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Blockage Revision

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Abstract Blockage revision is a version of descriptor revision, i.e. belief change in which a belief set K is changed with inputs whose success conditions are metalinguistic expressions containing the belief predicate $\mathfrak{B}$. This is a highly general framework that allows a single revision operator $\circ$ to take inputs corresponding to sentential revision ($K \circ \mathfrak{B} p$), contraction ($K \circ \neg \mathfrak{B} p$) as well as more complex and composite operations. In blockage revision, such an operation is based on a relation $\rightharpoondown$ of blockage among the set of potential outcomes. $X \rightharpoondown Y$ signifies that if X satisfies the success condition of a belief change, then Y cannot be its outcome. The properties of blockage revision are investigated, and conditions on the blocking relation are specified that characterize various properties of the resulting operation of change.

Keywords Belief change · Blockage revision · Descriptor revision · Monoselective choice function · Outcome set · Repertoire

1 Introduction

Descriptor revision was introduced in (Hansson 2014a) as an alternative, more general approach to belief change. Like most other belief change models it employs logically closed sets (belief sets) to represent an epistemic agent's state of belief, and a change in belief is represented by an input-driven transformation from one such belief set to another. These transformations have two major characteristics.

The first of these is the form in which the input is expressed. Traditional belief change models use sentences as inputs, and we distinguish between different types of

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operations according to what is expected be done with the input sentence. In belief contraction, the input sentence should be removed from the belief set, whereas in revision it should be added to the belief set. In other words a contraction has the *success condition* that a specified sentence should be removed, whereas the success condition of a revision is that a specified sentence should be added to the belief set. In descriptor revision, success conditions are expressed in a unified way, employing a metalinguistic operator $\mathfrak{B}$ representing belief. $\mathfrak{B}p$ denotes that p is believed, $\neg\mathfrak{B}p$ that p is not believed, $\mathfrak{B}p \vee \mathfrak{B}q$ that either p or q is believed, etc. With this generalized way to express success conditions we only need one operator of change, denoted $\circ$. A revision of the current belief set K by the sentence p can be written $K \circ \mathfrak{B}p$ and a contraction of the same set by the sentence q can be written $K \circ \neg\mathfrak{B}q$. Other more complex success conditions can also be expressed in this way, for instance the operation $K \circ (\neg\mathfrak{B}p \;\&\; \mathfrak{B}q)$ (or equivalently $K \circ \{\neg\mathfrak{B}p, \mathfrak{B}q\}$) in which p is replaced by q, the multiple operation $K \circ \{\neg\mathfrak{B}p_1, \ldots, \neg\mathfrak{B}p_n\}$ in which a set of sentences is simultaneously removed, and the operation $K \circ (\mathfrak{B}p \vee \mathfrak{B}\neg p)$ in which the agent makes up its mind concerning p. The formal construction of such composite descriptors follows a simple recursive pattern:

> $\mathcal{L}$ is the object language whose sentences are believed or disbelieved. An *atomic belief descriptor* is a metalinguistic expression $\mathfrak{B}p$ with $p \in \mathcal{L}$. A *molecular belief descriptor* is a truth-functional combination of atomic descriptors. A *composite belief descriptor* (descriptor; denoted by upper-case Greek letters) is a set of molecular descriptors.[1]
>
> $\mathfrak{B}p$ is *satisfied* by a belief set K if and only if $p \in K$. Satisfaction for molecular belief descriptors is defined inductively, hence K satisfies $\neg\alpha$ if and only if it does not satisfy α, and it satisfies $\alpha \vee \beta$ if and only if it satisfies either α or β. A composite belief descriptor is satisfied by K if and only if all its elements are satisfied by K.
>
> $K \Vdash \Psi$ means that K satisfies Ψ, and $\Psi \Vdash \Xi$ that all belief sets satisfying Ψ also satisfy Ξ. The corresponding equivalance relation is written $\dashv\Vdash$, hence $\Psi \dashv\Vdash \Xi$ holds if and only if both $\Psi \Vdash \Xi$ and $\Xi \Vdash \Psi$ hold.
>
> The *descriptor disjunction* $\veebar$ is defined by the relationship
>
> $\Psi \veebar \Xi = \{\alpha \vee \beta \mid (\alpha \in \Psi) \;\&\; (\beta \in \Xi)\}$.[2]

The second major characteristic of descriptor revision is that changes take place as choices among a given repertoire, or set of plausible outcomes. The repertoire is a set of belief sets and will be denoted $\mathbb{X}$. Hence, $K \circ \mathfrak{B}p$ is assumed to be an element X of $\mathbb{X}$ such that $X \Vdash \mathfrak{B}p$. In (Hansson 2014a) two ways to construct a descriptor revision $\circ$ were investigated:

[1] A finite composite descriptor can be replaced by the molecular descriptor that is the conjunction of all its elements. However, an infinite composite descriptor cannot in general be replaced by a single molecular descriptor.

[2] Descriptor disjunction will be used in Observation 3 and the discussion following it.

$\circ$ is a *monoselective descriptor revision* if and only if: (i) if there is some $X \in \mathbb{X}$ with $X \Vdash \Psi$, then $K \circ \Psi = \widehat{C}(\{X \in \mathbb{X} \mid X \Vdash \Psi\})$, where $\widehat{C}$ is a monoselective choice function,[3] and (ii) otherwise, $K \circ \Psi = K$.

$\circ$ is a *relational descriptor revision* if and only if there is a relation $\leqq$ on $\mathbb{X}$ with $K \leqq X$ for all $X \in \mathbb{X}$, and: (i) if there is some $X \in \mathbb{X}$ with $X \Vdash \Psi$, then $K \circ \Psi$ is the unique $\leqq$-minimal element of $\mathbb{X}$ that satisfies Ψ, and (ii) otherwise, $K \circ \Psi = K$.

It is the purpose of the present contribution to introduce another construction of descriptor revision. It is based on *blockage relations* that express how, when we choose among potential outcomes (elements of $\mathbb{X}$) satisfying the success condition Ψ, the presence of one such potential outcome can make another ineligible. To each descriptor revision $\circ$ we can associate a blockage relation $\rightharpoondown$ on its repertoire $\mathbb{X}$, defined as follows:[4]

For all $X, Y \in \mathbb{X}$: $X \rightharpoondown Y$ if and only if it holds for all descriptors Ψ that if $X \Vdash \Psi$ then $K \circ \Psi \neq Y$.

