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Modeling Optimal Allocation of Deficit Irrigation: Application to Crop Production in Saudi Arabia

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Abstract We develop an integrated dynamic programming—linear programming (LP) model to solve for optimal land exploitation for a given crop. The model applies deficit irrigation in order to increase the irrigated area at the expense of reducing the crop yield per unit area. The dynamic program guarantees that deficit irrigation is considered only when it is economically efficient. Moreover, it provides the best irrigation level for each growth stage of the crop, accounting for the varying impact of water stress overtime. The LP provides the best tradeoff between expanding the irrigated area and decreasing water share per hectare. The model objective is to maximize the total expected crop yield. The model is particularly applicable for regions suffering from irrigation water scarcity, such as Saudi Arabia. The implementation was made for crops in Al-Jouf Region, north of Saudi Arabia

Keywords Applied operations research · Dynamic programming · Linear programming · Deficit irrigation · Crop production

1 Introduction

1.1 Background

Irrigation scheduling consists on applying the right amount of water at the right time, usually to meet crop water requirements. This depends basically on the crop life stage. Typically, five crop growth stages are widely used in the literature (e.g. Vedula and Nagesh Kumar [17], Mannocchi and Mecarelli [10]). These are the establishment, vegetative, flowering, yield formation and ripening stages. Water requirement and duration of each stage differ as the growth rate differs from one stage to the other. The amount of water applied to crops during the irrigation periods has a major impact both on the growth of a crop and on its yield. Each crop has optimal amount of water that results in maximum growth and maximum yield.

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When water supply is sufficient, it is important to apply the right amount of water to satisfy crop requirement. In fact, crops growth and yield will suffer from a negative effect both when over-irrigated and when under-irrigated. In fact, applying excessive irrigation water will increase the depth of infiltrated water until no more water can be absorbed. This will prevent root aeration and necessitate installing a drainage system to get rid of the unabsorbed water. The drainage water will cause nutrient loss, which will affect negatively the crop yield. Moreover, the cost of the drainage system (operating, maintenance, labor, etc.) will be added to the overall cost of the crops reducing the corresponding profit. On the other hand, when water supply does not meet water requirements during some of the crop growth stages, the actual evapotranspiration will fall below maximum evapotranspiration. Under this condition, water stress will develop in the plants, which will adversely effect crop growth and ultimately crop yield.

The concept of deficit irrigation consists on deliberately applying water below crop requirement in order to increase the irrigated area in case of water shortage. This would decrease crop yield per unit area but may increase total yield over the entire cropped area. The effect of water stress on growth and yield depends on the magnitude and time of occurrence of water stress. It also differs with crop species and variety. When water deficit occurs during a given growth period of the crop, the yield response to water deficit can vary greatly depending on how sensitive the crop is to water stress at the particular growth period. In general, crops are more sensitive to water deficit during emergence, flowering and early yield formation than they are during early vegetative, after establishment and late growth periods (ripening). Therefore, in case of water shortage, irrigation planning should be directed towards meeting the full water requirements of the crop during the most sensitive growth stages rather than spreading the available limited supply to the crop equally over the total growing periods. For the non sensitive growth stages, the amount of water allocated must be sufficient enough to prevent productive tissue to die at any stage. Different crops resist differently to water shortages. Some may have a major yield reduction when it is subject to water deficit, while others may have a relatively minor yield reduction.

The effect of water deficit on crop growth and crop yield can be quantified by empirically derived values of what is called yield response factor, *ky* (Doorenbos and Kassam [5]). Large values of this factor indicate that the growth and yield will be greatly affected by water deficit and vice-versa. Under the same conditions of limited water that is spread equally over the total growing season, involving crops with different *ky* values, the crop with the higher *ky* value will suffer a greater yield loss than the crop with the lower *ky* values. Therefore, the available water supply should be directed towards meeting the full water requirement of the crops that have the highest *ky* values in a given period of time. However, for the crops with low *ky* values increasing the yield may be achieved by increasing the total cropped area without meeting necessarily the full water requirements.

