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Applications for Affine Invariant Descriptor and Affine Parameter Estimation Based on Two-Source ICA*

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Abstract. This paper proposes a new scheme based on two-source independent component analysis (ICA) for object recognition with affine transformation and for affine motion estimation between video frames. The idea is that, in two-source case, the indeterminate permutation matrix can be fixed. So an affine invariant description will be extracted by ICA algorithm. Simulation results show that ICA method has better performance comparing with traditional Fourier methods on object recognition and affine motion estimation.

Mathematical Subject Classification: 62H35, 62H25, 68U10, 68T45.

Key words: independent component analysis, affine invariant, affine estimation, object recognition.

1. Introduction

Independent Component Analysis (ICA) is a very useful method for blind signal separation [1, 2, 9], in which observed random vector $\boldsymbol{X}$ are considered as a linear combination of some independent sources of $\boldsymbol{S}$, $\boldsymbol{X} = \boldsymbol{AS}$. The ICA tries to separate the independent components from $\boldsymbol{X}$ in the case of unknown mixture matrix A and source signals $\boldsymbol{S}$ via

$$\boldsymbol{Y} = \boldsymbol{WX} = \boldsymbol{WAS} \approx \boldsymbol{S}, \boldsymbol{X} \in \boldsymbol{R}^m, \boldsymbol{Y} \in \boldsymbol{R}^n, \boldsymbol{W} \in \boldsymbol{R}^{m \times n} \tag{1}$$

Here the matrix $\boldsymbol{W}$ can be found by unsupervised learning in the case of the components of vector $\boldsymbol{Y}$ as independent as possible. It was shown that if the components of source signal $\boldsymbol{S}$ are statistical independent and no-Gaussian or at most one Gaussian distribution, then the matrix $\boldsymbol{W}$ exists and the sources can be separated by ascent algorithm of many kinds of cost functions [3, 10, 11]. Obviously, the order and scale of the random variables do not affect their independency, so $\boldsymbol{Y}$ can recover the independent component $\boldsymbol{S}$ up to constant scales and a permutation of components, when the components of $\boldsymbol{Y}$ become

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independent [14]. Some studies showed that when the observed random vector $\boldsymbol{X}$ is whitened to zero-mean and unit variance, $\boldsymbol{Y}$ recovers $\boldsymbol{S}$ only up to a permutation of components with sign indeterminacy [8, 15, 16]. In the case of two-source, the permutation matrix only has eight forms [15, 16].

The key problem of rigid object recognition is to find an affine invariant description of observed object. Because the equation of affine transform is more similar to ICA, a try of this paper is to use two-source ICA in affine object recognition and affine parameters estimation. Experimental results show that this idea is feasible and the performance of new scheme is better than other classical methods.

This paper is organized as follows: In Section 2 the ICA theory and algorithm that use in our paper are introduced, the method to determine permutation matrices and how to use it for affine object recognition is presented in Section 3, and affine motion estimation examples are presented in Section 4. The conclusion and discussion is presented finally.

2. Independent Component Analysis Theory

Let us consider the case with two observed signals. Assume that observed random variables $\{x_1\ (t), x_2\ (t)\}$ are generated as a linear mixture of independent components $\{s_1(t), s_2\ (t)\}$ written as

$$\boldsymbol{X} = (x_1(t), x_2(t))^T = \boldsymbol{A}(s_1(t), s_2(t))^T = \boldsymbol{AS} \tag{2}$$

Here t is the time or sample index, and $\boldsymbol{A}$ is an unknown mixture matrix, $\boldsymbol{A} \in \boldsymbol{R}^{2\times 2}$. If the independent random variables, $s_1(t)$ and $s_2(t)$, are non-Gaussian distribution or at most one is Gaussian distribution then exist lots of cost function to find a linear transformation matrix $\boldsymbol{W}$ such that the random variables y_i, $i = 1, 2$ as independent as possible.

$$\boldsymbol{Y} = (y_1(t), y_2(t))^T = \boldsymbol{W}(x_1(t), x_2(t))^T = \boldsymbol{WA}(s_1(t), s_2(t))^T = \boldsymbol{WAS} \tag{3}$$

If the linear transformation $\boldsymbol{W}$ satisfies $\boldsymbol{W} \approx \boldsymbol{A}^{-1}$, we have $\boldsymbol{Y} \approx \boldsymbol{S}$. ICA is to solve $\boldsymbol{S}$ and $\boldsymbol{A}$ for only given the observed random $\boldsymbol{X}$. In general, $\boldsymbol{W} \approx \boldsymbol{A}^{-1}$ condition cannot satisfy only using independent criteria. A wide solution is defined as following equation:

$$\boldsymbol{Y} = \boldsymbol{WX} = \boldsymbol{WAS} = \Lambda \boldsymbol{PS} \tag{4}$$

Here Λ is a diagonal matrix which changes the scale of random variables, and $\boldsymbol{P}$ is a permutation matrix in which only one element equals to 1 on each row or column. Permutation matrix only changes the order of random variables, y_i, $i = 1, 2$.

Because the independent criterion is related to probability density of random variables that cannot be exactly known previously, many approximate estimation methods have been proposed to test the independency of random variables such as minimum mutual information, Maximum likelihood, maximum entropy and the methods based on maximization of nongaussianity such as kurtosis and negentropy, etc [4, 7, 8].