$X \rightharpoondown Y$ can be interpreted as signifying that the plausibility of X is so high in relation to that of Y that the epistemic agent will not settle for Y as the outcome of a revision if X is available as a possible outcome of that revision. Importantly, this can be the case even if X is less plausible than Y. Examples can be found in which two potential outcomes seem to block each other although one of them is somewhat more plausible than the other.[5] For one such example, let p denote that Peter has stolen money from his mother and r that his sister Rebecca has done so. In my initial belief set K I believe in neither of these sentences, i.e. $K \Vdash \neg\mathfrak{B}p \,\&\, \neg\mathfrak{B}r$. However, I receive information that induces me to perform the revision $K \circ \mathfrak{B}(p \vee r)$. Let X and Y be two elements of $\mathbb{X}$, both of which satisfy $\mathfrak{B}(p \vee r)$ but such that $X \Vdash \mathfrak{B}(p\&\neg r)$ and $Y \Vdash \mathfrak{B}(r\&\neg p)$. Even if X is for some reason somewhat less plausible than Y, X can still be plausible enough to prevent me from adopting Y as my new set of beliefs. Consequently, X and Y block each other, and neither of them can become the outcome. Instead, the outcome can be some third belief set Z with $Z \Vdash \mathfrak{B}(p \vee r)$, $Z \nVdash \mathfrak{B}p$, and $Z \nVdash \mathfrak{B}r$.

If we define a blockage relation for a given revision operator, as above, then the resulting blockage relation may not contain sufficient information to reconstruct the revision operator from which it was derived.[6] In order to avoid such indeterminacy it is better to introduce the blockage relation as a primitive notion.

[3] I.e. $\widehat{C}(\mathbb{Y}) \in \mathbb{Y}$ whenever $\mathbb{Y} \neq \varnothing$.

[4] Blockage relations were first introduced in Hansoon (2013b) where they were used to construct a contraction operator.

[5] Cf. Hansoon (2013b, pp. 418–419) and Rott (2014).

[6] Two different monoselective descriptor revisions $\circ$ and $\circ'$ may give rise to the same blockage relation. Let $\mathbb{X} = \{K, X, Y, Z, W\}$, and let $\circ$ be based on a monoselective choice function $\widehat{C}$ such that $\widehat{C}(\mathbb{Y}) = K$ whenever $K \in \mathbb{Y} \subseteq \mathbb{X}$ and that $\widehat{C}(\{X, Y\}) = X$, $\widehat{C}(\{X, Z\}) = Z$, $\widehat{C}(\{X, W\}) = X$, $\widehat{C}(\{Y, Z\}) = Y$, $\widehat{C}(\{Y, W\}) = W$, $\widehat{C}(\{Z, W\}) = Z$, $\widehat{C}(\{X, Y, Z\}) = X$, $\widehat{C}(\{X, Z, W\}) = W$, $\widehat{C}(\{Y, Z, W\}) = Z$, $\widehat{C}(\{X, Y, W\}) = W$, and $\widehat{C}(\{X, Y, Z, W\}) = Y$. Furthermore, let $\widehat{C}'$ coincide with $\widehat{C}'$ with the sole exception that $\widehat{C}'(\{X, Y\}) = Y$, and let $\circ'$ be based on $\widehat{C}'$. Then $\circ$ and $\circ'$ give rise to the same blockage relation namely $\rightharpoondown = \{\langle K, X, \rangle, \langle K, Y\rangle, \langle K, Z\rangle, \langle K, W\rangle\}$. However, let Ψ be a descriptor that is satisfied by X and Y but not by K, Z, or W. Then $K \circ \Psi = X$ and $K \circ' \Psi = Y$.

2 The Basic form of Blockage Revision

We can define blockage revision as follows:

Definition 1 Let $\rightharpoondown$ be a binary relation on the set $\mathbb{X}$ of belief sets with $K \in \mathbb{X}$. The *blockage revision* on K generated by $\rightharpoondown$ is the operation $\circ$ such that for all descriptors Ψ:

(i) If X is the unique Ψ-satisfying element of $\mathbb{X}$ that is not blocked by any other Ψ-satisfying element of $\mathbb{X}$, then $K \circ \Psi = X$.
(ii) If there is no such unique Ψ-satisfying and unblocked element of $\mathbb{X}$, then $K \circ \Psi = K$.

Note that we can arrive at clause (ii) of the definition either because (1) there is no Ψ-satisfying element of $\mathbb{X}$, (2) all Ψ-satisfying elements of $\mathbb{X}$ are blocked within the Ψ-satisfying part of $\mathbb{X}$, or (3) at least two Ψ-satisfying elements of $\mathbb{X}$ are unblocked within the Ψ-satisfying part of $\mathbb{X}$.

The set $\mathbb{X}$ in Definition 1 is a repertoire, i.e. a set of potential (or viable) belief sets among which the outcomes of the $\circ$ operation have to be chosen. By the *outcome set* (Hansson 2013a) is meant the set of actually chosen outcomes, i.e. the set of belief sets X for which there exists a descriptor Ψ with $X = K \circ \Psi$. The following observation identifies the cases when the repertoire and the outcome set of a blockage revision coincide.

Observation 1 *Let $\circ$ be the blockage revision on K that is generated by the relation $\rightharpoondown$ on the set $\mathbb{X}$ of belief sets. Then $\mathbb{X}$ is the outcome set of $\circ$ if and only if $K \in \mathbb{X}$ and $\rightharpoondown$ satisfies irreflexivity within $\mathbb{X} \setminus \{K\}$.*

The proofs of this and all other formal results are given in an Appendix. (The proof of Observation 1 is based on a lemma showing that for each belief set there is a descriptor that is satisfied only by that belief set.)

In what follows it will be assumed that $\rightharpoondown$ is irreflexive and that consequently, the repertoire and the outcome set coincide. (Alternatively we could distinguish between the repertoire $\mathbb{X}$ and the outcome set which could then be identified as $\{K\} \cup \{X \in \mathbb{X} \mid X \not\rightharpoondown X\}$, but nothing important would be gained in that more complicated account.)

In order to compare blockage revision with monoselective and relational descriptor revision, the two operations mentioned in the previous section, we will have use for the following four postulates:

If $K \circ \Xi \Vdash \Psi$ then $K \circ \Psi \Vdash \Psi$ (regularity)
If $K \circ \Psi \Vdash \Xi$ then $K \circ \Psi = K \circ (\Psi \cup \Xi)$ (cumulativity)
If $K \circ \Psi \Vdash \Xi$ and $K \circ \Xi \Vdash \Psi$ then $K \circ \Psi = K \circ \Xi$ (reciprocity)
If $K \Vdash \Psi$ then $K \circ \Psi = K$ (confirmation)

Regularity holds for all monoselective descriptor revisions. The other three postulates hold for relational (but not in general for monoselective) descriptor revision. Blockage revision does not satisfy any of them:

Observation 2 *Let $\circ$ be the blockage revision on K that is generated by the relation $\rightharpoondown$ on its outcome set $\mathbb{X}$. Then:*

(1) It does not hold in general that $\circ$ *satisfies regularity.*
(2) It does not hold in general that $\circ$ *satisfies cumulativity.*
(3) It does not hold in general that $\circ$ *satisfies reciprocity.*
(4) It does not hold in general that $\circ$ *satisfies confirmation.*

On the other hand, the following observation introduces two postulates that are satisfied by all blockage revisions but not by all monoselective descriptor revisions.