Deficit irrigation principles appear to be very important in arid areas such as Saudi Arabia, which is the area of interest to this study. In fact, deficit irrigation enables to minimize the effects of water scarcity and may maximize the yield and therefore the profit for some restricted amount of water. While a lot of effort has been devoted in the literature to estimate the effect of water shortage on crop yield over the different growth periods of crops, little effort has been addressed toward generating optimal deficit irrigation policies accounting for the different sensitivities of crops to water stress over their different growth periods. In fact, for regions suffering from water scarcity, it is important to derive decision policies that guide growers to use efficiently their limited resource of water.

1.2 Problem Statement and Modeling Approach

The current study is concerned with developing optimal deficit irrigation policies. First, the study attempts to determine for a given stock of irrigation water and a given crop how much water to apply at each growth stage in order to maximize the expected yield. Next, as deficit irrigation calls for deliberately under-irrigating crops in order to expand the irrigated area, the study will also investigate the best tradeoff between the total area to be irrigated and the irrigation levels to be applied so that the total yield is maximized.

A dynamic program (DP) is proposed to solve the first version of the problem. For considering the water-land tradeoff, an integrated DP–linear programming (LP) model is suggested. Some illustrations will be given with discussion and analysis.

1.3 Paper Organization

The remainder of the paper is as follows. “Section 2” presents some literature review. “Section 3” discusses the models formulation. “Section 4” offers some illustrative examples with analysis. Finally, “Section 5” presents some conclusions and directions for future work.

2 Literature Review

The effect of deficit irrigation on crop yield has been largely investigated. Stewart and Hagan [14] conducted many experiments to determine the actual relations between yield and evapotranspiration, water purchased and applied, and field water supply. They developed a multiplicative formula for crop yield as a function of applied irrigation water. This formula is widely used in deficit irrigation. Doorenbos and Kassam [5] analyzed experimental data on crop yield response to water and empirically derived yield response factors, ky_t which represent the sensitivity indices for water stress in the specified growth stage t of the crop. Hart et al. [8] studied the relation between yield and water applied to determine the optimal amount of water that results in maximum yield. Hargreaves and Samani [7] analyzed various water-related factors of crop production. They tried to show how deficit irrigation influence profit and summarized the situation in which deficit irrigation is less desirable. They also introduced a number of mathematical yield models relating crop evapotranspiration and total water available to relative yields including the multiplicative and the additive forms. These models allow expressing analytically the actual yield in terms of the maximum yield when irrigation demand is not satisfied.

Various operations research (OR) techniques have been used for modeling optimal water allocation under deficit irrigation. Vedula and Nagesh Kumar [17] developed an integrated model based on seasonal inputs of reservoir inflow and rainfall in the irrigated area to determine the optimal reservoir release policies and irrigation allocations to multiple crops under water deficit. They considered the inflow and rainfall both to be stochastic. The model is conceptually made up of two modules. Module 1 is an intraseasonal allocation model to maximize the sum of relative yields of all crops, using LP. This module considers the crops growth stages, soil types, soil moisture, and irrigation policy for each crop. Module 2 is a seasonal allocation model to derive the steady state reservoir operating policy using stochastic dynamic programming. Vedula and Nagesh Kumar [17] was an extension of some old work made by Vedula and Mujumdar [16] that considered the same problem but under a deterministic framework. Vedula and Mujumdar [16] used a different approach in which they applied deterministic dynamic programming for the first module and a stochastic dynamic programming for the second module. Sunantra and Ramirez [15] proposed a two-stage decomposition approach to determine optimal seasonal multicrop irrigation water allocation and optimal stochastic intraseasonal (daily) irrigation scheduling. The first stage consists of two steps. In the first step, a relationship function between seasonal water availability and expected value of yields for each crop and for all possible initial soil moisture conditions was developed. In the second step, the optimal seasonal water allocation for each crop was defined so that total yield/ benefit from all crops was maximized. This was done using deterministic dynamic programming. The second stage used the optimal seasonal water allocation resulting from the first stage and generated optimal irrigation policies for each crop on a daily basis, using stochastic dynamic programming. Sarker et al. [12] used a LP when considering the problem of reaching maximum contribution by making the choice of “the right crops for the right type of land”, under the constraints of food demand, land availability and capital restrictions. Mainuddin et al. [9] suggested a monthly irrigation planning model in order to determine the optimal cropping pattern and groundwater withdrawal policy for a groundwater project in Thailand. Reddy et al. [11] developed a general computer software package called ZERO1 using 0–1 integer programming to obtain optimal rotational schedules of irrigation using lateral canals. Wardlaw and Barnes [18] used quadratic programming to build an optimization approach for determining optimal allocation of available water that maximized the total expected yield, while ensuring equity among water users in the system. They calculated the expected yield using the additive form of the yield formula (Hargreaves and Samani [7]). Other interesting OR models in the context of deficit irrigation can be found in Haouari and Azaiez [6], Azaiez and Hariga [3] Azaiez [1], Sarker and Quaddus [13], and Azaiez et al. [4] to cite only a few.

