisfies $\int_R \psi(t)dt = 0$.

A continuous-time wavelet transform of a signal $(f(t))$ *is* defined as

$$W_f(b,a) = |a|^{\frac{-1}{2}} \int_{-\infty}^{\infty} f(t)\,\psi^*\left(\frac{t-b}{a}\right)dt \tag{3}$$

where, the asterisk denotes a complex conjugate and multiplication of $|a|^{\frac{-1}{2}}$ is for the energy normalization purposes so that the transformed signal will have the same energy at every scale. Hence, the wavelets have adaptive nature, present a large time base for analyzing the low frequency components, and have a better time resolution for analyzing phenomena that are more transitory.

As a and b are continuous over $\boldsymbol{R}$ (over the real number), there is often redundant in CWT representation of the signal. A more compact representation can be found with a special case of WT, called the Discrete Wavelet Transform (DWT), where only the required wavelet coefficients for the reconstruction of $x(t)$ are kept. Substituting Eq. 3 into Eq. 2, DWT of a signal $f(t)$ is

$$DWT_\psi f(m,n) = \int_{-\infty}^{\infty} f(t)\,\psi^*_{(m.n)}(t)dt \tag{4}$$

where,

$$\psi_{m,n}(t) = 2^{-m}\psi\left(2^m t - n\right) \tag{5}$$

However, for computing Eq. 4, an infinite number of terms are required. Therefore, to overcome this problem, a new family of basis functions, called scaling functions ($\phi_{m,n}(t)$) was introduced, which are derived just similar to the wavelets:

$$\phi_{m,n}(t) = 2^{-m}\phi\left(2^m t - n\right) \tag{6}$$

where, ($\phi(t)$) is the mother scaling function. These scaling functions are complementary basis for wavelet function basis. Due to this, the multiresolution analysis (MRA) of signal is possible.

MRA means decomposition a signal into different frequency bands. For MRA, the average and the details features of the signal is found via scalar products with scaling signals and wavelets. The algorithm of wavelet signal decomposition is illustrated in Fig. 1.

At each step of DWT decomposition, there are two outputs: scaling coefficients $x^{j+1}(n)$ and the wavelet coefficients $y^{j+1}(n)$. These coefficients are given as

$$x^{j+1}(n) = \sum_{i=1}^{2n} g\left(2n - i\right) x^j(n) \tag{7}$$

and

$$y^{j+1}(n) = \sum_{i=1}^{2n} h\left(2n - i\right) x^j(n) \tag{8}$$

where, the original signal is represented by $x^0(n)$ and j shows the scaling number. Here $g(n)$ and $h(n)$ represent the low pass and high pass filters, respectively. The output of scaling function is input of next level of decomposition, known as the approximation coefficients. The approximation coefficients are low-pass filter coefficients, and high-pass filter coefficients are detailed coefficients of any decomposed signal. The relation between the low-pass and high-pass filter and the scalar function and the wavelet can be states as:

$$\phi(t) = \sum_k h(k)\phi\left(2t - k\right) \tag{9}$$

$$\psi(t) = \sum_k g(k)\psi\left(2t - k\right) \tag{10}$$

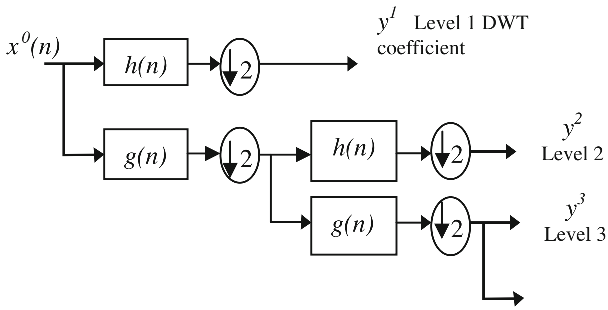


Fig. 1 Filter bank representation of DWT decomposition

These low-pass $h(n)$ and high filters $g(n)$ has mirror image at quadrature frequency, therefore filters satisfying this condition are known as the Quadrature Mirror Filters (QMF), commonly used in many engineering signal processing applications [25].