Observation 3 *(1) Let* $\circ$ *be the blockage revision on* K *that is generated by the relation* $\rightharpoondown$ *on its outcome set* $\mathbb{X}$. *Then* $\circ$ *satisfies:*

If $K \circ \Psi \neq K \neq K \circ (\Psi \cup \Xi)$ *and* $K \circ \Psi \Vdash \Xi$, *then* $K \circ \Psi = K \circ (\Psi \cup \Xi)$. *(peripheral cumulativity), and*

If $K \neq K \circ \Psi = K \circ \Xi$ *then* $K \circ \Psi = K \circ (\Psi \veebar \Xi)$ *(peripheral disjunctive identity).*

(2) Let $\circ$ *be a monoselective descriptor revision. It does not hold in general that* $\circ$ *satisfies peripheral cumulativity. Furthermore, it does not hold in general that* $\circ$ *satisfies peripheral disjunctive identity.*

The term "peripheral" restricts the scope of the postulates to the part of the outcome set that does not include the original belief set K. Peripheral cumulativity and peripheral disjunctive identity are restriction of the following postulates:

If $K \circ \Psi \Vdash \Xi$, then $K \circ \Psi = K \circ (\Psi \cup \Xi)$ (cumulativity)
If $K \circ \Psi = K \circ \Xi$ then $K \circ \Psi = K \circ (\Psi \veebar \Xi)$ (disjunctive identity)

Their analogues in sentential revision:

If $K * p \vdash q$, then $K * p = K * (p \& q)$ (cumulativity), and
If $K * p = K * q$ then $K * p = K * (p \vee q)$ (disjunctive identity)

both hold in transitively relational AGM revision.[7]

3 Additional Properties of Blockage Revision

In this section we will focus on the four postulates for monoselective and relational descriptor revision that were shown in Observation 2 not to hold in general for blockage revision. What properties must the blockage relation $\rightharpoondown$ satisfy in order for these postulates to hold? The following theorem answers that question for regularity:

[7] The sentential version of cumulativity seems to have appeared in the belief revision literature for the first time in Makinson and Gärdenfors (1991, p. 198). It has often been divided into two parts, "If $K * p \vdash q$ then $K * p \subseteq K * (p\&q)$" and "If $K * p \vdash q$ then $K * (p\&q) \subseteq K * p$" that are called cautious monotony respectively cut, due to their close relationships with patterns of nonmonotonic reasoning with the same names (Rott 1992, p. 49). On these postulates, see also Rott (2001). Disjunctive identity does not seem to have been referred to in the belief revision literature, but it is a trivial consequence of the postulate of disjunctive factoring (Either $K * (p \vee q) = K * p$ or $K * (p \vee q) = K * q$ or $K * (p \vee q) = K * p \cap K * q$) that holds for transitively relational AGM revision. (Disjunctive factoring was proved by Hans Rott and first reported in Gärdenfors (1988, pp. 57, 212, and 244).)

Theorem 1 *Let $\circ$ be the blockage revision on K that is generated by the relation $\rightharpoondown$ on its outcome set $\mathbb{X}$. Then $\circ$ satisfies regularity if and only if $\rightharpoondown$ satisfies the two postulates:*

If $K \nVdash \Psi$ but there is some $X \in \mathbb{X}$ with $X \Vdash \Psi$, then there is at least one unblocked element within the set of Ψ-satisfying elements of $\mathbb{X}$. (peripheral non-occlusion), and

If $X \neq Y$ and $X \neq K \neq Y$ then either $X \rightharpoondown Y$ or $Y \rightharpoondown X$ (peripheral weak connectivity)

These properties of $\rightharpoondown$ are also what it takes for a blockage revision to be a monoselective descriptor revision:

Observation 4 *Let $\circ$ be the blockage revision on K that is generated by the relation $\rightharpoondown$ on its outcome set $\mathbb{X}$. Then $\circ$ is a monoselective revision on K if and only if $\rightharpoondown$ satisfies peripheral non-occlusion and peripheral weak connectivity.*

The following characterization of cumulativity (If $K \circ \Psi \Vdash \Xi$ then $K \circ \Psi = K \circ (\Psi \cup \Xi)$) reveals an interesting connection between cumulativity and regularity:

Theorem 2 *Let $\circ$ be the blockage revision on K that is generated by the relation $\rightharpoondown$ on its outcome set $\mathbb{X}$. Then $\circ$ satisfies cumulativity if and only if $\rightharpoondown$ satisfies peripheral non-occlusion, peripheral weak connectivity, and:*

> *If $X \neq K \neq Y$ and $X \rightharpoondown Y \rightharpoondown K$, then* either *$X \rightharpoondown K \not\rightharpoondown X$* or *both $K \rightharpoondown X$ and $K \rightharpoondown Y$. (top adjacency).*

It follows from Theorems 1 and 2 that if a blockage revision satisfies cumulativity, then it satisfies regularity. However, the converse relationship does not hold.[8] The following theorem provides a condition that may be seen as the "missing part" that can be added to regularity in order to obtain cumulativity.

Theorem 3 *A blockage revision $\circ$ satisfies cumulativity if and only if it satisfies both regularity and:*

> *If $K \circ \Psi \Vdash \Xi$ and $K \circ \Xi \Vdash \Psi$, then $K \circ \Psi = K \circ \Xi$. (reciprocity)*

Reciprocity has a close analogue in sentential revision:

If $K * p \vdash q$ and $K * q \vdash p$ then $K * p = K * q$. (reciprocity)[9]

For descriptor revision, regularity and reciprocity are independent conditions:

Observation 5 *Let $\circ$ be the blockage revision on K that is generated by the relation $\rightharpoondown$ on its outcome set $\mathbb{X}$. Then: (1) It does not hold in general that if $\circ$ satisfies regularity then it satisfies reciprocity, and (2) it does not hold in general that if $\circ$ satisfies reciprocity then it satisfies regularity.*

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8 This follows from Theorem 3 and Observation 5.

9 Reciprocity seems to have been introduced independently in Alchourrón and Makinson (1982, p. 32) and Gärdenfors (1982, p. 97). It has been further discussed for instance in Makinson (1985, p. 354) where it was called the Stalnaker property, Makinson and Gärdenfors (1991, p. 198), Rott (2001, p. 110), and Rott (2014).

