J Log Lang Inf (2010) 19:485–503
DOI 10.1007/s10849-009-9119-4

Combinations of *Stit* and Actions

Ming Xu

Published online: 13 January 2010

Abstract We present a simple theory of actions against the background of branching time, based on which we propose two versions of an extended *stit* theory, one equipped with particular actions and the other with sets of such actions. After reporting some basic results of a formal development of such a theory, we briefly explore its connection to a version of branching ETL.

Keywords *Stit* · Action · Logic of agency · Logic of action

In the framework of *stit* theories, we present a way of talking about actions, and propose an extended *stit* theory called "*stit*-action logic". This "hybrid" theory comes with sentences of the form $[\alpha, e]A$, read "by doing e, α sees to it that A", where α is an agent term, e an action term, and A any sentence. The action terms display a main difference between this theory and the early ones because they have always taken actions or choices to be semantic notions only.

Stit theories and logic of actions (like PDL and ETL) both deal with consequences or outcomes of events/actions or choices, but they do so in quite different and virtually unrelated ways. As logic of agency, early *stit* theories emphasize connections between agents and the consequences of their choices, with actions to be absent from their languages. Meanwhile, theories like PDL, DEL and ETL emphasize connections between events/actions and their consequences, but there is no connection between agents and events/actions. The current project started after a conversation with Professor Johan van Benthem last year concerning the fact that the two fields above seem

M. Xu (✉)
Department of Philosophy, Wuhan University, 430072 Wuhan, People's Republic of China
e-mail: mingxu01@hotmail.com

M. Xu
Chinese Institute of Advanced Studies, Central University of Finance and Economics,
100081 Beijing, People's Republic of China

to have common goals but somehow have been "independent" for years. We need a junction of the two roads to logic theories of actions, which I believe lies in where we can directly connect actions, their agents and their consequences. The extended *stit* theory we propose here seems to provide such a connection.

There have been attempts to bring together different frameworks of formal studies of events/actions and knowledge. Some of these attempts have achieved a merge of various ideas related to dynamic logic (e.g., van Benthem and Pacuit 2006; van Benthem et al. 2009); others are toward a similar merge of ideas from *stit* and dynamic logic (e.g., Brown 2008a,b; Broersen et al. 2009).[1] This paper is to join their efforts, particularly the latter, with a focus on actions rather than epistemic notions.

We start with basic semantics for *stit*, followed by a simple theory of events in branching time. In Sect. 3, we present briefly a theory of actions as events, and then, in Sect. 4, we discuss a connection between actions by agents and possible choices for them. Our basic semantic theory for *stit*-action logic is given in Sect. 5, and a report of two basic systems for the logic in Sect. 6, with their computational aspect left open. Finally, in Sect. 7, we present a two way representation between a fragment of *stit*-action logic and a fragment of branching ETL.

1 *Stit* Theories

Stit theories start with the basic notion $[\alpha]A$ (read "α sees to it that A" or "α *stit* A"), where α is any agent term and A is any formula. There are quite a few versions of *stit*: *astit* (the achievement *stit*), *bstit*, *cstit*, *dstit* (the deliberative *stit*), and recently *xstit*.[2] The *cstit* and the *dstit* operators are directly relevant to our work here, for which we use $[\alpha]^{\mathsf{c}}$ and $[\alpha]^{\mathsf{d}}$ when we need to distinguish them. The interpretation of $[\alpha]A$ is roughly that A is guaranteed true due to a current choice made by agent α.[3]

A *stit structure* is a sequence $\langle T, <, \mathsf{Agent}, \mathsf{Choice}\rangle$ specified below. $\langle T, <\rangle$ is a tree-like frame, i.e., $T \neq \varnothing$ and $<$ is a strict partial ordering that is linearly ordered toward the past. We call maximal $<$-chains *histories* in $\langle T, <\rangle$, and use h, h' (H, H') etc. to range over (sets of) them. For each $m \in T$, we let $H_m = \{h : m \in h\}$, the set of all histories *passing through* m. For each $m \in T$, m is a $<$-*dead-end* if $m < m'$ for no $m' \in T$, and is a $>$-*dead-end* if $m' < m$ for no $m' \in T$. Agent is a nonempty set whose members α, β etc., are called *agents*.[4] Choice is a choice function that assigns

[1] I have not seen the last reference, but list it here for the readers' interests. Thanks to Johan van Benthem and Eric Pacuit for providing this reference.

[2] *Astit* is proposed in Belnap and Perloff (1988), *bstit* in Brown (1988), *cstit* in Chellas (1969), *dstit* in von Kutschera (1986) and Horty (1989) independently, and *Xstit* in Broersen (2008a). For detailed discussions on *astit* and *dstit*, the reader is referred to Belnap et al. (2001) and Horty (2001), and the references therein. Another *stit* operator is given in the work of Broerson and his colleagues, which reads $[\alpha]A$ as "α strategically sees to it that A" (see, e.g., Broersen et al. 2006a,b).

[3] *Astit* involves an earlier choice by the agent, while *bstit* deals with the ability of the agent. *Xstit* is similar to *dstit* in a way, and to *astit* in another. It can be show that on a discrete *stit* model, the *xstit* operator and $[\alpha]^{\mathsf{d}}$ (or $[\alpha]^{\mathsf{c}}$) are mutually definable at the presence of metric tense operators.

[4] In this paper, we use α, β etc. ambiguously both as agents and as terms for agents, for the sake of simplicity and easy comparison of *stit* with branching ETL in the last section. Later we will introduce actions and sets of actions, and will follow the same practice for the same reason.

to each $\alpha \in \mathsf{Agent}$ and each $m \in T$ a partition $\mathsf{Choice}^{\alpha}_{m}$ of the set H_m, and we use $\mathsf{Choice}^{\alpha}_{m}(h)$ with $h \in H_m$ for the member of $\mathsf{Choice}^{\alpha}_{m}$ to which h belongs. Members of $\mathsf{Choice}^{\alpha}_{m}$ are called *possible choices* (available) for α at m, and are subject to the following conditions:

- *No choice between undivided histories*: for all $m, m' \in T$ with $m < m'$, all $\alpha \in \mathsf{Agent}$ and all $h \in H_{m'}$, $H_{m'} \subseteq \mathsf{Choice}^{\alpha}_{m}(h)$.
- *Independence of agents*: for each $m \in T$ and each function f assigning to each $\alpha \in \mathsf{Agent}$ a possible choice $f(\alpha) \in \mathsf{Choice}^{\alpha}_{m}$, $\bigcap_{\alpha \in \mathsf{Agent}} f(\alpha) \neq \varnothing$.

Let $\mathfrak{F} = \langle T, <, \mathsf{Agent}, \mathsf{Choice}\rangle$ be any *stit* structure. A model on $\mathfrak{F}$ is a pair $\mathfrak{M} = \langle \mathfrak{F}, V\rangle$ (or a sequence $\langle T, <, \mathsf{Agent}, \mathsf{Choice}, V\rangle$) where V is a valuation assigning to each propositional letter a subset of $\{\langle m, h\rangle : m \in h\}$. The satisfaction relation $\mathfrak{M}, m/h \vDash A$ $(m \in h)$ is defined in the usual way, plus the following clauses, where $\Box$ is the operator of historical necessity:

$$
\begin{aligned}
\mathfrak{M}, m/h \vDash \Box A \quad &\text{iff} \quad \mathfrak{M}, m/h' \vDash A \quad \text{for all} \quad h' \in H_m.\\
\mathfrak{M}, m/h \vDash [\alpha]^{\mathsf{c}} A \quad &\text{iff} \quad \mathfrak{M}, m/h' \vDash A \quad \text{for all} \quad h' \in \mathsf{Choice}^{\alpha}_{m}(h).\\
\mathfrak{M}, m/h \vDash [\alpha]^{\mathsf{d}} A \quad &\text{iff} \quad \mathfrak{M}, m/h' \vDash A \quad \text{for all} \quad h' \in \mathsf{Choice}^{\alpha}_{m}(h),\\
&\qquad\qquad \text{and} \quad \mathfrak{M}, m/h'' \nvDash A \quad \text{for some} \quad h' \in H_m.
\end{aligned}
\tag{1}
$$

It is easy to see that each of the three operators $[\alpha]^{\mathsf{c}}$, $[\alpha]^{\mathsf{d}}$ and $\Box$ can be defined in terms of the other two: $[\alpha]^{\mathsf{d}}A$ can be defined as $[\alpha]^{\mathsf{c}}A \wedge \neg\Box A$, $[\alpha]^{\mathsf{c}}A$ as $[\alpha]^{\mathsf{d}}A \vee \Box A$, and $\Box A$ as $[\alpha]^{\mathsf{c}}A \wedge \neg[\alpha]^{\mathsf{d}}A$. It is also true that $\Box$ is definable in terms of $[\alpha]^{\mathsf{c}}$ and $[\beta]^{\mathsf{c}}$ if we fix α and β to be distinct agents: one can define $\Box A$ as $[\alpha]^{\mathsf{c}}[\beta]^{\mathsf{c}}A$ (see Balbiani et al. 2008).[5]

2 Outcomes, Transitions and Events

In order to combine *stit* with actions, we need a formal theory of the latter, and it seems natural to build this theory on a theory of events. Xu (1997) suggests that events can be characterized in terms of transitions against the background of branching time. Here we briefly present this theory of events.

Let $\langle T, <\rangle$ be any tree-like frame. For all $m, m' \in T$ and all subsets X, Y of T, we use the following abbreviations:

$$
\begin{aligned}
X < m \quad &\text{iff} \quad m' < m \quad \text{for all} \quad m' \in X.\\
m < X \quad &\text{iff} \quad m < m' \quad \text{for all} \quad m' \in X.\\
X < Y \quad &\text{iff} \quad m < m' \quad \text{for all} \quad m \in X \quad \text{and} \quad m' \in Y.\\
m \leqslant m' \quad &\text{iff} \quad m < m' \quad \text{or} \quad m = m'.
\end{aligned}
$$

[5] A basic axiomatization and the decidability of *cstit* and *dstit* logic, with $\Box$ present in the language, is given in Xu (1994), which is simplified in Balbiani et al. (2008), and a tableau calculus for the same logic is given in Wansing (2006). A system with *dstit* operators alone is given in Xu (1998), a system with tense operators is given in Wölfl (2002), and various systems connecting *stit* with epistemic logic or deontic logic are found in the recent work by Jan Broerson and his colleagues, e.g., in Broersen (2008a,b).