2.1 Beta Function and its Properties

Beta distribution function gives the compact support to continuous wavelet [33]. The Beta function is defined as [34–36]:

$$\beta\left(x, p, q, x_0,x_1\right) = \begin{cases} \left(\dfrac{x-x_0}{x_c-x_0}\right)^p\left(\dfrac{x_l-x}{x_l-x_c}\right)^q & \text{if } x\in[x_0,x_1] \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where, $x_c = \left(px_1+qx_0\right) / \left(p+q\right)$and also satisfies

$$\beta\left(x_0\right) = \beta\left(x_1\right) = 0 \quad \text{and} \quad \beta\left(x_c\right) = 1 \tag{12}$$

The derivatives of the Beta function are defined as:

$$\frac{d\beta\left(x\right)}{dx} = \frac{px_1+qx_0-\left(p+q\right)x}{\left(x-x_0\right)\left(x_1-x\right)}\beta\left(x\right) \tag{13}$$

$$\left.\frac{d\beta\left(x\right)}{dx}\right|_{x=x_c} = \left.\frac{d\beta\left(x\right)}{dx}\right|_{x=x_0} = \left.\frac{d\beta\left(x\right)}{dx}\right|_{x=x_1} = 0,\ \frac{p}{q} = \frac{x_c-x_0}{x_1-x_c} \tag{14}$$

and

$$\frac{d^2\beta\left(x\right)}{dx^2} = \beta\left(x\right)A\left(x\right) \tag{15}$$

where,

$$A\left(x\right)=\frac{1}{\left(x-x_0\right)\left(x_1-x\right)}\left[\frac{1}{\left(x_1-x\right)}-\frac{1}{\left(x-x_0\right)}-\left(p+q\right)\left(x+1\right)+px_1+px_0\right] \tag{16}$$

The n^{th} derivative of the Beta function is derived as:

$$\begin{aligned}\psi_n\left(x\right)=\frac{d^n}{dx^n}\beta\left(x\right)&=\left[(-1)^n\ \frac{n!\,p}{\left(x-x_0\right)}\ +\ \frac{n!\,q}{\left(x_1-x\right)^{n+1}}\right]\beta(x)\ +\ P_n(x)\ P_1\,\beta(x)\\ &+\sum C_n^i\left[(-1)^n\ \frac{(n-i)!\,p}{\left(x-x_0\right)^{n+1+i}}\ +\ \frac{(n-i)!\,q}{\left(x_1-x\right)^{n+1-i}}\right]P_1(x)\,\beta(x)\end{aligned} \tag{17}$$

where, $P_1(x) = \frac{p}{x-x_0} - \frac{q}{x_1-x}$.

In addition to these, Beta function also satisfies some other properties such as

(i) *Oscillation*: The average of Beta function and its derivatives are zero similar to other wavelet functions. In order to satisfies these conditions, the Beta wavelet is defined as [34]

$$\psi_\beta(x) = \beta\left(x\ ,\ p,\ q,\ x_0,\ x_1\right) - \beta\left(x\ ,\ p,\ q,\ x_1,\ x_2\right) \text{ where } x_0\ <\ x_1\ <x_2. \tag{18}$$

Table 1 Filter coefficients of Beta wavelet

Filter tap	Beta 2$^{\text{nd}}$ derivative	Beta 3$^{\text{rd}}$ derivative
1	−0.00000000000107	0.00000000000015
2	0.00000908751724	−0.00000097034562
3	−0.01520588663582	0.00196406188196
4	0.31545681509525	−0.08260135730784
5	0.59802880777005	−0.03659498793811
6	0.18893408149005	0.39544617271356
7	−0.08723905153544	0.54800796970913
8	0.00001614629973	0.18695253117361
9		−0.01317449742417
10		0.00000107753734

$$\int_{-\infty}^{\infty} \psi_\beta(x)\,dx = 0, \quad \int_{-\infty}^{\infty} \psi'_\beta(x)\,dx = 0 \quad \text{and} \quad \int_{-\infty}^{\infty} \psi''_\beta(x)\,dx = 0. \tag{19}$$

(ii) *Localization with Admissibility Condition*: The Beta function and its derivatives are localized as it has compact support (zero outside $[x_0, x_1]$) and also satisfies the property of wavelet, i.e. admissibility:

$$\int_{-\infty}^{\infty} \frac{\left|\hat{\psi}_\beta(\omega)\right|^2}{|\omega|}\,dw < \infty, \quad \int_{-\infty}^{\infty} \frac{\left|\hat{\psi}'_\beta(\omega)\right|^2}{|\omega|}\,dw < \infty \quad \text{and} \quad \int_{-\infty}^{\infty} \frac{\left|\hat{\psi}''_\beta(\omega)\right|^2}{|\omega|}\,dw < \infty \tag{20}$$

where, $\psi'_\beta(\omega) = \frac{d}{d\omega}\psi_\beta(\omega)$ and $\psi''_\beta(\omega) = \frac{d^2}{d\omega^2}\psi_\beta(\omega)$.

