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Anti-dynamics: presupposition projection without dynamic semantics

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Abstract Heim 1983 suggested that the analysis of presupposition projection requires that the classical notion of *meanings as truth conditions* be replaced with a dynamic notion of *meanings as Context Change Potentials*. But as several researchers (including Heim herself) later noted, the dynamic framework is insufficiently predictive: although it allows one to state that, say, the dynamic effect of *F and G* is to *first* update a Context Set C with F and *then* with G (i.e., C[F and G] = C[F][G]), it fails to explain why there couldn't be a 'deviant' conjunction *and** which performed these operations in the opposite order (i.e., C[F and* G] = C[G][F]). We provide a formal introduction to a competing framework, the Transparency theory, which addresses this problem. Unlike dynamic semantics, our analysis is *fully classical*, i.e., bivalent and static. And it *derives* the projective behavior of connectives from their bivalent meaning and their syntax. We concentrate on the formal properties of a simple version of the theory, and we prove that (i) full equivalence with Heim's results is guaranteed in the propositional case (*Theorem 1*), and that (ii) the equivalence can be extended to the quantificational case (for any generalized quantifiers), but only when certain conditions are met (*Theorem 2*).

Keywords Presupposition · Dynamic semantics · Trivalence · Presupposition projection

1 The projection problem and the dynamic dilemma

1.1 The projection problem

How are the presuppositions of complex sentences computed from the meanings of their component parts? This is the so-called 'Projection Problem,' which is illustrated in (1):

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(1) a. The king of Moldavia is powerful.
b. Moldavia is a monarchy and the king of Moldavia is powerful.
c. If Moldavia is a monarchy, the king of Moldavia is powerful.

(1)a presupposes (incorrectly) that Moldavia has a king. But the examples in (1)b–c presuppose no such thing; they only presuppose that *if Moldavia is a monarchy, it has a king* (a condition which is satisfied if one knows that Moldavia is in Eastern Europe and that Eastern-European monarchies are of the French type, i.e., that if they have a monarch, it is a king, not a queen). How can these facts be explained?

Minimally, a theory of presupposition projection should be *descriptively adequate* and thus provide an algorithm to compute the presuppositions of complex sentences. If possible, the theory should also be *explanatory* and thus derive the algorithm from independent considerations (though what counts as more or less explanatory may be a matter of debate).

1.2 Stalnaker's pragmatic approach

In what might be the single most influential proposal in presupposition theory, Stalnaker (1974) offered a brilliant analysis of presupposition projection in conjunctions. His theory included three main assumptions (see also Karttunen, 1974, who adopts versions of (i) and (ii) as well).

(i) The presupposition of an elementary clause imposes a condition on the 'Context Set' in which it is uttered, which can be seen as *the set of worlds compatible with what the speech act participants take for granted*. Specifically, if $\underline{p}p'$ is an elementary clause with presupposition p and assertion p', the felicity condition is that for each $w \in C$, $p(w) = 1$ (we will henceforth systematically write $\underline{q}q'$ for an elementary clause with a presupposition q and an assertive component q'). If this condition is not satisfied, a presupposition failure (denoted by #) is obtained. If we write as $C[\underline{p}p']$ the effect of asserting $\underline{p}p'$ in a Context Set C, we obtain the following rule:
$C[\underline{p}p'] = \#$ unless for each $w \in C$, $p(w) = 1$.

(ii) As a conversation develops, the Context Set does not remain fixed but rather *evolves dynamically* according to certain principles of rationality. In particular, if $\underline{p}p'$ is an elementary clause that does not trigger a presupposition failure, it has the effect of 'updating' the Context Set C with its assertive component:
If $\neq \#$, $C[\underline{p}p'] = \{w \in C, p'(w) = 1\}$.

(iii) Finally, Stalnaker assimilated the assertion of a conjunction to the successive assertion of each conjunct. The idea was that the assertion of *F and G* leads one to first update the Context Set C with F, and then with G:
$C[F \text{ and } G] = \#$ iff $C[F] = \#$ or ($C[F] \neq \#$ and $C[F][G] = \#$). If $\neq \#$, $C[F \text{ and } G] = C[F][G]$.

Since *Moldavia is a monarchy* does not contain any presupposition trigger, by Principles (i) and (ii) the effect of the first conjunct of (1)b is to transform the initial Context Set C into a new Context Set C′, with:

$C' = C[\text{Moldavia is a monarchy}] = \{w \in C: \text{Moldavia is a monarchy in } w\}$.

By Principle (iii), the presupposition that Moldavia has a king is evaluated not with respect to C, but rather with respect to C′. And the presupposition will indeed be satisfied just in case for each w ∈ C, if Moldavia is a monarchy in w, Moldavia has a king in w.

While it is beautiful and convincing, Stalnaker's analysis raises three problems (see also Moltmann, 1997, 2003 for a critique of the notion of 'intermediate contexts' and a different attempt to construct a theory of presupposition that eschews it):

1. First, it is not clear that one should equate the assertion of a conjunction with a succession of two assertions. After all, conjunctions may be embedded under other operators, as in *None of my students is rich and proud of it*. But here there is no clear sense in which either *is rich* or *(is) proud of it* is *asserted* to hold of anybody, which makes it difficult to apply Stalnaker's analysis.
2. Second, even if one restricts attention to unembedded connectives, it is unclear how Stalnaker's ideas can be extended beyond conjunction. Consider for instance the sentence *Moldavia is not a monarchy or (else) its king is powerful,* which presupposes only that *if Moldavia is a monarchy, it has a king*. Since the entire point of a disjunction is that one may assert it without being committed to either disjunct, it is difficult to see how Stalnaker's assertion-based analysis can be made to work (and it is also unclear why it is the *negation* of the first disjunct which serves to justify the presupposition of the second one).
3. Third, even in the case of unembedded conjunctions or even juxtaposed sentences in discourse, it is not clear *why* a rational agent should update the Context Set in the way prescribed by Stalnaker. The Context Set, after all, represents what is taken for granted by both speech act participants. But just because my interlocutor claims that *Intelligent Design is a serious alternative to Darwinism and* ..., it doesn't follow that I must take for granted the first conjunct (or the first sentence) at any point in the conversation. I certainly take for granted *that my interlocutor has pronounced (or asserted) these words,* but if my skepticism is complete I have no reason whatsoever to revise any of my other beliefs. This casts some doubt on the idea that the Context Set must be updated with the content of the sentences that are asserted in the course of a conversation.

1.3 Heim's semantic alternative

Heim (1983) (who develops ideas in Karttunen, 1974) addresses these problems (in particular Problems 1 and 2) by biting the bullet and abandoning the pragmatic inspiration of Stalnaker's theory. She posits instead that the very meaning of a linguistic expression is to update the Context Set in a particular way. Thus the traditional concept of 'meanings as truth conditions' is replaced with a dynamic notion of 'meanings as Context Change Potentials,' i.e., as functions from Context Sets to Context Sets. In this way Heim (1983) can have Stalnaker's cake and eat it too because she takes his update rules to be the result of the *dynamic semantics* of certain connectives rather than of some kind of pragmatic reasoning. This allows her to define Context Change Potentials for all sorts of operators that Stalnaker did not consider—notably, for quantifiers (Beaver, 2001 extends the analysis to disjunction).

But as Soames (1989) and Heim (1990, 1992) forcefully emphasized, this semantic move is not without cost. The problem is that Heim's dynamic semantics is just *too powerful*: it can provide a semantics for a variety of operators and connectives which are never found in natural language. To make the point concrete, it suffices to observe that in Heim's framework one could easily define a deviant conjunction *and** with the same classical content as *and* but a different projection behavior:

(2) C[F and* G] = # iff C[G] = # or (C[G] $\neq$ # and C[G][F] = #). If $\neq$ #, C[F and* G] = C[G][F].

It is immediate that C[F and* G] = C[G and F] (with the order of the conjuncts reversed), and that when neither *F* nor *G* contains any presupposition trigger, C[F and* G] = C[F and G] (because in this case the order of the conjuncts does not matter). If *and** were the conjunction we find in natural language, *Moldavia is a monarchy and* the king of Moldavia is powerful* would result in a presupposition failure; while *The king of Moldavia is powerful and* Moldavia is a monarchy* should be entirely acceptable—quite the opposite from what we actually find. But the question, of course, is *why does natural language have* and *but not* and*? It seems that Heim's framework is too powerful to offer an answer. To put it differently, her theory may be descriptively adequate, but it is not quite explanatory.

It should be added that the descriptive adequacy of Heim's theory has also been called into question by van der Sandt (1992) and Geurts (1999). They offer a representational alternative, developed within DRT, to Heim's purely semantic analysis. While their theory makes different predictions from Heim's, it shares its explanatory weaknesses, in the sense that it does not make strong predictions about connectives whose presuppositional behavior is not stipulated to begin with. In the rest of this paper, we concentrate on Heim's theory, leaving a discussion of van der Sandt and Geurts's analysis for future research.

2 The Transparency theory: main ideas[1]

As we saw, dynamic theories are faced with a dilemma. If their dynamic component is viewed as a process of belief revision, the account is explanatory for a subset of the cases (in particular, for conjunctions), but it fails to be sufficiently general and it is thus descriptively inadequate. By contrast, if the dynamic component is interpreted as being semantic, the account gains in descriptive adequacy, but it loses in explanatory strength. We locate the source of the dilemma in the first step of the analysis, which is common to Stalnaker and to Heim/Karttunen: we deny that the Context Set needs to be updated with the content of any clause in the course of presupposition computation. This does not mean that it cannot be (after all, one is at liberty to change what one takes for granted if one believes that one's interlocutor is truthful and well-informed); but we deny that dynamic update is what accounts for presupposition projection. Of course this is not to deny that *something* evolves dynamically in the course of a conversation. But we claim that the only information that needs to be updated concerns *the words that the speech act participants have pronounced*. From this trivial observation it certainly does *not* follow that 'meaning is dynamic' in any sense, as we will see shortly. Thus *meaning is not dynamic*, though sentence comprehension might well be.

[1] B. Geurts (p.c.) informs me that some early ideas in van der Sandt (1988) might bear an interesting relation to the theory developed here. I leave a comparison for future research.

2.1 Pragmatic motivation[2]

The theory we develop is stated within bivalent (=pre-dynamic) logic. We do preserve the notion of a Context Set, but we assume that it remains fixed throughout the computation of the meaning of a sentence or even of a discourse. We also do without any recourse to trivalence: a presupposition failure will simply come out as the violation of a certain pragmatic principle. Our theory is thus fully classical, both bivalent and static.

The intuition we pursue is that the presupposition p of a clause $\underline{p}p'$ is simply a distinguished part of a bivalent meaning, one which is conceptualized as a 'pre-condition' of the entire meaning.[3] We do not seek to explain how certain parts of the meaning of a constituent are conceptualized as being its 'pre-conditions.' This is another form of the old 'triggering problem' for presuppositions, i.e., the problem of determining how elementary clauses come to have presuppositions to begin with. Since we are interested in the projection problem rather than in the triggering problem, we simply stipulate in the syntax of the object language that a clause represented as $\underline{p}p'$ has the truth-conditional content of the conjunction p *and* p', but that p is conceptualized as being the pre-condition of the entire meaning. On the other hand our goal is to give an explanatory account of presupposition projection. The crucial intuition is that a general pragmatic principle (presumably a Gricean maxim of manner, which we call *Be Articulate!*) requires that, *if possible*, the special status of the pre-condition should be articulated, and thus that one should say p *and* $\underline{p}p'$ rather than just $\underline{p}p'$. To illustrate, the principle requires that, *if possible*, one should say *It is raining and John knows it* rather than just *John knows that it is raining*.[4]

If we were to stop here, we would make the absurd prediction that *John knows that p* is never acceptable unless immediately preceded by *p and* __. However there are independent pragmatic conditions that sometimes rule out the full conjunction. *It is precisely when these conditions are met that* John knows that *p is acceptable on its own.* In this paper we will only consider cases in which the full conjunction is ruled out *because the utterance of the first conjunct is certain to be dispensable no matter what the end of the sentence turns out to be* (see Schlenker, 2006b for a discussion of further conditions, with several new predictions). This constraint is motivated by facts that have nothing to with presupposition projection:

(3) a. Context: Everyone is aware that Pavarotti has cancer.
i. ?Pavarotti is sick and he won't be able to sing next week.
ii. Pavarotti won't be able to sing next week.

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[2] For other pragmatically-inspired theories of presupposition, see (among others) Abbott (2000), Abusch (2002), Grice (1981), Simons (2001), and Sperber and Wilson (1989). For a survey of several theories of presupposition, see Kadmon (2001).

