J Math Model Algor (2010) 9:311–342
DOI 10.1007/s10852-010-9130-4

Medium Term Production Management for Cyclic Deliveries

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Received: 23 August 2009 / Accepted: 22 December 2009 / Published online: 10 March 2010

Abstract The growing quality and delay requirements have catalyzed the emergence of new commercial paradigms, which have strongly modified the customer–supplier relationship. Customers and suppliers become more and more linked with contracts or global orders spanned over a relatively important period. This paper, examines a type of contract which specifies a fixed and cyclic delivery dates with delivery quantities varying between a min and a max values. The exact delivery quantities are usually known only few days before the delivery. A company which produces n items on a bottleneck facility is considered; each item is confronted to a cyclic demand and has an important holding cost in comparison to set-up costs. We propose heuristic approaches, to build, in a medium term level, cyclic production schedules. These schedules face the demand and minimize a total cost function composed of holding and set-up costs. An experiment is proposed in order to prove the effectiveness of our approaches.

Keywords Scheduling · Production management · Cyclic production · Cyclic delivery schedule

1 Introduction

Three main facts have called into question the existing production management methods: the supply chain integration, the just-in-time philosophy development and the emergence of new commercial paradigms. Companies have slowly moved from

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the classical and punctual order between a customer and a supplier with a quantity and a delay to the notion of a contract.

This paper considers a specific kind of contract where the supplier is linked to the customer through a cyclic delivery schedule with a quantity varying between a minimum and a maximum for each delivery. For many years, many automotive constructors have adopted the principle of cyclic deliveries with their subcontractors for standard components in a low variety. For this kind of products, the demand is generally stable and the cyclic delivery schedules make the organisation of these products around the order and the reception easier. The cyclic production constitutes then, a natural response for facing this kind of demand. The synchronization between the production and the deliveries respects totally the just in time principles. Moreover, cyclic schedules improve the apprenticeship as well.

The production cycle is defined by a duration and a fixed sequence of manufacturing orders. The lot sizes for each order are estimated and based on average demands but the definitive lot sizes are decided in a short term level and are based on the real demand. The production capacities are also fixed or adjusted to the real demand in a short term level.

The remainder of the paper is organized as below. In Section 2, we review the relevant literature in the cyclic scheduling domain. In Section 3, we propose a mathematical modelling of the global problem and we suggest a global resolution strategy of the problem based on three main steps. We then give in detail in Sections 4, 5 and 6 the different steps of our global strategy. In Section 7, we present a numerical example in order to illustrate our approach. Section 8 proposes an experimentation of our approach. Finally, in Section 9, we present our conclusion and some future researches.

2 Literature Review

Many works have focused mainly on cyclic scheduling. We can find a synthesis in [4, 16, 37]. The simplicity of planning, describing and managing cyclic schedules was mentioned in [9, 39, 40]. The gains in terms of quality and productivity, resulted from the repetition of the cycle, were mentioned in [10, 20, 36]. Many works have also insisted on the complexity of implementing cyclic production [3, 21, 23, 30, 40].

In order to face a constant and continuous demand, many researchers built production cycles [2, 6, 17, 18, 22, 30, 36, 45, 46]. A generalisation of these works in a multi-level case can be found in [1, 19, 28, 31, 38, 42]. The case of a dynamic demand was treated in [10] and [34]. The case of a repetitive demand correspondent to a product delivery at fixed interval was treated in [11, 14, 23, 44]. The problem of minimizing the total annual costs of the single-vendor single-buyer was studied by Hocque [24]. They suppose that successive batches of a lot are transferred to the buyer in a finite number of unequal and equal sizes and that the batch sizes increase by a fixed factor in a limited transport capacity context. The same authors [25, 26] developed a heuristic solution procedure to minimize the total cost of setup or ordering and inventory holding for an integrated inventory system under controllable lead-time between a vendor and a buyer. In a more recent work, Hoque

[27] studied the synchronization in the single manufacturer multi-buyer integrated inventory supply chain.

Some researchers build cyclic schedules based on mathematical programming and continuous time formulation [1, 48]. Che and Chu [15] treated the problem of two robots in a flow-shop. Brucker and Kampmeyer [8] used taboo search algorithms in order to construct cyclic schedules and reduce the cycle time. Cavory et al. [12] proposed a genetic approach to solve the problem of cyclic job shop scheduling with linear constraints. Hsu et al. [28] studied the problem of cyclic scheduling for flexible manufacturing systems.

Other researchers have built cyclic schedules for the joint replenishment problem (JRP) (a review can be found in [32]). Webb et al. [47] studied the performance of different cyclical production schedules and concluded that the additional costs incurred by adopting cyclical schedules, for the JRP with dynamic demands, are usually small. The dynamic-demand joint replenishment problem has been also studied by Boctor et al. [7] and Lee and Chew [35]. Boctor et al. [7] showed that the Fogarty–Barringer heuristic, even though it is relatively simple, offers the best performance. Lee and Chew [35] demonstrated that an important saving can be obtained with a periodic policy compared to a static review policy. More recently, the JRP has been studied by Nillson et al. [41] who proposed a heuristic based on a spreadsheet technique to find a balance between the replenishment and holding costs.

Many researchers demonstrated that the global problem is extremely complex [4, 23, 30, 40]. Moreover, our problem of cyclic scheduling is different from the acyclic scheduling problems because we search in the acyclic case a scheduling solution for a set of tasks with some constraints on tasks and resources from an initial state while for our problem, we have to calculate the production frequencies, the production sequence and to determine the positioning of the sequence in the time horizon in order to reach a stationary state.

We found in the literature, many works aiming to construct production cycles, or other inventory approaches trying to calculate production frequencies, or even approaches based on lean manufacturing where the set-up costs are deliberately neglected. Our first contribution in this work is to propose a global architecture defining production cycles, production frequencies and considering set-up and holding costs in the long, medium, and short terms. Indeed, we can distinguish three levels in the control and the supervision of cyclic production systems [4]:

- A long term level for helping contracts negotiation: the signing of a new contract engages the company in a long term level with all the modalities that will govern the customer–supplier relationship. Before engaging, the company must have tools which help it to know whether it has the capacity to fulfil this new contract, how profitable it would be, and if it will need some new investments.
- A medium term level for defining and adapting the production cycle: to face the engagements made in a long-term level, the company must:
  - Define replenishment and production cycles on the whole logistic chain and determine the correspondents' cyclic production schedules.
  - Follow the demand evolution and trigger if necessary the revision of the cyclic production plans. These evolutions can be foreseeable (new contracts

or planned demand evolution) or not foreseeable. A proactive supervision system will inform and alert the company to trigger a new calculation of the production cycle.

– A short term level for adjusting production quantities and capacities: We must, in a short term level, permanently adjust the launched quantities on production and the capacity production to face the effective deliveries quantities and the new previsions transmitted by the customers [3]. We must also establish a dynamic management of safety stocks to face the uncertainty on the real delivery quantities [3]. Finally, the problem of manpower adjustment must be considered [16].

This paper focuses on the medium term management in order to construct a well adapted cyclic production schedule The nature of the problem in a medium term depends strongly on the ratio between holding and set-up costs:

– If set-up costs are important comparatively to holding costs, the tendency is to stock products and the number of production launching will be kept as low as possible. One manufacturing order can, in this context, cover the needs of many delivery cycles. The main problem in this case is to determine this production period for each product. The sequencing and the phasing have an insignificant importance (due to low holding costs).
– If holding costs are important relatively to set-up costs, we do not stock products and we must launch a greater number of manufacturing orders on the workshop. Many manufacturing orders will cover one delivery cycle which is generally equal to a week. The main problem in this case is to organize the week by determining first, the number of manufacturing orders for each product and its respective quantity, and second, the production sequencing and phasing (the sequence positioning in the time horizon) of the sequence on the production line (due to high holding costs).

This paper focuses on the case where the holding costs are important The main difference between our work and the existent works is that we consider a delivery cycle with different delivery quantities in different dates and we intend to determine for each product a production cycle with different manufacturing orders and different lot sizes. In fact, the existent works consider, generally, a periodic delivery cycle with a fixed and unique interval (ex: a delivery each two days) and determine a production cycle multiple of the delivery interval (for example Sarker and Parija [44], Hill [23], Nori and Sarker [42]).

3 Mathematical Modeling of the Global Problem

We consider a factory which produces n items on a single production line (the bottleneck resource), and which is confronted to a cyclic delivery schedule with several independent deliveries of the same product in the cycle (for instance, we must deliver the finished product 1, 1,000 units each Monday, 2,000 each Tuesday and 5,000 each Friday, the delivery cycle, in this case, is the week). For each product, the delivery cycle length, the numbers of deliveries in every cycle are given and the delivery dates are fixed.

Our objective is to determine a cyclic scheduling with a cycle time and, for every finished product, a production frequency in the cycle and a lot size minimizing the total cost function which includes set-up and carrying costs.