(iii) *Scaling and Shifting*: In addition to the above two properties, the Beta wavelets also hold the property of scaling and shifting. The detailed discussion on the Beta wavelet and its derivative is given in [33–36] and the references therein.

In this paper, a new wavelet family based on Beta function and its derivatives is exploited for ECG signal compression. Table 1. shown the filter coefficients of Beta function derivatives and Fig. 2 represents the plot of those derivative filters.

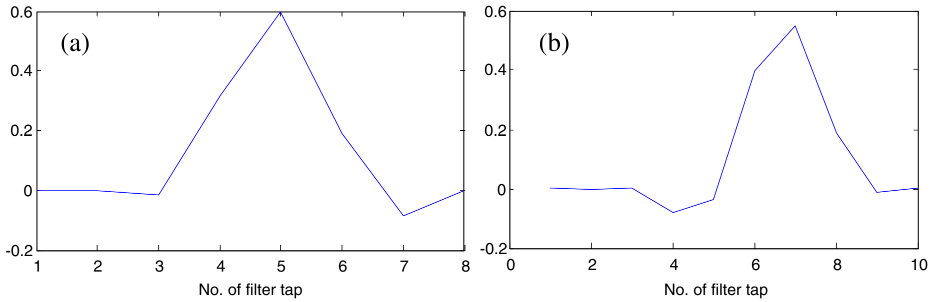


Fig. 2 Plot of filter coefficients **a** Beta 2$^{\text{nd}}$ Derivative and **b** Beta 3$^{\text{rd}}$ Derivative

3 Methodology for ECG Signal Compression

In this paper, the beta wavelet filters are used for the ECG signal compression. For ECG compression, three most commonly used steps are DWT decomposition, thresholding and Quantization, and Entropy encoding [26–28]. A typical block diagram of the methodology is depicted in Fig. 3.

Step 1: In this step, the mother wavelet is chosen, and then DWT decomposition is performed on the ECG signal. Several different criteria can be used for selecting the optimal wavelet filter. For example, the optimal wavelet filter must minimize the reconstructed error variance and also maximize signal to noise ratio (SNR). In general, the mother wavelets are selected based on the energy conservation properties in the approximation part of the wavelet coefficients. Then, decomposition level for DWT is selected which usually depends on the type of signal being analyzed or some suitable criteria such as entropy. Here, the beta wavelets are used as the mother wavelet and 4 level decomposition of DWT is applied on the ECG signal.

Step 2: After computing the wavelet transform of the ECG signal, compression involves truncating wavelet coefficients below a threshold value which make a fixed percentage of coefficients equal to zero. Two different approaches are available for calculating thresholds. The first type is known as Global Thresholding which involves taking the wavelet decomposition of the signal and keeping the largest absolute value coefficients. In this, the threshold value is set manually, this value is chosen from DWT coefficient $(0 \ldots x^{j}_{\max})$, where $x^{j}_{\max}$ is the maximum value of coefficients. The second approach is known as Level Thresholding in which the threshold value is calculated using Birge-Massart strategy [37]. In this paper, global thresholding is applied. Thereon, the quantization is performed on the truncated coefficients. In the quantization process, the wavelet coefficients are quantized using uniform step size which depends on three parameters: maximum $(M_{\max})$ and minimum $(M_{\min})$ values in the signal matrix, and the number of

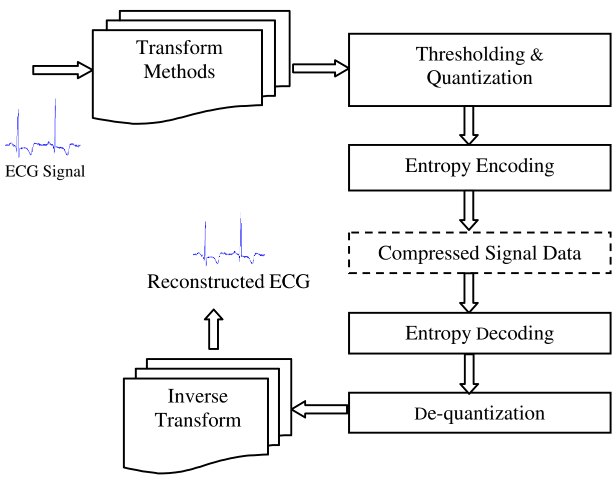


Fig. 3 Methodology for ECG signal compression

quantization level (L). Once these parameters are found, then step size (Δ) is computed by

$$\Delta = (M_{\max} - M_{\min}) / L \tag{21}$$

Then the input signal is divided into L + 1 level with equal interval size ranging from $M_{\min}$ to $M_{\max}$ to plot the quantization table. When the quantization is done, then quantized values are fed to the next stage of compression and these three parameters defined above are stored in a file as they are required for creating the quantization table during the reconstruction step. The actual compression is achieved at this stage. In this paper, 8-bit uniform quantization is employed for the ECG signal compression.