According to [8, 15, 16], the prewhiten of observed vector will fix the scale of recovered signal $\boldsymbol{Y}$ and simplify the relation between $\boldsymbol{Y}$ and $\boldsymbol{S}$. In this paper we adopt ICA algorithm with prewhiten process. There are two choices shown as follows.

In 1998 Xu lei proposed one-bit-matching conjecture and a criterion of weighed kurtosis to measure the independency. Minimizing the criterion can make all sources' separation. Recently they prove that when independent source $\boldsymbol{S}$, observed samples $\boldsymbol{X}$ and the output $\boldsymbol{Y}$ are all prewhitened with zero mean and identity covariance matrix, and the three order statistical value-skewness of y_i for $i = 1, 2 \ldots n$ equal zero or near equal zero, then minimizing this criterion is equal to minimizing mutual information. This algorithm is more precise, and can get results well, but it is time-consuming.

Other algorithm, called fast fixed-point algorithm (FastICA), was proposed by Hyvärinen in 2001 [8] based on Negentropy. The prewhiten process in FastICA is required, so the recovered source $\boldsymbol{Y}$ is almost equal to independent source $\boldsymbol{S}$ only orders' change, i.e., $\Lambda = \boldsymbol{I}$ in equation (4). FastICA has very fast convergent speed that can be easy to use in real world. Furthermore, it was proved that maximizing negentropy and minimizing mutual information are equivalent [8]. But in process of FastICA, each source would be computed separately and the linear transformation $\boldsymbol{W}$ needs to be orthogonalized, that resulted in slight error.

Both ICA algorithms are useful in our applications. Experiments of this paper are the results by FastICA. The cost function of FastICA, Negentropy is defined as follows

$$J(y_i) = H(v) - H(y_i) \tag{5}$$

where v is a Gaussian random variable, y_i is the random variable with any distribution. $H(\bullet)$ denotes the entropy of random variable. It is proved that when v and y_i have the same mean and variance, the Negentropy J is always non-negative. Maximizing the Negentropy J we can obtain each row vector of transformation $\boldsymbol{W}$. An approximating Negentropy is expressed as

$$J(y_i) = (<G(v)> - <G(y_i)>)^2, \quad i = 1, 2, \ldots n, \tag{6}$$

Where $G(y_i) = -\exp(-y_i^2/2)$, v and y_i are random variables with zero mean and unit variance.

The prewhiten process of observed signals is explained as follows: Suppose that the mean vector of observed vector $\boldsymbol{X}$ with two components is $\boldsymbol{m}_x$, and the centered observed vector $\boldsymbol{X} = \boldsymbol{X} - \boldsymbol{m}_x$, so in the following, $\boldsymbol{X}$ has zero mean vector, its covariance matrix $\boldsymbol{R}_x = E\{\boldsymbol{X}\boldsymbol{X}^T\}$. Let $\boldsymbol{U} = (u_1, u_2)$ be the matrix whose columns are the unit-norm eigenvector of the covariance matrix $\boldsymbol{R}_x$, and $\Sigma =$

$diag(\lambda_1, \lambda_2)$ be the diagonal matrix which is composed of the eigenvalues of covariance matrix $\boldsymbol{R}_x$, then the whitening random vector $\widetilde{\boldsymbol{X}}$ is given by

$$\widetilde{\boldsymbol{X}} = \sum\nolimits^{-1/2} \boldsymbol{U}^T \boldsymbol{X}, \tag{7}$$

Here $\widetilde{\boldsymbol{X}}$ satisfies $E\{\widetilde{\boldsymbol{X}}\widetilde{\boldsymbol{X}}^T\} = \boldsymbol{I}$. $\boldsymbol{I}$ is an identity matrix. Obviously, the whitening observed variables are uncorrelated each other. Considering that the scale of independent components s_j is mutable, we can also assume $E\{\boldsymbol{S}(t)\boldsymbol{S}(t)^T\} = \boldsymbol{I}$. We can maximize Negentropy J to get $\boldsymbol{W}$ from the prewhitened random vector $\widetilde{X}$, such that

$$\boldsymbol{Y} = \boldsymbol{W}\widetilde{\boldsymbol{X}} = \boldsymbol{WAS} = \boldsymbol{PS} \tag{8}$$

In two-source case, the permutation matrix $\boldsymbol{P}$ only have the following eight possible matrices [11].

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \tag{9}$$

In the following discussion, the symbol $\boldsymbol{X}$ denotes the prewritten vector $\widetilde{\boldsymbol{X}}$ for simplifying.

3. Applications for Affine Object Recognition Based on Two-source ICA

The fundamental difficulty in recognizing an object from three-dimension space is that the appearance of shape depends on the observing angle. In general, affine transform can be considered as a depiction observing from different angles for the same object. So objects recognition based on affine transform is a typical problem of rigid object recognition. In the affine transform, the points' coordinate (x,y) on the original object and the corresponding points' coordinate (x',y') on the transformed object satisfies the following relation:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \end{bmatrix} = \boldsymbol{ZX} + \boldsymbol{B} \tag{10}$$

Here z_{11}, z_{12}, z_{21}, z_{22}, b_x, b_y are affine parameters. Affine object recognition is to find an affine invariant descriptor in any affine parameters. Boundary curve of an object image, digitized by an integral grid, can be used to represent the object contour. Some affine invariant descriptors are based on object contour such as B-spline methods [6], Fourier descriptors [12, 13] etc.