Furthermore, the first three clauses above provide definitions of $X \leqslant m$, $m \leqslant X$ and $X \leqslant Y$ if we replace $<$ by $\leqslant$ on both sides. A *past* in $\langle T, <\rangle$ is an upper-bounded and nonempty initial segment p of a history in $\langle T, <\rangle$, i.e., a nonempty subset p of a history h such that $p \leqslant m \in h$ for some m, and $m'' < m' \in p$ only if $m'' \in p$. In particular, for each $m \in T$, we use $p_{(m)}$ for the past $\{m' \in T : m' \leqslant m\}$.

An *outcome* in $\langle T, <\rangle$ is a lower-bounded and nonempty subset O of T that is closed forward and connected backward, i.e., a nonempty subset O of T such that

- $m \leqslant O$ for some $m \in T$.
- $m < m'$ and $m \in O$ only if $m' \in O$.
- for all $m, m', m'' \in O$, $m' < m$ and $m'' < m$ only if either $m' \leqslant m''$ or $m'' \leqslant m'$.[6]

When a tree-like frame is clear in the context, we often omit mentioning it even though most notions we discuss are relative to the given tree-like frame.

Definition 2.1 A transition in $\langle T, <\rangle$ is a pair $\langle p, O\rangle$, where p is a past and O is an outcome such that $p < O$.

Let e be any set of transitions. We will call each member $\langle p, O\rangle$ of e an *e-transition*, and call its outcome O an *e-outcome*. For each history h, *e occurs in h* iff there is an e-outcome O such that $O \cap h \neq \varnothing$. e occurs in h *at most once* iff $\{\langle p, O\rangle \in e : O \cap h \neq \varnothing\}$ is either empty or a singleton.

Definition 2.2 An event in $\langle T, <\rangle$ is a nonempty set of transitions that occurs at most once in each history in $\langle T, <\rangle$.

A fuller presentation and a more detailed discussion of this theory of events can be found in Xu (1997). For similar or related theories of events, the reader is referred to von Kutschera (1993), Belnap (1995, 1996, 2005) and Wölfl (2005).

3 Actions

Our theory of actions aims at a characterization of them in terms of transitions, events and agents. For the purpose of the current paper, we introduce Act as a set of actions in $\langle T, <\rangle$, and content ourselves with the postulates on Act discussed in this section.

Postulate 3.1 *Each member of* Act *is an event.*

By Definition 2.1, an action is a nonempty set of transitions which occurs at most once in every history. Like particular events, particular actions do not occur repeatedly. If I knock on a door three times and we take each knock as a particular action, then

[6] There is another notion of outcome derived from Belnap (1995), which may be of philosophical importance: A *strict* outcome is an outcome O satisfying that for each $m \notin O$, there is an h such that $m \in h$ and $h \cap O = \varnothing$. Such a notion of outcome meets the informal constraint that once we are in the outcome, we will forever remain in it; but while we are not in the outcome, there is always a way to avoid it. For a discussion of this notion and its relation to the notion of free actions, see Xu (2009a).

there are three distinct actions, none of which repeats itself. Of course we may take the three knocks together as one particular action, in which case it is distinct from any prior or subsequent action of knocking n times at the door. To characterize this in terms of transitions, we require an action to be a set of transitions that occurs in each history at most once.

Postulate 3.2 *Each member of* Act *is associated with an agent.*

For each action, there is a performer or doer of the action. Now for each agent $\alpha \in \mathsf{Agent}$, we use Act_α for a set of events, presenting members of Act_α as actions associated with α, and let $\mathsf{Act} = \bigcup_{\alpha \in \mathsf{Agent}} \mathsf{Act}_\alpha$, the set of all actions we want to consider in the given context.

The purpose of associating an action with an agent is of course to pick out the performer of the action, but a mere association is hardly able to tell whether the associated agent is the performer rather than a mere recipient of the action. We will deal with this issue later, and for the moment, we only need a way of presenting such an association in our framework.

A transition $\langle p, O \rangle$ is an *immediate transition* if there is no m such that $p < m < O$, i.e., no moment is between its initial and its outcome. For the purpose of the current paper, an immediate transition is taken to be a transition $\langle p_{(m)}, O \rangle$ (recall that $p_{(m)}$ is a past with the last moment m) with no m' such that $m < m' < O$. This being the case, we may simply use $\langle m, O \rangle$ for the transition $\langle p_{(m)}, O \rangle$, and call m the *initial* of the transition.

Postulate 3.3 *Each member of* Act *is a set of immediate transitions.*

This postulate may be called the *no-part* condition, which amounts to saying that members of Act are *taken* primitive in the sense that in the given context, we are willing to ignore their detailed structures and the part-of relation among them. Note that this does not commit ourselves to the "primitive actions" in the usual sense (see, e.g., Davidson 1971).

For many reasons and in many contexts, we want or need to take certain actions to be "primitive", and take other actions as composed in terms of the "primitive" ones. For example, in a chess game, a player moved his knight from position X to position Y. When studying this game, we may take the player's move an action, and call it e. Surely e may be taken as a composition of a series of actions: taking the knight away from X, holding it and moving the hand through a sequence of spatial positions until over the position Y, and finally putting the knight down at Y and letting it go. But very often, these parts of e are not relevant to a study of the game, and therefore can be ignored in such a study or omitted from it.

To be willing to ignore the details of primitive actions amounts to saying that only the initials and the outcomes of their transitions are "important", and once we ignore such details, we can take primitive actions to be sets of immediate transitions. This means that we also take the time structure (not only details of actions) between initials and outcomes of primitive actions "unimportant" to the current study of actions.

For each moment m, let IT_m be the set of all immediate transitions with the same initial m. For each set e of immediate transitions, *e occurs at m* iff $\mathsf{IT}_m \cap e \neq \varnothing$ (i.e., there is an outcome O such that $\langle m, O\rangle \in e$), and *$e$ occurs at m/h* (*occurs at m in h*) iff there is an outcome O such that $\langle m, O\rangle \in e$ and $O \cap h \neq \varnothing$. We will use H_m^e for the set $\{h \in H_m : e \text{ occurs at } m/h\}$ (which is the same as $\{h \in H_m : e \cap \mathsf{IT}_m \text{ occurs in } h\}$). It is worth noting that to say e occurring at m/h (or occurring at m in h) is more than saying that it occurs at m and that it occurs in h, and likewise H_m^e is not in general the intersection of H_m and $\{h : e \text{ occurs in } h\}$. Here is our next postulate:

Postulate 3.4 *For all distinct actions associated with the same agent, they occur at the same time only if they occur at the time in different histories.*

The postulate above is saying that for all $e, e' \in \mathsf{Act}_\alpha$ with $e \neq e'$, if e and e' both occur at m, then they share no immediate transition $\langle m, O\rangle$, or in other words, that for all $e, e' \in \mathsf{Act}_\alpha$, $e \neq e'$ only if $e \cap e' = \varnothing$.

Like particular events, particular actions are taken to be objective processes, which are independent of any linguistic expressions of them. Suppose that John knocked at my door gently three times, Amy may describe it as John knocked at the door, Bob may describe it as John knocked at the door three times, and Cathy may describe it as John knocked at the door gently. But these are three descriptions of the same action, not of three different actions at the same time.

John presumably could have done something different from what he actually did (e), say he knocked at my door violently three times (e'), but the descriptions by Amy and Bob above still apply. Nevertheless, e is presumably different from e', and therefore each history in which e occurs should be different from all histories in which e' occurs. This is just what Postulate 3.4 is about.

Each postulate above involves only single agents, while our next one, "independence of actions", involves a relation between free actions by different agents at the same time. Before formulating this postulate, let us first consider the "rock-paper-scissers" games. In such a game, two players throw their hand signs (for rock, paper and scissers) at the same time, each such sign ties with the sign of the same kind, beats one of the other two, and loses to the other. The central feature in such a game is that at each time the players throw their hand signs "independently": at each time, no matter what hand sign a player throws, the other player can throw any of the three hand signs.[7] In other words, each of these throwings is considered free in a typical Libertarian sense, according to which an action is free just in case under the same circumstance when the action is performed, the same agent in question has the freedom of not performing the action (see, e.g., Campbell 1957, pp. 162–164).

Our postulate "independence of actions" is a generalization of the consideration above: actions free at the same time are "independent" if associated with different agents. In order to formulate this postulate, we need the following notions. Let m be any moment. e is *free at m* iff $\varnothing \neq \mathsf{IT}_m \cap e \neq \mathsf{IT}_m$, i.e., e occurs at m in some history passing through m but not in others. Let $\mathsf{C}_m^\alpha = \mathsf{IT}_m - \bigcup \mathsf{Act}_\alpha$ for each $m \in T$ and $\alpha \in \mathsf{Agent}$ (the "remaining" of IT_m when all e-transitions with $e \in \mathsf{Act}_\alpha$ are

[7] A close study of such hand-sign throwings in everyday life may show various tendencies (under various circumstances) or probability distributions. The issue at hand, however, concerns not whether one kind of throwings is more likely than the others, nor how much more it is likely than the others, but whether the three kinds of throwings are equally *possible*.

taken out). Provided that $\mathsf{C}_m^\alpha \neq \varnothing$, we call it the (*action*) *complement for* α *at* m. A *selection function at* m is a function f on Agent such that for each $\alpha \in \mathsf{Agent}$, $f(\alpha)$ is either an action in Act_α occurring at m, or the complement for α at m. Here is our postulate:

Postulate 3.5 *[Independence of actions] For each moment m and each selection function f at m,* $(\bigcap_{\alpha\in\mathsf{Agent}} f(\alpha)) \cap \mathsf{IT}_m \neq \varnothing$.