While a large variety of operations research models are encountered in water management problems, only a limited number of such models were devoted to deficit irrigation. Among these, many consider an aggregate yield response factor to all development stages, which does not allow for accounting for the tradeoff in water allocation so that insensitive growth stages to water stress can suffer the most from important deficit. Moreover, among those which explicitly allocate water to development stages based on their respective attitude to water stress, not a single model (to the best knowledge of the author) considers the case where land is not limiting. That is, none of the existing models allows answering the question of what would be the best tradeoff between expanding the irrigated area and opting for deficit irrigation over the different growth periods, in order to maximize the total expected yield. The current study investigates the answer to such a question.

3 Modeling Approach

As mentioned earlier, the crop life consists of a number of growth stages with the following characteristics of interest to us in this study.

- Each growth stage has its own water requirement. When full requirement is not satisfied, the crop is subjected to water stress leading to yield reduction.
- When water stress reaches a particular level (wilting point), the crop cannot survive any further. It is usually assumed that the wilting point occurs at 50–60% of full water requirement.

- It is possible for each growth stage to consider a number of possible irrigation levels that can be defined as the fraction of water applied to full water required. The minimum level should ensure crop survival (i.e., corresponding to the wilting point).
- The sensitivity of the crop to water stress depends on the specific growth stage t and could be measured empirically by the yield response factor ky_t.

When the available amount of irrigation water is enough to satisfy full irrigation water requirement, the problem would be simply to allocate water in time and quantity in order to satisfy full requirement at each growth stage. This will normally result in maximum yield per unit area cropped (once all other input factors are taken at their optimum level). If however the available amount of irrigation water does not satisfy full water requirement, then a decision should be taken on the level of irrigation to apply for each growth stage in order to maximize the expected yield of the corresponding crop per unit cropped area.

The decision on the selected irrigation level for each growth stage should take into account the sensitivity of each growth stage to water deficit such that highly sensitive periods receive relatively high share of irrigation water and low sensitive periods receive low shares. Therefore, the proposed model will determine, for a given amount of irrigation water, the irrigation level that should be applied to each growth stage of the crop life in order to maximize the expected yield per unit cropped area. Initially, the objective is to find the optimum water allocation over time of a given amount of irrigation water, for a given crop, in order to maximize the actual expected yield per unit cropped area. Equivalently, one could rather maximize the ratio of actual expected yield to the maximum expected yield (which is obtained when full water requirement is satisfied over all growth stages of the crop). Next, the problem will extend to determining the best irrigation water-land tradeoff. The first version of the problem will be approached using dynamic programming. This version will constitute the first-step of the solution procedure for the considered extension which will be solved using an integrated DP–LP approach. The following notation is given.

3.1 Notations

n total number of growth stages of the crop
t: growth stage index, $t=1,\ 2,\ \ldots\ n$
Y_m: maximum expected yield that results from applying full irrigation level for each growth stage of the crop life. It can be estimated and it is considered as an input to the model.
Y_a: expected actual yield
ky_t: yield response factor of the crop for growth stage t. It reflects the sensitivity of the growth stage to water deficit. Its value is empirically calculated and is available in the literature.
ET_{mt}: maximum evapotranspiration at growth stage t
ET_{at}: actual evapotranspiration at stage t
WR_t: quantity of water required at stage t.
WA_t: quantity of water applied at stage t.
S_t: state variable at each stage t consisting of the total amount of water applied over stages $t,\ t_{+}1,\ t_{+}2,..,\ n$
J_t total number of possible irrigation levels at growth stage t
IL_{t_j}: jth irrigation level in the tth growth stage of the crop life, $j=1,\ 2,..,J_t$.
$\zeta(S_t)$: optimal contribution of stages t to n to the ratio of actual to maximum yield to be used in the recursive equation of the dynamic program, DP.