[3] Thanks to D. Wilson for helpful remarks on this issue.

[4] The question of the relative order of the two conjuncts does not really arise because the second conjunct asymmetrically entails the first one, which makes the order *It is raining and John knows it* the only admissible one. Quite generally, it is possible to utter a conjunction whose second conjunct is stronger than the first, but the opposite is impossible:

(i) a. John reside in France and he lives in Paris.
b. #John lives in Paris and he resides in France.

b. Context: Nothing is assumed about Pavarotti's health.
i. #Pavarotti has cancer and he is sick and he won't be able to sing next week.
ii. Pavarotti has cancer and he won't be able to sing next week.

c. Context: Nothing is assumed about Pavarotti's health.
i. #If Pavarotti has cancer, he is sick and he won't be able to sing next week.
ii. If Pavarotti has cancer, he won't be able to sing next week.

The infelicitous examples are all cases in which one can determine as soon as one has heard *Pavarotti is sick and* that *no matter how the sentence will end*, these four words will have been uttered in vain because they could not possibly affect the truth-conditions of the sentence relative to the Context Set. Specifically, in a Context Set C in which it is assumed that Pavarotti has cancer, we can be sure that no matter what the second conjunct γ is, *Pavarotti is sick and* γ is equivalent in C to γ. We will say that given C these two sentences are *contextually equivalent* (i.e., $C \vDash$ (Pavarotti is sick and $\gamma) \Leftrightarrow \gamma$). Similarly, in any Context Set in which it is assumed that cancer is a disease, *Pavarotti has cancer and he is sick and* γ is contextually equivalent to *Pavarotti has cancer and* γ, and by the same reasoning, *If Pavarotti has cancer, he is sick and* γ is contextually equivalent to *If Pavarotti has cancer*, γ. In all these cases, then, one can ascertain as soon as one has heard *he is sick and* that these words were uttered in vain. Any reasonable pragmatics should presumably rule this out, as suggested by (3).

Note, however, that we don't want to make the prohibition against redundant material too strong. For it is sometimes permissible to include a conjunct that turns out to be dispensable, but just in case one may only determine *later* in the sentence that the conjunct in question was eliminable. This scenario is illustrated in (4):

(4) a. John resides in France and he lives in Paris.
b. If he is in Europe, John resides in France and he lives in Paris.

In both examples the contextual meaning of the sentence would be unaffected if we deleted the words *John resides in France and*. However this is something that can only be ascertained *after* one has heard the end of the sentence; thus in (4)b, if the end of the sentence had been ... *and he is happy*, the first conjunct would not have been redundant.

These observations lead us to the following definition[5]:

(5) **Definition of Transparency**
Given a Context Set C, a predicative or propositional occurrence of *d* is *transparent* (and hence infelicitous) in a sentence that starts with the string α (*d and* just in case for any constituent γ of the same type as *d* and for any sentence completion β[6],

$$C \vDash \alpha \text{ (d and } \gamma)\beta \Leftrightarrow \alpha\gamma\beta$$

[5] See Katzir (2006) and Singh (2006) for recent investigations which might help illuminate the statement of Transparency.

[6] As the terminology suggests, β is a *sentence completion* for the string α (*d and* γ) just in case the string α (*d and* $\gamma)\beta$ is a well-formed formula. It is immediate that if γ is a constituent of the same type as *d*, β is a sentence completion for α (*d and* γ) just in case β is a sentence completion for $\alpha\gamma$.

We note that the principle is rather sensitive to the expressive power of the language, since we quantify over syntactic expressions. We will systematically assume that the language is quite expressive, and in particular that it includes tautologies and contradictions.

Our observations in (3)–(4) can now be summarized by noting that α *(d and $\underline{d}d'$)*... is semantically deviant if *d* is transparent. Going back to the analysis of presupposition, it is clear that when *d* is transparent, a full conjunction *(d and $\underline{d}d'$)* will be systematically ruled out, which will leave $\underline{d}d'$ as the sole contender, and thus as the 'winner' in the competition process. Assuming for simplicity that Transparency is the *only* pragmatic principle that can rule out a full conjunction *(d and $\underline{d}d'$)*, we are finally led to our formula for presupposition projection:

(6) **Principle of Transparency**[7]
Given a Context Set C, a predicative or propositional occurrence of $\underline{d}d'$ is acceptable in a sentence that starts with the string α $\underline{d}d'$
if and only if the 'articulated' competitor α *(d and $\underline{d}d'$)* is ruled out because *d* is transparent,
if and only if for any constituent γ of the same type as *d* and for any sentence completion β,
$C = \alpha \text{ (d and } \gamma) \beta \Leftrightarrow \alpha\gamma\beta$

This immediately accounts for some simple facts of presupposition projection, which parallel the non-presuppositional data we observed in (3):

(7) a. *Context*: Everyone is aware that Pavarotti has cancer.
i. #Pavarotti is sick and he knows it.
ii. Pavarotti knows that he is sick.

b. *Context*: Nothing is assumed about Pavarotti's health.
i. #Pavarotti has cancer and he is sick and he knows it.
ii. Pavarotti has cancer and he knows that he is sick.

c. *Context*: Nothing is assumed about Pavarotti's health.
i. #If Pavarotti has cancer, he is sick and he knows it.
ii. If Pavarotti has cancer, he knows that he is sick.

In each case, the acceptability of (ii) is a consequence of the unacceptability of (i), which is due to the fact that *Pavarotti/he is sick* is transparent.

The rest of this paper is devoted to a demonstration that, under certain conditions, the Principle of Transparency suffices to derive the projection facts discussed in Heim (1983). The advantage of the Transparency theory is that it *predicts* the projection behavior of connectives from their classical (i.e., bivalent) meaning (together with their syntax). This will solve the over-generation problem that was discussed earlier: a purported connective with the behavior of our 'deviant' conjunction *and** will be ruled out on principled grounds, as is desired. This will also allow us to make predictions about connectives that Heim did not consider, as we show at the end of this paper.

[7] This principle was first stated in Schlenker (2006a), but no general equivalence with Heim's system was provided (only special examples were considered). As indicated in the text, the assumption that Transparency as stated is the *only* factor that could rule out a full conjunction is a simplification. See Schlenker (2006b) for a more fine-grained analysis, which makes different predictions from Heim (1983).

2.2 Formal motivation

The Principle of Transparency can also be motivated by formal considerations. The key observation is that Heim's semantics does in fact obey a kind of 'Dynamic Transparency,' which in her theory is a derived property rather than a principle. As before, let us write $\underline{d}d'$ for an atomic expression with presupposition d and assertive component d'. And for any formula F, let us call $F*$ the result of deleting from F all the underlined material. Thus if $F = \underline{p}p'$, $F^* = p'$; and if $F = (p \text{ and } \underline{q}q')$, $F^* = (p \text{ and } q')$. Then for any Context Set C and for any formula F, Heim's system guarantees that:

(8) If $C[F] \neq \#$, $C[F] = C[F^*]$ (Dynamic Transparency).

The proof is immediate once the Context Change Potentials of the connectives are defined, as is done below (Sect. 3). For the moment, it is enough to illustrate Dynamic Transparency in the case of atomic formulas and of conjunctions, whose dynamic semantics was defined as follows:

(9) a. $C[\underline{p}p'] = \#$ iff for some $w \in C$, $p(w) = 1$. If $\neq \#$, $C[\underline{p}p'] = \{w \in C: p'(w) = 1\}$.
b. $C[F \text{ and } G] = \#$ iff $C[F] = \#$ or ($C[F] \neq \#$ and $C[F][G] = \#$). If $\neq \#$, $C[F \text{ and } G] = C[F][G]$.

Dynamic Transparency is trivially satisfied at the atomic level. And if it is satisfied in any Context Set for F and G, $C[F] = C[F^*]$ and $C[F][G] = C[F^*][G] = C[F^*][G^*]$, whence $C[F \text{ and } G] = C[F^* \text{ and } G^*] = C[(F \text{ and } G)^*]$. The proof easily generalizes to other connectives and operators, as we will see in Sect. 3.

Seen in this light, the Transparency theory is an attempt to turn a derived property of Heim's dynamic system into the centerpiece of a classical analysis. However, the simplest implementation of this idea does not quite derive Heim's results. It is worth considering what the problems are, and how they naturally lead to the version of Transparency which was posited in (6).

– We could try to require that if a formula F is uttered in a Context Set C, C should satisfy:

(10) $C \vDash F \Leftrightarrow F^*$

It is immediate, however, that this fails to derive the asymmetric projective behavior of *and*. As we saw, *Moldavia is a monarchy and the king of Moldavia is powerful* only presupposes that *if Moldavia is a monarchy, it has a king*, whereas a different result appears to be obtained when the order of the conjuncts is reversed[8]. But since the rule in (10) is to be interpreted within classical logic, it is intrinsically incapable of accounting for the asymmetric behavior of conjunction.

– In order to account for the asymmetry, we required earlier that Transparency be checked as soon as an initial string of the form $\alpha\ \underline{p}p'$ is heard. But this measure is still insufficient. Consider the sentence *It is John who won*. It is usually analyzed as $\underline{p}p'$ with p = *Exactly one person won*, and p′ = *John won*. Since the clause is atomic, the requirement that we check Transparency 'as soon as' the clause is heard does not add anything, and we simply end up with the condition in (11):

(11) $C \vDash \underline{p}p' \Leftrightarrow p'$

[8] One could challenge this empirical assumption, but doing so would lead one away from Heim (1983). See Schlenker (2006b) for some initial discussion.

The left-to-right direction is satisfied no matter what C is. As for the right-to-left direction, it is satisfied if and only if:

(12) $C \vDash p' \Rightarrow p$

But in the case at hand this yields a result which is too weak: we predict that it should only be presupposed that *if John won, exactly one person won*—which in most cases is trivially satisfied. But as a matter of fact, *It is John who won* presupposes something stronger, namely that *someone won*.
– By contrast, the statement of Transparency that we obtained on pragmatic grounds in Sect. 2.1 yields precisely the desired results. The idea was that $\underline{d}d'$ is acceptable on its own if the more explicit competitor *(d and* $\underline{d}d'$) is ruled out because as soon as *(d and* has been uttered one can ascertain that these words were uttered in vain, i.e., if

(13) for every expression γ of the same type as *d* and every sentence completion β, $C \vDash (d \text{ and } \gamma)\beta \Leftrightarrow \gamma\beta$

It is immediate that the condition is satisfied if $C \vDash p$. Conversely, if the condition is satisfied, for null β and for some tautology γ, we have $C \vDash (p \text{ and } \gamma) \Leftrightarrow \gamma$ and thus $C \vDash p$, as is desired.

2.3 Examples

Before we turn to the formal development of the theory, it might be helpful to consider some examples. We just saw that for an (unembedded) atomic formula $\underline{p}p'$, Transparency requires that the Context Set entail *p*. In the case of conjunctions and conditionals (which, following Heim, 1983, we analyze as material implications), the following results are generally taken to be desirable (and they are indeed derived by Heim, 1983):

—A conjunction ($\underline{p}p'$ *and q)* presupposes *p*.
—A conjunction *(p and* $\underline{q}q'$) presupposes *(p* $\Rightarrow$ *q)*.
—A conditional *(if* $\underline{p}p'.q$) presupposes *p*.
—A conditional *(if p.* $\underline{q}q'$) whose antecedent is non-presuppositional presupposes *(p* $\Rightarrow$ *q)*.

Let us now see how these results are obtained in the Transparency theory.

(14) ($\underline{p}p'$ and q)

a. *Transparency* requires that for each clause γ and for each sentence completion β,
$C \vDash ((p \text{ and } \gamma)\beta \Leftrightarrow (\gamma\beta$

b. **Claim** *Transparency* is satisfied $\Leftrightarrow C \vDash p$

c. *Proof*
$\Rightarrow$: Suppose that *Transparency* is satisfied. In particular, taking γ to be a tautology and β to be of the form *and* δ*)* for some tautology δ,
$C \vDash ((p \text{ and } \gamma) \text{ and } \delta) \Leftrightarrow (\gamma \text{ and } \delta)$, hence
$C \vDash p$

$\Leftarrow$: Suppose that $C \vDash p$. Then for each clause γ, $C \vDash (p \text{ and } \gamma) \Leftrightarrow \gamma$, and *Transparency* follows.