It is easy to show that this postulate is equivalent to that for each m and each selection function f at m, there is a history $h \in \bigcap_{\alpha\in\mathsf{Agent}} H_m^{f(\alpha)}$. As a consequence of independence of actions (and Postulate 3.4), we have the following:

Proposition 3.6 *Each action free at a moment is associated with only one agent.*

Here is an argument. Suppose for reductio that an action e is free at m and is associated with different agents α and β. According to independence of actions, for each selection function f, there is a history h such that both $f(\alpha)$ and $f(\beta)$ occurs at m/h. Let f' be such a function that $f'(\alpha) = e$ (as an action associated with α) and $f'(\beta) =$ either a different action in Act_β occurring at m or the action complement for β at m (taking e to be associated with β). We then obtain a history h' such that both e and $f'(\beta)$ occur at m/h', which is impossible, by Postulate 3.4 and the definition of action complements.

We said earlier that the purpose of associating an action with an agent is to associate the action with its performer rather than a mere recipient, and whether our association does that needs some justification. A characterization of a mere recipient of an action obviously requires a more complex platform than what we have here, but whatever that characterization turns out to be, it seems plausible to require it imply the following:

- A necessary condition for α to be a mere recipient of an action e is that there is a different agent β who is the performer of e (associated with e, using our terminology).

Taking this for granted (or as another postulate), we have the following as an easy consequence of Proposition 3.6:

Proposition 3.7 *For each* $e \in \mathsf{Act}_\alpha$*, if e is free at a moment, then* α *is not a mere recipient of e.*

In Xu (2009a), we present a more detailed discussion of the postulates above, where we also discuss action series and group actions among other things. From now on, we call a sequence $\langle T, <, \mathsf{Agent}, \mathsf{Act}\rangle$ a *stit-action structure* if $\langle T, <\rangle$ is a tree-like frame, Agent is a set of agents, and Act is a set of actions subject to Postulates 3.1–3.5. In some contexts, *stit*-action structures can also be written as $\langle T, <, \mathsf{Agent}, \{\mathsf{Act}_\alpha\}_{\alpha\in\mathsf{Agent}}\rangle$.

4 Actions and Possible Choices

If an action is identified with the set of histories in which it occurs, it may roughly be identified with a possible choice for an agent at a moment. We say “roughly” because there is a difference between them. An action can be identified not only with a single possible choice for an agent at a moment, but also with several such possible choices, as long as they do not overlap. Neverthelss, each *stit*-action structure determines a unique *stit* structure, and each *stit* structure determines a unique *stit*-action structure as well.

Let $\mathfrak{F} = \langle T, <, \mathsf{Agent}, \mathsf{Act}\rangle$ be any *stit*-action structure, and let $\alpha \in \mathsf{Agent}$ and $m \in T$. We define $\mathsf{Choice}^{\alpha}_{m}$ as follows. If m is a $<$-dead-end, we let $\mathsf{Choice}^{\alpha}_{m} = \{H_m\}$. Suppose that m is not a $<$-dead-end. We know that each immediate transition $\langle m, O\rangle$ is either an e-transition for some $e \in \mathsf{Act}_\alpha$, or contained in the action complement for α at m. We know by Postulate 3.4 that for all $e, e' \in \mathsf{Act}_\alpha$, and for all $\langle m, O\rangle \in e$ and $\langle m, O'\rangle \in e'$, if $e \neq e'$ then $O \cap O' = \varnothing$. Letting $\mathbb{C}$ be $\{e \cap \mathsf{IT}_m : e \in \mathsf{Act}_\alpha \wedge e \cap \mathsf{IT}_m \neq \varnothing\}$, it is then easy to see that IT_m is partitioned by $\mathbb{C}$ if $\mathsf{C}^{\alpha}_{m} = \varnothing$ (recall that $\mathsf{C}^{\alpha}_{m} = \mathsf{IT}_m - \bigcup \mathsf{Act}_\alpha$), and that IT_m is partitioned by $\mathbb{C} \cup \{\mathsf{C}^{\alpha}_{m}\}$ if $\mathsf{C}^{\alpha}_{m} \neq \varnothing$ (if the action complement for α at m exists). It follows that H_m is partitioned by $\{H^e_m : (e \in \mathsf{Act}_\alpha \wedge e \cap \mathsf{IT}_m \neq \varnothing) \vee e = \mathsf{C}^{\alpha}_{m} \neq \varnothing\}$, and hence we can define $\mathsf{Choice}^{\alpha}_{m}$ to be this partition. Note that the condition of no choice between undivided histories is automatically satisfied due to a common feature of outcomes ($O \cap O' \neq \varnothing$ only if either $O \subseteq O'$ or $O' \subseteq O$). It is also easy to see that when $\mathsf{Choice}^{\beta}_{m}$ is thus defined for all $\beta \in \mathsf{Agent}$, they satisfy the condition of independence of agents, due to our postulate of independence of actions.

To sum up, for each *stit*-action structure $\mathfrak{F} = \langle T, <, \mathsf{Agent}, \mathsf{Act}\rangle$, we call $\mathsf{Choice} = \{\langle\langle \alpha, m\rangle, \mathsf{Choice}^{\alpha}_{m}\rangle : m \in T \wedge \alpha \in \mathsf{Agent}\}$ the choice function *determined by* Act, and call $\langle T, <, \mathsf{Agent}, \mathsf{Choice}\rangle$ the *stit* structure *determined by* $\mathfrak{F}$, where for each $\alpha \in \mathsf{Agent}$ and each $m \in T$,

$$\mathsf{Choice}^{\alpha}_{m} = \begin{cases} \{H_m\} & \text{if } m \text{ is a } <\text{-dead-end,} \\ \{H^e_m : e \cap \mathsf{IT}_m \neq \varnothing \wedge (e \in \mathsf{Act}_\alpha \vee e = \mathsf{C}^{\alpha}_{m})\} & \text{o.w.} \end{cases}$$

It is easy to verify the following:

Proposition 4.1 *Let* $\langle T, <, \mathsf{Agent}, \mathsf{Act}\rangle$ *be any stit-action structure, let* $\langle T, <, \mathsf{Agent}, \mathsf{Choice}\rangle$ *be the stit structure determined by* $\langle T, <, \mathsf{Agent}, \mathsf{Act}\rangle$*, and let* $m \in T$ *and* $e \in \mathsf{Act}_\alpha$ *with* $\alpha \in \mathsf{Agent}$. *Then for all* h *in* $\langle T, <\rangle$, $h \in H^e_m$ *only if* $\mathsf{Choice}^{\alpha}_{m}(h) = H^e_m$.

It is easier to turn a *stit* structure into a *stit*-action structure than the other way around—we only need to turn each possible choice H for α at m into an action by α that occurs exclusively at m/h with $h \in H$.

Let $\mathfrak{F} = \langle T, <, \mathsf{Agent}, \mathsf{Choice}\rangle$ be any *stit* structure. For each $m \in T$ that is not a $<$-dead-end, and for each $H \subseteq H_m$, we use $e_{m,H}$ for $\{\langle m, O\rangle \in \mathsf{IT}_m : \exists h \in H(O \cap h \neq \varnothing)\}$, i.e., the set of immediate transitions which occurs exclusively at m/h with $h \in H$. For each $\alpha \in \mathsf{Agent}$, let

$$\mathsf{Act}_\alpha = \{e_{m,H} : m \in T \wedge H \in \mathsf{Choice}^{\alpha}_{m} \wedge m \text{ is not a } <\text{-dead-end}\}.$$

We call $\mathsf{Act} = \bigcup_{\alpha \in \mathsf{Agent}} \mathsf{Act}_\alpha$ the set of actions *determined by* Choice, and call $\langle T, <, \mathsf{Agent}, \mathsf{Act}\rangle$ the *stit*-action structure *determined by* $\mathfrak{F}$. It is easy to verify that

Act defined above satisfies all postulates discussed in the last section, by way of no choice between undivided choices and independence of agents.

For each *stit*-action structure $\langle T, <, \mathsf{Agent}, \mathsf{Act}\rangle$, each $e \in \mathsf{Act}$ and each $m \in T$, if $e \cap \mathsf{IT}_m \neq \varnothing$ (i.e., e occurs at m), let us call it e-at-m. We know from the above discussion that in the *stit*-action structure determined by a *stit* structure, each action e occurring at m is the same as e-at-m, but this may not be the case for *stit*-action structures in general. By the same token, in such a structure, each action e in Act_α occurring at m corresponds to a unique $H \in \mathsf{Choice}^\alpha_m$, where e occurs at m/h with $h \in H$, but this may not be the case for *stit* -action structures in general. In other words, actions in *stit*-action structures in general may occur at different moments (though not in the same history), while each action determined by the choice function in a *stit* structure occurs at a unique moment.[8]

5 Semantics for *Stit* with Actions

In this section, we extend the semantics for *stit* formulas to "*stit*-action formulas", where a *stit-action formula* is a formula of the form $[\alpha, e]A$ or $[\alpha, \pi]A$, with α to be an agent, e an action, π a set of actions, and A any formula.[9] Such formulas may take either the *cstit* version $[\alpha, e]^{\mathsf{c}}A$ (or $[\alpha, \pi]^{\mathsf{c}}A$) or the *dstit* version $[\alpha, e]^{\mathsf{d}}A$ (or $[\alpha, \pi]^{\mathsf{d}}A$).