The first difficulty is related to the modeling of the cost function. Indeed, the calculation of the holding costs of the finished products inferred by a given scheduling solution is more complex in the general case.

On Fig. 1 a production planning of three products A, B and C is represented. We suppose that the machine is always in production and that, for every finished product, there is only one manufacturing order and one delivery:

To calculate volumes of average inventories inferred by production planning of Fig. 1, it is sufficient to determine, for each of the finished products, the integral of its inventory function on the duration of the cycle T and to divide the result by the same cycle. This integral calculation can be made either by modeling the function before calculating its integral, or by calculating directly the area of this function for the duration of the cycle. For that purpose, we need to define these notations:

T: duration of the production cycle.
dl_j: delivery date of product j in the cycle.
Q_j: delivered quantity of product j (equal to the produced quantity).
dd_j: production beginning date of product j in the cycle.
P_j: production rate of product j.
df_j: production ending date of product j in the cycle.

$$\forall j \text{ we have } Q_j = (df_j - dd_j) * P_j$$

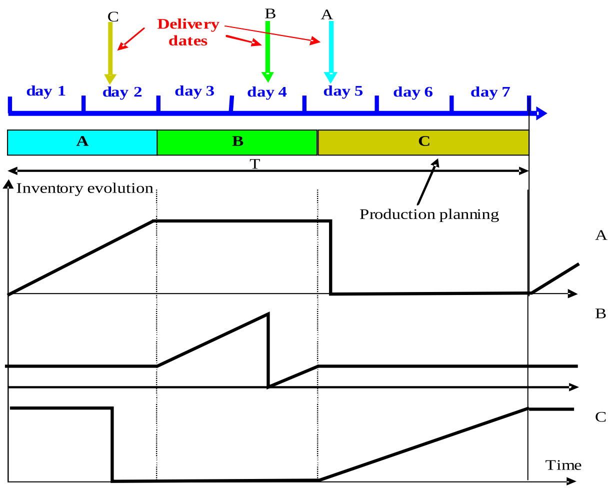


Fig. 1 Example 1 of inventory evolution function of time

Three different cases can happen and, for every case, the calculation of the average inventory is made in a different way:

Case 1 $dd_j < df_j < dl_j$ It is the case of product A, the average inventory is made as below:

$$AI_A = \frac{1}{T}\left(\frac{(df_A - dd_A) * Q_A}{2} + Q_A * (dl_A - df_A)\right)$$
$$= \frac{1}{T}\left(\frac{Q_A^2}{2 * P_A} + Q_A * (dl_A - df_A)\right) \quad (1)$$

Case 2 $dd_j < dl_j < df_j$

It is the case of product B, the initial inventory II_B that guarantees a positive inventory during the time horizon is equal to:

$$II_B = (df_B - dl_B) * P_B$$

The average inventory is equal to:

$$AI_B = \frac{1}{T}\left(T * SI_B + \frac{P_B((dl_B - dd_B)^2 - (df_B - dl_B)^2)}{2}\right) \quad (2)$$

Case 3 $dl_j < dd_j < df_j$

It is the case of product C, the initial inventory II_C that guarantees a positive inventory during the time horizon is equal to:

$$II_C = Q_C$$

The average inventory is equal to:

$$AI_C = \frac{1}{T}\left(\frac{(df_c - dd_c) * Q_C}{2} + Q_C * dl_C\right) = \frac{1}{T}\left(\frac{Q_C^2}{2 * P_C} + Q_C * dl_c\right) \quad (3)$$

In the general case, the problem of evaluation of the average inventories becomes much more complex. Indeed, for a defined cyclic scheduling, several manufacturing orders of the same product can exist on the same cycle and, furthermore, a manufacturing order can cover completely or partially several deliveries as shown in Fig. 2.

Figure 2 represents a production cyclic scheduling of a week for three finished products A, B and C with several deliveries in this cycle for each one of these products. Three curves of evolution of the inventory are represented. The difficulty to estimate the volumes of inventories by calculating the integrals of the inventory functions appears then clearly. Indeed, it is very difficult to establish in the general case a function that calculates the corresponding holding costs.

It is essential, however, to be able to estimate a scheduling solution in term of holding costs. For that reason, we use the principle of discretization of the time horizon in periods. It is then, a question of establishing relations that exist between the quantities in stock of every beginning of period. Indeed, from period i to the following one $i+1$, the stock increases if the product in question is in production

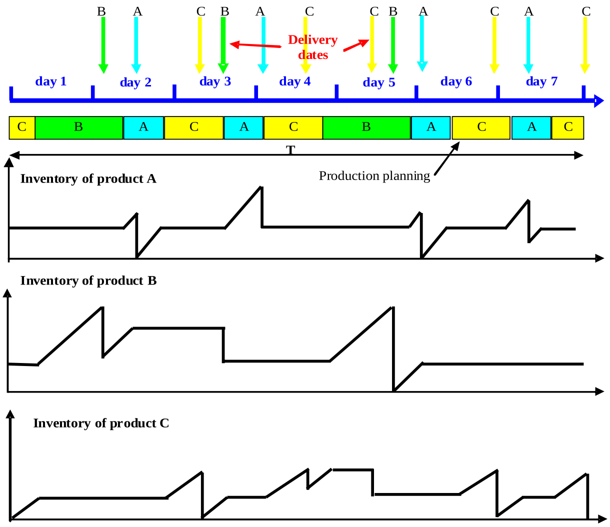


Fig. 2 Example 2 of inventory evolution function of time

during period i and decreases if the product is delivered during period i as shown in Fig. 3.

On Fig. 3, the inventory remains constant from the beginning of period 1 to period 8. In the beginning of period 9, the stock increases by a quantity equal to the quantity produced during period 8. In the beginning of period 10, the stock increases by the same quantity and decreases by a quantity equal to the quantity delivered during period 9.

We can estimate the average inventory by considering that, for every period, the value of the inventory remains constant and equal to the value of the beginning of

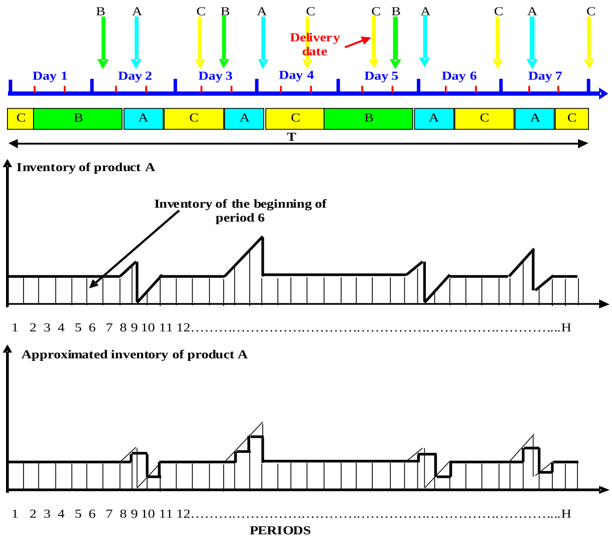


Fig. 3 Discretization of the production cycle

the period. If PI_t is the value of the stock during period t and if the number of periods is H in a cycle T, the average stock can be calculated as below:

$$AI = \frac{1}{H}\sum_{t=1}^{H} PI_t$$

The smaller the step of discretization is, the more precise the result will be. Indeed, we suppose that, for a given period, the product is exclusively in production or not. This estimate is acceptable if the step of discretization is small enough.

So we obtain a discreet formulation of the evolution of the inventory function of time at the level of the beginning of every period. It is on the basis of this discretization that our mathematical model (and in particular the holding cost function) will be established.

The mathematical model is constituted by a cost function that we must minimize under a set of constraints. The purpose is to determine an optimal cyclic scheduling in terms of holding and set-up costs. The holding costs are modelled thanks to the discretization of the production cycle as presented in the previous paragraph. In this model, we suppose that the decision variables are the cycle of production T as well as the periods of production for every finished product. The number of manufacturing orders will be deduced from it. The production cycle T is variable and will depends of the holding and set-up costs. If the set-up costs are equal to 0, the production cycle will be equal to the least common multiplier of the deliveries cycles. If the set-up costs are important, we can have production cycle more important but in all case it's a multiple of the least common multiplier of the deliveries cycles. The cycle of production will be discretized in a number H (which is variable because it depends on the length of the cycle T) of periods with a step of discretization defined and constant. The periods during which the deliveries take place are always supposed to be known whatever the value of H is. The considered unit of time is the step of discretization SP, the cycle of production T will be equal, thus, to H.

Let us consider this mathematical model:

H: total number of periods in the global production cycle.
SP: discretization step.
x_{jt}: product quantity of the product j which will be delivered in period t.
P_j: production rate of finished product j.
n: number of finished products (F.P.).
K_j: manufacturing set-up cost of finished product j.
h_j: holding costs of the finished product j by unit product and unit time.
Y'_j: maximal number of manufacturing orders of product j.
L_j: delivery cycle length of finished product j in periods.