(15) (p and $\underline{q}q'$)

a. *Transparency* requires that for each clause γ and each sentence completion β, $C \vDash (p \text{ and } (q \text{ and } \gamma)\beta \Leftrightarrow (p \text{ and } \gamma\beta$

b. **Claim** *Transparency* is satisfied $\Leftrightarrow C \vDash p \Rightarrow q$

c. *Proof*
$\Rightarrow$: Suppose that *Transparency* is satisfied. In particular, taking β to be) and γ to be some tautology, we have:
$C \vDash (p \text{ and } (q \text{ and } \gamma)) \Leftrightarrow (p \text{ and } \gamma)$, hence
$C \vDash (p \text{ and } q) \Leftrightarrow p$
and in particular
$C \vDash p \Rightarrow q$
$\Leftarrow$: Suppose that $C \vDash p \Rightarrow q$. Then for each clause γ, $C \vDash (p \text{ and } (q \text{ and } \gamma)) \Leftrightarrow (p \text{ and } \gamma)$. For syntactic reasons, the only acceptable sentence completion β in *(p and* $\gamma\beta$ is $\beta =$ *)*. The result follows directly.

(16) (if $\underline{p}p' \cdot q$)

a. *Transparency* requires that for each clause γ and for each sentence completion β,
$C \vDash (\text{if } (p \text{ and } \gamma)\beta \Leftrightarrow (\text{if } \gamma\beta$

b. **Claim** *Transparency* is satisfied $\Leftrightarrow C \vDash p$

c. *Proof*
$\Rightarrow$: Suppose that *Transparency* is satisfied. In particular, taking γ to be some tautology β to be . δ) for some contradiction δ, we have:
$C \vDash (\text{if } (p \text{ and } \gamma).\ \delta) \Leftrightarrow (\text{if } \gamma.\ \delta)$, hence
$C \vDash (\text{not } (p \text{ and } \gamma)) \Leftrightarrow (\text{not } \gamma)$, and
$C \vDash (p \text{ and } \gamma) \Leftrightarrow \gamma$. But since γ is a tautology,
$C \vDash p$
$\Leftarrow$: Suppose that $C \vDash p$. Then for each clause γ, $C \vDash (p \text{ and } \gamma) \Leftrightarrow \gamma$, and *Transparency* follows.

(17) (if $p \cdot \underline{q}q'$)

a. *Transparency* requires that for each clause γ and each sentence completion β, $C \vDash (\text{if } p.\ (q \text{ and } \gamma)\beta \Leftrightarrow (\text{if } p \cdot \gamma\beta$

b. **Claim** *Transparency* is satisfied $\Leftrightarrow C \vDash p \Rightarrow q$

c. *Proof*
$\Rightarrow$: Suppose that *Transparency* is satisfied. In particular, taking β to be *)* and γ to be some tautology, we have
$C \vDash (\text{if } p \cdot (q \text{ and } \gamma)) \Leftrightarrow (\text{if } p \cdot \gamma)$, hence
$C \vDash (\text{if } p \cdot q)$
$\Leftarrow$: Suppose that $C \vDash p \Rightarrow q$. Then for each clause γ, $C \vDash (\text{if } p \cdot (q \text{ and } \gamma)) \Leftrightarrow (\text{if } p \cdot \gamma)$.

For syntactic reasons, the only acceptable sentence completion β in $(if\ p \cdot (q\ and\ \gamma)\beta$ is $\beta =)$. The result follows directly.

Having motivated and illustrated the Transparency theory, we now turn to its formal development.

3 Formal systems

3.1 Syntax

To make our analysis precise, we define a syntax in which the presuppositions of atomic clauses are underlined. In order to keep the proofs as simple as possible, our object language does not contain variables, and we do not include Boolean operators on predicates (except for $\underline{P}P'$, which is taken to be a predicate with assertive component P' and presupposition P; thus *stopped smoking* has as its presupposition something like *smoked* and as its assertive component *doesn't smoke;* in the Transparency framework, $\underline{P}P'$ has the semantics of predicate conjunction). In order to state various principles in the meta-theory, we enrich the object language with individual variables and quantifiers, as well as symbols for material implication and equivalence. We also need a (non-recursive) rule of predicate conjunction, which in a more realistic—but more complex—analysis would be part of the object language. The rules that are not part of the object language are indicated in bold.

(18) **Syntax**
-Generalized Quantifiers: Q ::= Q_i
-Predicates: P ::= P_i| $\underline{P_i}P_k$|(**P_i and P_k**)
-Propositions: p ::= p_i| $\underline{p_i}p_k$
-Individual variables: d ::=d_i
-Formulas F ::= p | (not F) | (F and F) | (F or F) | (if F · F) | (Q_iP · P) | **P(d)** |**∀d F** |**∃d F** | **[F ⇒ F]** | **[F ⇔ F]** [9]

Terminology We will say that p_i, $\underline{p_i}p_k$ are 'atomic propositions' and that P_i, $\underline{P_i}P_k$ are 'atomic predicates.'

It will be helpful to have at our disposal the following Lemma, where a 'constituent' is a predicate or a formula:

(19) **Syntactic Lemma**
a. If α is the beginning of a constituent in a string F, then α is the beginning of a constituent in any well-formed string that contains α.
Proof Given the syntax in (18), the beginning of a complex constituent is always marked by a left bracket (the case of atomic constituents is trivial). The end of this constituent is the first point to the right at which an equal number of left and right brackets has been encountered.
b. If a formula F starts with $(s$, where s is a symbol different from a parenthesis, then the smallest initial string of F which is a constituent is F itself.
Proof Suppose this were not the case, and suppose that c is a proper initial substring of F which is a constituent. Given that c is a *proper* substring of F, it must have been concatenated with other symbols (to its right) by one of the

9 In practice, we sometimes omit outer brackets and parentheses for the sake of legibility.

object-language rules in (18). But each such rule would require that there be a left parenthesis before *c*, contrary to our assumption that *c* is initial.

3.2 Semantics

□ *Framework and interpretation of lexical items*

We define the semantics for a (possibly infinite) domain of possible worlds W, each of which has a domain of individuals D^w of a fixed finite size n. We write $[A \rightarrow B]$ to denote the set of functions with domain A and codomain B, and we use standard type-theoretic notation wherever useful (e.g., <s, t> is the type of propositions, i.e., of functions from possible worlds to truth values; and <s, <e, t>> is the type of properties, i.e., of functions from possible worlds to characteristic functions of sets).

(20) **Interpretation of Lexical Items**

We define a static interpretation function I.

For all $i \geq 0$,

a. Q_i is a generalized quantifier satisfying Permutation Invariance, Extension and Conservativity (Keenan, 1996). Its value is entirely determined by a numerical function f_i in $[|N \times |N \rightarrow \{0,1\}]$, which we call the 'tree of numbers' of Q_i (van Benthem, 1986).

Thus for all $w \in W$, $I_w(Q_i)$ is of type <<e, t>, <<e, t>, t>> and for all A, B of type <e, t>, $I_w(Q_i)(A)(B) = 1$ iff $f_i(|A\text{-}B|, |A \cap B|) = 1$.

b. $I_w(P_i) \in [W \rightarrow [D \rightarrow \{0,1\}]]$ (i.e. it is of type <s, <e, t>>)

c. $I_w(p_i) \in [W \rightarrow \{0,1\}]$ (i.e. it is of type <s, t>)

□ *Dynamic Semantics and Dynamic Transparency*

Next, we define a dynamic semantics which is precisely that of Heim (1983), augmented by the analysis of disjunction offered in Beaver (2001) (Heim did not discuss disjunction). For notational simplicity, we write F^w instead of $I_w(F)$. And when certain elements are optional, we place angle brackets (<>) around them and around the corresponding part of the update rules.

(21) **Dynamic (Trivalent) Semantics**

Let C be a subset of W.

$C[p] = \{w \in C: p^w = 1\}$

$C[\underline{p}p'] = \#$ iff for some $w \in C$, $p^w = 0$; if $\neq \#$, $C[\underline{p}p'] = \{w \in C: p'^w = 1\}$

$C[(\text{not } F)] = \#$ iff $C[F] = \#$; if $\neq \#$, $C[(\text{not } F)] = C - C[F]$

$C[(F \text{ and } G)] = \#$ iff $C[F] = \#$ or ($C[F] \neq \#$ and $C[F][G] = \#$); if $\neq \#$, $C[(F \text{ and } G)] = C[F][G]$

$C[(F \text{ or } G)] = \#$ iff $C[F] = \#$ or ($C[F] \neq \#$ and $C[\text{not } F][G] = \#$); if $\neq \#$, $C[(F \text{ or } G)] = C[F] \cup C[\text{not } F][G]$

$C[(\text{if } F.\ G)] = \#$ iff $C[F] = \#$ or ($C[F] \neq \#$ and $C[F][G] = \#$); if $\neq \#$, $C[(\text{if } F.G)] = C - C[F][\text{not } G]$

$C[(Q_i <\underline{P}>P'.<\underline{R}>R') = \#$ iff <for some $w \in C$, for some $d \in D$, $\underline{P}^w(d) = 0$> or <for some $w \in C$, for some $d \in D$, <$\underline{P}^w(d) = 1$ and> $P'^w(d) = 1$ and $\underline{R}^w(d) = 0$>. If $\neq \#$, $C[(Q_i <\underline{P}>P'.<\underline{R}>R') = \{w \in C: f_i(a^w, b^w) = 1\}$ with $a^w = |\{d \in D: P'^w(d) = 1$ and $R'^w(d) = 0\}|$, $b^w = |\{d \in D: P'^w(d) = 1$ and $R'^w(d) = 1\}|$

Remark The meta-language also includes a limited form of predicate conjunction [e.g., (P_1 *and* P_2)], as specified by the syntactic rules in (18). Predicate conjunction is interpreted in the usual way: if P_1 denotes A and P_2 denotes B (both of type <s, <e, t>>), (P_1 *and* P_2) denotes $\lambda w_s.\ \lambda d_e.\ A(w)(d) = B(w)(d) = 1$.

If the actual world w belongs to C, we can recover a static notion of truth from Heim's dynamic semantics:

(22) **Truth**
If $w \in C$,
F is a presupposition failure in w iff C[F] = #;
F is true in w iff C[F] ≠ # and $w \in C[F]$; and
F is false in w iff C[F] ≠ # and $w \notin C[F]$.

(See Bonomi, 2006 for a recent discussion of the case in which $w \notin C$).

Given the semantics in (21), it is immediate that Heim's system is 'dynamically transparent,' in the following sense:

(23) **Dynamic Transparency**
Let *F* be a formula, and let *F** be the result of deleting from *F* all underlined material. Then for any $C \subseteq W$, if C[F] ≠ #, then C[F] = C[F*].
Proof By induction on the construction of formulas, we observe that no underlined material appears in the 'if ≠ #' part of the rules in (21).

It is worth asking whether Dynamic Transparency is a *general* property of Heim's dynamic semantics, or whether it is a consequence of a particular choice of lexical items. In the absence of further constraints on the semantics of connectives, the second alternative is the correct one. To see this, consider a deviant disjunction *or**, whose dynamic semantics is given in (24):

(24) C[(F or* G)] = # iff C[F] = # or (C[F] ≠ # and C[not F][G] = #); if ≠ #:
if C[G] ≠ #, C[(F or* G)] = C[(not G)][F] ∪ C[(not F)][G]
if C[G] = #, C[(F or* G)] = C[F] ∪ C[(not F)][G].

According to this semantics, *or** has exactly the same dynamic effect as *or* when C[G] = #. However, when C[G] ≠ #, C[(F or* G)] = C[(not G)][F] ∪ C[(not F)][G]. This is in particular the rule that must be applied if *F* and *G* contain no presupposition triggers. Now consider the formula H = ((not p) or* $\underline{p}p'$), with the assumption that C[$\underline{p}p'$] = #. We can apply (24) to *H* and to *H** (where *H** is *H* with the underlined material deleted), and we obtain:

C[H] = C[(not p) or* $\underline{p}p'$] = C[not p] ∪ C[not (not p)][$\underline{p}p'$] = C[not p] ∪ C[p][$\underline{p}p'$] = C[not p] ∪ C[p][p′] = $\{w \in C: p^w = 0$ or $(p^w = 1$ and $p'^w = 1)\} = \{w \in C: p^w = 0$ or $p'^w = 1\}$.

C[H*] = C[(not p) or* p′] = C[(not p′)][(not p)] ∪ C[p][p′] = $\{w \in C: p'^w = p^w = 0$ or $p'^w = p^w = 1\}$.