Step 3: In this step, signal compression is further achieved by efficiently encoding the truncated small-valued coefficients. The quantized data contains same redundant data which is waste of space. To overcome this problem, Huffman encoding is used. In this, the probabilities of occurrence of the symbols in the signal are computed. After that, these are arranged according to the probabilities of occurrence in descending order and build a binary tree and codeword table. Form these codeword's eliminate the redundant data. Finally, the compressed ECG signal data values are obtained.

4 Results and Discussions

In this section, a wavelet based methodology has been used for the ECG signal compression. Different ECG records have been obtained from MIT-BIH Arrhythmia Database and the wavelet filters designed with 2nd and 3rd derivative of Beta function is exploited for ECG signal compression. Several examples are included to illustrate the effectiveness of the beta wavelet filters in the field of data compression. The performance of the beta wavelet filters can be evaluated by considering the fidelity of the reconstructed signal to the original signal. For this, the following fidelity assessment parameters are considered [8, 38, 39]:

- Compression ratio (*CR*):

$$\mathrm{CR} = \frac{\mathit{Length}\ \text{of Original Signal}}{\mathit{Length}\ \text{of Compressed signal}} \tag{22}$$

- Percent root mean square difference (*PRD*):

$$PRD = \left(\frac{\text{Reconstructed noise energy}}{\text{Origional signal energy}}\right)^{1/2} \times 100$$
$$= \sqrt{\frac{\sum \left[x(n) - y(n)\right]^2}{\sum x(n)^2}} \times 100 \tag{23}$$

- Mean square error (*MSE*):

$$MSE = \frac{1}{2} \sum_n |x(n) - y(n)|^2 \tag{24}$$

- Signal to noise ratio (*SNR*):

$$SNR = 10 \log_{10} \left\{ \frac{\sum x^2(n)}{\sum |x(n) - y(n)|^2} \right\} \tag{25}$$

Table 2 lists the simulation results obtained with beta wavelet filters (2nd and 3rd derivatives) at global thresholding (Threshold = 0.15). Figure 4 shows the plot of the original ECG signals (MIT-BIH Rec.100 M, 69) and its reconstructed version with beta wavelet filters.

It can be seen from Table 2 that a significant compression ratio is achieved with the wavelets filters based on beta function. The average compression ratio obtained with 2nd derivative beta wavelet filter is 4.34%, while in case of 3rd derivative is 4.33% for the single level decomposition. It is also evident that the good fidelity measures can be achieved with the beta wavelet within acceptable range [8]. The average *PRD* obtained with beta wavelet filters is 5.14% in 2nd derivative and 4.70% in 3rd derivative. PRD in range of 2–10% have been acceptable in practice [8, 38]. Other parameters (*MSE* and *SNR*) have also been improved. When beta wavelet filters (2nd and 3rd derivatives) are compared, the beta wavelet filter based on 3rd derivative gives better performance in term of compression ratio, while 2nd derivative

Table 2 Fidelity assessment parameters in proposed algorithm with use of Beta wavelet at single decomposition level

Signal	Beta wavelet filter	Original signal length	Compressed signal length	*CR*	*PRD*	*MSE*	*SNR*
MIT-BIH Rec. 100: M, 69	2nd	1014	972	4.17	4.17	1.09×10^{-4}	27.51
MIT-BIH Rec. 112: M, 54	2nd	1014	963	5.06	3.02	4.13×10^{-4}	30.42
MIT-BIH Rec. 117: M, 69	2nd	1014	974	3.96	8.79	2.50×10^{-3}	21.29
MIT-BIH Rec. 210, M, 89	2nd	1014	971	4.18	4.59	3.89×10^{-5}	26.80
MIT-BIH Rec. 100: M, 69	3rd	1014	972	4.17	3.24	6.65×10^{-5}	29.65
MIT-BIH Rec. 112: M, 54	3rd	1014	963	5.05	2.81	3.56×10^{-4}	31.06
MIT-BIH Rec. 117: M, 69	3rd	1014	974	3.96	8.53	2.40×10^{-3}	21.53
MIT-BIH Rec. 210: M, 89	3rd	1014	972	4.16	4.24	3.34×10^{-5}	27.46