Let us first consider formulas of the form $[\alpha, e]A$, and start with the *cstit* version. Let $\mathfrak{F} = \left\langle T, <, \mathsf{Agent}, \{\mathsf{Act}_\alpha\}_{\alpha\in\mathsf{Agent}}\right\rangle$ be any *stit*-action structure. A model on $\mathfrak{F}$ is a pair $\mathfrak{M} = \langle \mathfrak{F}, V\rangle$ where V is a valuation as before. Intuitively, $[\alpha, e]A$ is interpreted as that A is guaranteed true by α's current action e, and our truth definition of $[\alpha, e]^{\mathsf{c}}A$ is as follows, where $\mathfrak{M} = \langle \mathfrak{F}, V\rangle$ and $m \in h$:

$$\begin{aligned} \mathfrak{M}, m/h \vDash [\alpha, e]^{\mathsf{c}}A \quad \text{iff} \quad & e \in Act_\alpha \text{and } h \in H^e_m, \text{ and} \\ & \mathfrak{M}, m/h' \vDash A \quad \text{for all} \quad h' \in H^e_m. \end{aligned} \tag{2}$$

Original *stit* formulas $[\alpha]^{\mathsf{c}}A$ (and $[\alpha]^{\mathsf{d}}A$ below) are interpreted in the original way, except the choice function Choice is now determined by Act. It is easy to see from (2) that $\mathfrak{M}, m/h \vDash [\alpha, e]^{\mathsf{c}}\top$ iff $e \in \mathsf{Act}_\alpha$ and $h \in H^e_m$, and then by (2) and Proposition 4.1, the following are equivalent:

$$\begin{aligned} &\mathfrak{M}, m/h \vDash [\alpha, e]^{\mathsf{c}}A. \\ &e \in Act_\alpha \quad \text{and} \quad h \in H^e_m \quad \text{and} \quad \mathfrak{M}, m/h \vDash [\alpha]^{\mathsf{c}}A. \\ &\mathfrak{M}, m/h \vDash [\alpha, e]^{\mathsf{c}}\top \wedge [\alpha]^{\mathsf{c}}A. \end{aligned} \tag{3}$$

[8] A similar theory of actions is presented in Brown (2008a,b), which is also based on events and agents. A main difference between that theory and ours is that actions in Brown's theory are in general like e-at-m above, each occurring at a unique moment and definable in terms of a possible choice. A comparison between Brown's theory and ours would be interesting, but I have to leave it to another occasion.

[9] For convenience, we use e, e' etc. ambiguously both as actions and as terms for actions; likewise, we use π, π' etc. both as sets of actions and as terms for these sets.

Here is our truth definition of $[\alpha, e]^{\mathsf{d}} A$:

$$\mathfrak{M}, m/h \vDash [\alpha, e]^{\mathsf{d}} A \quad \text{iff} \quad e \in Act_\alpha \quad \text{and} \quad h \in H_m^e, \mathfrak{M}, m/h' \vDash A \\ \text{for all} \quad h' \in H_m^e, \quad \text{and} \quad \mathfrak{M}, m/h'' \nvDash A \\ \text{for some} \quad h'' \in H_m. \tag{4}$$

Other semantic notions are defined as usual. It is easy to verify by (2), (3) and (4) that the following are equivalent:

$$\begin{aligned} &\mathfrak{M}, m/h \vDash [\alpha, e]^{\mathsf{d}} A. \\ &\mathfrak{M}, m/h \vDash [\alpha, e]^{\mathsf{c}} A \wedge \neg\Box A. \\ &\mathfrak{M}, m/h \vDash [\alpha, e]^{\mathsf{c}} \top \wedge [\alpha]^{\mathsf{c}} A \wedge \neg\Box A. \\ &\mathfrak{M}, m/h \vDash [\alpha, e]^{\mathsf{c}} \top \wedge [\alpha]^{\mathsf{d}} A. \end{aligned} \tag{5}$$

Let $[\alpha, e]^* A$ be $[\alpha, e]^{\mathsf{c}} A \wedge \neg\Box[\alpha, e]^{\mathsf{c}} A$. It is easy to verify that $[\alpha, e]^* \top$ says not only that $e \in \mathsf{Act}_\alpha$ and e occurs at the current moment in the current history, but that e is free at the current moment. It is also easy to verify that $[\alpha, e]^{\mathsf{d}} A \leftrightarrow [\alpha, e]^* \top \wedge [\alpha]^{\mathsf{d}} A$ is valid.

Applying (5), we know that the following is valid:

$$[\alpha, e]^{\mathsf{d}} A \leftrightarrow [\alpha, e]^{\mathsf{c}} A \wedge \neg\Box A \tag{6}$$

which means that at the presence of $\Box$ and $[\alpha, e]^{\mathsf{c}}$, $[\alpha, e]^{\mathsf{d}}$ is definable. Furthermore, it is easy to see from the above that the following are valid:

$$\begin{aligned} &[\alpha, e]^{\mathsf{c}} A \leftrightarrow [\alpha, e]^{\mathsf{c}} \top \wedge ([\alpha]^{\mathsf{d}} A \vee \Box A) \\ &[\alpha, e]^{\mathsf{d}} A \leftrightarrow [\alpha, e]^{\mathsf{c}} \top \wedge ([\alpha]^{\mathsf{c}} A \wedge \neg\Box A) \end{aligned} \tag{7}$$

which means that at the presence of any two of $\Box$, $[\alpha]^{\mathsf{c}}$ and $[\alpha]^{\mathsf{d}}$, as well as $[\alpha, e]^{\mathsf{c}} \top$ (taking as a "constant" saying "α's current action is e"), $[\alpha, e]^{\mathsf{c}}$ and $[\alpha, e]^{\mathsf{d}}$ are all definable.

Next we turn to formulas of the form $[\alpha, \pi]A$, where π is a set of actions. In some applications, it seems more useful to talk about types of actions than particular actions themselves. What then constitute a type of actions? I do not know the answer to this question, but if a type of actions is a set of actions "similar to each other" in some sense, we may deal with sets of actions for now, and leave the rest for later studies, and this is what we do here.

Let $\mathfrak{F} = \langle T, <, \mathsf{Agent}, \mathsf{Act}, \mathsf{ASet}\rangle$ be a *stit*-action structure equipped with a nonempty set ASet, whose members π, π' etc. are nonempty sets of actions. We will call $\mathfrak{F}$ a *stit*-action structure *with types*. A valuation V on $\mathfrak{F}$ is as before. Intuitively, $[\alpha, \pi]A$ is interpreted as that A is guaranteed true by α's current action of type π, and the truth definition of $[\alpha, \pi]^{\mathsf{c}} A$ is as follows, where $\mathfrak{M} = \langle \mathfrak{F}, V\rangle$ and $m \in h$:

$$\mathfrak{M}, m/h \vDash [\alpha, \pi]^{\mathsf{c}} A \quad \text{iff} \quad \text{there is an} \quad e \in Act_\alpha \cap \pi \quad \text{such that} \quad h \in H_m^e, \quad \text{and} \quad \mathfrak{M}, m/h' \vDash A \quad \text{for all} \quad h' \in H_m^e. \tag{8}$$

Here is our truth definition of $[\alpha, \pi]^{\mathsf{d}} A$:

$$\mathfrak{M}, m/h \vDash [\alpha, \pi]^{\mathsf{d}} A \quad \text{iff} \quad \text{there is an} \quad e \in Act_\alpha \cap \pi \quad \text{such that} \quad h \in H_m^e, \mathfrak{M}, m/h' \vDash A \quad \text{for all} \quad h' \in H_m^e, \quad \text{and} \quad \mathfrak{M}, m/h'' \nvDash A \quad \text{for an} \quad h'' \in H_m. \tag{9}$$

Other semantic notions are defined as usual. From (8) it is easy to see that $\mathfrak{M}, m/h \vDash [\alpha, \pi]^{\mathsf{c}}\top$ iff there is an $e \in \mathsf{Act}_\alpha \cap \pi$ such that $h \in H_m^e$, and then by Proposition 4.1, the following are equivalent:

$$\begin{aligned}
&\mathfrak{M}, m/h \vDash [\alpha, \pi]^{\mathsf{c}} A.\\
&\text{there is an} \quad e \in \mathsf{Act}_\alpha \cap \pi \quad \text{such that} \quad h \in H_m^e \quad \text{and} \quad \mathfrak{M}, m/h \vDash [\alpha]^{\mathsf{c}} A.\\
&\mathfrak{M}, m/h \vDash [\alpha, \pi]^{\mathsf{c}}\top \wedge [\alpha]^{\mathsf{c}} A.
\end{aligned} \tag{10}$$

It is then easy to verify by (8), (9) and (10) that even with actions replaced by sets of actions, items displayed in (5) are still equivalent, and (6) and (7) still hold.

6 Two Axiomatic Systems of *Stit*-action Logic Based on Metric Tense Logic

We present here two basic axiomatic systems for *stit*-action logic based on metric tense logic for discrete tree-like frames, one of which takes "actions" to mean particular actions, and the other sets of particular actions.

A *discrete stit-action structure* (*with types*) is a *stit*-action structure (with types) $\langle T, <, \mathsf{Agent}, \mathsf{Act}\rangle$ ($\langle T, <, \mathsf{Agent}, \mathsf{Act}, \mathsf{ASet}\rangle$) with $\langle T, <\rangle$ to be a *discrete* tree-like frame, i.e., a tree-like frame in which for each history h and each $m \in h$, the following hold:

- if m is not a $<$-dead-end, there is a unique $m^* \in h$ such that $m < m^*$ and $m < m' < m^*$ for no m' (we will use m_h^+ for this m^*); and
- if m is not a $>$-dead-end, there is a unique m^* such that $m^* < m$ and $m^* < m' < m$ for no m' (we will use m^- for this m^*).

A model on such a structure (with types) $\mathfrak{F}$ is a pair $\mathfrak{M} = \langle \mathfrak{F}, V\rangle$, where V is a valuation as before.

Given a discrete *stit*-action structure $\langle T, <, \mathsf{Agent}, \mathsf{Act}\rangle$, our new language is generated as follows, where $\alpha, \beta \in \mathsf{Agent}$ and $e, e' \in \mathsf{Act}$:

$$q \mid \alpha = \beta \mid e = e' \mid \neg A \mid A \wedge B \mid FA \mid PA \mid \Box A \mid [\alpha]A \mid [\alpha, e]A$$

The connectives F ("next") and P ("last") are metric tense operators, and should not be confused with the ordinary "will-be" and "was" operators. When given a discrete *stit*-action structure with types $\langle T, <, \mathsf{Agent}, \mathsf{Act}, \mathsf{ASet}\rangle$, our language is also generated by the above, except that we replace e, e' by π, π', and Act by ASet. From now on, we use $[\alpha]$ for $[\alpha]^{\mathsf{c}}$, and $[\alpha, e]$ and $[\alpha, \pi]$ for $[\alpha, e]^{\mathsf{c}}$ and $[\alpha, \pi]^{\mathsf{c}}$.