By construction, C[H] ≠ #, but in general C[H] ≠ C[H*] (for instance C[H] may contain C-worlds for which $p^w = 0$ and $p'^w = 1$, but C[H*] cannot). The conclusion is that in Heim's semantics Dynamic Transparency is not really 'built in'; it depends on a particular choice of lexical entries for the connectives (or of certain constraints, to be specified, on what these lexical entries can be).

□ *Static Semantics*

Since our goal is to show that the results of Heim's dynamic semantics can be obtained in a fully classical logic, we must specify a classical interpretation for the language defined in Sect. 3.1. The semantics is bivalent, and thus we only give 'if and only if' conditions for truth (falsity conditions immediately follow). As is natural in this framework, a clause $\underline{p}p'$ (or a predicate $\underline{P}P'$) is interpreted as a simple conjunction. Thus it is only when the principle of Transparency is applied that the underlined material turns out to impose conditions on the Context Set.

(25) **Static (Bivalent) Semantics**
$w \vDash p$ iff $p^w = 1$
$w \vDash \underline{p}p'$ iff $p^w = p'^w = 1$
$w \vDash$ (not F) iff $w \nvDash F$
$w \vDash$ (F and G) iff $w \vDash F$ and $w \vDash G$
$w \vDash$ (F or G) iff $w \vDash F$ or $w \vDash G$
$w \vDash$ (if F. G) iff $w \nvDash F$ or $w \vDash G$
$w \vDash (Q_i <\underline{P}>P'.<\underline{Q}>Q'$ iff $f_i(a^w, b^w) = 1$ with $a^w = |\{d \in D: <\underline{P}^w(d) = 1$ and$> P'^w(d) = 1$ and $(<\underline{Q}^w(d) = 0$ or$> Q'^w(d) = 0)\}|$, $b^w = |\{d \in D: <\underline{P}^w(d) = 1$ and$> P'^w(d) = 1$ and $<\underline{Q}^w(d) = 1$ and$> Q'^w(d) = 1\}|$.

(As noted above, conditionals are analyzed for simplicity as material implications).

□ *Transparency*

We repeat in a more concise form our Principle of Transparency, which was already mentioned in (6):

(26) **Principle of Transparency**
For any initial string of the form $\alpha\ \underline{d}d$ of a sentence uttered in a background of assumptions C (where $\underline{d}d'$ is propositional or predicative), it should be the case that for any constituent γ of the same type as d and for any sentence completion β,
$C \vDash \alpha$ (d and $\gamma)\beta \Leftrightarrow \alpha\gamma\beta$

□ *Terminology*
We write Transp(C, F) if F satisfies Transparency in C. Instead of writing 'for any constituent γ of the same type as *d*', we will often say 'for any appropriate γ'.

4 Propositional case

We now prove that in the propositional case the Transparency theory is equivalent to Heim's system. As noted, we assume that the language is sufficiently expressive to include tautologies and contradictions.

Theorem 1 *Consider the propositional fragment of the language defined above. For any formula F and for any* $C \subseteq W$*:*

(i) *Transp(C, F) iff* $C[F] \neq \#$.

(ii) *If C[F] $\neq$ #, C[F] = {w $\in$ C: w $\vDash$ F}.*

We start with a useful lemma:

(27) **Transparency Lemma**

a. If for some formula *G* and some sentence completion δ, Transp(C, (G δ), then Transp(C, G).

Proof Suppose, for contradiction, that not Transp(C, G). Then for some initial string $\alpha\ \underline{d}d'$ of *G*, for some appropriate expression γ, for some sentence completion β, and for some w $\in$ C,

w $\nvDash$ α (d and $\gamma)\beta \Leftrightarrow \alpha\gamma\beta$.

Taking τ to be some tautology, this entails that w $\nvDash$ (α (d and $\gamma)\beta$ and τ) $\Leftrightarrow$ ($\alpha\gamma\beta$ and τ). But this means that not Transp(C, (G δ), contrary to hypothesis.

b. If for some formula *G* and some sentence completion δ, Transp(C, (if G. δ), then Transp(C, G).

Proof Suppose, for contradiction, that Transp(C, (if G. δ) but not Transp(C, G). Then for some initial string $\alpha\ \underline{d}d'$ of *G*, for some appropriate expression γ, for some sentence completion β, and for some w $\in$ C, w $\nvDash$ α (d and $\gamma)\beta \Leftrightarrow \alpha\gamma\beta$. Taking ε to be some contradiction, it follows that w $\nvDash$ (if α (d and $\gamma)\beta \cdot \varepsilon$) $\Leftrightarrow$ (if $\alpha\gamma\beta \cdot \varepsilon$). But this means that not Transp(C, (if G $\cdot\delta$)), contrary to our hypothesis.

We can now proceed to the proof of Theorem 1.

Proof (by induction on the construction of formulas):

a. F = p

(i) C[F] $\neq$ # and Transp(C, F).

(ii) It is also clear that C[F] = {w $\in$ C: $p^w = 1$} = {w $\in$ C: w $\vDash$ F}.

b. F = $\underline{p}p'$

(i) If Transp(C, F), for any formula γ and for any sentence completion β, C $\vDash$ (p and γ) $\beta \Leftrightarrow \gamma\beta$, hence in particular C $\vDash$ (p and δ) $\Leftrightarrow$ δ for some tautology δ, and thus C $\vDash$ p. Therefore C[F] $\neq$ #.

Conversely, if C[F] $\neq$ #, C $\vDash$ p and thus for any clause γ, C $\vDash$ (p and γ) $\Leftrightarrow$ γ. It follows that for any clause γ and for any sentence completion β, C $\vDash$ (p and $\gamma)\beta \Leftrightarrow \gamma\beta$.[10] But this shows that Transp(C, F).

(ii) If C[F] $\neq$ #, C $\vDash$ p and C[F] = {w $\in$ C: $p'^w = 1$} = {w $\in$ C: $p'^w = p^w = 1$} = {w $\in$ C: w $\vDash$ $\underline{p}p'$}.

c. F = (not G)

(i) Suppose that Transp(C, F) and suppose, for contradiction, that C[F] = #. Then C[G] = # and by the Induction Hypothesis not Transp(C, G), i.e., for some initial string $\alpha\ \underline{d}d'$ of *G*, for some appropriate expression γ, for some sentence completion β, and for some world w $\in$ C,

w $\nvDash$ α (d and $\gamma)\beta \Leftrightarrow \alpha\gamma\beta$

10 In fact, the syntax in (18) guarantees that the only acceptable sentence completion is one in which β is the null string.

But if so, w $\nvDash$ (not α (d and $\gamma)\beta) \Leftrightarrow$ (not $\alpha\gamma\beta$), and hence not Transp(C, F). Contradiction. For the converse, suppose that C[F] $\neq$ #. Then C[G] $\neq$ #, and by the Induction Hypothesis Transp(C, G). Now suppose, for contradiction, that not Transp(C, F). Then for some initial string α $\underline{d}d'$ of G, for some appropriate expression γ, for some sentence completion β, and for some w $\in$ C,

w $\nvDash$ (not α (d and $\gamma)\beta \Leftrightarrow$ (not $\alpha\gamma\beta$

By the Syntactic Lemma in (19) (part (b)), *(not α (d and $\gamma)\beta$* is the smallest initial string of itself which is a constituent. It follows that β is of the form δ), and thus:

w $\nvDash$ (not α (d and $\gamma)\delta) \Leftrightarrow$ (not $\alpha\gamma\delta$)

and therefore

w $\nvDash$ α (d and $\gamma)\delta \Leftrightarrow \alpha\gamma\delta$

But this shows that not Transp(C, G). Contradiction.

(ii) If C[F] $\neq$ #, C[F] = C − C[G]. By the Induction Hypothesis, C[G] = {w $\in$ C: w $\vDash$ G} and thus C[F] = C − {w $\in$ C: w $\vDash$ G} = {w $\in$ C: w $\vDash$ (not G)}.

d. F = (G and H)

(i) Suppose that Transp(C, F). By the Transparency Lemma (part (a)), Transp(C, G). By the Induction Hypothesis, C[G] $\neq$ #, and by the Induction Hypothesis (Part (ii)) C[G] = {w $\in$ C: w $\vDash$ G}. Calling C′ = {w $\in$ C: w $\vDash$ G}, we claim that Transp(C′, H). For suppose this were not the case. For some initial string α $\underline{d}d'$ of *H*, for some appropriate expression γ, for some sentence completion β, and for some world w′ $\in$ C′, we would have w′ $\nvDash$ α (d and $\gamma)\beta \Leftrightarrow \alpha\gamma\beta$. But then w′ would refute Transp(C, (G and H)) because we would have w′ $\nvDash$ (G and α (d and $\gamma)\beta) \Leftrightarrow$ (G and $\alpha\gamma\beta$) with w′ $\vDash$ G (since w′ $\in$ C′). So Transp(C′, H), and thus by the Induction Hypothesis (Part (i)) C′[H] $\neq$ #, i.e., C[G][H] $\neq$ #.

For the converse, suppose that C[F] $\neq$ #. Then C[G] $\neq$ # and C[G][H] $\neq$ #. By the Induction Hypothesis, Transp(C, G), C[G] = {w $\in$ C: w $\vDash$ G} (a set we call *C′*), and Transp(C′, H). Suppose, for contradiction, that not Transp(C, F), and let w $\in$ C satisfy w $\nvDash$ α (d and $\gamma)\beta \Leftrightarrow \alpha\gamma\beta$, where α $\underline{d}d'$ is an initial string of *(G and H)*.

—Let us first show that this occurrence of $\underline{d}d'$ is not part of *G*. For suppose, for contradiction, that it is. Then for some initial string α' of *G* we have w | $\neq$ (α' (d and $\gamma)\beta \Leftrightarrow (\alpha'\gamma\beta$. $\alpha'\underline{d}d'$ is the beginning of a constituent in *G*, and thus by the Syntactic Lemma (part (a)), it is the beginning of a constituent in ($\alpha'\underline{d}d'$ β; therefore α' *(d and γ)* is the beginning of a constituent in α' *(d and $\gamma)\beta$*. Let β' be the smallest initial string of β for which α' *(d and $\gamma)\beta'$* is a constituent. Since w $\nvDash$ (α' (d and $\gamma)\beta \Leftrightarrow (\alpha'\gamma\beta$, it must also be that w $\nvDash$ α' (d and $\gamma)\beta' \Leftrightarrow \alpha'$ $\gamma\beta'$. But this shows that not Transp(C, G), contrary to what was shown earlier.

—So this occurrence of $\underline{d}d'$ appears in *H*. Thus for some initial string α' $\underline{d}d'$ of *H*, for some appropriate expression γ and for some sentence completion β, we have:

w $\nvDash$ (G and α' (d and $\gamma)\beta \Leftrightarrow$ (G and $\alpha'\gamma\beta$

Since α' $\underline{d}d'$ is the beginning of a constituent in *H*, α' $\underline{d}d'$ is also the beginning of a constituent in α' $\underline{d}d'$ β (Syntactic Lemma, part (a)); and α' *(d and γ)* is the beginning of a constituent in α' (*d and* $\gamma)\beta$. Furthermore, since *G* is a constituent, *(G and α' (d and $\gamma)\beta$* and *(G and $\alpha'\gamma\beta$* must be of the form *(G and α' (d and $\gamma)\beta'$)* and *(G and*

$\alpha'\gamma\beta')$ respectively. It follows that G must be true at w, for otherwise both formulas would be false and they would thus have the same value at w, contrary to hypothesis. So w ⊨ G. Since w ⊭ (G and α' (d and $\gamma)\beta'$) ⇔ (G and $\alpha'\gamma\beta'$), it must be that w ⊭ α' (d and $\gamma)\beta'$ ⇔ α' $\gamma\beta'$. But then it follows that not Transp(C′, H), since w ∈ C′ and α' $\underline{d}d'$ is an initial string of H. However this contradicts our hypothesis. Thus Transp(C, (G and H)), i.e. Transp(C, F).

(ii) If C[F] ≠ #, C[F] = C[G][H] = {w ∈ C: w |= G}[H] = {w ∈ C: w ⊨ (G and H)}

e. F = (G or H)

(i) Suppose that Transp(C, F). Then by the Transparency Lemma (part (a)), it is also the case that Transp(C, G). By the Induction Hypothesis, C[G] ≠ #, and C[G] = {w ∈ C: w ⊨ G}. Therefore C[(not G)] = C − C [G] = {w ∈ C: w | ≠ G} (call this set C'). It follows that Transp(C′, H) because otherwise for some initial string α $\underline{d}d'$ of H, for some appropriate expression γ, for some sentence completion β and for some w′ ∈ C′, we would have:

w′ ⊭ α (d and $\gamma)\beta$ ⇔ $\alpha\gamma\beta$

But since w′ ⊭ G, w′ ⊭ (G or α (d and $\gamma)\beta$) ⇔ (G or $\alpha\gamma\beta$), and thus not Transp(C, (G or H)), contrary to hypothesis. So Transp(C′, H), and by the induction hypothesis C′[H] ≠ #, i.e., C[(not G)][H] ≠ #. By the dynamic semantics of *or*, C[(G or H)] ≠ #.