A new affine-invariant descriptor based on two-source ICA is proposed in this section. Suppose that an object contour is sampled N points for both original and its affine-transformed image, $\boldsymbol{X}$ and $\boldsymbol{X}'$.Their coordinate $\boldsymbol{X}(l) = [x(l)\ y(l)]^T$ and

$\boldsymbol{X}'(l) = [x'(l)\ y'(l)]^T$ can be considered as two observed random vectors with a linear combination of the same independent component $\boldsymbol{S}(l) = [s_x(l)\ s_y(l)]^T$ for different combination parameters matrix $\boldsymbol{A}$ and $\boldsymbol{A}'$ respectively. Because the object's contour is roughly smooth, so the observed random sequence sampling from object contour can satisfy subGaussion distribution (no-Gaussian distribution). In addition to the coordinate vector of sampling points in contour can be considered as random vector. According to ICA theory, there exists a linear transformation $\boldsymbol{W}$ such that the recovered independent components from $\boldsymbol{X}$ and $\boldsymbol{X}'$ can be estimated. From equation (10) if $\boldsymbol{B} = 0$, then both recovered independent vectors from $\boldsymbol{X}$ and $\boldsymbol{X}'$ are almost the same except components' order and sign. In the two-source case, this difference only has eight forms. If the recovered components' order and sign can be fixed, then object recognition is performed by using the recovered $\boldsymbol{S}(l)$. This method is based on the fact that a general curve has two independent invariants at each sampling point, and the curve can be determined uniquely by N sampling points, if sample's number satisfies sampling theorem [5, 17]. Because the corresponding sample points on both original and its affine one are unknown, the statistical properties of sampled points are used by two-source ICA.

The observed signals sampling from object contour are in two cases for ordinal sample and statistical sample, which have different processing methods as mentioned follows.

3.1. OBSERVED SIGNALS ARE OBTAINED BY ORDINAL SAMPLING ALONG THE OBJECT CONTOUR

If an object's contour curve is continuous and close, we can get the contour coordinates' data by ordinal sampling via any edge trace algorithm. Suppose that the object's contour is sampled N points for both original and its affine-transformed object with affine matrix $\boldsymbol{Z}$ and translation vector $\boldsymbol{B}$. Their coordinates $\boldsymbol{X}(l) = [x(l)\ y(l)]^T$ and $\boldsymbol{X}'(l) = [x'(l)\ y'(l)]^T$ satisfy

$$\boldsymbol{X}'(l) = \boldsymbol{Z}\boldsymbol{X}(l - l_0) + \boldsymbol{B} \tag{11}$$

Here l_0 is the difference of starting sampling point between contour $\boldsymbol{X}'$ and $\boldsymbol{X}$. It is obvious that $\boldsymbol{B} = 0$ when $\boldsymbol{X}'$ and $\boldsymbol{X}$ are set in their centroid coordinates, respectively, so in the following, we take $\boldsymbol{B} = 0$ for analytic convenience. Consider that $\boldsymbol{X}'$ and $\boldsymbol{X}$ are the linear combination of the same source $\boldsymbol{S}$ with $\boldsymbol{A}'$ and $\boldsymbol{A}$, according to (11) we have $\boldsymbol{X}'(l) = \boldsymbol{A}'\boldsymbol{S}'(l)$, $\boldsymbol{X}(l - l_0) = \boldsymbol{A}\boldsymbol{S}(l - l_0)$. By using ICA, we can adapt matrix $\boldsymbol{W}'$ and $\boldsymbol{W}$ to obtain $\boldsymbol{S}'(l)$ and $\boldsymbol{S}(l - l_0)$, $\boldsymbol{A}'$ and $\boldsymbol{A}$, that satisfy as

$$\begin{aligned} \boldsymbol{Y}'(l) &= \boldsymbol{W}'\boldsymbol{X}'(l) = \boldsymbol{W}'\boldsymbol{A}'\boldsymbol{S}'(l) \approx \boldsymbol{P}'\boldsymbol{S}(l) \\ \boldsymbol{Y}(l) &= \boldsymbol{W}\boldsymbol{X}(l - l_0) = \boldsymbol{W}\boldsymbol{A}\boldsymbol{S}(l - l_0) \approx \boldsymbol{P}\boldsymbol{S}(l - l_0). \end{aligned} \tag{12}$$

So $\boldsymbol{Y}$ and $\boldsymbol{Y}'$ denote recovered $\boldsymbol{S}$ and $\boldsymbol{S}'$ respectively, only order and sign are changed by permutation matrix. As mentioned above, the independent components satisfy:

$$E\left\{\boldsymbol{S}(l)\boldsymbol{S}(l)^T\right\} = I \approx E\left\{\boldsymbol{Y}(l)\boldsymbol{Y}(l)^T\right\} = E\left\{\boldsymbol{Y}'(l)\boldsymbol{Y}'(l)^T\right\} \tag{13}$$

For the case of $l_0 = 0$, using equations (12) and (13) we have

$$\boldsymbol{Y}'(l) = \left(\boldsymbol{P}'\boldsymbol{P}^{-1}\right)\boldsymbol{Y}(l)$$

$$\boldsymbol{Y}(l) = \left(\boldsymbol{P}'\boldsymbol{P}^{-1}\right)^{-1}\boldsymbol{Y}'(l) = \boldsymbol{M}\boldsymbol{Y}'(l) \tag{14}$$

The matrix $\boldsymbol{M}$ is the relation between $\boldsymbol{Y}$ and $\boldsymbol{Y}'$. From equation (9), the permutation matrix $\boldsymbol{P}$ and $\boldsymbol{P}'$ only have eight cases, so we have $M = \begin{vmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{vmatrix}$ for same order, and $M = \begin{vmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{vmatrix}$ for oppositional order.