The satisfaction relation $\mathfrak{M}, m/h \vDash A$ $(m \in h)$ is defined as usual, plus (1), (2), (8) and the following:

$$\begin{aligned} &\mathfrak{M}, m/h \vDash FA \quad \text{iff} \quad m \quad \text{is not a} \quad <\text{-dead end and} \quad \mathfrak{M}, m_h^+/h \vDash A. \\ &\mathfrak{M}, m/h \vDash PA \quad \text{iff} \quad m \quad \text{is not a} \quad >\text{-dead end and} \quad \mathfrak{M}, m^-/h \vDash A. \end{aligned} \tag{11}$$

Our first system for *stit*-action logic takes *modus ponens* and the following as rules of inference:

RM infer $FA \to FB$ and $PA \to PB$ from $A \to B$
RN$_\Box$ infer $\Box A$ from A

and takes as axioms (axiom schemes) all substitution instances of truth-functional tautologies and the following:

A1 $FA \wedge FB \to F(A \wedge B)$, $PA \wedge PB \to P(A \wedge B)$
A2 $F(A \vee B) \to FA \vee FB$, $P(A \vee B) \to PA \vee PB$
A3 $A \wedge FB \leftrightarrow F(B \wedge PA)$, $A \wedge PB \leftrightarrow P(B \wedge FA)$
A4 S5 axioms for $\Box$ and for each $[\alpha]$
A5 $P[\alpha]A \to \Box PA$[10]
A6 $A \wedge \neg F\top \to \Box A$
A7 $\Box A \to [\alpha]A$
A8 $\alpha = \alpha, \alpha = \beta \to (A \to A(\alpha/\beta))$
A9 $(\bigwedge_{0 \leqslant i \neq j \leqslant n} \neg(\beta_i = \beta_j)) \wedge (\bigwedge_{k \leqslant n} \Diamond[\beta_k]A_k) \to \Diamond(\bigwedge_{k \leqslant n} A_k)$ $(n > 0)$[11]
A10 $e = e, e = e' \to (A \to A(e/e'))$
A11 $[\alpha, e]A \leftrightarrow [\alpha](A \wedge [\alpha, e]\top)$
A12 $[\alpha, e]A \to \Box([\alpha, e]\top \to [\alpha, e]A)$
A13 $[\alpha, e]\top \to F\top$
A14 $P^k \Diamond F^n([\alpha, e]\top \wedge \neg\Box[\alpha, e]\top) \to ([\beta, e]\top \to \alpha = \beta)$
A15 $P^n[\alpha, e]\top \to \neg[\beta, e]\top$ $(n > 0)$
A16 $[\alpha, e]\top \wedge [\alpha, e']\top \to e = e'$

Our second system for the basic *stit*-action logic takes "actions" to mean sets of particular actions, and thus we use π, π' etc. in the language instead of e, e' etc. This system takes as rules of inference *modus ponens*, RM and RN$_\Box$, and takes as axioms all substitution instances of truth-functional tautologies, A1 –A9 and the following:

[10] This corresponds to the condition of no choice between undivided histories, which renders the usual axiom $P\Box\phi \to \Box P\phi$ provable.

[11] This can be replaced by $\mathrm{diff}(\alpha, \beta_0, \ldots, \beta_n) \wedge [\alpha](\bigvee_{k \leqslant n}[\beta_k]A) \to \Box A$, as shown in Balbiani et al. (2008), and when using $[\alpha]$ and $[\beta]$ to define $\Box$ (with the assumption of different agent terms for different agents), the S5 axioms for $\Box$ becomes redundant.

A10′ $\pi = \pi, \pi = \pi' \to (A \to A(\pi/\pi'))$
A11′ $[\alpha, \pi]A \leftrightarrow [\alpha](A \wedge [\alpha, \pi]\top)$
A12′ $[\alpha, \pi]\top \to F\top$

A completeness result can be found in Xu (2009b), which is based on the completeness proof in Xu (1994) for *dstit* logic and a straightforward completeness proof for the metric tense logic on discrete tree-like frames.

7 *Stit*-action Logic Versus Branching ETL

In this section we compare *stit*-action logic (with types) and a version of branching epistemic temporal logic (ETL), which can be found in Chap. 9 of van Benthem (2008). The reason why we pick up this version for comparison is that in this version, formulas are evaluated at state/history pairs rather than at states alone, which provides a common ground for the comparison.

There are at least two ways for our comparison to proceed: one way is to extend *stit*-action logic to deals with epistemic operators as ETL does, while the other way is to ignore the epistemic operators in ETL. We want to take the second way, not only because it is simpler,[12] but also because in my view it zooms in and gives us a close-up view at the main connection and the main difference between the two theories.

From now on, we call the fragment of ETL without epistemic operators a *temporal event logic*, or simply TEL. Its language is generated by the following, with $\mathbb{E}$ to be a set of "events":

$$q \mid \neg A \mid A \wedge B \mid F_e A \mid P_e A \quad \text{where } e \in \mathbb{E}$$

In our discussions below, all tree-like frames are discrete. In such a frame $\langle T, <\rangle$, each outcome O is $O_m = \{m' \in T : m \leqslant m'\}$ for a unique $m \in T$, and then each immediate transition is $\langle m, O_{m'}\rangle$ for some $m, m' \in T$ with $m <^1 m'$, i.e., $m < m'$ and $m < u < m'$ for no $u \in T$. Thus we will use $\langle m, m'\rangle$ for $\langle m, O_{m'}\rangle$ for convenience. This being the case, we have that

$$h \in H_m^e \quad \text{iff} \quad \langle m, m_h^+\rangle \in e \quad \text{iff} \quad m' \in h \quad \text{for some} \quad \langle m, m'\rangle \in e. \tag{12}$$

An TEL model is a sequence $\mathfrak{M} = \langle \mathbb{E}, \mathbb{S}, V\rangle$ defined as follows. $\mathbb{E}$ is a nonempty set of *events* (or event types), and $\mathbb{S}$ is a nonempty set of *states*, which are nonempty finite sequences of events closed under "prefixes". We use s, s' etc. to range over nonempty (finite or infinite) sequences of events. For all $s = \langle e_i\rangle_{i \leqslant \eta}$ and $s' = \langle e'_i\rangle_{i \leqslant \eta'}$ with $\eta, \eta' \leqslant \omega$, s is a *prefix* of s' (written $s \preceq s'$) iff $\eta \leqslant \eta'$ and $e_i = e'_i$ for all $i \leqslant \eta$. We will use $s * \langle e\rangle$ for $\langle e_0, \ldots, e_k, e\rangle$ when $s = \langle e_0, \ldots, e_k\rangle$, and use $s \prec s'$ for $s \preceq s'$ but $s \neq s'$. Maximal (possibly infinite) $\preceq$-chains in $\mathbb{S}$ constitute *histories* in $\mathbb{S}$ (which we also use h, h' to range over), but for convenience we often use $\langle e_0, e_1, \ldots\rangle$ instead of $\{s_0, s_1, \ldots\}$ for a history in $\mathbb{S}$ if $s_k = \langle e_0, \ldots, e_k\rangle$ for all $k \geqslant 0$. When $s \preceq h$ with

[12] Thanks to John Horty, who pointed out to me a certain complication resulted from a combination of *stit* and epistemic operators. See, e.g., Broersen (2008b).

$s \in \mathbb{S}$ and h to be a history in $\mathbb{S}$, we also say that h *passes through* s. Finally, V is a valuation assigning to each propositional letter q a subset $V(q)$ of $\{\langle s, h\rangle : h$ passes through $s\}$. The satisfaction relation $\mathfrak{M}, s, h \vDash A$ is defined as follows, where h passes through s:

$$\begin{array}{rcl}
\mathfrak{M}, s, h \vDash q & \text{iff} & \langle s, h\rangle \in V(q); \\
\mathfrak{M}, s, h \vDash \neg A & \text{iff} & \mathfrak{M}, s, h \nvDash A; \\
\mathfrak{M}, s, h \vDash A \wedge B & \text{iff} & \mathfrak{M}, s, h \vDash A \quad \text{and} \quad \mathfrak{M}, s, h \vDash B; \\
\mathfrak{M}, s, h \vDash F_e A & \text{iff} & (s * \langle e\rangle) \preceq h \quad \text{and} \quad \mathfrak{M}, s * \langle e\rangle, h \vDash A; \\
\mathfrak{M}, s, h \vDash P_e A & \text{iff} & s = s' * \langle e\rangle \quad \text{and} \quad \mathfrak{M}, s', h \vDash A;
\end{array}$$

Not only will ETL be restricted in our comparison, so will be the *stit*-action logic. There are a few reasons for this. First of all, ETL currently lacks a mechanism connecting agents and events, and therefore the connection in *stit*-action logic between agents and actions has no correspondent in ETL. Secondly, ETL does not have a necessity-like version of F_e, which makes the *stit*-action operators in *stit*-action logic appear too strong to find a match in ETL. Finally, there is a certain kind of situations that *stit* theories can express while ETL (and DEL) cannot, which makes an unconditional ETL representation of *stit* very difficult, if not impossible. The kind of situations is like this: an agent may have different, but the same type of, alternatives at a single moment, which may even have incompatible consequences. For example, at a moment Amy can help either Bob or Cathy, but not both; if she helps Cathy, both Bob and Cathy will be happy, and if she helps Bob, not both of them will be happy. Treating the two alternatives to be of the same type (say "helping someone"), ETL (and DEL) would not allow them both to occur at the same state.