For the converse, suppose that C[(G or H)] ≠ #. Thus C[G] ≠ # and C[(not G)][H] ≠ #. By the Induction Hypothesis, Transp(C, G) and Transp(C′, H) with C′ = C[(not G)]. Suppose, for contradiction, that not Transp(C, F), and let w ∈ C satisfy w ⊭ α (d and $\gamma)\beta$ ⇔ $\alpha\gamma\beta$, where α $\underline{d}d'$ is an initial string of*(G or H)*.

-Let us first show that this occurrence of $\underline{d}d'$ is not part of G. Suppose, for contradiction, that it is. Then for some initial string $\alpha'\underline{d}d'$ of G, for some appropriate expression γ, for some sentence completion β and for some w ∈ C, we have

w ⊭ (α' (d and $\gamma)\beta$ ⇔ ($\alpha'\gamma\beta$

$\alpha'\underline{d}d'$ is the beginning of a constituent in G, and thus by the Syntactic Lemma (part (a)), it is the beginning of a constituent in ($\alpha'\underline{d}d'$ β; and thus α' *(d and* γ*)* is the beginning of a constituent in *(*α' *(d and* $\gamma)\beta$. Let β' be the smallest initial string of β for which α' *(d and* $\gamma)\beta'$ is a constituent. Since w ⊭ (α' (d and $\gamma)\beta$ ⇔ ($\alpha'\gamma\beta$, it must also be that w ⊭ α' (d and $\gamma)\beta'$ ⇔ α' $\gamma\beta'$. But this shows that not Transp(C, G), contrary to what was shown earlier.

–So this occurrence of $\underline{d}d'$ appears in H. Thus for some initial string α' $\underline{d}d'$ of H, for some appropriate expression γ, for some sentence completion β and for some w ∈ C we have:

w ⊭ (G or α' (d and $\gamma)\beta$ ⇔ (G or $\alpha'\gamma\beta$

Since α' $\underline{d}d'$ is the beginning of a constituent in H, α' $\underline{d}d'$ is also the beginning of a constituent in α' $\underline{d}d$' β (Syntactic Lemma, part (a)). Furthermore, since G is a constituent, *(G or* α' *(d and* $\gamma)\beta$ and *(G or* $\alpha'\gamma\beta$ must be of the form *(G or* α' *(d and* $\gamma)\beta$*')* and *(G or* $\alpha'\gamma\beta')$ respectively. It follows that G must be false at w, for otherwise both formulas would be true and they would thus have the same value at w, contrary to hypothesis. So w ⊭ G. But since w ⊭ (G or α' (d and $\gamma)\beta'$) ⇔ (G or $\alpha'\gamma\beta'$), it must be that w ⊭ α' (d and γ) β' ⇔ α' $\gamma\beta'$. But then it follows that not Transp(C′, H), since w ∈ C′ and α' $\underline{d}d'$ is an initial string of H. But this contradicts our hypothesis that Transp(C′, H). Thus Transp(C, (G and H)), i.e. Transp(C, F).

(ii) If C[F] $\neq$ #, then C[G] $\neq$ #, C[(not G)][H] $\neq$ #, and C[F] = C[G] $\cup$ C[(not G)][H]. By the Induction Hypothesis, C[G] = {w $\in$ C: w $\vDash$ G}, C[(not G)] = {w $\in$ C: w $\nvDash$ G}, and C[(not G)][H] = {w $\in$ C: w $\nvDash$ G and w $\vDash$ H}. Therefore C[F]= {w $\in$ C: w $\vDash$ G} $\cup$ {w $\in$ C: w $\nvDash$ G and w $\vDash$ H} = {w $\in$ C: w $\vDash$ (G or H)}.

f. F = (if G. H)

(i) Suppose Transp(C, F). By the Transparency Lemma (part (b)), it must also be the case that Transp(C, G). Let us now show that Transp(C′, H) with C′ = C[G]. Suppose, for contradiction, that this is not the case. Then for some initial string α $\underline{d}d'$ of H, for some appropriate expression γ, for some sentence completion β, and for some w′ $\in$ C′,

w′ $\nvDash \alpha$ (d and $\gamma)\beta \Leftrightarrow \alpha\gamma\beta$

Since w′ $\in$ C′, it must also be the case that w′ $\nvDash$ (if G. α (d and $\gamma)\beta) \Leftrightarrow$ (if G. $\alpha\gamma\beta$), which shows that not Transp(C, (if G. H)), contrary to hypothesis. So Transp(C′, H), and thus C[G][H] $\neq$ #. Since C[G] $\neq$ # and C[G][H] $\neq$ #, C[(if G. H)] $\neq$ #.

For the converse, let us assume that C[F] $\neq$ #. Then C[G] $\neq$ # and C[G][H] $\neq$ #. By the Induction Hypothesis, Transp(C, G) and Transp(C′, H) with C′ = C[G]. Now suppose, for contradiction, that not Transp(C, (if G. H)). Then for some initial string α $\underline{d}d'$ of $G.H$, for some appropriate expression γ, for some sentence completion β, and for some w $\in$ C, we have

w $\nvDash$ (if α (d and $\gamma)\beta \Leftrightarrow$ (if $\alpha\gamma\beta$

-It couldn't be that this occurrence of $\underline{d}d'$ is in G because in that case we would have for some strings β' and β":

w $\nvDash$ (if α (d and $\gamma)\beta'$. β" $\Leftrightarrow$ (if $\alpha\gamma\beta'$. β"

where α *(d and* $\gamma)\beta'$ is a constituent. But this entails that w $\nvDash \alpha$ (d and $\gamma)\beta' \Leftrightarrow \alpha\gamma\beta'$, and hence that not Transp(C, G), contrary to what we showed earlier.

-Now suppose that this occurrence of $\underline{d}d'$ is in H. For some initial string α $\underline{d}d'$ of H, for some appropriate expression γ, for some sentence completion of the form β'*)*, and for some w $\in$ C,we have

w $\nvDash$ (if G. α (d and $\gamma)\beta'$) $\Leftrightarrow$ (if G. $\alpha\gamma\beta'$)

But then it must also be that w $\vDash$ G, for otherwise both sides of the biconditional would be true at w. Furthermore, it must be the case that α *(d and* $\gamma)\beta'$ is a constituent and that w $\nvDash \alpha$ (d and $\gamma)\beta' \Leftrightarrow \alpha\gamma\beta'$ [because otherwise we would have w $\vDash$ (if G. α (d and $\gamma)\beta'$) $\Leftrightarrow$ (if G. $\alpha\gamma\beta'$)]. But this shows that not Transp(C′, H), contrary to what we showed earlier. In sum, Transp(C, F).

(ii) If C[F] $\neq$ #, C[F] = C – C [G][not H]. But by the Induction Hypothesis C[G] = {w $\in$ C: w $\vDash$ G} and C[G][not H] = {w $\in$ C: w $\vDash$ G}[not H] = {w $\in$ C: w $\vDash$ (G and (not H))} and thus C[F] = {w $\in$ C: w $\nvDash$ (G and (not H))} = {w $\in$ C: w $\nvDash$ (if G.H)}.

5 Quantificational case

We now turn to the quantificational case, which we treat separately because it involves additional complications and leads to weaker equivalence results than the propositional case.

Heim's claim is that for any generalized quantifier Q, (i) $(Q\underline{P}P'.R)$presupposes that every individual in the domain satisfies P, and (ii) $(QP.\ \underline{R}R')$ presupposes that every individual in the domain that satisfies P also satisfies R.[11] We will find conditions under which these predictions are indeed derived from our system. We start by stating the conditions, and then we construct the proof in two steps: first, we obtain the desired result for quantificational formulas that are unembedded; second, we integrate the argument into a proof by induction that extends to all formulas of the language.

5.1 Non-triviality and Constancy

The equivalence with Heim's result turns out to be weaker than in the propositional case; it holds only when the Context Set satisfies additional constraints. To see why a weaker result is obtained, let us note that there could be a world w in which Transparency is satisfied because Q has a degenerate semantics. Consider the following scenario:

-In w, there are exactly 2 P-individuals, one of whom satisfies R and one of whom doesn't.
-The sentence uttered is $(QP.\underline{R}R')$ with Q = *less than three.*

Even though it is not the case that each P-individual satisfies R in w, Transparency is trivially satisfied with respect to w because for any predicative expression γ,

$$w \vDash (Q\ P.\ (R \text{ and } \gamma)) \Leftrightarrow (Q\ P.\ \gamma)$$

Of course the equivalence holds because *no matter what the nuclear scope Y is*, *(QP. Y)* is true at w: since there are exactly two P-individuals, *a fortiori* there are less than three individuals that satisfy both P and Y.

We will solve the problem by making two assumptions:

(i) First, we require that each quantificational clause should make a non-trivial contribution to the truth conditions (=*Non-Triviality*). Specifically, we require that for each initial string $\alpha\ A$ of any sentence uttered in a Context Set C, where A is a quantificational clause (i.e., a clause of the form $(Q_iG.H)$), there is at least one sentence completion β for which A makes a semantic contribution that could not be obtained by replacing A with a tautology T or a contradiction F. Thus Non-Triviality requires that for some sentence completion β,

$$C \nvDash \alpha\ A\ \beta \Leftrightarrow \alpha\ T\ \beta$$
$$C \nvDash \alpha\ A\ \beta \Leftrightarrow \alpha\ F\ \beta$$

If the Context Set only includes worlds with less than three P-individuals, Non-Triviality will automatically rule out any sentence of the form α $(Q\ P.\ \underline{R}R')\ \beta$ for Q = *less than three*. This is because when one has heard α $(Q\ P.\ \underline{R}R')$, one can determine that one can replace $(QP.\underline{R}R')$ with T without modifying the contextual meaning of the sentence, no matter how it ends.
(ii) This measure won't be enough, however. Suppose that C = {w, w′, w″}, where w is the world mentioned earlier in which there are exactly two P-individuals, while w′ and w″ are worlds that have exactly four P-individuals, with the following specifications:

[11] Heim (1983) observes that special provisions are needed for indefinites, which trigger extremely weak presuppositions. Thus *A fat man was pushing his bicycle* certainly doesn't presuppose that every fat man had a bicycle. We disregard this point in what follows (see Schlenker, 2006b for a remark on the treatment of indefinites in the Transparency framework).

w′: all *P*-individuals satisfy *R* and *R*′.
w″: all *P*-individuals satisfy *R* but no *P*-individual satisfies *R*′.

Consider the sentence ($QP.\underline{R}R'$). As before, Transparency is satisfied in w (despite the fact that in w some *P*-individual does not satisfy *R*). Furthermore, Transparency is also satisfied in w′ and w″ because in these worlds each *P*-individual satisfies *R*. Contrary to the case we considered in (i), however, *this situation is not ruled out by Non-Triviality*:

w′ $\nvDash$ (Q P. $\underline{R}$R′) $\Leftrightarrow$ T (the left-hand side is false, but the right-hand side is true),
w″ $\nvDash$ (Q P. $\underline{R}$R′) $\Leftrightarrow$ F (the left-hand side is true, but the right-hand side is false).

In this counter-example, however, it is crucial that the extension of *P* does not have the same size in w ($|P^w \vDash 2$) and in w′ and w″ ($|P^{w'} \vDash |P^{w''} \vDash 4$). We will see that that this property is indeed essential to construct the problematic examples, and that when Non-Triviality is combined with the requirement (=*Constancy*) that the size of the extension of each restrictor be fixed throughout the Context Set, the equivalence with Heim's theory can indeed be achieved. We will come back to this point at the end of this paper, where the empirical plausibility of Constancy is discussed.[12]

Before we prove our (limited) equivalence result, let us give a precise statement of Non-Triviality:

(28) **Definition of Non-Triviality**
Let C be a Context Set and let *F* be a formula. <C, F> satisfies Non-Triviality just in case for any initial string of the form α *A*, where *A* is a quantificational clause (i.e., a formula of the form ($Q_iG.H$)), there is a sentence completion β such that:
C $\nvDash$ α A β $\Leftrightarrow$ α T β
C $\nvDash$ α A β $\Leftrightarrow$ α F β
where *T* is a tautology and *F* is a contradiction.