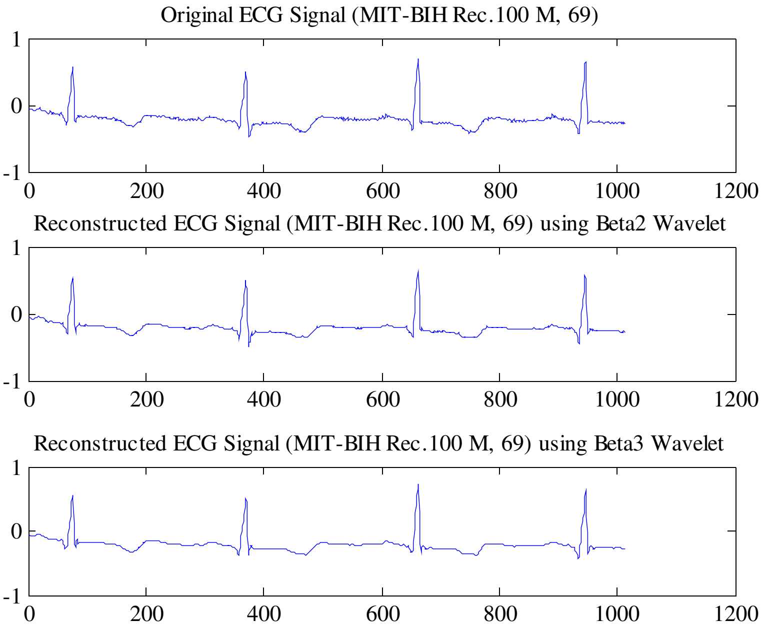


Fig. 4 The original ECG signals (MIT/BIH-100) and its reconstructed version based on the Beta wavelet with 2$^{\text{nd}}$ and 3$^{\text{rd}}$ derivative filter

performs better in terms of fidelity measures for higher level of signal decomposition form Table 3. Therefore, beta wavelet can be effectively used for the ECG signal compression.

Figures 5 and 6 depict the plots of performance obtained with beta wavelet at different global thresholding. As it can be observed from the simulation results included in the figures that the compression ratio (CR) is increased at higher threshold values, while the quality of reconstructed signal is degraded. Thus, by choosing appropriate threshold value, good compression ration can be achieved with preserving all clinical information.

A comparison of the developed wavelet filters with other existing wavelet filters is carried out and it is graphically illustrated in the Fig. 7. For this, ECG records (MIT-BIH Rec. 100: M 69) have been taken from MIT-BIH Database and the performance measures obtained in each wavelet filters using same methodology and at same threshold value is listed in Table 2. As, it can be seen that the performances of the beta wavelet filters are significantly improved as compared to earlier known wavelet filters in terms of all performance measuring parameters for the biomedical signal compression.

The simulated results compare with the other algorithms or methods [10, 38] results show the presented results of ECG compression based on Beta wavelet is better than the others. From the Fig. 4, it's clearly represented the reconstructed ECG signal after compression is identical to original signal based on Beta2 and Beta3

Table 3 A comparison of performance of different wavelet filters with ECG signal (MIT-BIH Rec. 100: M 69) at fourth decomposition level

Wavelet filters	*CR*	*PRD*	*MSE*	*SNR (dB)*
Db8	8.18	11.18	7.15×10^{-4}	19.26
Db10	7.94	11.98	8.15×10^{-4}	18.69
Coif5	7.64	8.33	4.13×10^{-4}	21.71
Sym5	8.53	9.54	5.38×10^{-4}	20.57
bior4.4	10.63	18.13	1.70×10^{-3}	15.52
Beta2	**8.46**	**8.13**	$\mathbf{3.88 \times 10^{-4}}$	**21.92**
Beta3	**8.65**	**8.88**	$\mathbf{4.69 \times 10^{-4}}$	**21.09**

The significance of bold data shows the improved performance of Beta wavelet as compared to other existing wavelets.