In accordance with these considerations, we restrict our *stit*-action logic to a fragment of it, with a single agent α and with $\top$ alone to be an argument to $[\alpha, \pi]$, and call this fragment a *restricted stit-action logic* (RSL). To be precise, RSL models are discrete *stit*-action models (with types) $\langle T, <, \{\alpha\}, \mathsf{Act}, \mathsf{ASet}, V\rangle$ (or simply $\langle T, <, \mathsf{Act}, \mathsf{ASet}, V\rangle$) with Act to be the same as Act_α, and RSL language is generated by the following, with α to be the only agent:

$$q \mid \neg A \mid A \wedge B \mid [\alpha, \pi]\top \mid FA \mid PA \qquad \pi \in \mathsf{ASet}$$

In the remaining of this section, we present two way representations between TEL and RSL. Let us start with the easy direction.

Given any TEL model $\mathfrak{M} = \langle \mathbb{E}, \mathbb{S}, V\rangle$, we define the "RSL transformation" of $\mathfrak{M}$ as follows. For all s and e such that $(s * \langle e\rangle) \in \mathbb{S}$, let $\varepsilon_{s*\langle e\rangle} = \{\langle s, s * \langle e\rangle\rangle\}$, and let $\mathsf{Act} = \mathsf{Act}_\alpha = \{\varepsilon_{s*\langle e\rangle} : (s * \langle e\rangle) \in \mathbb{S}\}$. For each $e \in \mathbb{E}$ such that $(s * \langle e\rangle) \in \mathbb{S}$ for some $s \in \mathbb{S}$, let $\pi_e = \{\varepsilon_{s*\langle e\rangle} : (s * \langle e\rangle) \in \mathbb{S}\}$. Finally let $\mathsf{ASet} = \{\pi_e : e \in \mathbb{E} \wedge \exists s((s * \langle e\rangle) \in \mathbb{S})\}$.

Let $\mathfrak{M}^\wedge = \langle \mathbb{S}, \prec, \mathsf{Act}, \mathsf{ASet}, V\rangle$ as defined above, with $\prec$ to be restricted to $\mathbb{S}$. We call $\mathfrak{M}^\wedge$ the RSL *transformation of* $\mathfrak{M}$. It is easy to see that $\langle \mathbb{S}, \prec\rangle$ is a tree-like frame, that each $\langle s, s * \langle e\rangle\rangle$ is an immediate transition in $\langle \mathbb{S}, \prec\rangle$, and that each $\varepsilon_{s*\langle e\rangle}$ is an action in $\langle \mathbb{S}, \prec\rangle$. It is also easy to verify that Act satisfies all postulates given in

Sect. 3, and that ASet is a nonempty set whose members are nonempty sets of actions. It follows that $\mathfrak{M}^\wedge$ is an RSL model. Note that for each s and e with $s * \langle e \rangle \in \mathbb{S}$, it is always true that $\varepsilon_{s*\langle e \rangle} \in \pi_e = \mathsf{Act}_\alpha \cap \pi_e$, and hence for each h passing through s, we have the following by (12) and truth definition (cf. the discussion following 9):

$$s * \langle e \rangle \in h \quad \text{iff} \quad h \in H_s^{\varepsilon_{s*\langle e \rangle}} \quad \text{iff} \quad \mathfrak{M}^\wedge, s/h \vDash [\alpha, \pi_e]\top. \tag{13}$$

For all TEL formulas A, we define $A^\wedge$ as follows:

$$\begin{aligned} q^\wedge &\triangleq q \\ (\neg A)^\wedge &\triangleq \neg(A^\wedge) \\ (A \wedge B)^\wedge &\triangleq A^\wedge \wedge B^\wedge \\ (F_e A)^\wedge &\triangleq [\alpha, \pi_e]\top \wedge F A^\wedge \\ (P_e A)^\wedge &\triangleq P([\alpha, \pi_e]\top \wedge A^\wedge) \end{aligned}$$

The following is our first representation theorem.

Theorem 7.1 *For each TEL model $\mathfrak{M} = \langle \mathbb{E}, \mathbb{S}, V \rangle$, each TEL formula A, and each $s \in \mathbb{S}$ and h in $\mathbb{S}$ passing through s, $\mathfrak{M}, s, h \vDash A$ iff $\mathfrak{M}^\wedge, s/h \vDash A^\wedge$.*

Proof We only show the case for F_e. Let $A = F_e B$. Then $A^\wedge = [\alpha, \pi_e]\top \wedge F B^\wedge$. Suppose that $\mathfrak{M}, s, h \vDash F_e B$. Then $s * \langle e \rangle \preceq h$ and $\mathfrak{M}, s * \langle e \rangle, h \vDash B$. By induction hypothesis, $\mathfrak{M}^\wedge, (s * \langle e \rangle)/h \vDash B^\wedge$. Because $s * \langle e \rangle \preceq h$, it follows that $s \prec^1 s * \langle e \rangle$ and $s * \langle e \rangle \in h$, and hence by (13), $\mathfrak{M}^\wedge, s/h \vDash [\alpha, \pi_e]\top \wedge F B^\wedge$, i.e., $\mathfrak{M}^\wedge, s/h \vDash (F_e B)^\wedge$. Suppose that $\mathfrak{M}^\wedge, s/h \vDash (F_e B)^\wedge$, i.e., $\mathfrak{M}^\wedge, s/h \vDash [\alpha, \pi_e]\top \wedge F B^\wedge$. Then by (13), $s * \langle e \rangle \in h$ (and hence s is not a $\prec$-dead-end) and $\mathfrak{M}^\wedge, s_h^+/h \vDash B^\wedge$. It follows that $s * \langle e \rangle \preceq h$ and $s_h^+ = s * \langle e \rangle$, and thus $\mathfrak{M}^\wedge, (s * \langle e \rangle)/h \vDash B^\wedge$, and hence by induction hypothesis, $s * \langle e \rangle \preceq h$ and $\mathfrak{M}, s * \langle e \rangle, h \vDash B$, i.e., $\mathfrak{M}, s, h \vDash F_e B$. □

Let $\mathfrak{M} = \langle T, <, \mathsf{Act}, \mathsf{ASet}, V \rangle$ be any rooted RSL model, i.e., an RSL model where each history h is either finite or of order type ω, and let $p = \{m_0, m_1, \ldots, m_n\}$ and $p' = \{u_0, u_1, \ldots, u_k\}$ be any pasts in $\langle T, < \rangle$. $p \approx p'$ iff $m_0 = u_0$ and $k = n$, and for each $i < k$, there is a $\pi \in \mathsf{ASet}$ such that $\langle m_i, m_{i+1} \rangle, \langle u_i, u_{i+1} \rangle \in \bigcup \pi$ (i.e., $\langle m_i, m_{i+1} \rangle \in e_1 \in \pi$ and $\langle u_i, u_{i+1} \rangle \in e_2 \in \pi$ for some e_1 and e_2). Intuitively, $p \approx p'$ means that p and p' have the same beginning, are of the same "length", and at each pair of corresponding "stages", the same kind of actions occur. Let h and h' be histories in $\langle T, < \rangle$. $h \trianglelefteq_{\mathfrak{M}} h'$ iff either of the following holds:

- there are $m \in T$ and $u \in h'$ such that $h = p_{(m)} \approx p_{(u)}$, or
- $h = \{m_0, m_1, \ldots\}$ and $h' = \{u_0, u_1, \ldots\}$ and $p_{(m_i)} \approx p_{(u_i)}$ for all $i \geqslant 0$.

It is easy to verify that the RSL transformation $\mathfrak{M} = \langle T, <, \mathsf{Act}, \mathsf{ASet}, V \rangle$ of a TEL model satisfies the following:

C1 Each immediate transition is a member of an action (i.e., no action complements).
C2 ASet is a partition of Act.
C3 Each history h is either finite or of order type ω.

C4 $h \trianglelefteq_{\mathfrak{M}} h'$ only if $h = h'$ for all histories h and h'.
C5 For each $\pi \in \mathsf{ASet}$ and $m \in T$, at most one action in π occurs at m.
C6 Each action is a singleton of an immediate transition.

Now we work toward the "representation theorem" of the other direction. Note that this "representation" is not unconditional because we will start with RSL models satisfying conditions C1–C4 above.

Let $\mathfrak{M} = \langle T, <, \mathsf{Act}, \mathsf{ASet}, V\rangle$ be any RSL model satisfying C1–C4. We need to transform $\mathfrak{M}$ into an RSL model $\mathfrak{t}(\mathfrak{M})$ to satisfy all C1–C6.

Let $m, m' \in T$. $m \approx m'$ iff $p_{(m)} \approx p_{(m')}$. Because of C2, $\approx$ is an equivalence relation. We then use $\mathfrak{t}(m)$, or simply $[m]$, for the $\approx$-equivalence class to which m belongs, and use $\mathfrak{t}(T)$ for $\{\mathfrak{t}(m) : m \in T\}$. Let $\mathfrak{t}(<)$ (also written as $\lhd$) be the following relation on $\mathfrak{t}(T)$: Let $[m] \lhd^0 [m']$ iff $[m] = [m']$, and for each $k \geqslant 0$, $[m] \lhd^{k+1} [m']$ iff there is an m_0 such that $[m] \lhd^k [m_0]$ and $u <^1 u'$ for some $u \in [m_0]$ and $u' \in [m']$. Finally, let $[m] \lhd [m']$ iff $[m] \lhd^k [m']$ for some $k > 0$. It is easy to verify that $[m] \lhd [m']$ iff for each $u' \in [m']$, there is a $u \in [m]$ such that $u < u'$, from which it follows that $\langle \mathfrak{t}(T), \mathfrak{t}(<)\rangle$ is a tree-like frame.

For each $h = \{m_0, m_1, \ldots\}$ in $\langle T, <\rangle$, let $\mathfrak{t}(h) = \{[m_0], [m_1], \ldots\}$. It is easy to see that

$$\text{for each} \quad m \in h, [m_h^+] = [m]^+_{\mathfrak{t}(h)}. \tag{14}$$

Obviously $\mathfrak{t}(h)$ is a history in $\langle \mathfrak{t}(T), \mathfrak{t}(<)\rangle$ whenever h is a history in $\langle T, <\rangle$. Note that there might be histories in $\langle \mathfrak{t}(T), \mathfrak{t}(<)\rangle$ that is not $\mathfrak{t}(h)$ for any h in $\langle T, <\rangle$. Fortunately, RSL formulas do not include those of the form $\Box A$, $[\alpha, \pi]A$ (except $[\alpha, \pi]\top$) or $[\alpha]A$, which is why we can still get through our proof below.