Let us now turn to the fourth of the postulates that were shown in Observation 2 not to hold in general for blockage revision, namely confirmation. It can be characterized as follows:

Theorem 4 *Let $\circ$ be the blockage revision on K that is generated by the relation $\rightharpoondown$ on its outcome set $\mathbb{X}$. Then $\circ$ satisfies confirmation if and only if $\rightharpoondown$ satisfies the postulate*

If $X \neq K \neq Y$ and $X \rightharpoondown K$ then (1) $K \rightharpoondown X$ and (2) either $K \rightharpoondown Y$ or $X \rightharpoondown Y$. (near-superiority)

Top adjacency and *near-superiority* represent different ways to ensure that K has a strong position within $\mathbb{X}$ in terms of $\rightharpoondown$. The following are stronger and somewhat simpler properties with similar effects:

If $K \neq X$ then $K \rightharpoondown X$ (superiority)
$X \not\rightharpoondown K$ (non-inferiority)

Observation 6 *Let $\rightharpoondown$ be an irreflexive relation on a set $\mathbb{X}$ with $K \in \mathbb{X}$. Then:*

(1) If $\rightharpoondown$ satisfies superiority, then it satisfies top adjacency and near-superiority.
(2) If $\rightharpoondown$ satisfies non-inferiority, then it satisfies top adjacency and near-superiority.
(3) It does not hold in general that if $\rightharpoondown$ satisfies top adjacency then it satisfies near-superiority.
(4) It does not hold in general that if $\rightharpoondown$ satisfies near-superiority then it satisfies top adjacency.

As the following two observations show, confirmation is independent of the other three postulates.

Observation 7 *Let $\circ$ be a blockage revision that satisfies cumulativity. It does not hold in general that $\circ$ satisfies confirmation.*

Observation 8 *Let $\circ$ be a blockage revision that satisfies confirmation. (1) It does not hold in general that $\circ$ satisfies regularity. (2) It does not hold in general that $\circ$ satisfies reciprocity.*

Finally, the combination of cumulativity and confirmation, and therefore also of the postulates that characterize them, is necessary and sufficient for a blockage revision to also be a relational descriptor revision, in the sense defined in Sect. 1.

Theorem 5 *Let $\circ$ be a descriptor revision on K. Then $\circ$ is a relational descriptor revision if and only if it is a blockage revision generated by a relation $\rightharpoondown$ that satisfies peripheral non-occlusion, peripheral weak connectivity, near-superiority, and top adjacency.*

4 Discussion

The results from the previous section are summarized in Fig. 1. They provide a series of classes of operations representing different degrees of orderliness. It may be of some

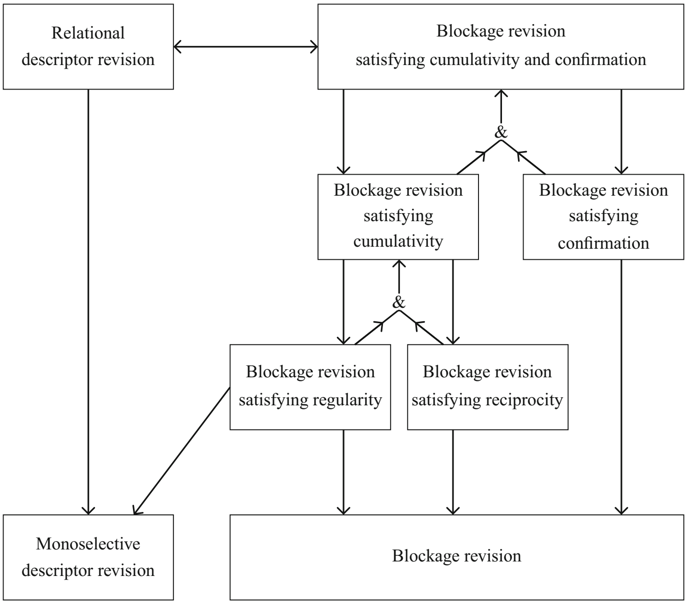

Fig. 1 *Relationships of implication among the classes of descriptive revision studied in this paper*

interest to compare them to Hans Rott's (2014) arguments for the need, in AGM theory, of "floors" between the (at least in some respects) implausibly weak level of the basic postulates and the exceptionally strong level of transitively relational operations. The classes of operations investigated here are not directly comparable to those proposed by Rott for the AGM framework, partly because descriptor revision represents a more general type of belief change than the AGM operations and partly because some of the basic postulates of AGM are not satisfied even by the strongest class of descriptor revision studied here.[10] Generally speaking, the "floors" introduced above are lower (logically weaker) than those discussed by Rott in relation to the AGM operations. However, the results obtained here can be summarized in terms of four floors of blockage revision, describable from the top downwards in the following series where each item is logically stronger than those that follow below it:

- Blockage revision satisfying regularity, reciprocity, cumulativity, and confirmation.
- Blockage revision satisfying regularity, reciprocity, and cumulativity.
- Blockage revision satisfying regularity.
- Blockage revision (the general case).

Important issues concerning blockage revision remain to investigate. In particular, it remains to characterize blockage revision axiomatically, and a wider range of properties of the $\rightharpoondown$ relation than those studied here should be investigated.

[10] This applies in particular to the success conditions for contraction and sentential revision that can be translated into $K \circ \neg\mathfrak{B}p \Vdash \neg\mathfrak{B}p$ respectively $K \circ \mathfrak{B}p \Vdash \mathfrak{B}p$. It also applies to the recovery postulate for contraction that can be translated into $(K \circ \neg\mathfrak{B}p) \circ \mathfrak{B}p \subseteq K$ and to the vacuity property for sentential revision that can be translated into: If $K \nVdash \mathfrak{B}\neg p$ then $K \circ \mathfrak{B}p = \mathrm{Cn}(K \cup \{p\})$.

Appendix: Proofs

Definition 2 (Hansson 2014a) For any belief set X, we denote by Π_X the descriptor $\{\mathfrak{B}p \mid p \in X\} \cup \{\neg\mathfrak{B}p \mid p \notin X\}$.

Definition 3 For any set $\mathbb{X}$ of belief sets and any descriptor Ψ:

$$[\![\Psi]\!]_{\mathbb{X}} = \{X \in \mathbb{X} \mid X \Vdash \Psi\}.$$

The index can be omitted if no ambiguity follows from doing so, i.e. we can then write $[\![\Psi]\!]$ instead of $[\![\Psi]\!]_{\mathbb{X}}$.

Lemma 1 *If a binary relation satisfies non-occlusion then it satisfies acyclicity. If it satisfies acyclicity then it satisfies asymmetry. If it satisfies asymmetry then it satisfies irreflexivity.*

Proof of Lemma 1: Left to the reader. □

Lemma 2 $K \Vdash \Psi \veebar \Xi$ *holds if and only if either* $K \Vdash \Psi$ *or* $K \Vdash \Xi$ *holds.*

Proof of Lemma 2: See Hansson (2014b, p. 85). □

Proof of Observation 1: For one direction, let $K \in \mathbb{X}$ and let $\rightharpoondown$ satisfy irreflexivity within $\mathbb{X} \setminus \{K\}$. It follows directly from $K \in \mathbb{X}$ and Definition 1 that the outcome set is a subset of $\mathbb{X}$. We also have to show that each element of $\mathbb{X}$ is an element of the outcome set. For each $X \in \mathbb{X} \setminus \{K\}$, $\{X\}$ is the set of Π_X-satisfying elements of $\mathbb{X}$, and due to the irreflexivity of $\rightharpoondown$, X is the unique unblocked element of $\{X\}$, thus $K \circ \Pi_X = X$. Furthermore, $K = K \circ \neg\mathfrak{B}\top$ (where $\top$ is a tautology) due to clause (ii) of Definition 1. Thus all elements of $\mathbb{X}$ are elements of the outcome set.