The decision variables in this program are:

T: the production cycle which will be divided in H periods ($t = 1..H$) with a duration of SP for each period. The considered unit time is the period. T is a multiple of the least common multiplier of the deliveries cycles L_j.
δ_{jt}: 1 if the product j is produced in period t.
0 else.
PI_{jt}: product j inventory at the end of period t.
NO_j: number of manufacturing orders of the product j.

The objective function can be modelled as below:

$$Min\left(\frac{1}{H}\left(\underbrace{\sum_{t=1}^{H}\sum_{j=1}^{n}h_jPI_{jt}}_{Holding-costs}+\underbrace{\sum_{j=1}^{n}NO_jK_j}_{Set-up-costs}\right)\right) \tag{4}$$

Uder the following constraints:

$$\forall\, j, t,\ PI_{jt} \geq 0 \tag{5}$$

$$for\, t = 1,\ PI_{j1} = PI_{jH} - x_{jH} + \delta_{jH}P_j \tag{6}$$

$$for\, t > 1,\ PI_{jt} = PI_{j(t-1)} - x_{j(t-1)} + \delta_{j(t-1)}P_j \tag{7}$$

$$\forall j,\ \sum\nolimits_{t=1}^{H} x_{jt} = \sum\nolimits_{t=1}^{H} \delta_{jt} * P_j \tag{8}$$

$$I\begin{cases}\forall j,\ NO_j = \delta_{jH}\overline{\delta_{j1}} + \sum_{t=1}^{H-1}\delta_{jt}\overline{\delta_{j(t+1)}} & (9)\\ \forall j,\ NO_j \leq Y'_j & (10)\\ \forall t,\ \sum_{j=1}^{n}\delta_j t \leq 1 & (11)\end{cases}$$

$$II\begin{cases}\forall j,\ NO_j = (\delta_{jH}\sum_{j'=1 and \neq j}^{n}\delta_{j'1}) + \sum_{t=1}^{H-1}(\delta_{jt}\sum_{j=1 and \neq j}^{n}\delta_{j'(t+1)}) & (12)\\ \forall j,\ NO_j \leq Y'_j & (13)\\ \forall t,\ \sum_{j=1}^{n}\delta_{jt} = 1 & (14)\end{cases}$$

The developed mathematical model contains two variants, a first variant with the block I of expressions and a second variant with block II.

(4): objective function composed of two terms:

The first one represents the total holding cost by unit of time of all the finished products. We suppose that the inventory of a product j remains constant during period t (equal to the inventory of the beginning of the period). This term is obtained by adding, over all the periods of the cycle, the values of the inventories of all the products, which we multiply by the corresponding holding cost (the unit time is supposed to be equal to the period),

The second one represents the set-up costs by unit of time. It is obtained by adding, on all the manufacturing orders and for all the finished products, the corresponding set-up costs (the number of orders for every finished product are deduced by means of expression 9 or 12) and by dividing the obtained result by the production cycle.

(5): Constraint imposing the fact that the inventory for all the products has to remain always positive.

(6), (7): Expressions defining the evolution of the inventory, from one period to another, for all the finished products.

(8): Constraint indicating that the total produced quantity during the cycle is equal to the total delivered quantity

(9): Function of calculation of the number of manufacturing orders for a finished product j. This calculation is made by detecting the changes of state of the product j. Indeed, if we move from a production state for period t to a non production state for period $t+1$, we move from $\delta jt = 1$ to $\delta j(t+1) = 0$. In fact, this expression does not represent a constraint but models the calculation of the manufacturing order number.

(10): Constraint of limitation of the maximal number of manufacturing orders.

(11): Constraint expressing the fact that we can have during a period t at most only one product in production.

(12): Function calculating in a different way the number of manufacturing orders of a finished product j. Indeed, this time, we detect the change of product on the production line. However, this method implies that whatever the period is, one product, at least, has to be in a state of production. This expression, as for expression 9, does not represent a constraint.

(13), (14): The same constraints of the expressions (10 and 11) but by using the second method of calculation of the manufacturing orders number.

The developed models are based on a discretization of the time horizon in a set of periods of constant duration. During a given period, the inventory is supposed to remain constant and equal to the inventory of the beginning of the period. This approximation is valid only if the step of discretization is small enough.

Furthermore, these models contain several non linearities which result from the following variables:

- The production cycle T.
- The quantity in stock for a product j and for the period t: PI_{jt}.
- The calculation of the number of manufacturing orders NO_j for the first model.
- The calculation of periods of production for the second model.

We will prove that this problem is NP-complete by showing that our problem is a generalization of a well known scheduling problem and proven to be NP-complete. In fact, if we consider that we have not set-up costs, the production cycle can be easily fixed to the least common multiplier of the different deliveries cycles. We can also simplify our problem by considering that we will consider a manufacturing order for each delivery, we have then a set of tasks and for each task a duration, a holding cost and a due date. We obtain a scheduling problem well known in the literature as the single machine scheduling to minimize weighted earliness subject to no tardy jobs. The weighted earliness represents the inventory costs and the delivery dates represent the due dates. Chand et al. [13] prove that this problem is NP-complete; we can conclude that our problem which is more complex is also NP-complete. Even if we consider the problem of only one product, or only one delivery by product, we have always a set of tasks with inventory costs and due dates, so the same scheduling problem which is proven that it is an NP-complete problem.

The developed mathematical models are NP-complete and we propose in this paragraph a global strategy composed of a set of heuristics in order to construct a well adapted cyclic production schedule in a medium term management level and where the holding costs are important. The global resolution strategy is composed of three main phases:

- Ph1. The research of the production frequencies of each product separately in the delivery cycle: this research is based on forecasted demands and consists of determining the number of manufacturing orders for each product in the production cycle that minimizes the sum of holding and set-up costs. We have developed two approaches:
  - S.ph1. The first is based on a cost function modelling grouping the set-up and the holding costs. The optimal production frequency is obtained by deriving the cost function relative to the production frequency.
  - S.ph2. The second approach consists of enumerating all production frequency possibilities ranging between one manufacturing order for all the deliveries and one manufacturing order for each delivery and, then, choosing the best frequency relative to the set-up and holding costs.
- Ph2. The generation of production sequences: Consists of generating global production sequences integrating all the products. These production sequences are based on the production frequencies determined in the precedent phase.
- Ph3. The determination of the cyclic production planning: Consists of finding, for each feasible production sequence, the best positioning in the time horizon (phasing), regarding the holding costs, by moving the sequence circularly. After that, we choose the sequence and the position that give the best holding and set-up costs.

4 Ph1: The Research of the Production Frequencies of each Product Separately

We consider a factory that produces n items on a single production line (the bottleneck resource) and which is confronted to a cyclic delivery schedule with several independent deliveries of the same product in the cycle (for instance, we must deliver the finished product 1, 1,000 units each Monday, 2,000 each Tuesday and 5,000 each Friday, the delivery cycle in this case is the week). For each product, the delivery cycle length, the number of deliveries in every cycle is given and the delivery dates are fixed.

We aim at determining a production frequency for each finished product separately in its delivery cycle which minimizes a total cost function regrouping holding and set-up costs.

We have developed two approaches: the first one consists of modelling a cost function regrouping set-up and holding costs and the second one consists of enumerating some solutions.

4.1 First Approach

The first approach consists of building a total cost function grouping holding, set-up and ordering costs. The production frequency (number of manufacturing order for each product in the production cycle) is then obtained by deriving this function. The following notations are used:

n: number of finished products (F.P.).
L_j: delivery cycle length of finished product j.
P_j: production rate of finished product j.
h_j: holding cost of finished product j by products unit and time unit.
K_j: manufacturing set-up cost of finished product j.
SD_j: manufacturing set-up duration of finished product j.
w_j: delivery number of finished product j in the cyclic delivery schedule.
dl_{vj}: the v^{th} delivery date in the delivery cycle of product j.
x_{uj}: the u^{th} delivery quantity of finished product j in the cyclic delivery schedule.
l_{uj}: time interval between the u^{th} delivery and the $(u-1)^{th}$ one for finished product j. with $l_{(W_j+1)j} = L_j - dl_{(W_j)j}$.
Q_j: batch size of finished product j.
T_j: time interval between two productions of finished product j.
NO_j: production frequency or number of manufacturing order of finished product j in a cycle L_j.
X_j: represents the total delivery quantity by delivery cycle:

$$X_j = \sum_{u=1}^{w_j} x_{uj}$$

We are looking for:

Q_j: lot size of a manufacturing order of finished product j.
NO_j: production frequency or number of manufacturing orders of finished product j during the delivery cycle L_j.

We assume that we produce, at a fixed frequency, the same quantity of products. We need this simplification to build the total cost function. In fact, we use this model only for determining the frequency, the calculus of the production dates will be treated in a second step. The set-up durations are not considered in this step but will be considered in the global production sequence scheduling.