Let $\mathfrak{t}(\mathsf{Act}) = \{\{\langle [m], [m']\rangle\} : m <^1 m'\}$. Clearly, each member of $\mathfrak{t}(\mathsf{Act})$ is a singleton of an immediate transition in $\langle \mathfrak{t}(T), \mathfrak{t}(<)\rangle$. For each immediate transition $\langle m, m'\rangle$ in $\langle T, <\rangle$, we know by C1–C2 that $\langle m, m'\rangle \in \bigcup \pi$ for a unique $\pi \in \mathsf{ASet}$, and if $u <^1 u'$ with $u \in [m]$ and $u' \in [m']$, it must be the case that $\langle u, u'\rangle \in \bigcup \pi$. Now for each $\pi \in \mathsf{ASet}$, let $\mathfrak{t}(\pi) = \{\{\langle [m], [m']\rangle\} : m <^1 m' \wedge \langle m, m'\rangle \in \bigcup \pi\}$, and let $\mathfrak{t}(\mathsf{ASet}) = \{\mathfrak{t}(\pi) : \pi \in \mathsf{ASet}\}$. Note that by definition, the following holds for all $m, m' \in T$ with $m <^1 m'$:

$$\langle m, m'\rangle \in \bigcup \pi \quad \text{iff} \quad \langle [m], [m']\rangle \in \bigcup \mathfrak{t}(\pi). \tag{15}$$

Finally, let $\mathfrak{t}(\mathfrak{M}) = \langle \mathfrak{t}(T), \mathfrak{t}(<), \mathfrak{t}(\mathsf{Act}), \mathfrak{t}(\mathsf{ASet}), \mathfrak{t}(V)\rangle$, where $\mathfrak{t}(V)$ is a valuation such that for each propositional letter q, each $m \in T$ and each $h \in H_m$, $\langle \mathfrak{t}(m), \mathfrak{t}(h)\rangle \in \mathfrak{t}(V)(q)$ iff $\langle m, h\rangle \in V(q)$.[13] We call $\mathfrak{t}(\mathfrak{M})$ the $\mathfrak{t}$-*transformation of* $\mathfrak{M}$, and conclude from the observations above that $\mathfrak{t}(\mathfrak{M})$ is an RSL model satisfying all C1–C6.

Using $A(\mathfrak{t})$ for the result of replacing each π in A by $\mathfrak{t}(\pi)$, we have:

[13] For histories h^* in $\langle \mathfrak{t}(T), \mathfrak{t}(<)\rangle$ that are not $\mathfrak{t}(h)$ for any histories h in $\langle T, <\rangle$, we let $\langle [m], h^*\rangle \notin \mathfrak{t}(V)(q)$ for every $m \in T$ and every q. This will ensure a unique transformation.

Lemma 7.2 *Let* $\mathfrak{M} = \langle T, <, \mathsf{Act}, \mathsf{ASet}, V\rangle$ *be any RSL model satisfying* C1–C4*, and let* $\mathfrak{t}(\mathfrak{M})$ *be the* $\mathfrak{t}$*-transformation of* $\mathfrak{M}$*. Then for each RSL formula* A*, each* $m \in T$ *and each* $h \in H_m$, $\mathfrak{M}, m/h \models A$ *iff* $\mathfrak{t}(\mathfrak{M}), \mathfrak{t}(m)/\mathfrak{t}(h) \models A(\mathfrak{t})$.

Proof We only show the case for $[\alpha, \pi]\top$: $\mathfrak{M}, m/h \models [\alpha, \pi]\top$ iff $h \in H^e_m$ for some $e \in \pi$ iff $\langle m, m^+_h\rangle \in \bigcup \pi$ iff (by 15) $\langle [m], [m^+_h]\rangle \in \bigcup \mathfrak{t}(\pi)$ iff $\langle [m], [m^+_h]\rangle \in e' \in \mathfrak{t}(\pi)$ for some e' iff (by 14) $\langle [m], [m]^+_{\mathfrak{t}(h)}\rangle \in e' \in \mathfrak{t}(\pi)$ for some e' iff $\mathfrak{t}(h) \in H^{e'}_{[m]}$ for some $e' \in \mathfrak{t}(\pi)$ iff $\mathfrak{t}(\mathfrak{M}), [m]/\mathfrak{t}(h) \models [\alpha, \mathfrak{t}(\pi)]\top$. □

Now let $\mathfrak{M} = \langle T, <, \mathsf{Act}, \mathsf{ASet}, V\rangle$ be any RSL model satisfying all C1–C6. We define the "$\mathfrak{u}$-transformation" of $\mathfrak{M}$ as follows. Let us fix $\Pi = \{m \in T : m$ is a $>$-dead-end in $T\}$, let $\mathbb{E}_{\mathfrak{M}} = \mathsf{ASet} \cup \Pi$, and let $\mathbb{E}^*$ be the set of all (finite or infinite) sequences of members of $\mathbb{E}_{\mathfrak{M}}$. For each history h in $\langle T, <\rangle$, there are (distinct) actions $e_1, e_2, \ldots$ and a maximal $\preceq$-chain $\mathfrak{u}(h)$ in $\mathbb{E}^*$ satisfying the following:

- $h = \{m_0, m_1, \ldots\}$ with $\langle m_k, m_{k+1}\rangle \in e_{ki+1}$ for all $k \geqslant 0$, and
- $\mathfrak{u}(h) = \big\langle m_0, \pi_{[e_1]}, \pi_{[e_2]}, \ldots\big\rangle$, where $\pi_{[e]}$ is the unique member of ASet to which e belongs. (recall C2: ASet is a partition of Act.)

We call such a $\mathfrak{u}(h)$ *the correspondent of* h. For each $m \in T$, there is a sequence $\mathfrak{u}(m) \in \mathbb{E}^*$ such that for some $k \geqslant 0$ and some actions $e_1, \ldots, e_k$, the following hold:

- $p_{(m)} = \{m_0, \ldots, m_k\}$ with $m = m_k$, and $\langle m_i, m_{i+1}\rangle \in e_{i+1}$ for all $i < k$, and
- $\mathfrak{u}(m) = \big\langle m_0, \pi_{[e_1]}, \ldots, \pi_{[e_k]}\big\rangle$, where $\pi_{[e]}$ is the unique member of ASet to which e belongs.

We call such an $\mathfrak{u}(m)$ *the correspondent of* m.[14] Using $\mathbb{H}$ for the set of all correspondents of histories in $\langle T, <\rangle$, we let $\mathbb{S}_{\mathfrak{M}}$ be the set of all finite (nonempty) prefixes of histories in $\mathbb{H}$. It is easy to see that each $s \in \mathbb{S}_{\mathfrak{M}}$ is a correspondent of some $m \in T$ (i.e., $s = \mathfrak{u}(m)$ for some $m \in T$). Finally, let $\mathfrak{u}(\mathfrak{M}) = \langle \mathbb{E}_{\mathfrak{M}}, \mathbb{S}_{\mathfrak{M}}, \mathfrak{u}(V)\rangle$ where for each propositional letter q, each $m \in T$ and each $h \in H_m$, $\langle m, h\rangle \in V(q)$ iff $\langle \mathfrak{u}(m), \mathfrak{u}(h)\rangle \in \mathfrak{u}(V)(q)$. We call $\mathfrak{u}(\mathfrak{M})$ the $\mathfrak{u}$-*transformation of* $\mathfrak{M}$, and it is easy to see that $\mathfrak{u}(\mathfrak{M})$ is a TEL model.

Assuming that ASet is finite. For RSL formulas A, we define A° as follows:

$$\begin{aligned}
q^\circ &\triangleq q\\
(\neg A)^\circ &\triangleq \neg(A^\circ)\\
(A \wedge B)^\circ &\triangleq A^\circ \wedge B^\circ\\
([\alpha, \pi]\top)^\circ &\triangleq F_\pi \top\\
(FA)^\circ &\triangleq \textstyle\bigvee_{\pi \in \mathsf{ASet}} F_\pi(A^\circ)\\
(PA)^\circ &\triangleq \textstyle\bigvee_{\pi \in \mathsf{ASet}} P_\pi(A^\circ)
\end{aligned}$$

14 It is easy to see that correspondents of moments can be defined recursively: For each $>$-dead-end m in T, $\mathfrak{u}(m) = \langle m\rangle$; for each m such that $u <^1 m$ for some $u \in T$, $\mathfrak{u}(m) = \mathfrak{u}(u) * \langle \pi\rangle$, where π is the unique member of ASet to which $\{\langle u, m\rangle\}$ belongs.

Lemma 7.3 *Let $\mathfrak{M} = \langle T, <, \mathsf{Act}, \mathsf{ASet}, V\rangle$ be any RSL model satisfying all* C1–C6*, and let $\mathfrak{u}(\mathfrak{M})$ be the $\mathfrak{u}$-transformation of $\mathfrak{M}$. Suppose that* ASet *is finite. Then for all RSL formula A, all m and h, $\mathfrak{M}, m/h \vDash A$ iff $\mathfrak{u}(\mathfrak{M}), \mathfrak{u}(m), \mathfrak{u}(h) \vDash A^\circ$.*

Proof Using $\mathfrak{M}^*$ for $\mathfrak{u}(\mathfrak{M})$ and h^* for $\mathfrak{u}(h)$, we show the cases for $[\alpha, \pi]$ and F. Let $A = [\alpha, \pi]\top$. Suppose that $\mathfrak{M}, m/h \vDash [\alpha, \pi]\top$. Then there is an $e \in \mathsf{Act} \cap \pi$ such that $h \in H_m^e$, which implies that $\langle m, m_h^+\rangle \in e \in \pi$, and hence by definition above, $\mathfrak{u}(m) * \langle\pi\rangle = \mathfrak{u}(m_h^+)$. Clearly $m_h^+ \in h$, and hence $(\mathfrak{u}(m) * \langle\pi\rangle) \preceq h^*$. Since obviously $\mathfrak{M}^*, \mathfrak{u}(m) * \langle\pi\rangle, h^* \vDash \top$, it follows that $\mathfrak{M}^*, \mathfrak{u}(m), h^* \vDash F_\pi\top$. Suppose that $\mathfrak{M}^*, \mathfrak{u}(m), h^* \vDash F_\pi\top$. Then $(\mathfrak{u}(m)*\langle\pi\rangle) \preceq h^*$, and consequently $e = \{\langle m, m_h^+\rangle\} \in \pi$ and hence $h \in H_m^e$, from which it follows that $\mathfrak{M}, m/h \vDash [\alpha, \pi]\top$.