The measure of performance in the DP is the ratio of the expected to the maximum yield. This ratio can be calculated using various methods. In this study, we focus on two different approaches, which are the most commonly used. The first is referred to as the multiplicative form, and the second as the additive form. Further discussion will be offered below. The following development will consider the multiplicative form. The additive form will have a similar procedure of calculation and will be briefly discussed later.

3.2 The Multiplicative Form

The fraction of the expected to maximum yield can be calculated by the following formula, which was widely used in the literature (e.g., Stewart and Hagan [14], Azaiez and Hariga [3], Haouari and Azaiez [6]).

$$\frac{Y_a}{Y_m} = \prod_{t=1}^{n} \left(1 - ky_t \left(1 - \frac{ET_{at}}{ET_{mt}} \right) \right) \tag{3.1}$$

This formula has been proven to give a good estimation of the yield for most crops (Doorenbos and Kassam [5]). Note that the contribution of a given period t to the reduction in yield increases with the corresponding yield response factor ky_t and vanishes when no deficit is incurred in that period (in which case the ratio of actual to maximum evapotranspiration is 1. Note also that this equation deals only with the quantitative reduction in yield. However, deficit irrigation could affect the yield qualitatively too (when the deficit is considerable). One way to account for the qualitative degradation due to deficit irrigation is to reduce the set of alternative levels of deficit irrigation to those that can still result in yields of acceptable quality.

The maximum and the actual evapotranspiration can be represented by the ratio of water applied to that of water required. It is plausible to assume that the ratio of actual to maximum crop evapotranspiration is the same as the ratio of water applied to water required (Wardlaw and Barnes [18]). That is,

$$\frac{ET_{at}}{ET_{mt}} = \frac{WA_t}{WR_t} \tag{3.2}$$

Therefore, the ratio of actual to maximum evapotranspiration will be referred to in this study as the irrigation level.

$$IL_t = \frac{ET_{at}}{ET_{mt}} = \frac{WA_t}{WR_t} \tag{3.3}$$

It should be noted that each stage may have a number of candidate irrigation levels that may differ from one stage to the other for the same crop as well as for different crops. Therefore, we will add another index to the irrigation level notation to designate the irrigation level in the stage under consideration. The new notation (consistent with the one given in the notation section) is IL_{t_j}.

The irrigation level at each growth stage will vary from a minimum value (IL_{t1}) (below which the crop would not survive) to full irrigation level (IL_{tJ_t}; in which case the water applied is equal to water required).

Substituting Eq. 3.3 in Eq. 3.1 for pre-selected j's yields the following formula:

$$\frac{Y_a}{Y_m} = \prod_{t=1}^{n} \left(1 - ky_t\left(1 - IL_{t_j}\right)\right), \tag{3.4}$$

According to the above formula, the fraction of the expected to maximum yield, which is the suggested measure of performance in this particular model, will differ according to the selected irrigation levels over each growth stage of the crop life. Therefore, the decision variable would be the irrigation level at each growth stage. The problem of allocating the available water over the different growth stages in order to maximize the ratio of actual to maximum yield is a knapsack problem and can therefore be approached adequately through dynamic programming.