$QP(t)$represents the cumulative produced quantity function of time t and $QD(t)$ the cumulative delivered quantity function of time t. The cumulative produced and delivered quantities evolutions are represented during a delivery cycle in the figure below:

The cumulative delivered and produced quantities functions for a finished product j can be formulated function of time t and during a cycle length $L_j(0 \leq t \leq L_j)$ as below:

$$\left\{ QD(t) = \sum_{u=0}^{v} x_{uj} \quad \begin{array}{l} dl_{vj} \leq t < dl_{(v+1)j} \text{ for } 0 < v < w_j \\ with\, x_{0j} = 0,\ dl_{0j} = 0\, and\, dl_{(w_j+1)j} = L_j \end{array} \right. \tag{15}$$

$$\begin{cases} QP(t) = P_j\left(t + \frac{iX_j}{P_j + NO_j} - \frac{iL_j}{NO_j}\right) \ \frac{iL_j}{NO_j} \leq t \leq \frac{iL_j}{NO_j} + \frac{X_j}{P_j * NO_j}\ for\, i\, from\, 0\, to\, NO_{j-1}. & (16) \\ QP(t) = \frac{(i+1)*X_j}{NO_j}\ \frac{iL_j}{NO_j} + \frac{X_j}{P_j * NO_j} \leq t \leq \frac{(i+1)L_j}{NO_j}\ for\, i\, from\, 0\, to\, NO_j - 1. & (17) \end{cases}$$

We suppose that $P_jL_j - X_j \geq 0$. Indeed, the delivery cycle duration should allow the production of the global quantity delivered in the cycle.

The average inventory AI_j during a delivery cycle L_j is equal to:

$$
\begin{aligned}
AI_j &= \frac{1}{L_j}\int_0^{L_j}(QP(t)-QD(t))dt \\
&= \left(\frac{1}{L_j}\sum_{i=0}^{NO_j-1}\int_{\frac{iL_j}{NO_j}}^{\frac{iL_j}{NO_j}+\frac{X_j}{P_{j*NO_j}}} P_j\left(t+\frac{iX_j}{Pj*NO_j}-\frac{iL_j}{NO_j}\right)dt\right) \\
&\quad + X_j\left(L_j-\frac{X_j}{P_j}\right)\frac{NO_j+1}{2NO_j} - \left(\frac{1}{L_j}\sum_{v=0}^{w_j}\sum_{u=0}^{v}\int_{dl_{vj}}^{dl_{(v=1)j}} dt\right) \Longrightarrow AI_j \\
&= \left(\frac{X_j}{2}+\frac{X_j(P_jL_j-X_i)}{2NO_jL_jP_j}-\frac{1}{L_j}\sum_{v=1}^{w_j} l_{(v+1)j}\sum_{u=1}^{v}x_{uj}\right)
\end{aligned} \tag{18}
$$

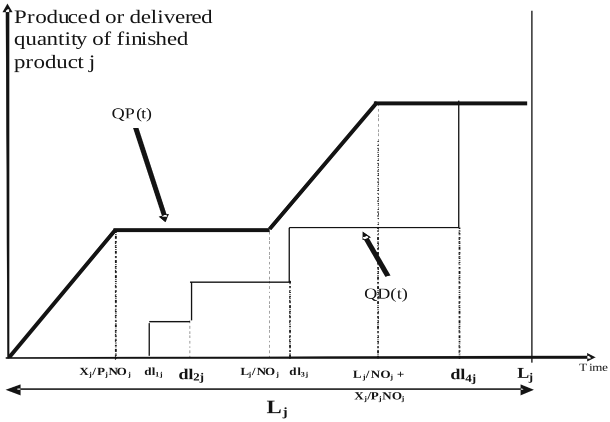
The holding costs of the finished products can be defined as below (Fig. 4):

$$
HC_j = AI_j * h_j \Longrightarrow HC_j = \left(\frac{X_j}{2}+\frac{X_j(P_jL_j-X_j)}{2NO_jL_jP_j}-\frac{1}{L_j}\sum_{v=1}^{w_j} l_{(v+1)j}\sum_{u=1}^{v}x_{uj}\right)h_j
$$

We can model the set-up and ordering costs by unit-time for each finished product j as below:

$$
OC_j = \frac{1}{L_j/NO_j}(K_j) \Longrightarrow OC_j = \frac{NO_j}{L_j}(K_j)
$$

Fig. 4 Produced QP and delivered QD quantities evolution



The total cost function $TC_j$for a finished product j can be evaluated as the sum of the precedent costs as below:

$$TC_j = \left(\frac{X_j}{2} + \frac{X_j(P_jL_j - X_j)}{2NO_jL_jP_j} - \frac{1}{L_j}\sum_{v=1}^{w_j} l_{(v+1)j}\sum_{u=1}^{v} x_{uj} \right) h_j + \frac{NO_j}{L_j}K_j \tag{19}$$

$$\begin{aligned} TC_j = {} & \frac{NO_j}{L_j}K_j + \frac{1}{2NO_jL_jP_j}(X_j(P_jL_j - X_j)h_j) \\ & + \left(\frac{X_j}{2} - \frac{1}{L_j}\sum_{v=1}^{w_j} l_{(v+1)j}\sum_{u=1}^{v} x_{uj} \right) h_j \end{aligned} \tag{20}$$

Let us derive two times TC_j relatively to NO_j:

$$\frac{d^2TC_j}{d^2NO_j} = \frac{1}{L_jP_jNO_j{}^3}(X_j(P_jL_j - X_j)h_j) \Longrightarrow \frac{d^2TC_j}{d^2NO_j} \geq 0$$

TC_j is a convex function, it is two times derivable and $\frac{d^2TC_j}{d^2NO_j} \geq 0$. The minimum of this function is obtained when $dTCj/dNOj = 0$. We can obtain the optimal NO_j as below:

$$\begin{aligned} \frac{d^2TC_j}{d^2NO_j} = 0 \Longrightarrow \frac{1}{L_j}(K_j) - \frac{1}{(2P_jL_jNO_j)^2}(X_j(P_jL_j - X_j)h_j) = 0 \Longrightarrow NO_{jopt} \\ = \left(\frac{(X_j(P_jL_j - X_j)h_j)}{2P_jK_j} \right)^{\frac{1}{2}} Q_{jopt} = \frac{X_j}{NO_{jopt}} = \left(\frac{2X_jP_jK_j}{(P_jL_j - X_j)h_j} \right)^{\frac{1}{2}} \end{aligned} \tag{21}$$

The cost function is valid only for NO_j entire. As a consequence, we must test the two entire values around NO_j and choose the best value in terms of cost. We made the assumption that we produce the same quantity in each manufacturing order, and regularly all along the production cycle which means that the production dates are fixed as soon as the frequencies are. This approach can be used only to estimate the frequency production for each product very rapidly. It does not guarantee that the inventory remains positive but we can remedy that by fixing an initial stock. Indeed, during a cycle (the total produced quantity is equal to the delivered quantity) if the inventory is negative in the first cycle, a sufficient initial stock IS can cover this cumulative negative inventory and at the end of the first cycle, we will have an excess of stock of exactly IS, which will cover the negative inventory of the second cycle and so on. We can also calculate a max limit NO_{jmax} of the number of manufacturing orders leading to the number of set-ups that we can perform in the remaining time of the cycle $(L_j - \frac{X_j}{P_j})$:

$$NO_{jmax} = Ent\left(\frac{L_j - \frac{X_j}{P_j}}{SD_j} \right) \tag{22}$$

We choose after that the minimum between this limit and the frequency founded with the approach (Eq. 21).

4.2 Second Approach

In the first approach, we have considered that for a finished product, the lot sizes are the same for all the manufacturing orders. We have also supposed that the first manufacturing order starts always in the beginning of the production cycle. These two strong hypotheses could affect the results and that is why we have developed a second approach for the production frequency and lot sizing determination. The main idea of this approach consists of calculating for each product the holding and set-up costs for a frequency NO_j, which will vary from one production for all the delivery cycle to one production for each delivery. For each frequency, we must test all covering combinations of the deliveries by the productions. This approach is based on three steps:

- The determination of all the possible solutions by varying the production frequency and the covering of the deliveries by the M.O.
- The calculation for each possible solution, the corresponding set-up and holding costs. The set-up costs are easy to calculate because for a possible solution, the number of M.O. is known.
- The choice of the best solution in terms of set-up and holding costs. Fig. 5 Example 1 of covering the deliveries of the product P1 by three manufacturing orders

In Fig. 5, we have considered a product P1 that have to be delivered four times a week (day 2, 4, 6, and 7). As a consequence, we must test a production planning with a frequency production varying between one and four times a week. For a production frequency, we test all the possibilities of covering the deliveries by the manufacturing orders. In our example (Fig. 5), we have three manufacturing orders, the first one covers one delivery, the second one covers one delivery and the third one covers two

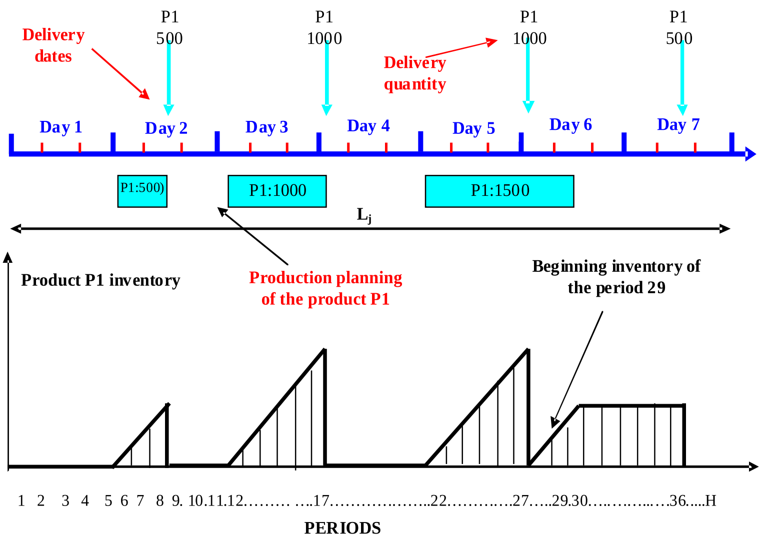


Fig. 5 Example 1 of covering the deliveries of the product P1 by three manufacturing orders

Table 1 Different possibilities of covering the deliveries of product P1 by 3 M.O.