An immediate consequence of the definition will turn out to be useful:

(29) **Non-Triviality Corollary**
Let Q_i be a generalized quantifier with the associated tree of numbers f_i. Consider a formula ($Q_iG.H$) evaluated in a Context Set C. Then:
(i) If <C, (Q_i G . H) > satisfies Non-Triviality and if in C the domain of individuals is of constant finite size n,

[12] One additional condition is worth emphasizing. In our discussions, we have restricted attention to the case in which the domain of objects is finite and of constant size throughout the Context Set. This is in part to keep the discussion manageable, but also to avoid essential problems that arise with infinite domains. Consider the sentence ($QP.\underline{R}R'$) with Q = *infinitely many*. We claim that as long as $P^w - R^w$ is finite, any world w guarantees that for any predicative expression γ and for any sentence completion β,

(i) w $\vDash$ (Q P. (R and γ) β $\Leftrightarrow$ (Q P. $\gamma\beta$

For syntactic reasons, β can only be *)*. The left-to-right direction is then immediate. Now suppose that w $\vDash$ (Q P. γ), and that $P^w - R^w$ is finite. Since w $\vDash$ (Q P. γ), $|P^w \cap \gamma^w \vDash \infty$. But $|P^w \cap \gamma^w \vDash |(P^w \cap R^w) \cap \gamma^w| + |(P^w - R^w) \cap \gamma^w|$. Since $P^w - R^w$ is finite, $|(P^w \cap R^w) \cap \gamma^w \vDash \infty$ and w $\vDash$ (Q P. (R and γ)). This proves the right-to-left direction. By restricting attention to the case in which the domain of objects is finite and of constant size, we avoid this problem in the present paper. We leave it for future research to find conditions under which our results can be generalized when this hypothesis is relaxed.

$\{f_i(a, b)$: a, b $\in \mathbb{N}$ and $a+b \leq n\} = \{0, 1\}$,

(ii) If <C, (Q_i G . H)> satisfies Non-Triviality and if in C the extension of *G* is of constant finite size g,

$\{f_i(a, b)$: a, b $\in \mathbb{N}$ and $a+b = g\} = \{0, 1\}$.

Proof (i) If $\{f_i(a, b)$: a, b $\in \mathbb{N}$ and $a+b \leq n\} = \{v\}$ for $v \in \{0, 1\}$, and if the domain has size n throughout C, then $(Q_iG.H)$ must have value v throughout C, which violates Non-Triviality. (ii) If $\{f_i(a, b)$: a, b $\in \mathbb{N}$ and $a+b = g\} = \{v\}$ for $v \in \{0, 1\}$, and if the extension of *G* has size g throughout C, $(Q_iG.H)$ must have value v throughout C, which violates Non-Triviality. □

5.2 Unembedded quantificational sentences

We come to a proof of equivalence with Heim's results for unembedded quantificational sentences.

Lemma 1 *Let Q_i be a generalized quantifier with the associated tree of numbers f_i.*

(i) *Suppose that (a) throughout C, the domain of individuals is of constant finite size n,*
(b) any property over the domain can be expressed by some predicate, and
(c) $\{f_i(a, b)$: a, b $\in \mathbb{N}$ and $a+b \leq n\} = \{0, 1\}$.
Then Transp(C, (Q_i $\underline{P}P'$. R)) iff C $\vDash \forall d\, P(d)$.

(ii) *Suppose that (a) throughout C, the extension of P is of constant finite size p,*
(b) any property over the domain can be expressed by some predicate,
and
(c) $\{f_i(a, b)$: a, b $\in \mathbb{N}$ and $a+b = p\} = \{0, 1\}$.

Then Transp(C, (Q_i P . $\underline{R}R'$)) iff C $\vDash \forall d\, [P(d) \Rightarrow R(d)]$.

Remark By the Non-Triviality Corollary:

(i) c can be replaced with: <*C*, (Q_i $\underline{P}$ P'. *R*)> *satisfies Non-Triviality*, and
(ii) c can be replaced with: <C, (Q_i*P*. $\underline{R}$ R')> *satisfies Non-Triviality.*

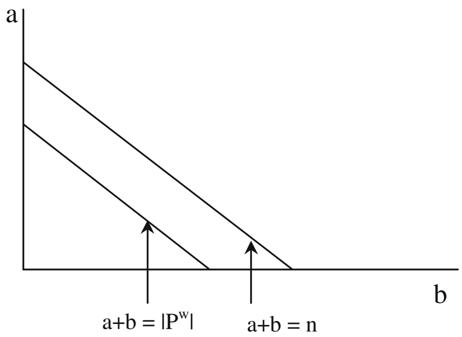
Proof We note that in (i) and (ii) the *if* part is immediate (for (ii), because of Conservativity), and we just discuss the *only if* part. We will make crucial use of the fact that the semantics of Q_i can be represented in terms of the tree of numbers $f_i(a, b)$, for variable *a* and *b*.

Part (i): Presupposition Trigger in the Restrictor
Let us suppose, for contradiction, that for some world w $\in$ W, w $\vDash \exists$d (not P(d)). We show that this suffices to falsify Transparency in w: there exist predicates *X* and *Y* for which

$$w \nvDash (Q_i(P \text{ and } X).Y)) \Leftrightarrow (Q_iX.Y)$$

Note that by our assumption that any property is expressible, it is enough to construct X^w and Y^w.

The semantic situation can be represented in a graph in which *a* (i.e., the number of elements that satisfy the restrictor but not the nuclear scope) is on the vertical axis and *b* (i.e., the number of elements that satisfy both the restrictor and the nuclear scope) is on the horizontal axis. As before, we write P^w for the value of *P* in w, and *n* for the (fixed) size of the domain $D^{w'}$ at any world w′.



By assumption, $f_i(a, b)$ is not constant throughout the larger triangle, which represents the set of points (a, b) for which $a+b \leq n$. We now distinguish two cases, and show that in either case Transparency is refuted.

Case 1. $f_i(a, b)$ is not constant throughout the smaller triangle (including the segment $a+b = |P^w|$). We claim that (i) or (ii) holds:

(i) for some (a, b) for which $a+b \leq n$ with $a \geq 1$, $f_i(a-1, b) \neq f_i(a, b)$, or
(ii) for some (a, b) for which $a+b \leq n$ with $b \geq 1$, $f_i(a, b-1) \neq f_i(a, b)$

The claim follows because the negation of (i) and (ii) would allow us to prove by finite iteration that, throughout the smaller triangle, $f_i(a, b) = f(0, 0)$: the negation of (i) entails that all the vertical lines in the triangle correspond to the same value for $f_i(a, b)$; and the negation of (ii) entails that all the horizontal lines correspond to the same value.

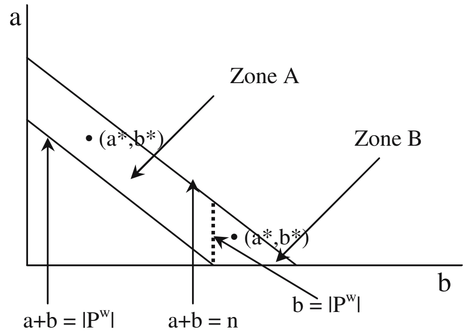
In both cases, we construct X^w by taking the union of

(1) (a+b−1) elements of P^w, together with
(2) one element of $D^w - P^w$ — which is possible since by assumption $w \vDash \exists d$ (not P(d)).

—In Case (i), we construct Y^w by taking b elements from (1). By construction, $|(X^w \cap P^w) - Y^w \vDash a-1$, $|X^w - Y^w \vDash a$, $|(X^w \cap P^w) \cap Y^w \vDash |X^w \cap Y^w \vDash b$, and thus *X*, *Y* refute Transparency in w: $w \nvDash (Q_i(P \text{ and } X).Y)) \Leftrightarrow (Q_i X.Y)$

—In Case (ii), we construct Y^w by taking (b-1) elements from (1), together with the lone element of (2). By construction, $|(X^w \cap P^w) - Y^w \vDash |X^w - Y^w \vDash a$, $|(X^w \cap P^w) \cap Y^w \vDash b-1$, and $|X^w \cap Y^w \vDash b$. Here too, *X*, *Y* refute Transparency in w.

Case 2. $f_i(a, b)$ is constant throughout the smaller triangle (including the segment $a+b = |P^w|$). Since by assumption $f_i(a, b)$ is not constant throughout the larger triangle, there must be a point (a*, b*) for which (i) (a*, b*) belongs to the rest of the larger triangle, and (ii) $f_i(a^*, b^*)$ is different from the (constant) value obtained throughout the smaller triangle. This gives rise to the following picture, were Zone A and Zone B are separated by the vertical line $b = |P^w|$:



-If (a*, b*) is in Zone A, there is a point of the line a+b = $|P^w|$ with the same *b*-coordinate as (a*, b*) (this is the vertical projection of (a*, b*) onto the line). In other words, for some c satisfying $0 < c \leq |D^w\text{-}P^w|$, $f_i(a^* - c, b^*) \neq f_i(a^*, b^*)$. We construct X^w by taking the union of

(1) $(a^* + b^* - c)$ elements from P^w (this is possible because (a*, b*) is in Zone A, hence $a^* - c \geq 0$), together with
(2) c elements from $D^w\text{-}P^w$.

We construct Y^w by taking b* elements from (1) (again, this is possible because $a^* - c \geq 0$, and thus $a^* + b^* - c \geq b^*$).

By construction, $|(X^w \cap P^w) - Y^w \vDash a^* - c$, $|X^w - Y^w \vDash a^*$, and $|(X^w \cap P^w) \cap Y^w \vDash |X^w \cap Y^w \vDash b^*$. Thus *X*, *Y* falsify Transparency at w:

w $\nvDash$ (Q_i(P and X).Y)) $\Leftrightarrow$ (Q_iX.Y)

-If (a*, b*) is in Zone B, there is a point of the line $a + b = |P^w|$ with the same *a*-coordinate as (a*, b*) (this is the horizontal projection of (a*, b*) onto the line). Thus for some c satisfying $0 < c \leq |D^w - P^w|$, $f_i(a^*, b^* - c) \neq f_i(a^*, b^*)$. We construct X^w by taking

(1) $(a^* + b^* - c)$ elements from P^w (this is possible because (a*, b*) is in Zone B, and thus $b^* - c \geq 0$), and
(2) c elements from $D^w - P^w$.

We construct Y^w by taking the union of $(b^* - c)$ elements from (1) and all c elements from (2). By construction, $|(X^w \cap P^w) - Y^w \vDash |X^w - Y^w \vDash a^*$, $|(X^w \cap P^w) \cap Y^w \vDash b^* - c$, and $|X^w \cap Y^w \vDash b^*$. Here too, *X*, *Y* falsify Transparency at w.

Part (ii): Presupposition Trigger in the Nuclear Scope

We assume that the size p of the extension of *P* is constant over C, and we suppose, for contradiction, that for some w ∈ C, w $\nvDash \forall$ d [P(d) $\Rightarrow$ R(d)]. We show that for a certain *Y*, Transparency is refuted at w:

w $\nvDash$ (Q_i P. (R and Y)) $\Leftrightarrow$ (Q_i P. Y).

Since $\{f_i(a, b)$: a, b ∈ | N and a+b = p} = {0, 1}, for some $b^* \geq 0$, $f_i(p - b^*, b^*) \neq f_i(p - b, b)$ with $b = |P^w \cap R^w|$.

Case 1. b > b*

There must be some b' < b for which $f_i(p - b', b') \neq f_i(p - (b' + 1), b' + 1)$ (otherwise one could prove that for all b' < b, $f_i(p - b', b') = f_i(p - b, b)$).

We construct Y^w by taking

(1) b' elements from $P^w \cap R^w$, and
(2) one element from $P^w - R^w$ — which is possible because w $\nvDash \forall$ d [P(d) $\Rightarrow$ R(d)].

By construction, $|P^w \cap Y^w \cap R^w \vDash b'$ and $|P^w \cap Y^w \vDash b'+1$, and thus

$w \nvDash (Q_i\ P.\ (R \text{ and } Y)) \Leftrightarrow (Q_i\ P.\ Y)$

Case 2. $b < b^*$

We construct Y^w by taking

(1) all b elements from $P^w \cap R^w$, and

(2) $b^* - b$ elements from $P^w - R^w$ – which is possible because $|P^w - R^w \vDash p - b$ and, since $b < b^* \leq p$, $b^* - b \leq p - b$.