As mentioned above, N is the number of sample point in both contours. Suppose that $[\boldsymbol{Y}'] = [\boldsymbol{Y}'(1)\ \boldsymbol{Y}'(2)\ \ldots\ \boldsymbol{Y}'(N)]$ and $[\boldsymbol{Y}] = [\boldsymbol{Y}(1)\ \boldsymbol{Y}(2)\ \ldots\ \boldsymbol{Y}(N)]$, we have $E\{\boldsymbol{Y}'(l)\boldsymbol{Y}'(l)^T\} = [\boldsymbol{Y}'][\boldsymbol{Y}']^T = N\boldsymbol{I}$. From equations (13) and (14) matrix $\boldsymbol{M}$ can be computed as follows

$$\boldsymbol{M} = \frac{1}{N}[\boldsymbol{Y}]\left[\boldsymbol{Y}'\right]^T \tag{15}$$

Suppose that $\boldsymbol{Y}$ is an affine-invariant descriptor, for any affine transformed contour $\boldsymbol{Y}'$, the corresponding matrix $\boldsymbol{M}$ is obtainable by equation (15), so their affine-invariant descriptor $\boldsymbol{Y}$ is easily solved.

For the case of $l_0 \neq 0$, a cost function J_τ is considered to determine the shift l_0, that is

$$J_\tau = abs\left(\left|[\boldsymbol{Y}]\left[\boldsymbol{Y}'_\tau\right]^T\right|\right) = abs\left(\left|\boldsymbol{M}\left[\boldsymbol{Y}'_{l_0}\right]\left[\boldsymbol{Y}'_\tau\right]^T\right|\right)$$
$$= abs\left(\left|R_{y'}(\tau - l_0)\right|\right), \tag{16}$$

Where $[\boldsymbol{Y}'_i]$ denotes the $[\boldsymbol{Y}']$ with ring shifting $[i]$. We obtain the shift l_0 by

$$l_0 = \arg_\tau(\max J_\tau). \tag{17}$$

We also can obtain the matrix $\boldsymbol{M}$ by

$$\boldsymbol{M} = \frac{1}{N}[\boldsymbol{Y}][\boldsymbol{Y}'_{l_0}]^T. \tag{18}$$

As the same to equation (15) $\boldsymbol{Y}(l) = \boldsymbol{M}\boldsymbol{Y}'(l - l_0)$

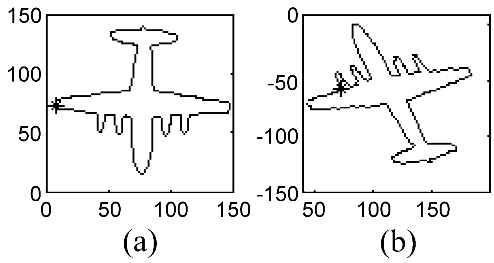

Figure 1. (a) airplane's counter; (b) affined shape of (a).

Object recognition is achieved by comparing $\boldsymbol{Y}(l)$ with $\boldsymbol{MY}'(l - l_0)$. Following simulation examples show that the proposed scheme is effective.

Example 1. Figure 1(a) is an airplane's contour, which is extracted by using any image processing method, and its affine transform shape is shown in Figure 1(b). Here the affine matrix $\boldsymbol{Z} = \begin{bmatrix} 1 & 0.5 \\ 0.2 & -1 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 40 \\ 140 \end{bmatrix}$. Both contours are sampled 220 points with equal interval. The difference of start sample index for the two contours is 200 signed by asterisk in Figure 1(a,b), respectively. Figure 2 shows the recovered independent variables for Figure 1(a,b) on x and y axis by using the proposed method.

From Figure 2, we can see that the random variables of the object and its affined shape are same. The affine invariant descriptor composed of two random components of $\boldsymbol{Y}$ is shown in Figure 3. For any kinds of affine transform of object in Figure 1(a), the affine invariant descriptor is the same by using our method.