	M.O.1	M.O.2	M.O.3
Deliveries	1	2	3;4
	2	3	4;1 of next cycle
	4	1	2;3
	1;2	3	4

deliveries. Other combinations are possible; all the possibilities are summarized in Table 1.

For each combination,we calculate the latest starting dates of each manufacturing order relatively to the deliveries that it will cover (these starting dates will define the productive and non productive periods used in the approach of the next paragraph) and then, we calculate the total cost function composed of holding and set-up costs. We choose after that the combination with a minimal total cost.

If we know for a manufacturing order the deliveries that it covers, we can calculate its latest starting date on the last machine as below:

$$\forall i \; LSD_i = Min_{u=1\to U_i}(dl_i - \textstyle\sum_{v=1}^{u} \frac{x_{vi}}{P_i}) \tag{23}$$

With:

- LSD_i: the latest starting date of the manufacturing order i.
- P_i: production rate of the manufacturing order *i*.
- x_{vi}: v^{th} delivery quantity covered by the M.O. manufacturing order *i*.
- dl_{ui}: date of the u^{th} delivery covered by the manufacturing order *i*.
- U_i: number of deliveries covered by the manufacturing order *i*.

The cost function is difficult to build in the general case, so we used the principle of discretization of the time horizon (we decompose the time horizon in H sub-periods; the sub-period may be an hour for example) and we propose an algorithm to calculate the holding cost of one particular solution. This algorithm will be executed for each combination in which the productive periods are fixed and so the set-up costs are known. The aim of the algorithm is to calculate for each combination the total inventories that will ensure the deliveries (the productive periods and the delivery dates are fixed for a combination). After that, we can calculate the induced holding costs.

The following notations are used:

SP: discretization step.
L: delivery cycle divided into H periods t (t = 1, .., H) with a duration SP. The unit time is the period.
x_t: product quantity which has to be delivered in period t. These quantities are fixed by the customer and known.
P: production rate per period t (the unit time is the period with a duration SP).
h: holding costs of the finished product by unit product.
K: manufacturing set-up cost of the finished product.
NO: Number of manufacturing orders or production frequency of the finished product.

δ_t: 1 if the product is produced in period t (fixed and known for a combination).
0 Else.
$InMin$: the minimal value of the inventory.
PI_t: product inventory at the end of period t.

The inventories at the end of each period are calculated as below:

– Calculate the inventory at the end of the first period :
  $PI_1 = \delta_1 * P - x_1$
– Calculate the inventory at the end of the other periods :
  For t from 2 to H do
  $PI_t{=}PI_{(t-1)} - x_t + \delta_t P$
  End For
– Calculate the minimal inventory :
  $InMin = Min_{t=1\rightarrow H} PI_t$
– Correct the inventories if there are negative inventories:

```
If      InMin <0
        For      t form 1 to H do
                 PIt= PIt - InMin
        End      For
End     If
```

The average inventory is calculated as below:
$AI = \frac{(\sum_{t=1}^{H} PI_t)}{H}$
The total cost by unit time is equal to:

$$TC = \frac{h\sum_{t=1}^{H} PI_t}{H} + \frac{NO}{H}K \tag{24}$$

If we consider Fig. 5, we have three manufacturing orders, we can calculate their latest starting dates regarding to their deliveries and we obtain the productive period: from period 6 to 8, 12 to 17, 22 to 30. The delivery dates (periods 8, 17, 28, 36) and the delivery quantities for these periods are fixed and known, the q_t for these periods corresponds to the delivery quantity and is equal to 0 for the other periods. For this specific combination, the algorithm will calculate the minimum inventories that will ensure the deliveries. In fact, in the first phase of the algorithm we consider an initial inventory equal to 0. After that, we calculate the minimum $InMin$ of the inventories of all the periods. If this minimum is negative we correct the inventory by adding to the inventory of all periods ($-InMin$). In fact, $InMin$ represents the minimum value of the initial inventory at period 0 to ensure that the inventory will always remain positive. The case where we must have an initial inventory is represented in Fig. 6.

In Fig. 6, we have another example of a product P2 with four deliveries and two manufacturing orders. The first M.O. will cover the deliveries 2 and 3; the second M.O. will cover delivery 4 of this cycle and delivery 1 of the next cycle. The latest starting dates of the M.O. will be calculated in relation with their delivery dates (periods 5, 20, 29, 39) and we obtain the productive periods: 16 until 22 and 35 until 41. At the end of period H, we have a remaining inventory which will be the initial inventory of the next cycle and will cover delivery 1. The proposed algorithm will

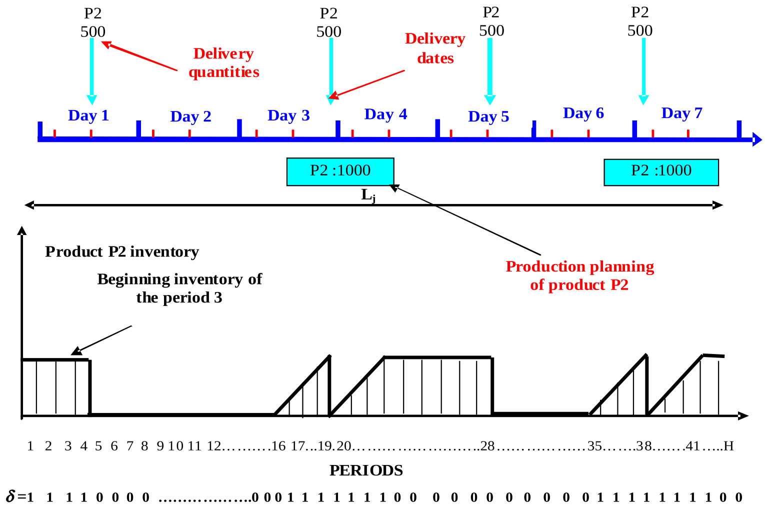


Fig. 6 Example 2 of covering the deliveries of product P2 by two manufacturing orders

calculate this minimum initial inventory that will ensure the deliveries and so the induced holding costs. Remarks:

- The productive and non-productive periods (δt) are determined by calculating, for each manufacturing order, the latest starting date with regard to the covered deliveries. This starting date or period is the first productive period; the last productive period can be calculated according to the production rate and the lot size. In Fig. 5, the third manufacturing order covers two deliveries, the latest starting date will be period 22 and the last productive period concerning this manufacturing order will be period 30. In Fig. 6, the first M.O. covers deliveries 2 and 3, so the latest starting date will be period 16 which is enough to cover, in time, delivery 2 in period 20.
- This program will be executed for the different possible production frequencies and covering combinations and for each product separately. The best solution for each product will be kept for the next stage. The complexity of this phase is polynomial and depends on the number of products, the number of deliveries.

This approach can be summarized as below:

For each finished product j Do
    For $NO_j = 1$ to w_j (delivery number) Do
        For each covering possibility of the deliveries by the M.O. Do
            Calculates the total cost (set-up and holding costs)
        End For
    End For
    Choose the best combination which gives the lowest total cost.
End For

The two presented approaches for determining production frequencies consider, in a medium term, that the company has globally the required capacity (on the basis of average demand). The capacity problems due to the increasing of the delivery quantities are considered only in a short term level.