Fig. 5 Performance based on the Beta wavelet filter with 2nd Derivative

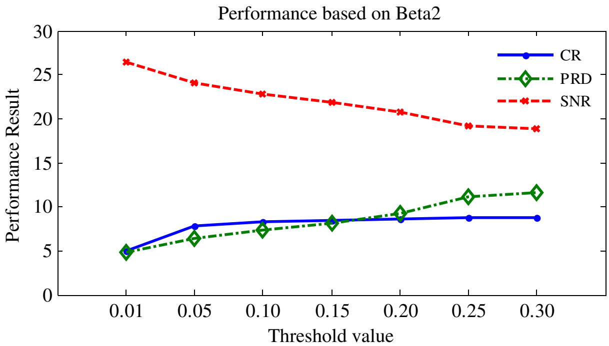
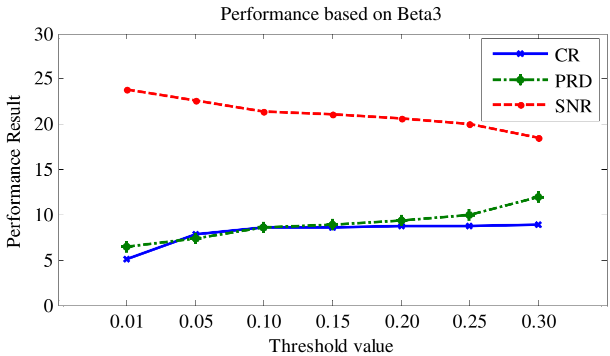


Fig. 6 Performance based on the Beta wavelet filter with 3rd Derivative



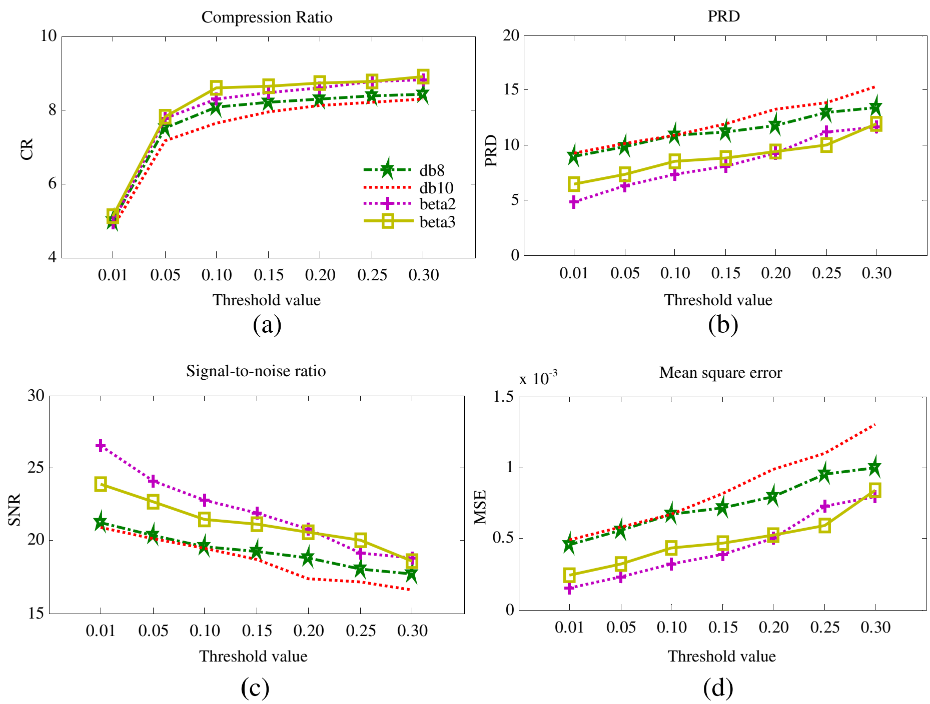


Fig. 7 Comparison of performance of the Beta wavelet filters (2nd and 3rd derivatives) and other existing wavelet filters. **a** Compression ratio (*CR*) **b** *PRD* **c** *SNR* **d** *MSE*

wavelets at the obtained fidelity parameters (PRD, SNR and MSE). Its means the applying wavelet is optimal for the biomedical signal compression with preserving signal information.

5 Conclusions

A wavelet based methodology is presented for the ECG signal compression. In this methodology, beta wavelet filters are exploited for signal compression, which are derived using beta function and its derivatives. Simulation results included in this paper clearly show the key advantageous features of the beta wavelet filters over others in the field of biomedical signal processing. It is found that the beta wavelet filters yields more compression with preserving all clinical information. All fidelity measuring parameters are improved. Therefore, it is concluded that it can be very effectively used in ECG signal compression.

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