Let $A = FB$. Suppose that $\mathfrak{M}, m/h \vDash FB$. Then m is not a $<$-dead-end and $\mathfrak{M}, m_h^+/h \vDash B$, and hence by induction hypothesis, $\mathfrak{M}^*, \mathfrak{u}(m_h^+), h^* \vDash B^\circ$. Clearly, $e = \{\langle m, m_h^+\rangle\} \in \mathsf{Act}$ by C1 and C6. Applying C2, $e \in \pi'$ for some $\pi' \in \mathsf{ASet}$, and hence by definition above, $\mathfrak{u}(m_h^+) = \mathfrak{u}(m) * \langle\pi'\rangle$, from which it follows that $(\mathfrak{u}(m) * \langle\pi'\rangle) \preceq h^*$ and $\mathfrak{M}^*, \mathfrak{u}(m) * \langle\pi'\rangle, h^* \vDash B^\circ$, i.e., $\mathfrak{M}^*, \mathfrak{u}(m), h^* \vDash F_{\pi'}B^\circ$, and hence $\mathfrak{M}^*, \mathfrak{u}(m), h^* \vDash \bigvee_{\pi\in\mathsf{ASet}} F_\pi B^\circ$. Suppose that $\mathfrak{M}^*, \mathfrak{u}(m), h^* \vDash \bigvee_{\pi\in\mathsf{ASet}} F_\pi B^\circ$. Then for some $\pi \in \mathsf{ASet}$, $\mathfrak{M}^*, \mathfrak{u}(m), h^* \vDash F_\pi B^\circ$, and hence $\mathfrak{u}(m) * \langle\pi\rangle \preceq h^*$ and $\mathfrak{M}^*, \mathfrak{u}(m) * \langle\pi\rangle, h^* \vDash B^\circ$. Because $\mathfrak{u}(m) * \langle\pi\rangle \preceq h^*$ and h^* is the correspondent of h, we know that there is an $m' \in h$ with $m <^1 m'$ and $\{\langle m, m'\rangle\} \in \pi$. Clearly, $m' = m_h^+$, and then $\mathfrak{u}(m) * \langle\pi\rangle = \mathfrak{u}(m_h^+)$, and hence $\mathfrak{M}^*, \mathfrak{u}(m_h^+), h^* \vDash B^\circ$. By induction hypothesis, $\mathfrak{M}, m_h^+/h \vDash B$, and m is obviously not a $<$-dead-end, and hence $\mathfrak{M}, m/h \vDash FB$. □

Let $\mathfrak{M} = \langle T, <, \mathsf{Act}, \mathsf{ASet}, V\rangle$ be any RSL model satisfying C1–C4. We know that $\mathfrak{t}(\mathfrak{M})$ satisfies all C1–C6, and that $\mathfrak{u}(\mathfrak{t}(\mathfrak{M}))$ is a TEL model. We let $\mathfrak{M}^\circ$ be $\mathfrak{u}(\mathfrak{t}(\mathfrak{M}))$, and call it the TEL *transformation of* $\mathfrak{M}$. For each $m \in T$, we use m° for $\mathfrak{u}(\mathfrak{t}(m))$, and for each h in $\langle T, <\rangle$, we use h° for $\mathfrak{u}(\mathfrak{t}(h))$. Finally, for each RSL formula A, we use $A(\mathfrak{t})$ for the result of replacing each π in A by $\mathfrak{t}(\pi)$. Appying Lemma 7.2 and 7.3, we can establish the following.

Theorem 7.4 *For each RSL model $\mathfrak{M} = \langle T, <, \mathsf{Act}, \mathsf{ASet}, V\rangle$ satisfying* C1–C4 *and that* ASet *is finite, each RSL formula A, each $m \in T$ and each $h \in H_m$, $\mathfrak{M}, m/h \vDash A$ iff $\mathfrak{M}^\circ, m^\circ, h^\circ \vDash (A(\mathfrak{t}))^\circ$.*

References

Balbiani, P., Herzig, A., & Troquard, N. (2008). Alternative axiomatics and complexity of deliberative stit theories. *Journal of Philosophical Logic, 37*, 387–406.

Belnap, N. (1995). Outcomes on branching histories. Unpublished manuscript, Department of Philosophy, University of Pittsburgh.

Belnap, N. (1996). The very idea of an outcome. *Serbian Scientific Review, 19*(20), 15–16.

Belnap, N. (2005). A theory of causation: Causae causantes (originating causes) as inus conditions in branching space-times. *British Journal of the Philosophy of Science, 56*, 221–253.

Belnap, N., & Perloff, M. (1988). Seeing to it that: A canonical form for agentives. *Theoria, 54*, 175–199.

Belnap, N., Perloff, M., & Xu, M. (2001). *Facing the future: Agents and choices in our indeterminist world*. Oxford: Oxford University Press.

Broersen, J. (2008a). A complete stit logic for knowledge and action, and some of its applications. In M. Baldoni, T. C. Son, M. B. van Riemsdijk, & M. Winikoff (Eds.), *Declarative agent languages and technologies VI, DALT 2008* (Vol. 5397 of Lecture Notes in Computer Science, pp. 47–59). Berlin: Springer.
Broersen, J. (2008b). A logical analysis of the interaction between 'obligation-to-do' and 'knowingly doing'. In R. van der Meyden, & L. van der Torre (Eds.) (pp. 140–154).
Broersen, J., Herzig, A., & Troquard, N. (2006a). Embedding alternating-time temporal logic in strategic stit logic of agency. *Journal of Logic and Computation, 16*(5), 559–578.
Broersen, J., Herzig, A., & Troquard, N. (2006b). A stit-extension of ATL. In M. Fisher (Ed.), *Proceedings tenth European conference on logics in artificial intelligence, JELIA 06* (Vol. 4160 of Lecture Notes in Artificial Intelligence, pp. 69–81). Berlin: Springer.
Broersen, J., Herzig, A., & Troquard, N. (2009). What groups *do*, *can do*, and *know they can do*: A normal modal logic analysis. *Journal of Applied Non-Classical Logics* (to appear).
Brown, M. (1988). On the logic of ability. *Journal of Philosophical Logic, 17*, 1–26.
Brown, M. (2008a). Acting, events and actions. In R. van der Meyden & L. van der Torre (Eds.) (pp. 19–33).
Brown, M. (2008b). Acts and actions in branching time. Manuscript, Philosophy Department, Syracuse University. Presented at the 36th annual meeting of the society for exact philosophy.
Campbell, C.A. (1957). *On selfhood and godhood*. New York: George Allen & Unwin Ltd., The MacMillan Company.
Chellas, B. (1969). *The logical form of imperatives*. Stanford: Perry Lane Press.
Davidson, D. (1971). Agency. In R. B. Robert Binkley, & A. Marras (Eds.), *Agent, action and reason* (pp. 43–61). Toronto: University of Toronto Press (Reprinted in Davidson 1980).
Davidson, D. (1980). *Essays on actions and events*. Oxford: Oxford University Press.
Governatori, G., Hodkinson, I., & Venema, Y. (Eds.). (2006). *Advances in modal logic* (Vol. 6). College Publications, Department of Compuer Science, Strand, London WC2R 2LS, UK.
Horty, J. F. (1989). An alternative *stit* operator. Manuscript, Department of Philosophy, University of Maryland.
Horty, J. F. (2001). *Agency and deontic logic*. Oxford: Oxford University Press.
van Benthem, J. F. A. K. (2008). *Logical dynamics of information and interaction*. Manuscript.
van Benthem, J., Gerbrandy, J., Hoshi, T., & Pacuit, E. (2009). Merging frameworks for interaction. *Journal of Philosophical Logic, 38*, 491–526.
van Benthem, J., & Pacuit, E. (2006). *The tree of knowledge in action: Towards a common perspective* (pp. 87–106) (in Governatori et al. 2006).
van der Meyden, R., & van der Torre, L. (Eds.). (2008). *Deontic logic in computer science: 9th International conference, DEON 2008, Luxembourg, Luxembourg, July 2008, proceedings* (Vol. 5076 of Lecture Notes in Artificial Intelligence). Berlin: Springer.
von Kutschera, F. (1986). Bewirken. *Erkenntnis, 24*, 253–281.
von Kutschera, F. (1993). Causation. *Journal of Philosophical Logic, 22*, 563–588.
Wansing, H. (2006). *Tableaux for multi-agent deliberative stit logic* (pp. 503–520) (in Governatori et al. 2006).
Wölfl, S. (2002). Propositional Q logic. *Journal of Philosophical Logic, 31*, 387–414.
Wölfl, S. (2005). Events in branching time. *Studia Logica, 79*, 255–282.
Xu, M. (1994). Decidability of deliberative *stit* theories with multiple agents. In D. M. Gabbay, & H. J. Ohlbach (Eds.), *Temporal logic, first international conference, ICTL'94, Bonn, Germany, proceedings* (Vol. 827 of Lecture Notes in Artificial Intelligence, pp. 332–348). Berlin: Springer.
Xu, M. (1997). Causation in branching time (I): Transitions, events and causes. *Synthese, 112*, 137–192.
Xu, M. (1998). Axioms for deliberative *stit*. *Journal of Philosophical Logic, 27*, 505–552.
Xu, M. (2009a). Actions as events. Unpublished manuscript, Department of Philosophy, Wuhan University.
Xu, M. (2009b). Axioms for *stit* with actions based on metric tense logic in branching time. Unpublished manuscript, Department of Philosophy, Wuhan University.