The recursive equation of the proposed dynamic program will be derived from Eq. 3.4 as follows:

The contribution to the measure of performance at a single stage t (which is the ratio of actual to maximum yield) can be written in the following form:

$$\left(1 - ky_t\left(1 - IL_{t_j}\right)\right) \tag{3.5}$$

Therefore, for the last stage $t{=}n$, this contribution is optimized as follows:

$$\zeta(S_n) = \max{}_{IL_{n_1}, IL_{n_2}, \ldots\ldots, IL_{n_{J_n}}} \left[1 - ky_t \times \left(1 - IL_{n_j}\right)\right] \tag{3.6}$$

Subject to:

$$S_n \geq WR_n \times IL_{n_j} \tag{3.7}$$

Equation 3.6 will select the best irrigation level for the last stage while not exceeding the available amount S_n of irrigation water for that stage (constraint Eq. 3.7). Since the multiplicative form of the measure of performance is used, the contribution of previous stage ($t{=}n{-}1$) is calculated by Eq. 3.5 for each irrigation level and multiplied by the contribution of the current stage (n). Therefore, the recursive formula for any stage ($t<n$) is given by

$$\zeta(S_t) = \max{}_{IL_{t_1}, IL_{t_2}, \ldots\ldots, IL_{tJ_t}} \left\{\left[1 - ky_t \times \left(1 - IL_{t_j}\right)\right] \times \zeta(S_{t+1})\right\} \tag{3.8}$$

Subject to:

$$S_t \geq WR_t \times IL_{tj} + S_{t+1}, \text{ for all } j \tag{3.9}$$

When $t{=}1$, the recursive formula given by Eqs. 3.8 and 3.9 will provide the optimal ratio of actual to maximum expected yield, as given in Eqs. 3.10 and 3.11 below.

$$\frac{Y_a}{Y_m} = \zeta(S_1) = \max{}_{IL_{1_1}, IL_{1_2}, \ldots\ldots, IL_{1_{J_1}}} \left\{\left[1 - ky_1 \times \left(1 - IL_{1_j}\right)\right] \times \zeta(S_2)\right\} \tag{3.10}$$

Subject to:

$$S_1 \geq WR_1 \times IL_{1_j} + S_2 \tag{3.11}$$

Moreover, the optimal water allocation over time to the crop can be determined by obtaining successively the selected irrigation level IL_{t_j} at each stage t of the DP.

3.3 The Additive Form

The additive form of the objective function (representing the decrease in yield when applying deficit irrigation) was also used in the literature (i.e. Vedula and Nagesh Kumar [17], Sunantra and Ramirez [15], Wardlaw and Barnes [18]). This form also gives a good estimation of the ratio of actual to maximum yield for a variety of crops. The formula is given by:

$$\frac{Y_a}{Y_m} = 1 - \sum_{t=1}^{n} Ky_t \left(1 - \frac{ET_{at}}{ET_{mt}}\right) \tag{3.12}$$

When using the additive version of the objective function, the recursive formula will slightly change to take the following form:

$$\zeta(S_t) = \mathrm{Min}_{IL_{t_1}, IL_{t_2}, \ldots\ldots, IL_{t_{J_t}}} \left[ky_t \times (1 - IL_{t_j})\right] + \zeta\left(\sum_{i=t+1}^{n} WR_i \times IL_{i_j}\right) \tag{3.13}$$

Subject to:

$$S_t \geq WR_t \times IL_{t_j} + S_{t+1} \tag{3.14}$$

3.4 Example

The model is discussed for two crops namely grain maize and onion at Al-Jouf region in north of Saudi Arabia. Full water requirement for grain maize is assessed to be 9,500 m^3/ha/season and maximum expected yield is 12 ton/ha. These results are given from the agricultural company at which the model was tested. We consider 11 irrigation levels. These levels range between the wilting point (assumed to be 50% of full water requirement as commonly used) and the full water requirement with 5% increment. Table 1 below gives the results on actual yield and individual irrigation levels for each of the possible aggregate

Table 1 Optimal irrigation levels over the different growth stages for grain maize