For the other direction, we assume that either (i) $K \notin \mathbb{X}$ or (ii) $X \rightharpoondown X$ for some $X \in \mathbb{X} \setminus \{K\}$. In the first case it follows from $K \circ \neg\mathfrak{B}\top = K$ that the outcome set has an element that is not in $\mathbb{X}$. In the second case $K \circ \Psi \neq X$ for all Ψ, thus X is not in the outcome set although it is in $\mathbb{X}$. □

Proof of Observation 2: Part 1: Let $\mathbb{X} = \{K, X, Y\}$ and:

$$\rightharpoondown = \{\langle K, X\rangle, \langle K, Y\rangle, \langle X, Y\rangle, \langle Y, X\rangle\}.$$

Then $K \circ \Pi_X \Vdash \Pi_X \veebar \Pi_Y$ but $K \circ (\Pi_X \veebar \Pi_Y) \nVdash \Pi_X \veebar \Pi_Y$.

Part 2: Let $\mathbb{X} = \{K, X, Y\}$ and $\rightharpoondown = \{\langle K, X\rangle, \langle X, Y\rangle, \langle Y, K\rangle\}$. Then $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y) = K$ and $K \circ ((\Pi_K \veebar \Pi_X \veebar \Pi_Y) \cup (\Pi_K \veebar \Pi_Y)) = K \circ (\Pi_K \veebar \Pi_Y) = Y$, thus $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y) \Vdash \Pi_K \veebar \Pi_Y$ but $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y) \neq K \circ ((\Pi_K \veebar \Pi_X \veebar \Pi_Y) \cup (\Pi_K \veebar \Pi_Y))$.

Part 3: Let $\mathbb{X} = \{K, X, Y\}$ and $\rightharpoondown = \{\langle K, X\rangle, \langle X, Y\rangle, \langle Y, K\rangle\}$. Then $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y) \Vdash \Pi_K \veebar \Pi_Y$ and $K \circ (\Pi_K \veebar \Pi_Y) \Vdash \Pi_K \veebar \Pi_X \veebar \Pi_Y$ but $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y) \neq K \circ (\Pi_K \veebar \Pi_Y)$.

Part 4: Let $\mathbb{X} = \{K, X\}$ and let $\rightharpoondown = \{\langle X, K\rangle\}$. Then $K \Vdash \Pi_K \veebar \Pi_X$ but $K \circ (\Pi_K \veebar \Pi_X) = X$. □

Proof of Observation 3: Part 1, peripheral cumulativity: Let $K \circ \Psi \neq K \neq K \circ (\Psi \cup \Xi)$ and $K \circ \Psi \Vdash \Xi$. It follows from Definition 1 and $K \neq K \circ \Psi$ that $K \circ \Psi \Vdash \Psi$, thus $K \circ \Psi \Vdash \Psi \cup \Xi$, i.e. $K \circ \Psi \in [\![\Psi \cup \Xi]\!]$. It also follows from $K \neq K \circ \Psi$ that $K \circ \Psi$ is unblocked within $[\![\Psi]\!]$, and since $[\![\Psi \cup \Xi]\!] \subseteq [\![\Psi]\!]$ it is then also unblocked within $[\![\Psi \cup \Xi]\!]$. Thus $K \circ \Psi$ is $\Psi \cup \Xi$-satisfying and unblocked within $[\![\Psi \cup \Xi]\!]$. Since $K \neq K \circ (\Psi \cup \Xi)$ there is exactly one belief set with that property, namely $K \circ (\Psi \cup \Xi)$. It follows that $K \circ \Psi = K \circ (\Psi \cup \Xi)$.

Part 1, peripheral disjunctive identity: Since $K \neq K \circ \Psi$, all elements of $[\![\Psi]\!] \setminus \{K \circ \Psi\}$ are blocked within $[\![\Psi]\!]$, and similarly all elements of $[\![\Xi]\!] \setminus \{K \circ \Psi\}$ are blocked within $[\![\Xi]\!]$. Since $[\![\Psi \veebar \Xi]\!] = [\![\Psi]\!] \cup [\![\Xi]\!]$, all elements of $[\![\Psi \veebar \Xi]\!] \setminus \{K \circ \Psi\}$ are blocked within $[\![\Psi \veebar \Xi]\!]$. Since $K \circ \Psi$ is unblocked both within $[\![\Psi]\!]$ and within $[\![\Xi]\!]$, it is unblocked within $[\![\Psi \veebar \Xi]\!]$. Thus $K \circ \Psi$ is the only unblocked element within $[\![\Psi \veebar \Xi]\!]$, thus $K \circ \Psi = K \circ (\Psi \veebar \Xi)$.

Part 2, peripheral cumulativity: Let $\mathbb{X} = \{K, X, Y, Z\}$ and let $\circ$ be based on a monoselective choice function $\widehat{C}$ such that $\widehat{C}(\{X, Y, Z\}) = \{Y\}$ and $\widehat{C}(\{Y, Z\}) = \{Z\}$. Then $K \circ (\Pi_X \veebar \Pi_Y \veebar \Pi_Z) = Y$ and $Y \Vdash \Pi_Y \veebar \Pi_Z$ but $K \circ ((\Pi_X \veebar \Pi_Y \veebar \Pi_Z) \cup (\Pi_Y \veebar \Pi_Z)) = K \circ (\Pi_Y \veebar \Pi_Z) = Z$.

Part 2, peripheral disjunctive identity: Let $\mathbb{X} = \{K, X, Y, Z\}$ and let $\circ$ be based on a monoselective choice function $\widehat{C}$ such that $\widehat{C}(\{X, Y\}) = \widehat{C}(\{Y, Z\}) = Y$ and $\widehat{C}(\{X, Y, Z\}) = X$. Then $K \circ (\Pi_X \veebar \Pi_Y) = Y$, $K \circ (\Pi_Y \veebar \Pi_Z) = Y$, and $K \circ ((\Pi_X \veebar \Pi_Y) \veebar (\Pi_Y \veebar \Pi_Z)) = X$. □

Proof of Theorem 1: From regularity to peripheral non-occlusion: Let $K \notin [\![\Psi]\!]$ and $X \in [\![\Psi]\!]$. Due to Observation 1 and our assumption that $\rightharpoondown$ is irreflexive there is some Ξ with $X = K \circ \Xi$. Then $K \circ \Xi \Vdash \Psi$ and regularity yields $K \circ \Psi \Vdash \Psi$. From this and $K \circ \Psi \neq K$ it follows according to Definition 1 that $K \circ \Psi$ is an unblocked element within $[\![\Psi]\!]$.