5 Ph2: The Generation of Production Sequences

In the precedent phase, we calculated the optimal production frequency for each finished product. In this phase, our purpose is to generate, according to the production frequencies determined in the precedent phase, a production sequence that includes all the finished products. The production sequence generation is based on the works of Pinto-Mabert (Kim et al. [33, 34]). We first generate an initial production sequence, then, we generate the "equivalent sequences" by inverting the products with the same product frequencies. The initial sequence is generated using this algorithm:

- Calculate the total number of batches in a repetitive cycle b=$\sum_j f_j$.
- Calculate the number of sub-cycles $u = LCM$ (Least Commun Multiplier) of the production frequencies NO_j.
- Calculate the number of cells in each sub-cycle $l = [b/u]^+$
- Sort the items in the ascending order of NO_j and the processing time per batch.
- Do until all the batches (M.O.) are assigned to cells:
  - For each product, assign the first M.O. to the emptiest sub-cycle.
  - The other M.O. of the same product are assigned to the next cells by respecting a distance counted in sub-cycles equal to $SI_j = (u/NO_j)$.
  - The M.O. is assigned to the first free cell in the considered sub-cycle.
- End DO

Let us take an example of three products P1, P2, and P3; we suppose that the obtained production frequency in the phase 1 are as represented in Table 2.

The delivery cycle is the week for all the products. The global production cycle will also be the week and if we suppose that the production rate is the same for all the products, the first part of the algorithm gives:

- $b = 10$ (total number of the manufacturing orders)
- $u = 4$ (sub-cycles number)
- $l = 3$ (cells number = b / u)

The items are sorted as below:

P2, P1, P3.
The last part of the algorithm is illustrated in Fig. 7.

Table 2 Characterization of example 1

Finished product	P1	P2	P3
NOj	**4**	**2**	**4**

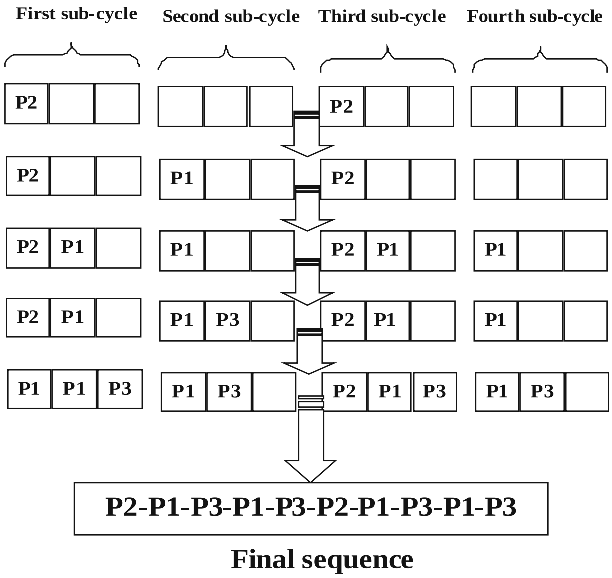


Fig. 7 Production sequence generation

The initial production sequence for the products (P1, P2 and P3) is as below:

| P2 | P1 | P3 | P1 | P3 | P2 | P1 | P3 | P1 | P3 |

One "equivalent sequence" can be generated by inverting the products that have the same production frequency (P1, P3):

| P2 | P3 | P1 | P3 | P1 | P2 | P3 | P1 | P3 | P1 |

This algorithm will not optimize but will create a production sequence by trying to distribute in a logical manner the manufacturing orders of a same product in the sequence and function of its frequency. The complexity of this generation depends on the number of products and of the production frequencies and is bounded by b!.

6 Ph3: The Determination of the Cyclic Production Planning

After generating the dominant production sequences, we must find the best phase for each sequence (the phasing consists of placing the repetitive sequence in the time horizon) by using a heuristic approach. The best phase is the one that minimizes the holding costs. For this reason, we must minimize the necessary inventories that ensure that the stock of all the products remains positive at any time. The sequencing and the phasing are in this case crucial, the number of the manufacturing orders is important, so it is essential to synchronize the production and the deliveries. It is important to note that, for one sequence, the duration of the different manufacturing orders are given and computed according to the algorithms of Section 4.

Figure 8 represents a delivery and production cycle equal to one week for three products P1, P2, P3. Three production schedules are represented (solution 1, 2, 3),

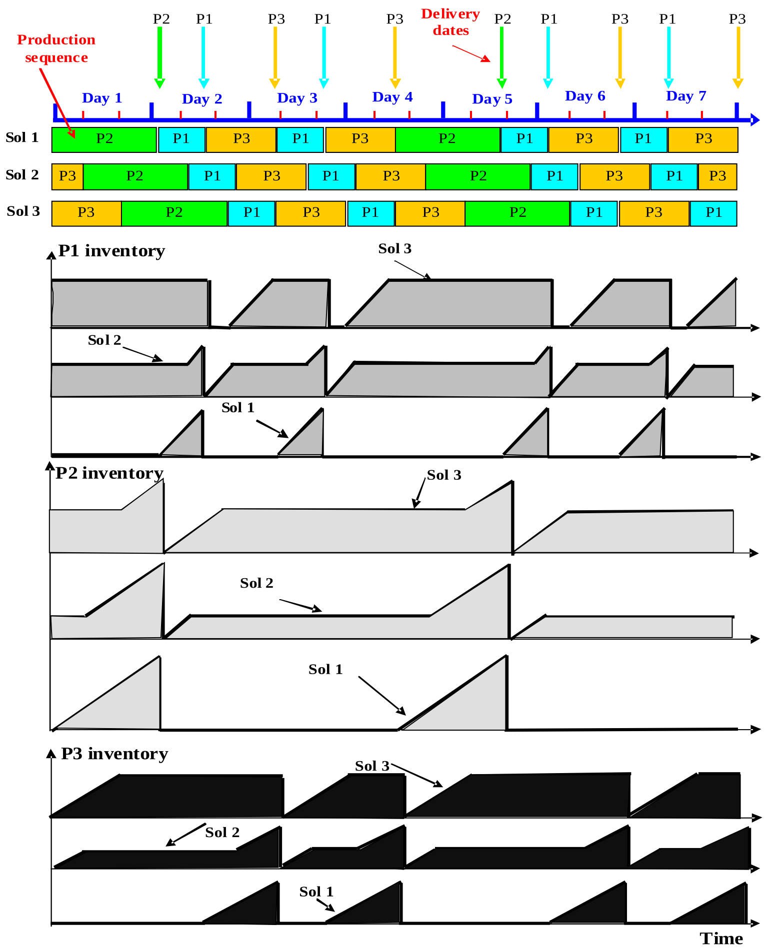


Fig. 8 Inventory evolution function of phasing

the second plan is moved circularly to the right by one subdivision and the third one is moved on two subdivisions. The inventory evolution for the three products is represented. We notice that the phasing of the sequence can have a big incidence on the inventory volumes. Indeed, if we take the example of product P1, the third solution induces an average inventory that is extremely greater than the average inventory induced by the first solution. The more the delivery number is covered by a manufacturing order, the more the incidence of the manufacturing order position is on the holding costs. For each possible couple (sequence, phase), we must calculate the necessary inventory for each product to guarantee the deliveries continuity during the cycle. The time horizon will be subdivided in periods, the passage from one phase to another will be done by moving all the production planning circularly by one period to the right.

In Fig. 8, we must test, for the first production sequence: P2-P1-P3-P1-P3-P2-P1-P3-P1-P3, 21 possible phases if the week is subdivided in 21 periods. The same treatment must be done for all the generated sequences, (in this example we must also test the sequence: P2-P3-P1-P3-P1-P2-P3-P1-P3-P1).

In order to evaluate the necessary inventory volumes for each sequence and phase, we developed an approach similar to the approach developed in paragraph 5.2 except that in this case the approach is multi-product.

Let us consider these notations:

L: delivery cycle divided into H periods $t(t = 1, .., H)$ with a duration SP. The considered unit time is the period.
n: finished product number.
P_j: production rate of the product j per period t.
x_{jt}: product quantity of the product j which will be delivered in period j.
hj: holding costs of the finished product j by unit product.
K_j: manufacturing set-up cost of the finished product j.
NO_j: number of manufacturing orders or production frequency of the finished product j.
δ_{jt}: 1 if the product j is produced in period j (fixed and known for a production sequence and phase).
0 else.
PI_{tj}: product j inventory at the end of period t.

The inventories at the end of each period and for each finished product j are calculated as below:

```
For       j = 1 to n do
          Calculate   the inventory at the end of the first period :
                      PI_j1 = P_j * δ_j1 − x_j1
          Calculate   the inventory at the end of the other periods :
                      For     t from 2 to H do
                              PI_jt = PI_j(t−1) − x_jt + δ_jt P_j
                      End     For
          Calculate   the minimal inventory :
                      InMin = Min_{t=1→H} PI_jt
          Correct     the inventories if there are negative inventories:
                      If      InMin < 0
                              For     t form 1 to H do
                                      PI_jt = PI_jt − InMin
                              End     For
                      End     If
End       For
```

The holding costs are calculated as below:

$$HC = \frac{\sum_{j=1}^{n} h_j \sum_{t=1}^{H} PI_{jt}}{H}$$

The total cost by unit time is the sum of holding costs and ordering costs:

$$TC = \frac{\sum_{j=1}^{n} h_j \sum_{t=1}^{H} PI_{jt}}{H} + \sum_{j=1}^{n} \frac{NO_j}{H} K_j \tag{25}$$

The aim of this algorithm is to calculate the necessary inventories of the different products that will ensure the deliveries (the productive periods and the delivery dates are fixed). After that, we can calculate the induced holding costs. The productive and non productive periods (δ_{jt}) are determined according to the considered production sequence and phase. In fact, for each finished product, the algorithm considers in a first phase an initial inventory equal to 0. After that, we calculate the minimum $InMin$ of the inventories of all the periods for each product. If this minimum is negative we correct the inventory by adding to the inventory of all periods ($-InMin$). In fact, ($-InMin$) represents the minimum value of the initial inventory at period 0 for a finished product j, which ensure that the inventory will always remain positive.