Level	Water consumed (m^3/ha)	Percentage of full requirement	Expected yield (ton/ha)	Detailed irrigation level allocation per growth stage			
				1	2	3	4
1	4,750	0.50	1.62	0.5	0.5	0.5	0.5
2	5,220	0.55	3.83	0.65	0.7	0.5	0.5
3	5,691	0.60	6.33	0.8	0.9	0.5	0.5
4	6,107	0.65	8.19	1	1	0.5	0.55
5	6,555	0.70	8.74	1	1	0.6	0.55
6	7,099	0.75	9.45	1	1	0.75	0.5
7	7,548	0.80	9.99	1	1	0.85	0.5
8	7,996	0.85	10.53	1	1	0.95	0.5
9	8,476	0.90	11.04	1	1	1	0.6
10	8,988	0.95	11.52	1	1	1	0.8
11	9,500	1.00	12.00	1	1	1	1

irrigation levels as obtained by the proposed DP. From the output of Table 1, it is clear that optimal water allocation for grain maize gives the priority to growth stages 2, 1, 3, and then 4 in the respective order. In addition, unless growth stages 2 and 1 receive their full water requirement, water allocation to stages 3 and 4 will remain at its minimum level (50%). Moreover, the additional improvement in crop yield when stage 4 obtains full water requirement compared to the minimum water requirement is by far smaller that that of stage 2 or 1. These results are found helpful to the company managers to give insight on how to allocate water for grain maize when water shortage occurs.

The other crop considered is onion. Full water requirement for onion is assessed to be 11,000 m^3/ha/season and maximum expected yield is 40 ton/ha at the region of application. The results of DP are given in Table 2 below.

The results show that growth stages 3 and then 1 are the most sensitive to water stress. Moreover, there is a great yield reduction when water deficit is applied. For instance, if 80% of water requirement is applied, then about 70% of maximum yield could be obtained while a similar reduction in water availability for grain maize would generate about 83% of maximum yield. This indicates that onion seems to be unfavorable for applying deficit irrigation. This will be confirmed in the next section.

3.5 Integrated DP-LP for Irrigation Water-land Tradeoff

The problem of interest in this study is to determine for a given crop how much irrigation water to apply per ha and how much irrigated area to use in order to maximize total expected yield. Note here that both the set of decision variables corresponding to alternative yields (based on the irrigation levels to be allocated overtime) and the land to be allocated to the given crop interact in a multiplicative form, which give rise to a nonlinear program. To avoid this, we suggest opting for an integrated DP–LP to solve for the optimal tradeoff between irrigation allocation overtime and land allocation to the given crop, so that optimal yield per unit area for each possible set of irrigation level (over all growth stages of the crop) will be an input parameter of the LP, which in turn was obtained as an output of the DP. In fact, when the stock of available water is limited, applying full water requirement may be achieved at the expense of reducing the cropped area. On the other hand, expanding the irrigated area could be made by opting for optimal deficit irrigation using the output of

Table 2 Optimal irrigation levels over the different growth stages for onion

Level	Water consumed (m3/ha)	Percentage of full requirement	Expected yield (ton/ha)	Irrigation level at growth stage			
				1	2	3	4
1	5,500	0.50	12.25	0.5	0.5	0.5	0.5
2	6,014	0.55	14.70	0.5	0.5	0.65	0.5
3	6,569	0.60	17.29	0.6	0.5	0.75	0.5
4	7,148	0.65	20.17	0.55	0.5	0.95	0.5
5	7,678	0.70	23.02	0.8	0.5	0.95	0.5
6	8,230	0.75	25.89	0.9	0.55	1	0.5
7	8,796	0.80	28.67	1	0.5	1	0.75
8	9,341	0.85	31.32	1	0.6	1	0.85
9	9,886	0.90	34.08	1	0.7	1	0.95
10	10,414	0.95	36.75	1	0.95	1	0.8
11	11,000	1.00	40.00	1	1	1	1

the DP for alternative deficit irrigation levels. The best tradeoff for levels of irrigation water and irrigated area is the solution of the following LP:

$$\begin{array}{ll} \max & \sum_{i=1}^{11} \text{yield}_i * \text{land}_i \\ s.t. & \sum_{i=1}^{11} \text{level}_i * \text{land}_i \leq \text{Water\_total} \\ & \sum_{i=1}^{11} \text{land}_i \leq \text{land\_total} \\ & \text{land}_i \geq 0, \quad 1 \leq i \leq 11 \end{array}$$

The decision variables land_i are the land areas to be irrigated at level_i, $1 \leq i \leq 11$. The 11 irrigation levels are the levels that range between the wilting point and the full water requirement with 5% increment, as in the example of the previous section. Water_total corresponds to the total amount of water available and Land_total corresponds to the total land available for the crop. Yield_i corresponds to the optimal yield as calculated by DP for irrigation level_i. Thus, the output of the DP for each candidate irrigation level will serve as input to the LP. The LP is of small size (11 decision variables and two constraints) for each crop to be considered. The implementation was made using LINGO 8.0 on a Pentium 4 PC. Lingo offers the option of coding a DP. The LP retrieves the output of the DP from a created file on LINGO and uses it as input. The computer time is negligible given the size of the problem. The implementation and analysis of this integrated model is presented in the next section.