Example 2. Figure 4(a) shows 10 known airplane models that represent 10 different objects. From different view angle, we will get different affine contour images. One kind of the affine models corresponding to 10 distorted images, with 40° skewed on horizontal and 10° on vertical direction, is shown in Figure 4(b), which is taken as test images. Table I shows the maximal similarity and

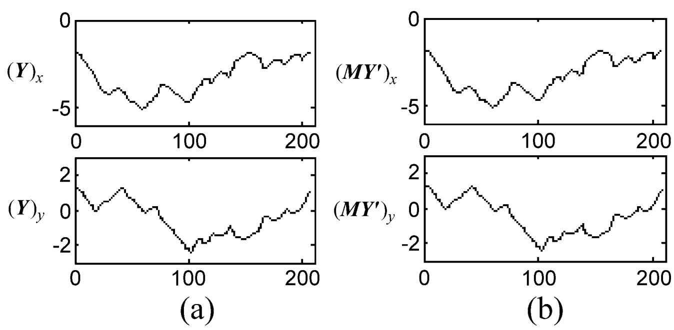

Figure 2. The results, $\boldsymbol{Y}$ and $\boldsymbol{MY}'$ by using the proposed method for $\boldsymbol{X}$ and $\boldsymbol{X}'$. (a) Two random variables of $\boldsymbol{Y}$; (b) Two random variables of $\boldsymbol{MY}'$.

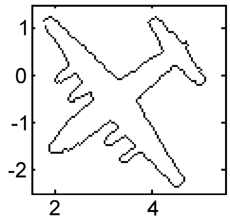

Figure 3. The invariant descriptor of Figure 1.

recognition results (shown in the brackets) by using our method and the Fourier method. The result shows that our method can successfully discriminate objects in the case of high skew, and it has better performance than the Fourier method.

3.2. OBSERVED CONTOUR DATA GOT BY RANDOM SAMPLING

In some practical applications, for the existence of illumination and noise, object's contour will be disconnected; so we could not use edge trace algorithm directly. In this case classical Fourier method does not work, but the proposed ICA method still work well under random sampling case.

Suppose that object's contour $\boldsymbol{X}$ had been random sampled as N discrete points. Each point is denoted a coordinate vector $\boldsymbol{X}(l)$. Note that the sampling points should cover all the contour, so the number of sampling point in random case should be more than in ordinal case. Let $\boldsymbol{X}'(l')$ be an affine transform from $\boldsymbol{X}(l)$ with affine matrix $\boldsymbol{Z}$ and translation vector $\boldsymbol{B}$ that can be written as

$$\boldsymbol{X}'(l') = \boldsymbol{Z}\boldsymbol{X}(l) + \boldsymbol{B}, \tag{19}$$

Here, l and l' are different because of the difference sampling on the contour $\boldsymbol{X}$ and $\boldsymbol{X}'$. In the following, we also take $\boldsymbol{B} = 0$ for analytic convenience.

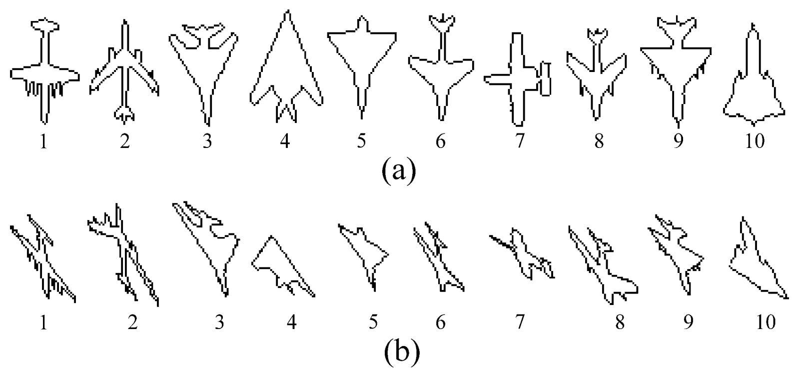

Figure 4. (a) Ten known airplane models. (b) The affined images correspond to the model images in (a), respectively.

Table I. The recognized results of test image in Figure 4(b) using our method and the Fourier method, respectively

Test image	1	2	3	4	5	6	7	8	9	10
Ordinal ICA Method	0.9969(1)	0.9741(2)	0.9628(3)	0.9739(4)	0.9782(5)	0.9872(6)	0.9980(7)	0.9768(8)	0.9798(9)	0.9863(10)
Fourier method	0.9955(1)	0.9281(2)	0.9280(3)	0.9746(4)	0.8922(4)	0.9784(6)	0.9447(4)	0.9571(8)	0.9115(4)	0.9261(6)

If $\boldsymbol{X}(l)$ is sampled on the contour $\boldsymbol{X}$ randomly, $\boldsymbol{X}(l)$ is a random vector. Consider that $\boldsymbol{X}'$ and $\boldsymbol{X}$ are the linear combination of the same source $\boldsymbol{S}$ with $\boldsymbol{A}'$ and $\boldsymbol{A}$, then $\boldsymbol{X}'(l') = \boldsymbol{A}'\boldsymbol{S}'(l')$ and $\boldsymbol{X}(l) = \boldsymbol{AS}(l)$. By using ICA, we can adapt matrix $\boldsymbol{W}'$ and $\boldsymbol{W}$ to obtain recovered $\boldsymbol{S}'(l')$ and $\boldsymbol{S}(l)$, $\boldsymbol{A}'$ and $\boldsymbol{A}$, that satisfy

$$\begin{aligned} \boldsymbol{Y}'(l') &= \boldsymbol{W}'\boldsymbol{X}'(l') = \boldsymbol{W}'\boldsymbol{A}'\boldsymbol{S}'(l') \approx \boldsymbol{P}'\boldsymbol{S}(l') \\ \boldsymbol{Y}(l) &= \boldsymbol{WX}(l) = \boldsymbol{WAS}(l) \approx \boldsymbol{PS}(l) \end{aligned} \tag{20}$$

According to property of ICA, $\boldsymbol{Y}'(l')$ and $\boldsymbol{Y}(l)$ is almost the same on the statistical characteristic, only the order of their elements and sampling points are difference.