This program must be executed for each couple (sequence, phase) in order to choose the best solution. The smaller the step of discretization is, the more precise the result will be. Indeed, we suppose that, for a given period, the product is exclusively in production or not and the inventory is constant. This estimate is acceptable if the step of discretization is small enough. In an industrial real case, we think that an hour could be a good choice for the discretization step. The complexity of this phase is polynomial and depends on the number of periods.

7 Numerical Example

Our approach is illustrated by a numerical example. Let us consider a company which produces four different products (A, B, C and D) with the characteristics defined by Table 3.

In order to be more realistic with the industrial context, we introduced set-up durations in our example. In fact, they are considered as an occupation of the machine without producing and we consider them when we calculate the starting dates of the different manufacturing orders of the sequence.

The considered time unit is the year for the set-up durations and the production rates. The delivery cycle is the week for all the products and the delivery dates are given in days. We apply the first approach to determine the production frequencies (Eq. 21) and we obtain the results reported in the Table 4.

After that, we determine the production frequencies according to the second approach. In Fig. 9, the product D is delivered four times a week, the delivery cycle is the week. The production frequency will vary from one production for each delivery to one production for the four deliveries.

For a fixed production frequency, we test a set of deliveries covering possibilities by the manufacturing orders. For instance, in case we have one M.O. for the product D (4 deliveries by cycle), we have to test all these covering combinations:

- M.O.1: Deliv.1, 2, 3 and 4.
- M.O.1: Deliv.2, 3, 4 and 1 of the next cycle.
- M.O.1: Deliv.3, 4 and 1, 2 of the next cycle.
- M.O.1: Deliv.4, and 1, 2, 3 of the next cycle.

Table 3 Characteristics of example 2

Finished product	Kj	hj	Lj (days)	Lj(year)	SDj(year)	Pj	x1j	dl1j	x2j	dl2j	x3j	dl3j	x4j	dl4j
A	88	7	7	0.01917	0.00009	620,900	1,300	2	1,750	7				
B	119	7.1	7	0.01917	0.00018	400,915	400	3	500	7				
C	82	10.2	7	0.01917	0.00018	510,238	500	2	450	6				
D	106	4.2	7	0.01917	0.00018	255,000	450	2	500	4	600	6	550	7

Table 4 Production frequencies and lot sizes obtained with the first approach

	Product A	Product B	Product C	Product D
Number of manufacturing orders in a week	1	1	1	1
Lot size of the manufacturing orders	3,050	900	950	2,100

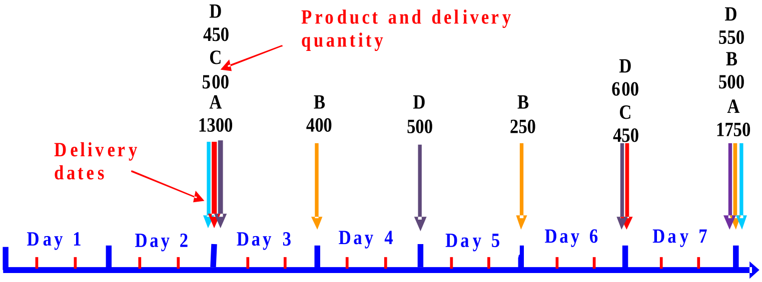


Fig. 9 Weekly cyclic schedule delivery

Table 5 Different covering possibilities of the deliveries of product D

	One M.O.	Two M.O.		Three M.O.			Four M.O			
Covering possibilities of deliveries	M.O.1	M.O.1	M.O.2	M.O.1	M.O.2	M.O.3	M.O.1	M.O.2	M.O.3	M.O.4
	1 2 3 4	1	2 3 4	1	2	3 4	1	2	3	4
	2 3 4 1	2	3 4 1	2	3	4 1	2	3	4	1
	3 4 1 2	3	4 1 2	3	4	1 2	3	4	1	2
	4 1 2 3	4	1 2 3	4	1	2 3	4	1	2	3
		1 2	3 4	1	2 3	4				
		2 3	4 1	2	3 4	1				
		3 4	1 2	3	4 1	2				
		4 1	2 3	4	1 2	3				
		1 2 3	4	1 2	3	4				
		2 3 4	1	2 3	4	1				
		3 4 1	2	3 4	1	2				
		4 1 2	3	4 1	2	3				

Table 6 Production frequencies and lot sizes obtained with the second approach

	Product A	Product B	Product C	Product D
Number of manufacturing orders in a week	2	1	1	1
Lot size of the manufacturing orders (M.O.)	M.O. 1 : 1,300 Covers deliv. 1. M.O. 2 : 1,750 Covers deliv. 2	M.O. 1 : 900 Covers deliv. 1, 2.	M.O. 1 : 950 Covers deliv. 1, 2.	M.O. 1 : 2,100 Covers deliv. 1, 2, 3,4

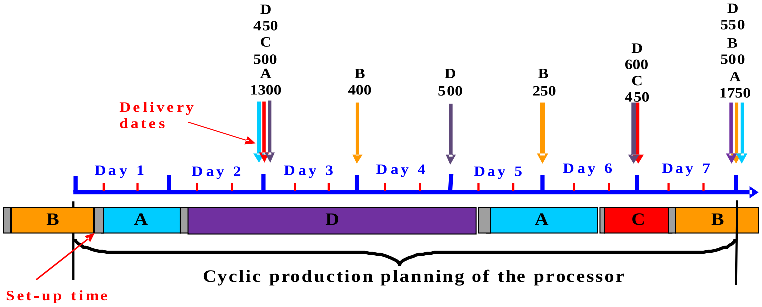


Fig. 10 Cyclic production planning of the processor

All the possibilities for product D are summarized in the Table 5.

For each combination:

– We calculate the latest starting dates of each manufacturing order relative to the deliveries that will cover (it will calculate the productive and non productive periods for the program of calculating holding costs).
– We calculate then, the total cost function composed of the holding and set-up costs using the approach of Section 4.2. For this purpose, we divide the time horizon in periods, each period lasts 8 h.

Then, we chose the combination with a minimal total cost. We obtain the results described by Table 6.

We generate then, the initial production sequence as described in Section 5, we obtain this production sequence: **| B | D | A | C | A |_**.

We can obtain five other equivalent sequences by inverting the position of the products having the same frequency (B, D and C): **| D | B | A | C | A |_ , | C| D | A | B | A |_ , | B | C | A | D | A |_ , | D | C | A | B | A |_ , | C | B | A | D | A |_** .

We divide then, the time horizon in periods. Each period is composed of 8 h, so we divide the week in 21 phases. We execute then the program that calculates the holding and ordering costs, 21 times for each production sequence. From phase to phase, we move circularly the production sequence to the right by one period. The best solution is represented in Fig. 10 (the production of B begins in the precedent cycle).

8 Experimentation

An experiment was carried out and concerned a set of 50 examples. The general conditions of this experimentation are:

All the examples concern the manufacturing of a certain number of finished products (between two and four).

– The demand of all the products is cyclic. The delivery cycle is the week and the number of deliveries by week varies between one and five deliveries.
– All the hypothesis taken in the different approaches are respected. For example, all the products are chosen in a manner that the holding costs dominate the set-up costs.

– All the calculated costs are annual.
– We considered that we are in a medium term level and the load is adjusted generally to the capacity on a bottleneck facility and there is not an important idle time. Nevertheless, if the idle time is important, we have to distribute it in the production sequence in order to minimize the holding costs. This problem is complex and we will work in a future work.

The construction of realistic examples was not easy; this fact explains the relatively low number of examples. Indeed, we were obliged to calculate the production cycles of the products before deciding if the example corresponds to our initial hypothesis because all the products should have a production cycle inferior or equal to a week (the holding costs must be chosen according to the set-up costs in a manner that the optimal production cycle does not exceed a week). We did our best to select representative examples by varying the number of deliveries in a wide area (between one and five deliveries for each product) which could allow us to better study the limits of the first approach in determining the production frequencies. We also, tried to select examples where the production load is near the capacity and we varied the ratio between the holding costs and the set-up costs in order to evaluate the importance of the phasing. We divided the experimentation in two parts:

1. The first part consists of calculating the production frequencies according to the two approaches described in Section 5.
2. The second part consists of calculating the annual holding costs for each couple (sequence-phase). It consists of generating the cyclic production schedules using the production frequencies obtained by the enumeration approach. The week is divided in 21 periods of 8 h each; the generated equivalent production sequences for each example vary between one and six sequences. For each couple (sequence, phase), a program is executed to determine the correspondent holding costs (for each sequence we have tested 21 phases).