By construction, $|P^w \cap Y^w \cap R^w \vDash b$ and $|P^w \cap Y^w \vDash b^*$, and thus

$w \nvDash (Q_i\ P.\ (R \text{ and } Y)) \Leftrightarrow (Q_i\ P.\ Y)$

5.3 Inductive step

In the preceding section, we found conditions under which Transparency is equivalent to Heim's results for unembedded quantificational sentences. We must now combine this result with the equivalence proof developed for the propositional case to yield a result that holds of quantificational languages. We will do so in two steps:

(i) First, we show in Lemma 2 that if <C, F> satisfies Non-Triviality, then all the pairs <C′, F″ > which must be 'accessed' (in a sense to be made precise) in the computation of C[F] also satisfy Non-Triviality.

(ii) Second, we combine the results of Lemma 1 and Lemma 2 to provide a general equivalence result between Transparency and Heim's results for quantificational languages.

We start by defining the pairs <C, F> which must be 'accessed' in the computation of C[F].

Definition Let C be a Context Set and F be a formula. We simultaneously define the relation *<C′, F′ > is accessed by <C, F>* and *<C″, F″ > is a parent of <C′, F′ >* by the following induction:

(i) <C, F > is accessed by <C, F>.

(ii) If <C′, (not F')> is accessed by <C, F>, then <C′, F′ > is accessed by <C, F> and <C′, (not F')> is the parent of <C′, F′ >.

(iii) If <C′, (G and H)> is accessed by <C, F>, then <C′, G> is accessed by <C, F> and <C′, (G and H)> is the parent of <C′, G>; and if C′[G] is defined, <C′[G], H> is accessed by <C, F> and <C′, (G and H)> is the parent of <C′[G], H>.

(iv) If <C′, (G or H)> is accessed by <C, F>, then <C′, G> is accessed by <C, F> and <C′, (G or H)> is the parent of <C′, G>; and if C′[G] is defined, <C′[(not G)], H> is accessed by <C, F> and <C′, (G or H)> is the parent of <C′[(not G)], H>.

(v) If <C′, (if G . H)> is accessed by <C, F>, then <C′, G> is accessed by <C, F> and <C′, (if G . H)> is the parent of <C′, G>; and if C′[G] is defined, <C′[G], H> is accessed by <C, F> and <C′, (if G . H)> is the parent of <C′[G], H>.

Lemma 2 *Suppose that <C, F> satisfies Non-Triviality. Then if <C′, F′> is accessed by <C, F>, <C′, F′> satisfies Non-Triviality as well.*

Proof We show by induction that if <C′, F′> is accessed by <C, F> and violates Non-Triviality, either <C′, F'>=<C, F>, or <C′, F′ > has a parent that also violates Non-Triviality. A trivial induction on the definition of pairs <C′, F′> that are accessed by <C, F> will then yield the Lemma.

(i) The case <C′, F’>=<C, F> is trivial.
(ii) If <C′, (not F’)> is the parent of <C′, F′> and <C′, F′> violates Non-Triviality, it is immediate that <C′, (not F’)> violates Non-Triviality as well.
(iii) a. Suppose that <C′, (G and H)> is the parent of <C′, G>, and that <C′, G> violates Non-Triviality. Then for some initial string α *A* of *G*, where *A* is a quantificational clause, for every sentence completion β,

$C' \vDash \alpha \text{ A } \beta \Leftrightarrow \alpha \text{ T } \beta$ or $C' \vDash \alpha \text{ A } \beta \Leftrightarrow \alpha \text{ F } \beta$

Now suppose, for contradiction, that <C′, (G and H)> satisfies Non-Triviality. In particular, there must be some sentence completion β’ for which

$C' \nvDash (\alpha \text{ A } \beta' \Leftrightarrow (\alpha \text{ T } \beta'$
$C' \nvDash (\alpha \text{ A } \beta' \Leftrightarrow (\alpha \text{ F } \beta'$

Since α *A* is the beginning of a constituent in *G*, it is the beginning of a constituent — call it α *A* γ- in (α *A* β’. But this entails that

$C' \nvDash \alpha \text{ A } \gamma \Leftrightarrow \alpha \text{ T } \gamma$
$C' \nvDash \alpha \text{ A } \gamma \Leftrightarrow \alpha \text{ F } \gamma$

However this refutes our assumption about α *A*.

b. Suppose that <C′, (G and H)> is the parent of <C′[G], H>, and that <C′[G], H> violates Non-Triviality. Then for some initial string α *A* of *H*, where *A* is a quantificational clause, for every sentence completion β,

$C'[G] \vDash \alpha \text{ A } \beta \Leftrightarrow \alpha \text{ T } \beta$ or $C'[G] \vDash \alpha \text{ A } \beta \Leftrightarrow \alpha \text{ F } \beta$

Now suppose, for contradiction, that <C′, (G and H)> satisfies Non-Triviality. In particular, there must be some sentence completion β' for which

$C' \nvDash (\text{G and } \alpha \text{ A } \beta' \Leftrightarrow (\text{G and } \alpha \text{ T } \beta'$
$C' \nvDash (\text{G and } \alpha \text{ A } \beta' \Leftrightarrow (\text{G and } \alpha \text{ F } \beta'$

Thus:

for some $w \in C'$, $w \nvDash (\text{G and } \alpha \text{ A } \beta' \Leftrightarrow (\text{G and } \alpha \text{ T } \beta'$
for some $w' \in C'$, $w' \nvDash (\text{G and } \alpha \text{ A } \beta' \Leftrightarrow (\text{G and } \alpha \text{ F } \beta'$

It follows that *G* is true at w (otherwise the left-hand side and the right-hand side of the biconditional would both be false), and by similar reasoning *G* is true at w′.

Since α *A* is the beginning of a constituent in *H*, it is the beginning of a constituent - call it α *A* γ- in *(G and* α *A* β'. This entails that

$w \nvDash \alpha \text{ A } \gamma \Leftrightarrow \alpha \text{ T } \gamma$
$w' \nvDash \alpha \text{ A } \gamma \Leftrightarrow \alpha \text{ F } \gamma$

Since $w \vDash$ G and $w' \vDash$ G, it follows that

$C'[G] \nvDash \alpha \text{ A } \gamma \Leftrightarrow \alpha \text{ T } \gamma$
$C'[G] \nvDash \alpha \text{ A } \gamma \Leftrightarrow \alpha \text{ F } \gamma$

But this refutes our hypothesis about α *A*.

(iv) a. Suppose that <C′, (G or H)> is the parent of <C′, G>, and that <C′, G> violates Non-Triviality. By the same argument as in (iii)a, it can be shown that <C′, (G or H)> violates Non-Triviality as well.

b. Suppose that <C′, (G or H)> is the parent of <C′[(not G)], H>, and that <C′[(not G)], H> violates Non-Triviality. By the same type of argument as in (iii)b, it can be shown by contradiction that <C′, (G or H)> violates Non-Triviality as well. In a nutshell, we take α *A* to be responsible for a violation of Non-Triviality in *H*relative to C′[(not G)], and we show in the end that for some worlds w, $w' \in C$,

$w \vDash (\text{not } G)$
$w' \vDash (\text{not } G)$
$w \nvDash \alpha\ A\ \gamma \Leftrightarrow \alpha\ T\ \gamma$
$w' \nvDash \alpha\ A\ \gamma \Leftrightarrow \alpha\ F\ \gamma$

It follows that

$C[(\text{not } G)] \nvDash \alpha\ A\ \gamma \Leftrightarrow \alpha\ T\ \gamma$
$C[(\text{not } G)] \nvDash \alpha\ A\ \gamma \Leftrightarrow \alpha\ F\ \gamma$

But this refutes our hypothesis that α *A* is responsible for a violation of Non-Triviality in *H* relative to C′[(not G)].

(v) a. Suppose that <C′, (if G . H)> is the parent of <C′, G>, and that <C′, G> violates Non-Triviality. By the same argument as in (iii)a, it can be shown that <C′, (if G . H)> violates Non-Triviality as well.

b. Suppose that <C′, (if G . H)> is a parent of <C′[G], H>, and that <C′[G], H> violates Non-Triviality. Then for some initial string α *A* of H, where *A* is a quantificational clause, for every sentence completion β,

$C'[G] \vDash \alpha\ A\ \beta \Leftrightarrow \alpha\ T\ \beta$ or $C \vDash \alpha\ A\ \beta \Leftrightarrow \alpha\ F\ \beta$

Now suppose, for contradiction, that <C′, (if G . H)> satisfies Non-Triviality. In particular, there must be some sentence completion β’ for which

$C' \nvDash (\text{if } G.\ \alpha\ A\ \beta' \Leftrightarrow (\text{if } G\ .\ \alpha\ T\ \beta'$
$C' \nvDash (\text{if } G\ .\ \alpha\ A\ \beta' \Leftrightarrow (\text{if } G\ .\ \alpha\ F\ \beta'$

Thus:
for some $w \in C'$, $w \nvDash (\text{if } G\ .\ \alpha\ A\ \beta' \Leftrightarrow (\text{if } G\ .\ \alpha\ T\ \beta'$
for some $w' \in C'$, $w' \nvDash (\text{if } G.\ \alpha\ A\ \beta' \Leftrightarrow (\text{if } G\ .\ \alpha\ F\ \beta'$

It follows that *G* is true at w (otherwise the left-hand side and the right-hand side of the bi-conditional would both be true), and by similar reasoning *G* is true at w′.

Since α *A* is the beginning of a constituent in *H*, it is the beginning of a constituent - call it α *A* γ - in *(if G . α A β’*. This entails that

$w \nvDash \alpha\ A\ \gamma \Leftrightarrow \alpha\ T\ \gamma$
$w' \nvDash \alpha\ A\ \gamma \Leftrightarrow \alpha\ F\ \gamma$

Since $w \vDash G$ and $w' \vDash G$, it follows that

$C'[G] \nvDash \alpha\ A\ \gamma \Leftrightarrow \alpha\ T\ \gamma$
$C'[G] \nvDash \alpha\ A\ \gamma \Leftrightarrow \alpha\ F\ \gamma$

But this refutes our hypothesis about α *A*. □

We are finally in a position to prove Theorem 2.

Theorem 2 *Let C be a Context Set and let F be a formula. Suppose that* (*i*) *the domain of individuals is of constant finite size over C, and* (*ii*) *the extension of each restrictor that appears in F is of constant size over C, and* (*iii*) *<C, F> satisfies Non-Triviality. Then for every < C′, F′ > which is accessed by <C, F>* (*including <C, F> itself*)*:*

(i) *Transp*(C', F') *iff* $C'[F'] \neq \#$.
(ii) *If* $C'[F'] \neq \#$, $C'[F'] = \{w \in C' : w \vDash F'\}$.

Proof The argument is by induction on the construction of F'. It is similar to the proof of Theorem 1, with some additions to Steps (a) through (f) and one additional step (Step (g)).
(a)–(b): no addition is needed

(c): F′ = (not G)

If <C′, F′> is accessed by <C, F>, <C′, G> is accessed by <C, F> as well, and we can use the Induction Hypothesis and proceed as in the proof of Theorem 1.

(d) F′ = (G and H)

Suppose that <C′, F′> is accessed by <C, F>. If <C′, G> and <C′[G], H> are accessed by <C, F>, the proof proceeds as in the propositional case. Otherwise, it must be that <C′, G> is accessed by <C, F>, and C′[G] = #. By the dynamic semantics of conjunction, C′[F′] = #. All that remains to be shown is that not Transp(C′, F′).

Since <C′, G> is accessed by <C, F>, by Lemma 2 <C′, G> satisfies Non-Triviality. Therefore we can apply the Induction Hypothesis to <C′, G> and obtain the result that not Transp(C′, G) (since C′[G] = #). But this entails that not Transp(C′, (G and H)) (by the Transparency Lemma, part (a)).

(e) F′ = (G or H)

The argument is the same as in (d): if both <C′, G> and <C′[(not G)], H> are accessed by <C, F>, the proof proceeds as in the propositional case. If not, it must be that <C′, G> is accessed and C′[(not G)] = #. By the dynamic semantics of disjunction, C′[F′] = # and we only have to show that not Transp(C′, F′).

Since <C′, G> is accessed by <C, F>, <C′, G> satisfies Non-Triviality, and we can apply the Induction Hypothesis to show that not Transp(C′, G). But this entails that not Transp(C′, (G or H)) (by the Transparency Lemma, part (a)).

(f) F′ = (if G . H)

The argument is the same as in (d): if both <C′, G> and <C′[G], H> are accessed by <C, F>, the proof proceeds as in the propositional case. If not, it must be that <C′, G> is accessed and C′[G] = #. By the dynamic semantics of conditionals, C′[F′] = # and we only have to show that not Transp(C′, F′).