From regularity to peripheral weak connectivity: Let $X, Y \in \mathbb{X}$, $X \neq Y$ and $X \neq K \neq Y$. Due to Observation 1 and our assumption that $\rightharpoondown$ is irreflexive there is some Ξ with $X = K \circ \Xi$. Thus $K \circ \Xi \Vdash \Pi_X \veebar \Pi_Y$. Regularity yields $K \circ (\Pi_X \veebar \Pi_Y) \Vdash \Pi_X \veebar \Pi_Y$, thus $K \circ (\Pi_X \veebar \Pi_Y) \in \{X, Y\}$, from which it follows that either $X \rightharpoondown Y$ or $Y \rightharpoondown X$.

From peripheral non-occlusion and peripheral weak connectivity to regularity: Let $K \circ \Xi \Vdash \Psi$.

First case, $K \nVdash \Psi$: Due to peripheral non-occlusion it follows from $K \circ \Xi \Vdash \Psi$ and $K \nVdash \Psi$ that $[\![\Psi]\!]$ has at least one non-blocked element. It follows from peripheral weak connectivity that it has at most one such element. Due to clause (i) of Definition 1, that element is equal to $K \circ \Psi$, thus $K \circ \Psi \Vdash \Psi$.

Second case, $K \Vdash \Psi$: According to Definition 1, $K \circ \Psi$ is either an element of $[\![\Psi]\!]$ or equal to K. In both cases, $K \circ \Psi \Vdash \Psi$. □

Proof of Observation 4: For one direction, let $\circ$ be a monoselective descriptor revision. Then $\circ$ satisfies regularity, and we can conclude from Theorem 1 that $\rightharpoondown$ satisfies peripheral non-occlusion and peripheral weak connectivity.

For the other direction, let $\rightharpoondown$ satisfy peripheral non-occlusion and peripheral weak connectivity, and let $\circ$ be the revision operator generated from $\rightharpoondown$. It follows from Theorem 1 that $\circ$ satisfies regularity. Let $\mathbb{X} = \{X \mid (\exists \Psi)(X = K \circ \Psi)\}$ and let $\widehat{C}$ be a function such that (a) when $\varnothing \neq \mathbb{Y} \subseteq \mathbb{X}$, then $\widehat{C}(\mathbb{Y}) = K \circ \Psi$ for all Ψ such that

$\{X \mid X \Vdash \Psi\} = \mathbb{Y}$ and (b) otherwise $\widehat{C}(\mathbb{Y})$ is undefined. We need to show (1) that $\widehat{C}$ is a monoselective choice function on $\mathbb{X}$, and (2) that $\circ$ is the monoselective descriptor revision on $\mathbb{X}$ that is based on $\widehat{C}$.

For (1) it is sufficient to show that $\widehat{C}$ is indeed a function. This can be done by noting that since $\circ$ is a blockage relation it satisfies the uniformity postulate:

If $K \circ \Xi \Vdash \Psi$ iff $K \circ \Xi \Vdash \Psi'$ for all Ξ, then $K \circ \Psi = K \circ \Psi'$

For (2) we need to consider the two cases referred to in Sect. 1. In case (i) there is some $X \in \mathbb{X}$ with $X \Vdash \Psi$. It follows directly from regularity that $K \circ \Psi$ is in this case indeed identical to $\widehat{C}(\{X \in \mathbb{X} \mid X \Vdash \Psi\})$. In case (ii) there is no $X \in \mathbb{X}$ with $X \Vdash \Psi$. It follows from Definition 1 that $K \circ \Psi = K$, as required. □

Proof of Theorem 2: From cumulativity to peripheral non-occlusion: Suppose to the contrary that $K \notin [\![\Psi]\!] \neq \varnothing$ and $[\![\Psi]\!]$ has no unblocked element. Let $X \in [\![\Psi]\!]$. Then $K \circ \Psi = K$, $K \circ \Psi \Vdash \Pi_K \veebar \Pi_X$, and $K \circ (\Psi \cup (\Pi_K \veebar \Pi_X)) = K \circ \Pi_X = X$, contrary to cumulativity.

From cumulativity to peripheral weak connectivity: Suppose to the contrary that there are $X, Y \in \mathbb{X}$ such that $X \neq Y \neq K \neq X$ and $X \not\rightharpoondown Y \not\rightharpoondown X$. Then $K \circ (\Pi_X \veebar \Pi_Y) = K$, thus $K \circ (\Pi_X \veebar \Pi_Y) \Vdash \Pi_K \veebar \Pi_Y$, but $K \circ ((\Pi_X \veebar \Pi_Y) \cup (\Pi_K \veebar \Pi_Y)) = K \circ \Pi_Y = Y$, contrary to cumulativity.

From cumulativity to top adjacency: Let $\circ$ be a blockage revision based on $\rightharpoondown$, and furthermore suppose that top adjacency does not hold. We are going to show that cumulativity does not hold. Let $X, Y \in \mathbb{X} \setminus \{K\}$ and $X \rightharpoondown Y \rightharpoondown K$. Since top adjacency does not hold, if $K \rightharpoondown X$ then $K \not\rightharpoondown Y$, and furthermore, if $K \not\rightharpoondown X$ then $X \not\rightharpoondown K$. We therefore have the following two cases:

Case 1, $K \rightharpoondown X$ and $K \not\rightharpoondown Y$: Then there is no unblocked element within $\{K, X, Y\}$ but there is a unique unblocked element within $\{K, Y\}$, namely Y. It follows that $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y) = K$, thus $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y) \Vdash \Pi_K \veebar \Pi_Y$, but $K \circ ((\Pi_K \veebar \Pi_X \veebar \Pi_Y) \cup (\Pi_K \veebar \Pi_Y)) = Y$, contrary to cumulativity.

Case 2, $K \not\rightharpoondown X$ and $X \not\rightharpoondown K$: We have $X \rightharpoondown Y$ and by applying peripheral non-occlusion (that we have just proved) to $\{X, Y\}$ we obtain $Y \not\rightharpoondown X$. Thus $\{K, X, Y\}$ has a unique unblocked element namely X, whereas $\{K, X\}$ has the unblocked elements K and X. It follows that $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y) = X$, thus $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y) \Vdash \Pi_K \veebar \Pi_X$, but $K \circ ((\Pi_K \veebar \Pi_X \veebar \Pi_Y) \cup (\Pi_K \veebar \Pi_X)) = K$, contrary to cumulativity.

Thus in both cases, if top adjacency does not hold, then neither does cumulativity.

From peripheral non-occlusion, peripheral weak connectivity, and top adjacency to cumulativity: Let $K \circ \Psi \Vdash \Xi$.

Case 1, $[\![\Psi]\!] = \varnothing$: Then $[\![\Psi \cup \Xi]\!] = \varnothing$, and we have $K \circ \Psi = K$ and $K \circ (\Psi \cup \Xi) = K$.