The results of only five examples (on a total of 50 examples) are summarized in Table 7, for each example we listed the number of products, the number of deliveries for each product in the week, the production frequencies obtained with the two approaches, the lot sizes (for the first approach the lot size is the same for all the manufacturing orders of a product) and the lowest, highest and average holding costs (function of phasing):

For the first part of the experimentation, Fig. 11 gives the production frequencies obtained with the two approaches. The complexity of the first approach in determining of the production frequencies is very low since we have just applied a formula to find out the frequencies. The results of the experimentation suggest that the first approach can be applied and give good results when the delivery number of the cycle is important and when the deliveries are dispatched uniformly on the cycle. Indeed, generally, we obtain for this case the same production frequencies with the two approaches. In other cases, the results obtained with the enumeration approach should be considered. In fact, if the deliveries are grouped in a part of the cycle (like the example of Fig. 12, where there are two deliveries grouped in days 5 and 7), the first approach will give a very important number of manufacturing orders (in Fig. 12 we have 6 M.O.). This problem is due to the fact that the production starts always in the beginning of the production cycle (day 1 in the example of Fig. 12), so in order to decrease the holding costs we will obtain too many M.O.

Table 7 Results of the experimentation for five examples

	EXAMPLE 1			EXAMPLE 2			EXAMPLE 3			EXAMPLE 4			EXAMPLE 5	
Number of products	3			3			3			3			2	
	A	B	C	A	B	C	A	B	C	A	B	C	A	B
Number of deliveries in the week	1	3	3	3	4	4	3	3	3	5	2	5	5	2
Frequency production (obtained with the first approach)	2	5	4	5	6	5	2	2	1	2	2	2	4	2
Lot sizes (first approach)	500	300	250	300	250	200	750	500	1,000	5,900	3,000	9,625	5,950	3,000
Frequency production (obtained with the second approach)	1	2	1	2	2	1	2	2	1	1	2	2	3	2
Lot sizes (second approach)	1,000 500	1,000	1,000 1,000	500 500	1,000	1,000 500	1,000 500	500	1,000 11,800	11,800 3,000	3,000 12,050	7,200 5,000 6,300	12,500 3,000	3,000
LC: The lowest cost	370.2			872.2			9,460.5			7,825.2			128,045.5	
HC: The highest cost	790.7			1,399.5			20,595.6			12,507.5			377,950.2	
Difference in % between LC and HC	113 %			60 %			118 %			60 %			195 %	
AC : The average cost	597.2			1,162.5			14,940.2			9,950.2			261,580.1	
Difference in % between LC and AC	61 %			33 %			58 %			27 %			104 %	
Difference in % between AC and HC	32 %			20 %			38 %			26 %			44 %	
Ratio = Num.M.O. (2nd approach) / Num. of deliveries	4/7 = 0.57			5/11 = 0.45			5/9 = 0.55			5/12 = 0.41			5/7 = 0.71	

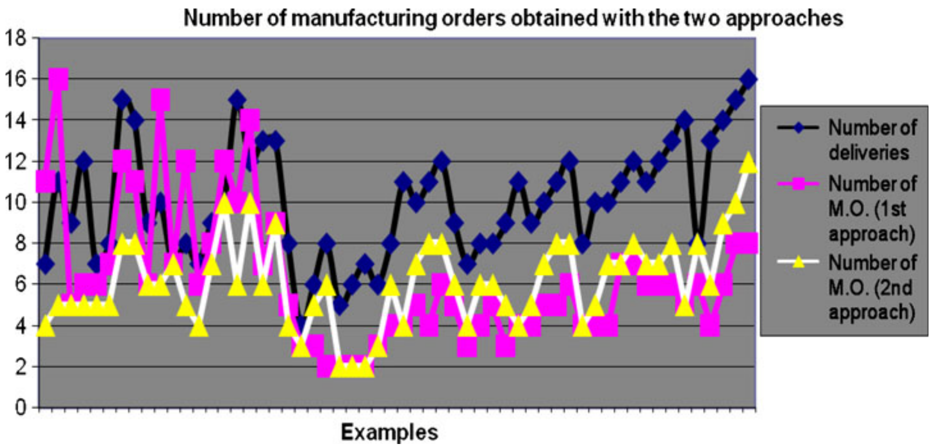


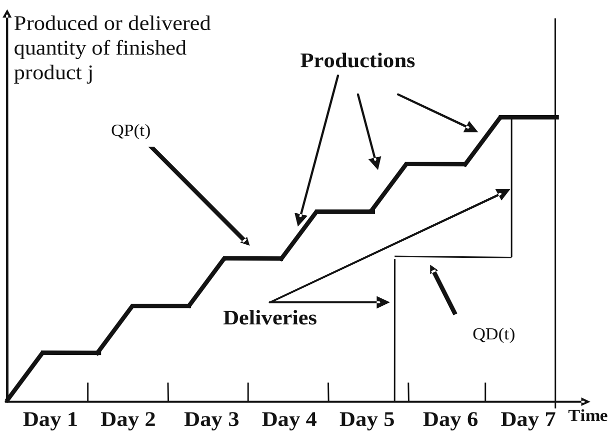
Fig. 11 The number of manufacturing orders obtained with the two approaches for the different examples

However, if the deliveries are grouped in some days in the cycle, we can ameliorate the obtained results by the first approach by moving the beginning of the production cycle near this group of deliveries (in day 3 for instance in Fig. 12).

For the second part of the experimentation, we calculated the holding costs for each couple (sequence, phase); the obtained results show that:

- The difference between the lowest and the highest cost is important (between 54% and 264%). This fact shows the interest to find out the best couple (sequence-phase).
- The obtained profit, in terms of total cost, varies in a wide area. Indeed, the profit depends on the ratio between the number of manufacturing orders and the number of deliveries. The nearer to 1 this ratio is, the more the incidence of the position of the manufacturing orders is important. In this case, the expected profit by looking for the best couple (sequence-phase) will be important.

Fig. 12 Produced and delivered quantities evolution



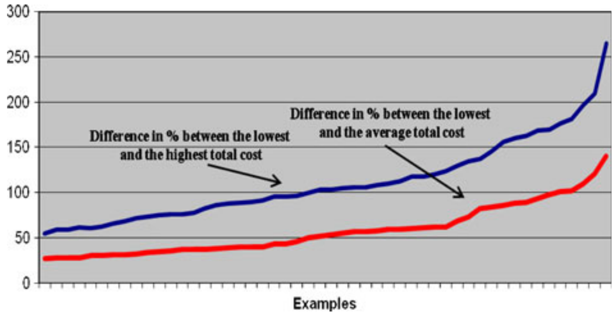


Fig. 13 The evolution of the difference between the lowest, the average and the highest cost for the determination of the best couple (sequence, phase)

Figure 13 traces the evolution of the difference in % between the lowest, the average and the highest total cost (the difference is classified in the increasing order). This difference between the lowest and the average cost shows the interest of this approach in comparison with a random solution.

9 Conclusion and Perspectives

In this paper, we have studied the case where companies are confronted with cyclic demand and we have shown the benefits of implementing cyclic production. We have, then, presented a global methodological framework adapted to the cyclic context in three levels: a long-term level for helping contracts negotiation, a medium term level for defining and adapting the production cycle and a short medium term level for adjusting production quantities and capacities.

In a medium term level, we have focused on the building of cyclic production schedules to face cyclic demand in the case where holding costs are important in relation to set-up costs. First, we developed a global mathematical model and showed that the model is NP-hard. Then we proposed a global strategy to solve the problem in three main phases. The first phase consists of determining the number of manufacturing orders for each product in the delivery cycle and with which quantity. We have presented two approaches for determining the production frequencies. The experiment shows that the first one can be used when the number of deliveries is important and is uniformly dispatched on the cycle. The second approach is more complex but gives good results in the majority of cases. The second phase consists of generating the production sequences. We calculate the cyclic production planning in the last phase. We have exposed for this an approach to determine the best sequence and phase of our cyclic schedule. We compared in the experiment for each treated example the lowest, the highest and the average total cost of the cyclic production planning. The results confirm the interest of our approach. However, we should compare our work with other existent works, even if they are generally more restrictive, to demonstrate more the efficiency of our approaches.

This work has tried to give original and simple solutions to build cyclic production schedules in the context of cyclic deliveries. This work considers that the load is adjusted to the capacity in a medium term level and do not manage the idle time. If there is an important idle time, we have to develop an approach to distribute judiciously this idle time in the production sequence in order to minimize the holding costs. Moreover, a supplementary work should be done in order to study more deeply the relation between the length of the period and the step of discretization. We will also extend this work to multistage and multi-resource production systems.

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