Since <C′, G> is accessed by <C, F>, we can apply the Induction Hypothesis to show that not Transp(C′, G). By the Transparency Lemma (part (b)), this entails that Transp(C′, (if G. H)).

(g) F′ = (Q_iG. H)

Suppose that <C′, F′> is accessed by <C, F>. Since <C, F> satisfies Non-Triviality, by Lemma 2 <C′, F′> does as well. Furthermore, it was shown in Lemma 1 that if <C′, F′> satisfies Non-Triviality,

(a) Transp(C′, (Q_i $\underline{P}P'$. R)) iff C′ ⊨ ∀d P(d)
(b) If the size of the extension of *P* is constant over C,

Transp(C′, (Q_i P . $\underline{R}R'$)) iff C′ ⊨ ∀ d [P(d) ⇒ R(d)]
Parts (i) and (ii) of the Theorem will now follow easily.

Part (i)
Given (a) and (b), the only remaining cases we need to consider are:
F′ = (Q_i P . R): this is trivial.
F′ = (Q_i $\underline{P}P'$. $\underline{R}R'$). Here we show successively that:

1. If Transp(C′, F′), then $C' \vDash \forall d\, P(d)$ and $C' \vDash \forall d\, [P(d) \Rightarrow R(d)]$ (as in Lemma 1).
2. If $C' \vDash \forall d\, P(d)$ and $C' \vDash \forall\, d\, [P(d) \Rightarrow R(d)]$, then Transp(C′, F′) (immediate).

Part (ii)

This is immediate as well. Let $F' = (Q_i\ <\underline{P}>P'.<\underline{R}>R')$ (where the material inside angle brackets is optional). If $C'[F'] \neq \#$,

$C'[F'] = \{w \in C: f_i(a^w, b^w)=1\}$ with $a^w = |\{d \in D: P'^w(d) = 1 \text{ and } R'^w(d) = 0\}|$, $b^w = |\{d \in D: P'^w(d) = 1 \text{ and } R'^w(d) = 1\}|$ (by the definition of our dynamic semantics; note that $\underline{P}$ and $\underline{R}$ play no role)

$= \{w \in C: w \vDash (Q_i P'.\ R')\}$
$= \{w \in C: w \vDash (Q_i\ <\underline{P}>P'.<\underline{R}>R')\}$ because $C'[F'] \neq \#$, and thus $C' \vDash \forall d\, P(d)$ and $C' \vDash \forall d\, [P(d) \Rightarrow R(d)]$ □

6 Conclusion

The main result of this investigation is that Heim's projection results can be derived from a theory which is fully classical, and in which the projection behavior of connectives and quantifiers is predictable from their (static) truth-conditional contribution and their syntax. In the propositional case Heim's results can be derived in full generality. In the quantificational case, the equivalence holds under the conditions of Constancy and Non-Triviality.

We conclude with some general comments and some perspectives for future research.

1. An immediate advantage of the Transparency theory is that it solves the overgeneration problem that plagued Heim's proposal. To illustrate, let us consider the connective *unless*, which is not discussed in Heim 1983. As a first approximation, the bivalent content of *Unless F, G* is the same as that of *If not F, G*. Of course from this it *does not follow* that the dynamic meaning of *unless* is that of *if not*. Specifically, all of the lexical rules in (32) below make exactly the same predictions when *F* and *G* contain no presupposition triggers (they are bivalently equivalent), but when this is not so the predictions diverge:

(32) a. C[unless F, G] = # iff C[F] = # or (C[F] $\neq$ # and C[(not F)][G] = #).
If $\neq$ #, C[unless F, G] = C − C [(not F)][(not G)]
b. C[unless F, G] = # iff C[F] = # or C[F][G] = #
If $\neq$ #, . . . (as in (a)).
c. C[unless F, G] = # iff C[F] = # or C[G] = #
If $\neq$ #, . . . (as in (a)).
d. C[unless F, G] = # iff C[G] = # or (C[G] $\neq$ # and C[(not G)][F] = #).
If $\neq$ #, C[unless F, G] = C − C [(not G)][F]

Consider now the sentence in (33):

(33) Unless John didn't come, Mary will know that he is here.

(33) seems to presuppose that *if John came, he is here*. This is exactly the prediction made by (32)a: since the *unless*-clause contains no presupposition trigger, the presupposition is that C[(not F)][G] $\neq$ # with F = *John didn't come* and G = *John is here*. The result follows immediately. By the same reasoning, (32)b predicts that the sentence

should presuppose that *if John didn't come, he is here*. This is squarely incorrect. And (32)c predicts a presupposition that *John is here*, which is probably too strong. Now (32)b and (32)c could potentially be ruled out by somehow requiring that the formulas of the form C′[*F*'] that appear in the definedness conditions be the same as those that appear in the update rules themselves. But this strategy won't suffice to rule out (32)d: building on the equivalence between *if not F, G* and *if not G, F*, we have given in (32)d definedness-cum-update rules that are entirely natural. The expected presupposition is in this case that *John is here*, arguably an incorrect result. Heim's theory fails to explain *why* (32)a is correct but (32)d isn't. For the Transparency theory, the explanation is immediate. *Unless* has essentially the same syntax and the same bivalent contribution as *if-not*, and therefore it should have the same projection behavior.[13] This is indeed what we find. The prediction is entirely general: if two combinations of connectives have the same syntax and the same bivalent contribution, they should have the same projective behavior. Dynamic semantics makes no such prediction.

2. Heim's theory requires an additional mechanism of *accommodation*, which comes in two varieties:

-*Global accommodation* occurs when a presupposition is not satisfied in the initial Context Set, which is thus *modified* so as to prevent the sentence from being infelicitous. Even if you don't know that I have siblings, I may tell you that *my sister is pregnant* without disrupting communication: being cooperative, you simply 'add' to the initial Context Set the information that I have a sister (Lewis, 1979). This mechanism is called 'global' accommodation because it involves a modification of the initial Context Set, which represents what the speech act participants initially take for granted. An analogous mechanism can easily be motivated within the Transparency framework: unless the initial Context Set satisfies certain conditions, the sentence uttered will end up violating the Principle of Transparency—and ultimately, *Be Articulate*. Since this is undesirable, it is only natural that a rational speech act participant should sometimes be willing to adapt his beliefs to avoid such a violation.

-*Local accommodation* is an additional—and considerably more dubious—device introduced in Heim (1983). Its *raison d'être* is entirely empirical. I may tell you (correctly) that *John doesn't know that Moldavia is a monarchy because it isn't!*. Adapting the initial Context Set to ensure that one takes for granted that Moldavia is a monarchy won't help, as this will make the sentence a (contextual) contradiction. What we want is for the sentence to mean *It is not the case that Moldavia is a monarchy and that John knows it, because*.... This is obtained in Heim's theory by postulating that under duress one may tinker with the *local* Context Set with respect to which an expression is evaluated. Unlike global accommodation, however, local accommodation cannot be imported into the Transparency theory because we do not have a notion of 'local Context Set' to begin with. On the other hand we may stipulate that under duress one may *fail* to apply the Principle of Transparency, thus leaving the speech act participants with an unadorned bivalent meaning. Since for us *John knows that p* just means something like *p and John believes that p*, we immediately obtain the correct result in this case.[14]

13 This is a simplification. Due to our bracketing conventions, *(unless F . G)* does not have exactly the same syntax as *(if (not F) . G),* since the latter formula involves two more parentheses. Still, it can be shown that we predict the same projection behavior for both cases.

14 Geurts (1999) and van der Sandt (1992) have argued that, in some cases, one must allow for 'intermediate accommodation', i.e. tinkering with a Context Set which is neither the global nor the local one. A purported example of this is the sentence *Every German loves his Mercedes*, from which one

3. If one is still enamored with dynamic semantics, one may use the Transparency theory to solve the over-generation problem *within* Heim's framework. The idea (suggested by K. Shan and I. Heim) is that one can go 'full circle' and use the Transparency theory to constrain the Context Change Potentials of primitive connectives and quantifiers. Thus there might be a 'meta-constraint' on the lexicon that requires that, say, the Context Change Potential of *and* should guarantee equivalence with Transparency. While this is certainly a line worth exploring, it will work best in the propositional case. In the quantificational case, as we saw, the equivalence with Heim's results is not complete, and it is only with additional assumptions that Heimian Context Change Potentials can be 'derived' from Transparency.

4. In our statement of Transparency, *linear order* plays a crucial role. This raises two further questions.

-First, what happens when the syntax becomes more complex, or when we allow for the possibility of structural ambiguities? This is currently an open question.

-Second, one could try to replace 'linear order' with 'order in which the expressions are processed,' which may turn out to be a rather different notion (we could also explore what happens when 'linear order' is replaced with structural notions, e.g., 'order of c-command'). This would free the theory from a direct reliance on linear order, which might turn out to be too simplistic in the general case (see Schlenker 2006a for some preliminary discussion).[15]

5. One should also consider the *empirical* points on which our analysis of quantified statements diverges from Heim's. Unfortunately the predictions are rather hard to test. For cases that involve restrictors whose extension is not of fixed finite size, we may try to assess the following:

(34) *Context* We are discussing a soccer match with Frenchmen on both teams.
-Team A includes 4 Frenchmen, who have all decided to retire. Some of them might conceivably reconsider their decision if their team wins.
-Team B includes 2 Frenchmen, only one of whom has decided to retire.
Before the game, I say:
No matter what happens, less than 3 Frenchmen of the winning team will reconsider their decision to retire.

Heim's prediction is that in every world of the Context Set, every Frenchman of the winning team has decided to retire. We make no such prediction because in the situation as described, it is guaranteed that *Less than 3 Frenchmen of the winning team* (*have decided to retire and* γ) is always equivalent to *Less than 3 Frenchmen of the winning team* γ. To see this, note that there are just two outcomes - call them world w and world w′:

-world w: Team A wins. In w, every Frenchman on the winning team has decided to retire, and therefore

does not infer that *every German has a Mercedes*, but rather that *every German who has a Mercedes loves his Mercedes*. The 'DRT' alternatives to Heim's semantics can obtain this reading. However it has been argued by other researchers (e.g., Beaver, 2001) that the reading we get is the result of an implicit domain restriction on quantifiers rather than of intermediate accommodation. We cannot do justice to this empirical debate in the present context, but it is clear that its outcome matters for the Transparency theory.

15 In Schlenker (2006b), we consider an alternative statement of Transparency which is symmetric (neither linear order nor any other type of order plays any role). But this requires a radical departure from the empirical assumptions of Heim (1983), and thus we leave this point for another occasion.

w $\vDash$ *Less than 3 Frenchmen of the winning team* (*have decided to retire and* γ) $\Leftrightarrow$ *Less than 3 Frenchmen of the winning team* γ

-world w′: Team B wins. But in w′, there are less than three Frenchmen on the winning team to begin with. Hence *Less than 3 Frenchmen of the winning team (have decided to retire and* γ) is true, as is *Less than 3 Frenchmen of the winning team* γ. In sum, here too we have

w′ $\vDash$ *Less than 3 Frenchmen of the winning team (have decided to retire and* γ) $\Leftrightarrow$ *Less than 3 Frenchmen of the winning team* γ

I leave an empirical evaluation of this and related sentences for future research.[16]

6. Finally, as noted by an anonymous referee, it would be interesting to determine whether the strategy applied here to derive Heim's results can be modified to match the (rather different) predictions of rival theories developed within Discourse Representation Theory (Geurts, 1999; van der Sandt, 1992).

These issues, no doubt, are intricate. Despite quite a few open questions, however, I hope to have shown that the Transparency theory offers a serious alternative to the dynamic analysis of presupposition projection, and that it presents a clear advantage if one is looking for a theory which is *explanatory*.

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[16] Concerning examples that have to do with infinite domains (see fn. 12), we can test sentences like the following:

(i) Infinitely many integers are unaware that they are lucky not to be the number 13.

Heim's theory predicts that the integers that are quantified over are all different from 13 (because all integers must satisfy the presupposition introduced by the nuclear scope *unaware that*...). We predict no such thing. This is because the sentence is of the form ($QP. \underline{Q}Q'$) with
Q = infinitely many
P = integers
Q = to be different from 13
Q′ = to not believe that one is lucky to be different from 13
As it happens, in any world w, $P^w - Q^w$ is a singleton, and it is thus a finite set. This predicts that Transparency should automatically be satisfied. But it is clear that a longer discussion would be needed to determine what the facts are, and how they should be analyzed.

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