4 Model Implementation and Analysis

We reconsider the previous example. The implementation is made based on different scenarios of land availability for a given stock of water. For the agricultural company at Al-Jouf region, land is considered as abundant. However, the managers are currently opting for full irrigation of a restricted area of land. The managers consider that expanding this area while applying deficit irrigation is a feasible but "risky" alternative. It should be clear that changing water availability is equivalent to expanding or reducing land availability. Eleven irrigation levels are considered. These levels range between the wilting point (assumed to correspond to 50% of full requirement) and the full water requirement with 5% increment, as in the above example.

We consider first grain maize. We assume that the available amount of water is 475,000 m^3, which is fixed for all the runs. This amount of water is enough to irrigate one farm of 50 ha of grain maize satisfying full water requirement. In that case, the total expected yield would be 600 ton. If the land could be expanded using the same amount of water, then deficit irrigation will be applied. The integrated model will specify how much land to use and how to allocate water for different values of total available land. The results are given in Table 3. The results grossly say that applying deficit irrigation for grain maize is more beneficial than using full irrigation at restricted land. It is better off expanding land as available until some peak (78 ha). Table 3 specifies how much land to irrigate with how much water in order to generate the largest yield.

For instance, if 55 ha are available, the optimal cropping policy suggests using the entire land and irrigating 9 ha with full irrigation requirement (level 11) and 46 ha with 90% of the requirement (irrigation level 9). If two more hectares are available, then the optimal policy suggests using again all the available land irrigating 40 ha with 90% of the requirement (level 9) and 17 ha with 85% of the requirement (level 8). This will generate 6 additional tons of maize compared to the previous case. The details on how to allocate

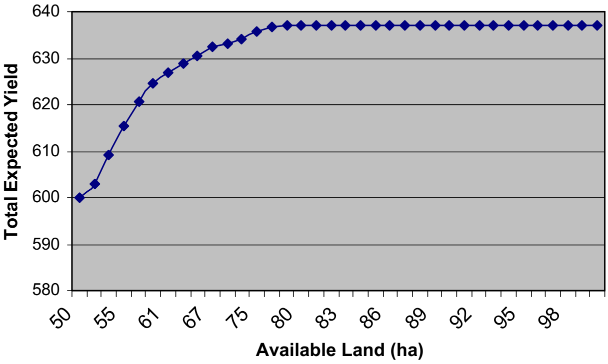
Table 3 Output of the integrated model for different scenarios of land availability for Grain Maize

Scenario	Available land (ha)	Land used (ha)	Irrigation level	Total expected yield (ton)
1	50	50	No Deficit, 11	600
2	53	28	Deficit, 9	609
		25	No Deficit, 11	
3	55	46	Deficit, 9	615
		9	No Deficit, 11	
4	57	17	Deficit, 8	621
		40	Deficit, 9	
5	61	14	Deficit, 6	627
		47	Deficit, 8	
6	65	50	Deficit, 6	631
		15	Deficit, 8	
7	67	1	Deficit, 4	632
		66	Deficit, 6	
8	69	15	Deficit, 4	633
		54	Deficit, 6	
9	73	44	Deficit, 4	635
		29	Deficit, 6	
10	75	58	Deficit, 4	636
		17	Deficit, 6	
11	77	72	Deficit, 4	637
		5	Deficit, 6	
12	≥ 78	78	Deficit, 4	637

water overtime for the proposed irrigation levels are given in Table 1 of the previous section. Moreover, if the available land exceeds 78 ha, then the optimal policy calls for using only 78 ha of land and irrigating this land with the same level of 65% of the requirement (i.e., level 4). When land is not limiting, then increasing the irrigated area from 50 ha to 78 ha will increase the yield by 37 tons using the same amount of water. These optimal policies as specified by the suggested model are far from being intuitive. Thus, this integrated model can help a lot the managers improve the company productivity, as stated by the head of the company. Figure 1 below depicts the relationship between land availability and the corresponding yield for grain maize at optimal exploitation.