Case 2, $K \notin [\![\Psi]\!] \neq \varnothing$: It follows from peripheral non-occlusion and peripheral weak connectivity that $[\![\Psi]\!]$ has exactly one unblocked element, and due to Definition 1 that element is equal to $K \circ \Psi$. It follows that $K \circ \Psi \Vdash \Psi$ and we already have $K \circ \Psi \Vdash \Xi$, so $K \circ \Psi \Vdash \Psi \cup \Xi$. Since $K \circ \Psi$ is unblocked within $[\![\Psi]\!]$, it is also unblocked within its subset $[\![\Psi \cup \Xi]\!]$. It follows from $K \notin [\![\Psi]\!]$ and $[\![\Psi \cup \Xi]\!] \subseteq [\![\Psi]\!]$ that $K \notin [\![\Psi \cup \Xi]\!]$. It follows from peripheral non-occlusion and peripheral weak connectivity that $[\![\Psi \cup \Xi]\!]$ has exactly one unblocked element, and then $K \circ \Psi$ is that element. Due to Definition 1, $K \circ \Psi = K \circ (\Psi \cup \Xi)$.

Case 3, $K \in [\![\Psi]\!]$ and $K \neq K \circ \Psi$: It follows from Definition 1 that $K \circ \Psi$ is the unique unblocked element of $[\![\Psi]\!]$. Since $K \circ \Psi \Vdash \Xi$ we have $K \circ \Psi \in [\![\Psi \cup \Xi]\!]$. Since $K \circ \Psi$ is unblocked within $[\![\Psi]\!]$, it is also unblocked within its subset $[\![\Psi \cup \Xi]\!]$.

Case 3A, $K \notin [\![\Psi \cup \Xi]\!]$: It follows from peripheral non-occlusion and peripheral weak connectivity that $[\![\Psi \cup \Xi]\!]$ has exactly one unblocked element, and then $K \circ \Psi$ is that element. Due to Definition 1, $K \circ \Psi = K \circ (\Psi \cup \Xi)$.

Case 3B, $K \in [\![\Psi \cup \Xi]\!]$: Since $K \circ \Psi$ is the unique unblocked element of $[\![\Psi]\!]$, there is some $X \in [\![\Psi]\!]$ with $X \rightharpoondown K$. Since $K \circ \Psi$ is unblocked within $[\![\Psi]\!]$ we also have $X \not\rightharpoondown K \circ \Psi$ and $K \not\rightharpoondown K \circ \Psi$. Due to peripheral weak connectivity, $K \circ \Psi \rightharpoondown X$. We conclude from top adjacency that $K \circ \Psi \rightharpoondown K$.

Next, let $Z \in [\![\Psi \cup \Xi]\!] \setminus \{K, K \circ \Psi\}$. Since $K \circ \Psi$ is unblocked within $[\![\Psi \cup \Xi]\!]$ it follows from peripheral weak connectivity that $K \circ \Psi \rightharpoondown Z$. Thus $K \circ \Psi$ is the only unblocked element within $[\![\Psi \cup \Xi]\!]$, thus $K \circ \Psi = K \circ (\Psi \cup \Xi)$.

Case 4, $K \in [\![\Psi]\!]$ and $K = K \circ \Psi$: It follows from $K \circ \Psi \Vdash \Xi$ that $K \in [\![\Psi \cup \Xi]\!]$.

Case 4A, K is unblocked within $[\![\Psi \cup \Xi]\!]$: If K is the only unblocked element within $[\![\Psi \cup \Xi]\!]$, then $K = K \circ (\Psi \cup \Xi)$ due to clause (i) of Definition 1. If it is one of at least two unblocked elements within $[\![\Psi \cup \Xi]\!]$, then $K = K \circ (\Psi \cup \Xi)$ due to clause (ii) of the same definition.

Case 4B, K is blocked within $[\![\Psi \cup \Xi]\!]$: Due to peripheral non-occlusion and peripheral weak connectivity there is some $Y \in [\![\Psi \cup \Xi]\!] \setminus \{K\}$ such that $Y \rightharpoondown X \not\rightharpoondown Y$ for all $X \in [\![\Psi \cup \Xi]\!] \setminus \{K, Y\}$. It follows that $K \circ (\Psi \cup \Xi)$ is either Y or K. We are going to show that it is not Y. Suppose that it is. Then clearly $K \not\rightharpoondown Y$.

Case 4Ba, $Y \not\rightharpoondown K$: Since K is blocked within $[\![\Psi \cup \Xi]\!]$ there is then some $X \in [\![\Psi \cup \Xi]\!] \setminus \{K, Y\}$ such that $X \rightharpoondown K$. We then have $Y \rightharpoondown X \not\rightharpoondown Y$, $K \not\rightharpoondown Y \not\rightharpoondown K$ and $X \rightharpoondown K$, contrary to top adjacency.

Case 4Bb, $Y \rightharpoondown K$: Due to peripheral non-occlusion and peripheral weak connectivity there is some $Z \in [\![\Psi]\!] \setminus \{K\}$ such that $Z \rightharpoondown V \not\rightharpoondown Z$ for all $V \in [\![\Psi]\!] \setminus \{K, Z\}$. Since K is blocked (within $[\![\Psi \cup \Xi]\!]$ and therefore also) within $[\![\Psi]\!]$ and $K \circ \Psi \neq Z$, Z is blocked within $[\![\Psi]\!]$, thus $K \rightharpoondown Z$. Since by assumption $K \not\rightharpoondown Y$, $Y \neq Z$. We then have $Z \rightharpoondown Y \not\rightharpoondown Z$, $K \not\rightharpoondown Y \rightharpoondown K$ and $K \rightharpoondown Z$, contrary to top adjacency.

Thus, in neither subcase can $K \circ (\Psi \cup \Xi)$ be equal to Y. We can conclude that $K \circ (\Psi \cup \Xi) = K$, thus $K \circ \Psi = K \circ (\Psi \cup \Xi)$ in case 4B as well. □

Proof of Theorem 3: From cumulativity to regularity: Directly from Theorems 1 and 2.

From cumulativity to reciprocity : Let $K \circ \Psi \Vdash \Xi$ and $K \circ \Xi \Vdash \Psi$. Then cumulativity yields $K \circ \Psi = K \circ (\Psi \cup \Xi) = K \circ \Xi$.

From regularity and reciprocity to cumulativity: Let $K \circ \Psi \Vdash \Xi$. There are two cases.

Case (i), $K \circ \Psi \nVdash \Psi$: Regularity yields $K \circ (\Psi \cup \Xi) \nVdash \Psi$, thus $K \circ (\Psi \cup \Xi) \nVdash \Psi \cup \Xi$. It follows from Definition 1 that $K \circ \Psi = K = K \circ (\Psi \cup \Xi)$.