For the unknown sample order of the contour data, we couldn't compare $\boldsymbol{Y}'(l')$ with $\boldsymbol{Y}(l)$ directly. A measure value called as Centroid-Contour Distance (CCD) was proposed for test $\boldsymbol{Y}'(l')$ and $\boldsymbol{Y}(l)$ which defines as:

$$d_Y(l) = \sqrt{Y_x(l)^2 + Y_y(l)^2} \qquad l = 1, 2, \ldots, N \tag{21}$$

Here $Y_x(l)$ and $Y_y(l)$ are the components of $\boldsymbol{Y}(l)$ on x and y axis. Then sort the $d_y(l)$, we get $d_{\text{sort}}(t)$, which satisfies:

$$d_{\text{sort}}(t_1) < d_{\text{sort}}(t_2) < \ldots < d_{\text{sort}}(t_N), t_1 < t_2 < \ldots < t_N \tag{22}$$

It is obvious that whatever the order of samples or components is the curve $d_{\text{sort}}(t)$ is affine invariant. In view of different curve length of actual object contour, before recognition we should resample the sorted curve $d_{\text{sort}}(t)$ to keep the same length. The re-sampled $d_{\text{sort}}(t)$ is also an affine invariant curve.

Example 3. Figure 5(a) shows an airplane's contour extracted by any image processing method, and its affine transform shape is shown in Figure 5(b). Figure 6 is their independent variables on x and y axis, in random sampling case, extracted from Figure 5(a,b) by ICA.

Figure 6 shows four random variables after ICA, $(\boldsymbol{Y}_x)$,$(\boldsymbol{Y})_y$,$(\boldsymbol{Y}')_x$,$(\boldsymbol{Y}')_y$ in which we cannot see any object shapes. Put both two random variables (coordinates on x and y axes) on 2-D plane, we can get two shapes shown in Figure 7. After re-sampled the sorted CCD curve of $\boldsymbol{Y}(l)$ and $\boldsymbol{Y}'(l')$ with 256 sample points are

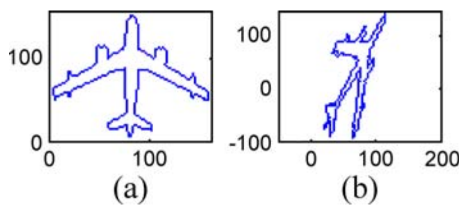

Figure 5. (a) airplane's counter; (b) skewed shape of (a).

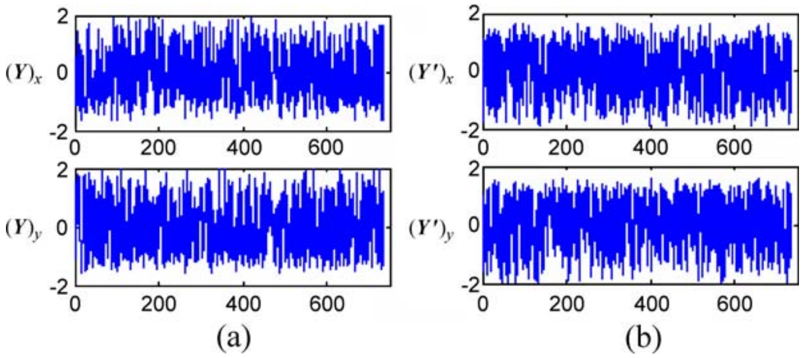

Figure 6. The ICA results, $\boldsymbol{Y}$ and $\boldsymbol{Y}'$ for $\boldsymbol{X}$ and $\boldsymbol{X}'$ by random sampling. (a) Two random variables of $\boldsymbol{Y}$; (b) Two random variables of $\boldsymbol{Y}'$.

shown in Figure 8. It is obvious that their CCD curve is the same, which can be used as an affine invariant curve.

To test the robustness of the proposed method, we add random noise on the sampled shape. It can see that the sorted CCD curve with noise has slight difference from affine invariant curve, but this change is small. Figure 9 shows the correlation coefficient between noised and affine invariant CCD curve of Figure 8 in different noise magnitude.

From Figure 9 we can see that the correlation coefficient's change is in the range of 0.2% even the noise magnitude reach 10 pixels.

Example 4. We use real airplane images in Example 2 to test random sample method. Table II shows the maximal correlation coefficient between current test image and each model (the recognition result is shown in the brackets) by using two-source ICA method in random sample case.

From Table II we can see that the proposed method only has one wrong comparing with result in Table I for the case of continuous and close contour. Ordinal sample case has better performance than in random case. Whatever the sampling methods (ordinal or random), the performance of ICA method is superior to Fourier Method. Especially in case of disconnected contour, Fourier method does not work, for different sampling order of coordinate data will change its data in frequency domain.