We consider now another crop with a different behavior with respect to deficit irrigation, namely onion. From the example on "Section 3", full water requirement for onion is

Fig. 1 Total expected yield vs. available land for Grain Maize



assessed to be 11,000 m^3/ha/season and maximum expected yield is 40 ton/ha. Assume that the available amount of water is 550,000 m^3, which is enough to satisfy full water requirement of a farm of 50 ha of onion. When more than 50 ha of land are available, the optimal policy calls for using only 50 ha and opting for full water requirement. This says that deficit irrigation is not appropriate for onion. To understand better this suggested policy, Fig. 2 below depicts the behavior of deficit irrigation on the onion yield when using the entire land available and following the water allocation as suggested by the DP.

This is consistent with a statement in the *Food and Agriculture Organization report 33* [4], which specifies that deficit irrigation is not applicable to all crops. Such a result is derived from field experiments. In the current study, the proposed model is able to confirm it quantitatively. More specifically, our model can determine which crops are profitable when submitted to deficit irrigation. It can also specify how much deficit irrigation to apply overtime and how much land to use in order to maximize crop yield.

5 Conclusions and Directions for Future Research

In this study, a dynamic programming model is proposed to solve for optimal water allocation over the different growth stages of a given crop, accounting for water shortage. The idea is to let the most sensitive growth stages to water stress receive the largest water shares while the least sensitive ones be submitted to most of the stress in order to maximize crop yield. The yield response factor at a given growth stage represents the measure of sensitivity to water stress. Two versions of the DP are given depending on the adopted formula for assessing the actual yield as a fraction of the maximum yield (when no deficit occurs); namely the multiplicative and the additive formulas.

The results of the DP model say in particular that it may be profitable to deliberately under irrigate a given crop in order to expand the irrigated area. This suggests determining when to stop. That is, the problem is extended to consider the best tradeoff between expanding the irrigated area and reducing deficit irrigation. A LP is proposed. The input to the LP is based on the DP outputs. The integrated model is solved and discussed for grain maize and onion at Al-Jouf region, north of Saudi Arabia. The outcome of this study is particularly interesting to arid regions suffering from water scarcity where large fractions of available land remains uncropped because of water unavailability.

As a direction for future work, the author is investigating the possible extension of this model to a multiple crop context where several crops compete for land and water. The problem is further complicated by the seasonal aspect of water supply and by the various

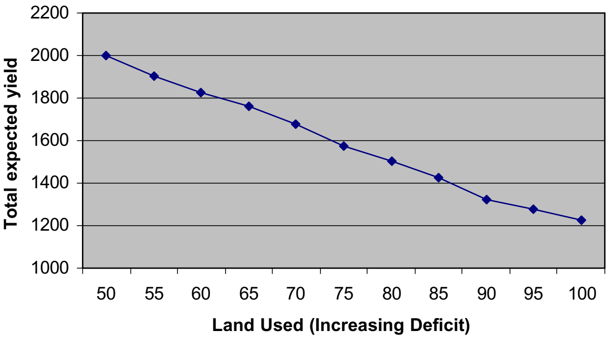


Fig. 2 Total expected yield vs. land used for onion

cropping seasons of different crops. The model also accounts, for each candidate crop, for the starting and ending growth season, land occupancy, potential crop predecessors and successors, in addition to other factors, such as market requirements and risk attitude of growers (Azaiez [2]).

A second avenue of investigation consists in considering the stochastic supply of water. In fact, for the current study, the region under consideration has very limited rainfalls, which results in low fluctuations in the total supply. Consequently, the water availability was reasonably treated as deterministic. This however may not be the case in other applications. One may consider a stochastic DP integrated with a chance-constrained program to attempt to model the stochastic behavior in water supply.

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