Case (ii), $K \circ \Psi \Vdash \Psi$: Then $K \circ \Psi \Vdash \Psi \cup \Xi$. Regularity yields $K \circ (\Psi \cup \Xi) \Vdash \Psi \cup \Xi$. We thus have $K \circ \Psi \Vdash \Psi \cup \Xi$ and $K \circ (\Psi \cup \Xi) \Vdash \Psi$, and reciprocity yields $K \circ \Psi = K \circ (\Psi \cup \Xi)$. □

Proof of Observation 5: Part 1: Let $\mathbb{X} = \{K, X, Y\}$ and $\rightharpoondown = \{\langle X, Y\rangle, \langle Y, K\rangle\}$. It follows from Theorem 1 that regularity is satisfied. We have $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y) = X$ and $K \circ (\Pi_K \veebar \Pi_X) = K$, thus $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y) \Vdash \Pi_K \veebar \Pi_X$ and $K \circ (\Pi_K \veebar \Pi_X) \Vdash \Pi_K \veebar \Pi_X \veebar \Pi_Y$ but $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y) \neq K \circ (\Pi_K \veebar \Pi_X)$, which shows that reciprocity does not hold.

Part 2: Let $\mathbb{X} = \{K, X, Y\}$ and $\rightharpoondown = \{\langle K, X\rangle, \langle K, Y\rangle\}$. In order to show that reciprocity holds it is sufficient to that there are no Ψ and Ξ such that either (1) $K \circ \Psi = K$, $K \Vdash \Xi$, $K \circ \Xi = X$, and $X \Vdash \Psi$, (2) $K \circ \Psi = K$, $K \Vdash \Xi$, $K \circ \Xi = Y$, and $Y \Vdash \Psi$, or (3) $K \circ \Psi = X$, $X \Vdash \Xi$, $K \circ \Xi = Y$, and $Y \Vdash \Psi$.

Suppose that (1) holds. Due to our construction of $\circ$ it follows from $K \circ \Xi = X$ that $X \Vdash \Xi$, $K \nVdash \Xi$, and $Y \nVdash \Xi$. But we also have $K \Vdash \Xi$, thus (1) does not hold. A symmetrical proof shows that (2) does not hold. Suppose that (3) holds. It then follows from $K \circ \Psi = X$ that $X \Vdash \Psi$, $K \nVdash \Psi$, and $Y \nVdash \Psi$, but we also have $Y \Vdash \Psi$, so that this case is impossible as well. Thus there are no Ψ and Ξ that satisfy either (1), (2), or (3), thus reciprocity holds. It follows from $K \circ \Pi_X = X$, $X \Vdash \Pi_X \veebar \Pi_Y$, and $K \circ (\Pi_X \veebar \Pi_Y) = K$ that regularity does not hold. □

Proof of Theorem 4: From confirmation to near-superiority: Let $X \rightharpoondown K$. Confirmation yields $K \circ (\Pi_K \veebar \Pi_X) = K$, which would not be the case if $X \rightharpoondown K \not\rightharpoondown X$. Thus $K \rightharpoondown X$.

If also follows from confirmation that $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y) = K$. Since $X \rightharpoondown K$, $K \circ (\Pi_K \veebar \Pi_X \veebar \Pi_Y)$ cannot follow from clause (i) of Definition 1, so it must be based on clause (ii). We already have $K \rightharpoondown X$, thus Y must be blocked by either K or X.

From near-superiority to confirmation: Let $K \Vdash \Psi$. If $[\![\Psi]\!]$ does not have exactly one unblocked element, then clause (ii) of Definition 1 yields $K \circ \Psi = K$. It remains to treat the case when $[\![\Psi]\!]$ has exactly one unblocked element. Suppose that element is not K. Then there is some $X \in [\![\Psi]\!]$ with $X \rightharpoondown K$. It follows from near-superiority that all elements of $[\![\Psi]\!]$ are blocked (either by K or by X). This contradicts the assumption that the only unblocked element of $[\![\Psi]\!]$ is not K. We conclude that it is K and that therefore $K \circ \Psi = K$ in this case as well. □

Proof of Observation 6: Parts 1 and 2 are left to the reader.

Part 3: Let $\mathbb{X} = \{K, X, Y\}$ and $\rightharpoondown = \{\langle X, Y\rangle, \langle X, K\rangle, \langle Y, K\rangle\}$.

Part 4: Let $\mathbb{X} = \{K, X, Y\}$ and $\rightharpoondown = \{\langle X, Y\rangle, \langle Y, X\rangle, \langle Y, K\rangle, \langle K, Y\rangle\}$.

Proof of Observation 7: Let $\mathbb{X} = \{K, X, Y\}$ and $\rightharpoondown = \{\langle X, Y\rangle, \langle X, K\rangle, \langle Y, K\rangle\}$. It follows from Theorem 2 that $\circ$ satisfies cumulativity and from $K \circ (\Pi_K \veebar \Pi_X) = X$ that it does not satisfy confirmation. □

Proof of Observation 8: Part 1: Let $\mathbb{X} = \{K, X, Y\}$ and $\rightharpoondown = \{\langle K, X\rangle, \langle K, Y\rangle\}$. It follows from Theorem 4 that $\circ$ satisfies confirmation and from $K \circ \Pi_X = X$, $X \Vdash \Pi_X \veebar \Pi_Y$, and $K \circ (\Pi_X \veebar \Pi_Y) = K$ that it does not satisfy regularity.

Part 2: Let $\mathbb{X} = \{K, X, Y, Z, V\}$ and:

$$\rightharpoondown = \{\langle K, X\rangle, \langle K, Y\rangle, \langle K, Z\rangle, \langle K, V\rangle, \langle X, Z\rangle, \langle Z, Y\rangle, \langle Y, V\rangle, \langle V, X\rangle\}.$$

It follows from Theorem 4 that $\circ$ satisfies confirmation. However, $K \circ (\Pi_X \veebar \Pi_Y \veebar \Pi_Z) = X$ and $K \circ (\Pi_X \veebar \Pi_Y \veebar \Pi_V) = Y$. Since $K \circ (\Pi_X \veebar \Pi_Y \veebar \Pi_Z) \Vdash \Pi_X \veebar \Pi_Y \veebar \Pi_V$ and $K \circ (\Pi_X \veebar \Pi_Y \veebar \Pi_V) \Vdash \Pi_X \veebar \Pi_Y \veebar \Pi_Z$, it follows that reciprocity is not satisfied. □

Proof of Theorem 5: From relational revision to blockage revision: Let $\rightharpoondown$ be the strict part of $\leqq$. Note that near-superiority and top adjacency are satisfied vacuously. (Since $\leqq$ is antisymmetric, $\rightharpoondown$ is irreflexive. Since it also has the property that $K \leqq X$ for all $X \in \mathbb{X}$, there is no X with $X \rightharpoondown K$.)

From blockage revision to relational revision: It was shown in Hansson (2014a) that $\circ$ is a relational descriptor revision if it satisfies extensionality (If $\Psi \dashv\Vdash \Psi'$ then $K \circ \Psi = K \circ \Psi'$), closure ($K \circ \Psi = \mathrm{Cn}(K \circ \Psi)$), relative success ($K \circ \Psi \Vdash \Psi$ or $K \circ \Psi = K$), regularity, cumulativity, and confirmation. It follows straight-forwardly from Definition 1 that the first three of these conditions are satisfied. That the last three are satisfied follows from Theorems 1, 2, and 4. □

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