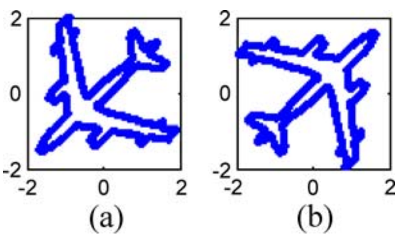

Figure 7. The recomposed shapes. (a) $\boldsymbol{Y}$, and (b) $\boldsymbol{Y}'$.

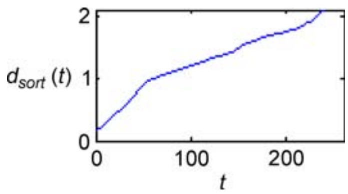

Figure 8. The affine invariant curve of $\boldsymbol{X}$ and it's all affine shapes.

Example 5. In practical applications of object recognition, it is difficult to extract object's contour perfectly because of incorrect segmentation or some obstructs front of object etc. Figure 10 is the 2# plane in Figure 4(a) whose left aerofoil had been lost. By using our method, the correlation coefficients of sorted CCD curves between misshapen plane and 10 planes in Figure 4(a) are shown in Table III.

Table III shows that the misshapen plane still has the highest correlation with the 2# plane. This example means that ICA method in random sample case also has a well performance to recognize misshapen objects.

4. Applications for Affine Transform Parameter Estimation

Motion estimation (i.e., the computation of the motion parameters of objects) plays an important role in image processing and communication. Here the affine motion estimation algorithm using two-source ICA is proposed.

Suppose that the object contour is sampled N points for both original and its affine-transformed object with affine matrix $\boldsymbol{Z}$ and translation vector $\boldsymbol{B}$, $\boldsymbol{X}$ and $\boldsymbol{X}'$, which satisfied equation (11). Centered the two observed data $\boldsymbol{X}$ and $\boldsymbol{X}'$, i.e., $\boldsymbol{B} = 0$. By using the same process as Section 3, we have

$$\begin{aligned} \boldsymbol{Y}'(l) &= \boldsymbol{W}'\boldsymbol{X}'(l) = \boldsymbol{W}'\boldsymbol{Z}\boldsymbol{X}(l - l_0) \\ \boldsymbol{Y}(l - l_0) &= \boldsymbol{W}\boldsymbol{X}(l - l_0) \end{aligned} \tag{23}$$

According to equations (12)–(18), $\boldsymbol{M}$, $\boldsymbol{Y}$, $\boldsymbol{Y}'$, l_0 can be computed, so the affine matrix $\boldsymbol{Z}$ can be estimated as

$$\boldsymbol{Z} = \boldsymbol{W}'{-}\boldsymbol{M}^{-1}\boldsymbol{W} \tag{24}$$

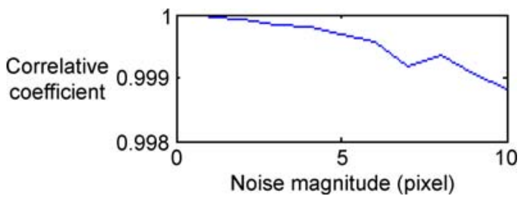

Figure 9. Correlation coefficient under different noise magnitude.

Table II. The recognized results for each test image in Figure 4(b) using our method.

Test image	1	2	3	4	5	6	7	8	9	10
Random ICA Method	0.9995(1)	0.9986(2)	0.9976(3)	0.9978(5)	0.9971(5)	0.9987(6)	0.9998(7)	0.9980(8)	0.9988(9)	0.9987(10)

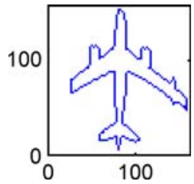

Figure 10. A misshapen plane corresponds to 2# in Figure 6(a).

Then the translation vector $\boldsymbol{B}$ can be obtained by

$$\boldsymbol{B} = E\{\boldsymbol{X}'(l) - \boldsymbol{Z}E\{\boldsymbol{X}(l - l_0)\}\} \tag{25}$$

Here E(•) denotes expectation.

Example 6. A new test sequence is built from standard image sequence "Claire'' (Figure 11) to estimate the proposed algorithm. The new test sequence is a natural head but it contains considerable amounts of rotational, scaling and affine transform (Figure 12). In Figure 12, Frame 1 contains the contour of original head; Frame 2 shows the magnified Frame 1 with size at two times, Frame 3 is rotation ($\theta = \pi/6$) of Frame 2, Frame 4 is Frame 3's affine transformation $\left(Z = \begin{bmatrix} 0.8 & 0.2 \\ 0 & 1.25 \end{bmatrix}\right)$. Four sample pictures from the sequence are shown in Figure 12. Table IV gives the estimation result comparing with Fourier method.

From Table IV we can see that both ordinal ICA Method and Fourier Method can estimation affine parameters accurately for the case of none noise on contour and $l_0 = 0$.

Example 7. In this example we use the proposed method and Fourier method respectively to estimate the affine motion between shapes with noise and start index delay. The original shape is in Figure 1(a), the result is shown in Table V. Here, we set start index delay $l_0 = 200$, and the random noise's magnitude is 3 pixels.

Table V shows that the results obtained by proposed method are very close to simulated actual motion and it has better performance than Fourier method in motion estimation with noise and start index delay.

The average error of ICA and Fourier method for different magnitude noise at 1,000 times is shown in Figure 13. It shows that the proposed method still work well in strong noised case.

Example 8. Two real key images taken from different angles are shown in Figure 14(a,b). In Figure 14(c,d) the real line's contour is taken from Figure 14(a). The dotted contour in Figure 14(c,d) are the result of motion estimation from Figure 14(a,b) by using proposed method and Fourier method, respectively. It is obvious that the proposed method can estimate the affine parameters more

Table III. Correlation coefficients between misshapen plane and models in Figure 4(a).

Plane index	1	2	3	4	5	6	7	8	9	10
Correlative coefficient	0.9898	0.9984	0.9798	0.9818	0.9744	0.9956	0.9793	0.9959	0.9928	0.9877

Figure 11. Frame 194 of the Claire sequence.

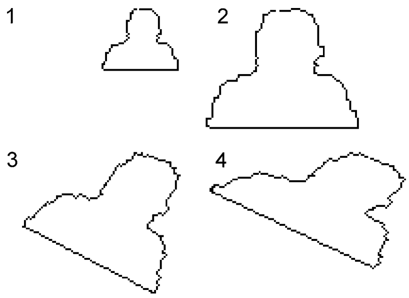

Figure 12. Contours corresponding to the transformations indicated.

precisely than Fourier method. The reason is that, in real application, the images taken from different angles do not satisfy affine transform relation strictly, and they are affected by illumination and noise, so the affine invariant descriptor deduced in the frequency domain or other transform domain does not work well, and our proposed method works in the spatial domain and the linear transform matrix $\boldsymbol{W}$ can adaptively modify to tally with real cases.

Table IV. Results of estimation of the parameters for the case of Figure 12 ($l_0 = 0$).

Actual motion				Ordinal ICA method				Fourier method			
z_{11}	z_{12}	z_{21}	z_{22}	z_{11}	z_{12}	z_{21}	z_{22}	z_{11}	z_{12}	z_{21}	z_{22}
2	0	0	2	2	0	0.0	2	2	0	0	2
0.866	0.5	−0.5	0.866	0.866	0.5	−0.5	0.866	0.866	0.5	−0.5	0.866
0.8	0.2	0	1.25	0.8	0.2	0.0	1.25	0.8	0.2	0	1.25

Table V. Actual motion and estimated parameters.

Affine parameter	l_0	z_{11}	z_{12}	z_{21}	z_{22}
Actual motion	200	0.6800	1.4300	−0.4700	0.9900
ICA method	200	0.6825	1.4270	−0.4698	0.9909
Fourier method	200	0.6742	1.4367	−0.4835	1.0056

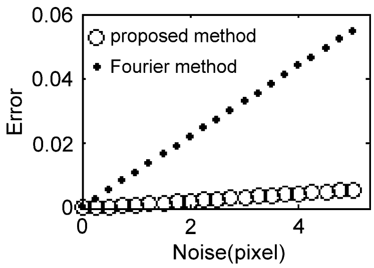

Figure 13. The comparative result of ICA method and Fourier method.

Example 9. To test ICA method's feasibility in real application, we get 1,000 groups random shape with different number of contour points, and test the computational time of affine motion estimation using ICA method and Fourier method. The result is shown in Table VI, from which we can see that the estimation of the start sample point by using ring shift algorithm will spend more time for ICA method, but it still satisfy requirement of real world processing. And the time of Fourier method is almost the same whether the start sample point is same or not.

A motion estimation example by using ICA is expressed as follows. For an affine motion object with 128 sample points on the contour in different starting sample point, it could estimate 35 frames per second according to Table VI, 1/ 0.02844 = 35.16 frames/second. It is enough to use in real video compression standard, such as MPEG4 and MPEG7.

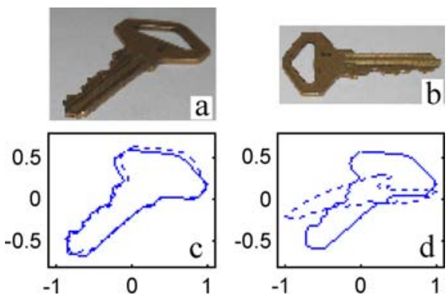

Figure 14. Affine parameters estimation. (a,b) Real key image taken from different angles. (c) The result by our method. (d) The result by Fourier method.

Table VI. The affine motion estimation time under different number of contour points.

	Start sample point	Contour points		
		64 s	128 s	256 s
ICA method	Same	0.00234	0.00250	0.00344
	Differ	0.01484	0.02844	0.06590
Fourier method	Same	0.0469	0.0501	0.0530
	Differ	0.0470	0.0502	0.0563

5. Conclusions

A new affine object recognition and affine motion estimation method are proposed based on two-source ICA. Assume that the coordinations of sample points from object contour are random sequences and their distribution are subGaussion. They can be considered as observed random vector in ICA theory. So the recovered independent sources by ICA can be considered as affine-invariant descriptor and the relation matrix between original object and its affine versions, $\boldsymbol{M}$, can be obtained. Because of the statistical properties ICA method has more robustness than other classical methods. We propose two different schemes to find the affine-invariant descriptor after ICA in the ordinal and random sample. Experimental results show that whatever in the ordinal or random samples the proposed methods have better performance than classical method such as Furious method.

ICA method also can be used in affine motion estimation. Experimental results show that the performance of the proposed method is better than other traditional invariant methods, especially, in real world applications.

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